A Novel Equivalent Circuit for Induction Motors under Voltage Fluctuation Conditions

M. Ghaseminezhad[i *]; A. Doroudi[ii] and S.H. Hosseinian[iii]

ABSTRACT

Voltage fluctuation is one of the important types of power quality aspects originated from heavy fluctuating loads. This phenomenon can influence the behavior of induction motors, produce excessive vibration and increase the ohmic losses. In this paper a simplified model to evaluate the electrical characteristics of three phase induction motors under voltage fluctuation conditions will be presented and verified using small and large signal analysis. The model consisting of two equivalent circuits gives the total current and torque perturbations and additional loss increase when induction motors are subjected to regular sinusoidal voltage fluctuations. Furthermore, the dependency of the electrical characteristics of induction motors on the frequency and depth of voltage Fluctuations is also examined.

KEYWORDS

Voltage fluctuations, Flicker, Induction motors, Equivalent circuit, Small signal analysis

1. INTRODUCTION

The rapid expansion of power systems and the vast usage of modern load types have caused the area of power quality to become a major issue from an obscurity within few years [1]. Amongst many power quality problems voltage fluctuations has gained a growing concern from utilities and end users. Voltage fluctuation is a common term used to describe systematic variations in the voltage envelope or a series of random voltage changes which can cause light intensity fluctuations with lamp especially with incandescent lamp [2]-[4]. If these fluctuations exceed certain magnitude for certain repetition frequencies, it is observed by human eyes as a disturbing unsteadiness of light intensity. This unsteadiness is referred to as flicker. Loads such as arc furnaces, rolling mills and etc. which exhibit continuous and rapid variations in their current can cause voltage fluctuations (or voltage flicker). Due to load characteristics, the flicker behavior can be cyclic, chaotic and stochastic. Nevertheless, in a short period, the voltage flicker can be approached by an amplitude modulations formula [5]. With this assumption a voltage flicker signal can be represented by the following equation:

$$v(t) = V_p[1+\sum_m \sin(2\pi f_m t + \phi_m)]\cos(2\pi f_b t) \qquad (1)$$

where f_b is the fundamental frequency of the ac voltage, f_m is the modulating (flicker) frequency, V_P is the line to neutral and k_m peak voltage is the modulating depth.

Causing visible lighting flicker, noise in television sets and some effects on ICU and CCU systems (these critical systems make different reports by different voltage source amplitude), voltage fluctuations can influence on behavior of the other electric devices such as induction motors. In fact, voltage fluctuations at the terminal of an induction motor produce slip and torque ripples and consequently excessive motor vibrations occur, which can be transferred to the loads and to the supports. This reduces in turn the life of the mechanical components such as bearings and joints and lead to variations in final quality of products. Apart from these mechanical effects, voltage fluctuations may produce some loss increase. The losses may lead to overheating and motor life reduction. However, the induction motor as the most popular one in the industry (over 85%), it is important to carry out studies about the effects of voltage flicker in the efficiency and behavior of the three phase induction motors. The losses may lead to overheating and motor life reduction.

Only a few studies regarding this topic have been presented so far. One of them is the work has been done in [6],[7] using small signal model to evaluate the behavior

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of the induction motor to regular fluctuations in the terminal voltage. The objective of the paper is to examine the dynamic behavior of induction motors in relation to the flicker attenuation. Another study within this topic [8] covered the induction motor behavior, when the motors are tested by superimposing a second frequency component. Other papers relevant to this work are [9], [10] where an experimental analysis was performed to evaluate the effects of rectangular voltage amplitude modulations on the three phase induction motor behavior. In [11] this problem also has been addressed through several simulations performed using the EMTP/ATP tool.

This paper, fully studies the effects of voltage fluctuations over the efficiency and other characteristics of induction motors in steady state mode of operation. Via a simple model the motor behavior is computed. This proposed model mainly consists of modified equivalent circuits which permit to explain why the motor behavior is altered. In order to validate the proposed model, both small and large signal analysis will be used. Specifically, our goals are: i) proposing a simplified model to evaluate the induction motors response in occurrence of input voltage amplitude modulations; ii) synthesis and testing of the simplified model; iii) obtaining power losses and other characteristics of the induction motor when magnitude and frequency of voltage fluctuations are varied.

The paper is organized as follows: In section 2 the fundamental concepts of the small signal modeling of induction motors and voltage fluctuations are studied and analysis of the model will be presented. Section 3 introduces the proposed equivalent circuit based model and further gives stator current perturbations response to fluctuating voltage supply. Section 4 compares the results from the proposed model and small and large signal models. The operational characteristics of an induction motor are obtained from the proposed model in section 5 and finally conclusions are given in section 6.

2. SMALL SIGNAL ANALYSIS

Small signal analysis using linear techniques provides valuable information about the inherent dynamic characteristics of the induction machine. In this model, the effects of magnetic saturation, eddy currents, and slotting are neglected. On the other hand, disturbances are considered to be small; therefore, the equations that describe the resulting response of the induction machine may be linearized for the purpose of analysis. The voltage fluctuation is a small perturbation that superimposed on the fundamental voltage. This fluctuation is generally considered sufficiently small for linearization of system equations. In the following, small signal model suitable for the analysis of induction motor behavior subjected to the regular voltage fluctuations will be presented.

A. Induction motors

Small-signal models covered in Appendix A are used to analyze the behavior of the three phase induction motors. The machine variables in Appendix A are expressed as per unit quantities using the following base values [12]:

V_{base}: Line to neutral peak voltage

I_{base} : Peak line current

Linearized voltage equations and the torque-speed relationship given in Appendix A, describe the small-signal model of a three phase induction motor that drives a fan-type load and can be expressed in state space form as:

$$\dot{x} = Ax + Bu \ .$$

B. Voltage Fluctuations

In flicker studies, it can be assumed that source voltage is a sinusoidal amplitude modulated signal. In this case sinusoids superimposed on the fundamental voltage are assumed to be the fluctuating components. With one modulation frequency, the line to neutral voltage of phase "a" is defined as:

$$v_a(t) = V_p[1 + k\sin(\omega_m t)]\cos(\omega_b t) \qquad (2)$$

where k is the modulation depth. The other phases can be expressed in the same way except in that the phase angle of fundamental frequencies in the other two phases should be modified by -120 and +120, respectively. The three phase voltages can be transformed into d-q frame to obtain Δv_{ds} , Δv_{qs} . These excursions expressed as per unit quantities are given by:

$$\begin{aligned} \Delta v_{ds} &= 0 \\ \Delta v_{qs} &= k\sin(\omega_{\rm m} t) \end{aligned} \qquad (3)$$

C. Small signal Analysis

The model presented in Appendix A is implemented in Matlab to facilitate calculations. A 500 hp three phase squirrel cage induction motor that drives a fan type load is considered for simulation. Motor ratings and its parameters are given in Appendix B [13]. It is assumed that the induction motor operates at its rated capacity and the modulating depth (k) is equal to 0.05. Time step of 1 millisecond is chosen for the simulations.

Figs. 1 and 2 show the variations of speed fluctuations and stator current perturbations for a 15 Hz flicker frequency, respectively. As seen in the figures, although the amplitude of speed fluctuations is relatively small, it has a strong influence on the stator current. Hence, through the simulations carried out, it has been noted that the variations of speed fluctuations can be ignored though the total fluctuating current should be considered.

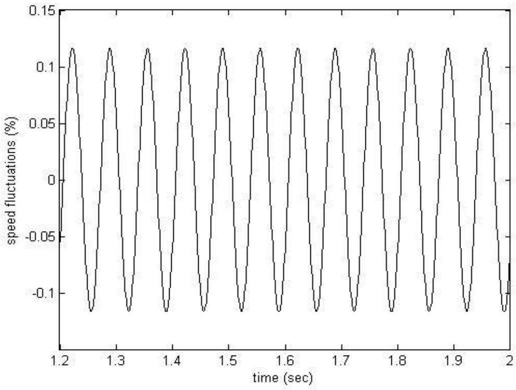

Figure 1: Speed fluctuations for $f_m = 15Hz$ and $k = 0.05$

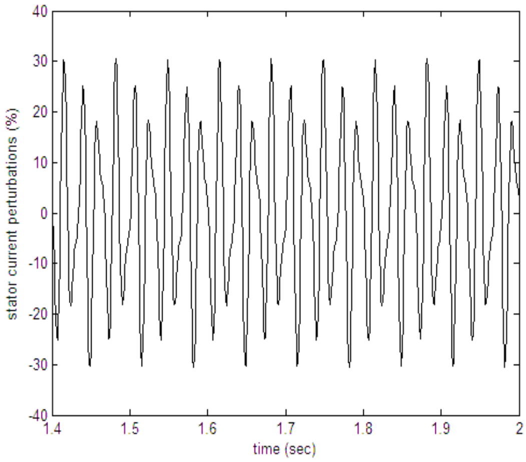

Figure 2: Stator current perturbations for $f_m = 15Hz$ and $k = 0.05$

Fig. 3 shows the variation of the peak value of the speed fluctuations as a percentage of the steady state speed for the 500 hp motor against flicker frequency at $k = 0.05$. The flicker frequency was changed between 1 Hz to 35 Hz which covering the perceptible flicker frequency range. The figure indicates that the speed fluctuations are varied with respect to the flicker frequency and it has a local maximum. The maximum point corresponds to one of the natural frequencies of the state matrix (matrix A) [7].

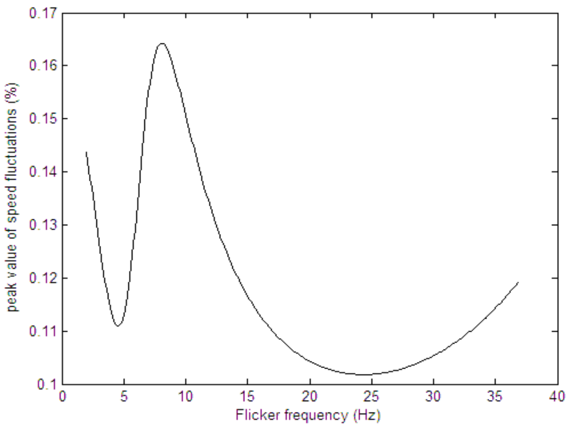

Figure 3: Peak value of $\Delta\omega_r$ versus flicker frequency

3. EQUIVALENT CIRCUIT OF INDUCTION MOTOR WHICH SUBJECTED TO REGULAR VOLTAGE FLUCTUATIONS

While equations stated in section 2 can be used directly to determine the induction motors performance subjected to regular voltage fluctuations, it is a common practice to use equivalent circuits to provide visual description of the motor model. In this section, an equivalent circuit based model is developed to present a complete electrical characteristic of the induction motor when regular voltage fluctuations exist at the terminals of the motor. The model consists of three equivalent circuits named as fundamental frequency, upper frequency and lower frequency equivalent circuits. Induction motor characteristics such as stator current, torque and copper losses are simply obtained using these equivalent circuits. Then, by assuming superposition principle, the total (resulting) quantity will be the sum of each individual component. To achieve this goal, we first rewrite (2) as:

$$\begin{aligned} v_a &= V_p \cos(\omega_b t)[1 + k \sin(\omega_m t)] = \\ & V_p \cos(\omega_b t) + V_p k \cos(\omega_b t)\sin(\omega_m t) = \\ & V_p \cos(\omega_b t) + \frac{V_p k}{2}\sin((\omega_b + \omega_m)t) + \frac{V_p k}{2}\sin((\omega_b - \omega_m)t) \end{aligned} \quad (4)$$

The first term in (4) is the fundamental frequency voltage. The second and third terms are the variation voltage fluctuations, superimposed to the fundamental frequency voltage and their frequency of them are $f_b + f_m$ (upper frequency) and $f_b - f_m$ (lower frequency).

We assumed that the superposition principle is valid for the analysis of induction motors subjected to the main frequency carrying superimposed frequencies. Consequently, the induction motor response can be obtained when the induction motor is subjected to each of the above individual components (fundamental, upper and lower frequencies).

A. Fundamental frequency voltage

When the first term in (4) is applied to the terminals of an induction motor, the machine response can be easily obtained by the well-known classical equivalent circuit of induction motors.

B. Upper frequency component

Induction motor response when subjected to the second term of (4) i.e. $\frac{V_p k}{2}\sin(\omega_m + \omega_b)t$ can be obtained with the equivalent circuit shown in Figure 4. In this case, magnitude of input voltage equals to $\frac{V_p k}{2}$. Since the frequency of input voltage is $\omega_b + \omega_m$, all of the inductances should be multiplied by this factor. Upper frequency slip s_p can be given by:

$$s_p = \frac{n_{sp} - n_r}{n_r} \tag{5}$$

where n_r is the rotor speed which is assumed constant under fluctuation conditions and n_{sp} is the upper frequency synchronous speed expressed as:

$$n_{sp} = \frac{120 \times (f_b + f_m)}{P} \tag{6}$$

where P is number of poles.

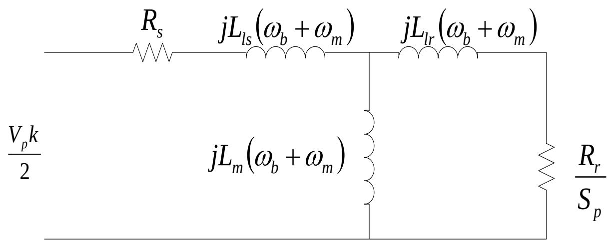

Figure 4: Upper frequency equivalent circuit of induction motor

Fig. 5 shows the variations of the upper frequency slip versus modulating frequency. As seen from this figure, increase in the modulating frequency raises the upper frequency slip. It is obvious that if f_m varies from 1 Hz to 35 Hz, the rotor speed is always less than n_{sp}. In other words, the rotor speed is always lower than the speed of the rotating field produced by this component. Hence, with different modulating frequencies, s_p is ever positive and the machine always works at motoring mode. We can conclude that the torque produced by the upper frequency component, apart from the modulating frequency is ever positive and adds to the fundamental frequency (main) torque.

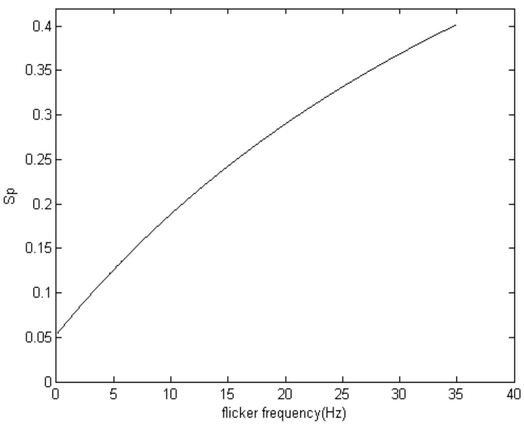

Figure 5: Upper frequency Slip versus modulating frequency

C. Lower frequency component

Induction motor response when subjected to the third term of (4) i.e. $\frac{V_p k}{2}\sin(\omega_b - \omega_m)t$ can be obtained by the equivalent circuit shown in Figure 6. In this case, the magnitude of input voltage is the same as the upper frequency component while the frequency of input voltage is $\omega_b - \omega_m$. The lower frequency slip (s_n) can be obtained as follows:

$$s_n = \frac{n_{sn} - n_r}{n_r} \tag{7}$$

where n_{sn} given in (8) is the lower frequency synchronous speed.

$$n_{sn} = \frac{120 \times (f_b - f_m)}{P} \tag{8}$$

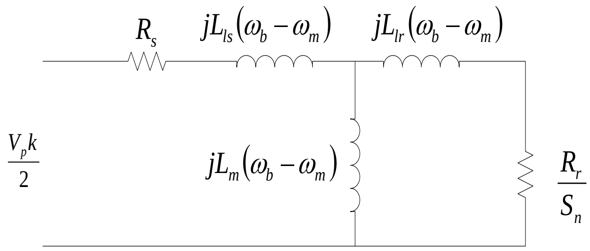

Figure 6: Lower frequency equivalent circuit of induction motor

Fig. 7 shows the variations of lower frequency slip versus modulating frequency. Depending on the modulating frequency and if n_{sn} is greater than n_r, s_n is positive and machine works in motoring mode. In this case, modulating frequency will be very low. As an example, for $f_b = 60Hz$, $p = 4$ and $n_r = 1773rpm$, s_n becomes positive if $f_m \le 0.9Hz$. As f_m increases, the rotor speed exceeds n_{sn} and s_n becomes negative. Negative slip means machine works in generating mode and giving rise to negative torque which opposes the main torque.

As a conclusion, it can be said that, in very low modulating frequencies, s_n is positive and the torque produced by the lower frequency component is also positive (adds to the main torque). However over a wide range of modulating frequencies, s_n and the produced torque becomes negative (opposes to the main torque). In the later case, the rotor speed is greater than the speed of rotating field produced by the lower frequency component.

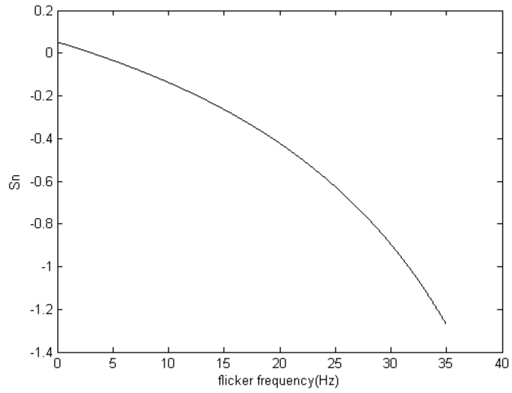

Figure 7: Lower frequency slip versus modulating frequency

4. MODEL VALIDITATION

A conventional dq domain large signal model simulated in Matlab/Simulink and small signal models covered in Appendix A and section II are used to verify the results of the proposed model.

To calculate the stator current under fluctuating supply conditions with the small signal model, it is necessary to examine the motor response to small voltage perturbations described by (3). This can be accomplished by formulating a relationship between small perturbation in the amplitude of the stator voltage (Δv_s) and the resulting perturbation in the stator current (Δi_s) using the linearized machine equations given in Appendix A.

The d-q axes small displacement stator currents $(\Delta i_{ds}, \Delta i_{qs})$ are related to the flux linkages as :

$$\begin{bmatrix} \Delta i_{ds} & \Delta i_{qs} \end{bmatrix}^T = Cx$$

$$C = \frac{1}{D}\begin{bmatrix} L_{rr} & 0 & -L_m & 0 & 0 \\ 0 & L_{rr} & 0 & -L_m & 0 \end{bmatrix} \quad (9)$$

$$x = \begin{bmatrix} \Delta\psi_{ds} & \Delta\psi_{qs} & \Delta\psi_{dr} & \Delta\psi_{qr} & \Delta\omega_r \end{bmatrix}$$

After calculating $\Delta i_{ds}, \Delta i_{qs}$, they should be transformed to the phase domain to establish the stator current variations in phase "a".

The stator current variations given in (10) can be obtained by the proposed model based on the superposition principle, i.e. the total stator current perturbations are sum of the stator current variations obtained from the upper frequency equivalent circuit (ΔI_p) and the lower frequency component (ΔI_n).

$$\begin{aligned} \Delta I_t = & \left|\Delta I_p\right| \sin\left((\omega_b + \omega_m)t + \angle\Delta I_p\right) + \\ & \left|\Delta I_n\right| \sin\left((\omega_b - \omega_m)t + \angle\Delta I_n\right) \end{aligned} \quad (10)$$

The calculated stator current perturbations versus time at f_m =10 Hz and k = 0.05 for 500 hp machine is compared with the results of small and large signal model in Figure 8. As it is shown there is a good agreement among the three models and it allows us to take in this consideration on the remaining operational characteristics. The difference between the proposed model and the small and large signal analysis (especially in the peak regions) is due to different rotor speed considered in the calculations. In the small and large signal analysis, the variation of the rotor speed is taken into account whereas in the proposed model a constant value is assumed for it.

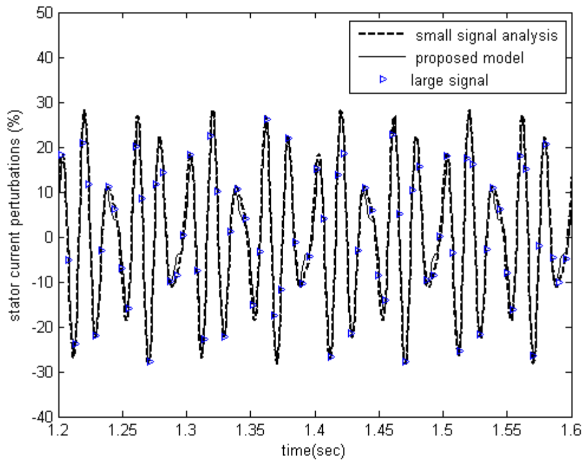

Figure 8: Comparison of stator current perturbations obtained from the proposed model, small signal analysis and large signal.

5. COMPUTATION RESULTS

This section presents the use of the proposed model to determine the induction motor operational characteristics under voltage fluctuations conditions.

Fig. 9 illustrates the variation of the magnitudes of the two frequency components in the current perturbation (lower and upper frequency components) at k = 0.05 versus modulating frequency for the 500 hp motor. It is evident from this figure that at low modulation frequencies, the magnitude of the stator current perturbations is relatively small. However, as f_m increases the upper and lower frequency components of current perturbations tend to increase except the upper one tends to stabilize at a near constant level when f_m is greater than about 4 Hz.

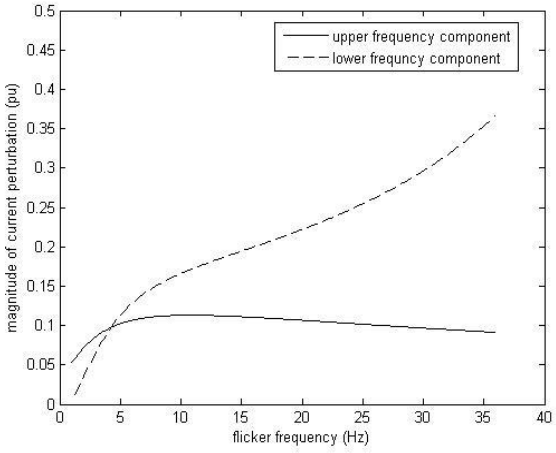

Figure 9: Current perturbations amplitude of upper and lower component versus flicker frequency.

The electromagnetic torque perturbations produced by the upper and lower frequency equivalent circuits versus the flicker frequency at k = 0.05 are plotted in Figure 10 for 500 hp motor. The resultant torque is the sum of the upper and lower frequency components. As shown in Figure 10, at low flicker frequencies the resultant torque is positive and leading to the main torque magnification. However, as f_m increases the resultant torque becomes negative and reduces the main torque.

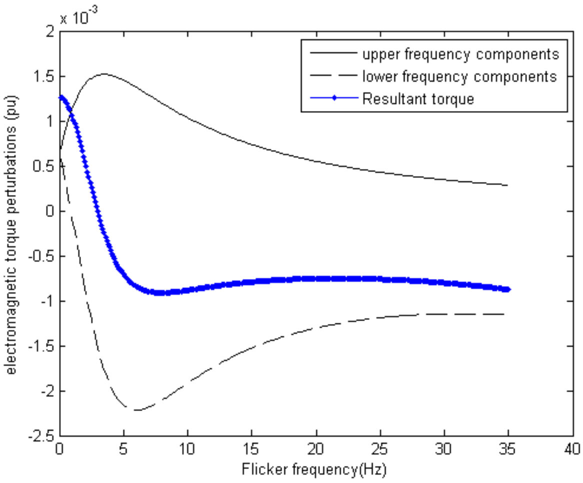

Figure 10: Electromagnetic torque perturbations of upper and lower equivalent circuit versus modulation frequency

Fig. 11 shows the variations of the additional copper losses of upper and lower frequency components of 500 hp motor at k = 0.05 against the flicker frequency. The figure depicts that over a wide range of flicker frequencies, the upper and lower frequency components produce different copper losses where the lower frequency component is seen to produce a higher extent compared to the upper one for $f_m \geq 4\,\text{HZ}$. Furthermore, for this region, as f_m increases the upper frequency component decreases, the lower one increases.

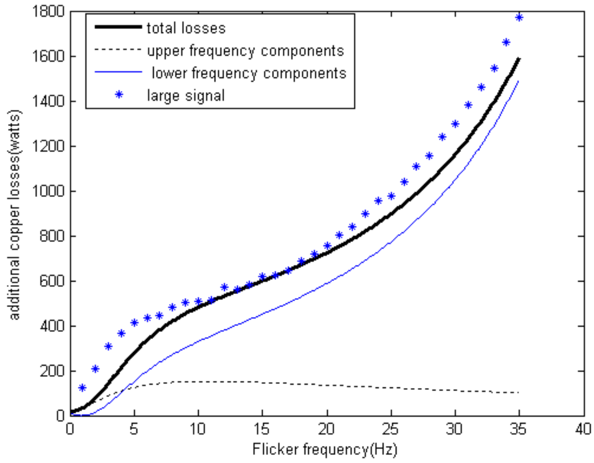

Figure 11: Additional copper losses of induction motor when subjected to voltage fluctuations versus flicker frequency.

On the other hand, these loss values were compared with the results obtained from the large signal analysis in order to investigate the accuracy of the proposed model. As shown in Figure 11, the agreement is high. To determine the losses by large signal analysis, a pure sinusoidal voltage was first applied to the induction motor and the copper losses were computed. Then the motor subjected to the fluctuated voltage and the copper losses were determined again. The difference between these two cases yields additional losses.

However, as seen, a small percentage of voltage fluctuations will result in additional copper losses. These additional losses reduce the efficiency of the motor and may cause an important increase in the motor heating. So, should voltage be fluctuated, the rated horsepower of the motor should be multiplied by a derating factor to reduce the possibility of motor damage. Derating factor is the fraction (or percent) of the nominal rating to which a specified quantity must be reduced due to unusual operating conditions.

Fig. 12 gives the percentage increase of copper losses of 500 hp and 2250 hp motors versus modulation depth and compares them with the results of the large signal analysis (Ratings and parameters of 2250 hp induction motor are also given in Appendix B). It can be noticed that the needed derating factor is comparatively more important for low power induction motors. Also, the figure depicts that, the superposition principle used for deriving the proposed model becomes increasingly valid at lower modulation depths which are common in flicker analysis.

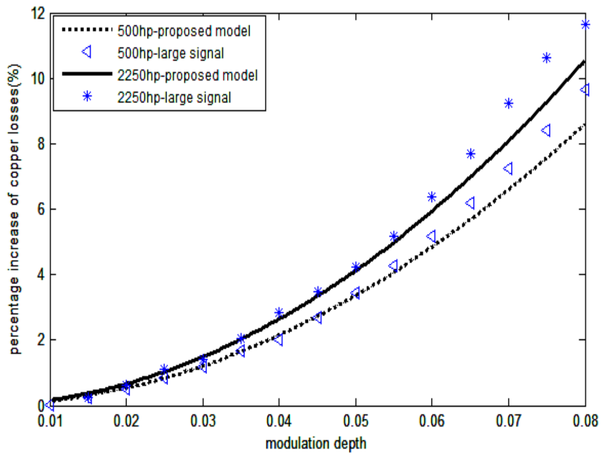

Figure 12: Percentage increase of copper losses for two motors versus the modulation depth.

6. CONCLUSIONS

The electrical characteristics of induction motors subjected to regular voltage fluctuations in the flicker frequency range has been investigated using an equivalent circuit based model. Small and large signal models were used to validate the proposed model. Based on this model the response of a 500 hp induction motor operating under fluctuating supply conditions was examined. It is seen that a small percentage of voltage fluctuations can lead to additional copper losses and hence reduce the efficiency of the induction motor. Therefore, to avoid overheating, the motor rated horsepower should be reduced so as to maintain the temperature within thermal class limits. It has been further shown that the required derating factor is comparatively more important for low power induction motors.

7. APPENDIXES

APPENDIX A: LINEARIZED MOTOR EQUATIONS

The per unit voltage equations for an induction motor operating at steady state balanced condition can be expressed in synchronously rotating reference frame as follows [13]:

$$\begin{aligned} v_{ds} &= R_s i_{ds} - \psi_{qs} + \dot{\psi}_{ds}\frac{1}{\omega_b} \\ v_{qs} &= R_s i_{qs} + \psi_{ds} + \dot{\psi}_{qs}\frac{1}{\omega_b} \\ v_{dr} &= R_r i_{dr} - s\psi_{qr} + \dot{\psi}_{dr}\frac{1}{\omega_b} \\ v_{qr} &= R_r i_{qr} + s\psi_{dr} + \dot{\psi}_{qr}\frac{1}{\omega_b} \end{aligned} \tag{A.1}$$

The relations between flux linkages and currents are given by:

$$\begin{aligned} i_{ds} &= \frac{1}{D}\left(L_{rr}\psi_{ds} - L_m\psi_{dr}\right) \\ i_{dr} &= \frac{1}{D}\left(-L_m\psi_{ds} + L_{ss}\psi_{dr}\right) \\ i_{qs} &= \frac{1}{D}\left(L_{rr}\psi_{qs} - L_m\psi_{qr}\right) \\ i_{qr} &= \frac{1}{D}\left(-L_m\psi_{qs} + L_{ss}\psi_{qr}\right) \end{aligned} \tag{A.2}$$

where:

$$\begin{aligned} L_{ss} &= l_{aa} + L_m \\ L_{rr} &= L_{AA} + L_m \\ D &= L_{ss}L_{rr} - L_m^{\ 2} \end{aligned} \tag{A.3}$$

The motion equations of an induction motor can be described by rotational inertia equation which shows the effect of unbalance between the electromagnetic torque and the mechanical torque of motor as follows:

$$\begin{aligned} T_e &= \frac{L_M}{D}\left(\psi_{dr}\psi_{qs} - \psi_{qr}\psi_{ds}\right) \\ T_e &= \frac{2H}{\omega_b}\dot{\omega}_r + T_L \end{aligned} \tag{A.4}$$

The load torque varies with speed. A commonly used expression for the load torque is:

$$T_L = T_0\omega_r^{\ m} \tag{A.5}$$

Hence, from (A.4) and (A.5) we have:

$$\frac{L_M}{D}\left(\psi_{dr}\psi_{qs} - \psi_{qr}\psi_{ds}\right) = \frac{2H}{\omega_b}\dot{\omega}_r + T_0\omega_r^{\ m} \tag{A.6}$$

Equations (A.1) to (A.6) can be linearized around an operating point. The linearized of voltage equations are given by (A.7).

$$\begin{bmatrix} \Delta V_{ds} \\ \Delta V_{qs} \\ \Delta V_{dr} \\ \Delta V_{qr} \end{bmatrix} = \begin{bmatrix} \frac{R_s L_{rr}}{D} + \frac{1}{\omega_b}\frac{d}{dt} & -1 & \frac{-R_s L_m}{D} & 0 & 0 \\ 1 & \frac{R_s L_{rr}}{D} + \frac{1}{\omega_b}\frac{d}{dt} & 0 & \frac{-R_s L_m}{D} & 0 \\ \frac{-R_r L_m}{D} & 0 & \frac{R_s L_{ss}}{D} + \frac{1}{\omega_b}\frac{d}{dt} & -s_0 & \psi_{qro} \\ 0 & \frac{-R_r L_m}{D} & s_0 & \frac{R_s L_{ss}}{D} + \frac{1}{\omega_b}\frac{d}{dt} & -\psi_{dro} \end{bmatrix} \begin{bmatrix} \Delta\Psi_{ds} \\ \Delta\Psi_{qs} \\ \Delta\Psi_{dr} \\ \Delta\Psi_{qr} \\ \Delta\omega_r \end{bmatrix} \tag{A.7}$$

where $s_0 = \dfrac{\omega_b - \omega_r}{\omega_b}$.

Using (A.5) and (A.6), the electromagnetic torque is obtained as (m=2):

$$\Delta T_e = \frac{L_M}{D}\left(\psi_{dr0}\Delta\psi_{qs} + \psi_{qs0}\Delta\psi_{dr} - \psi_{qr0}\Delta\psi_{ds} - \psi_{ds0}\Delta\psi_{qr}\right)$$

$$\Delta T_L = 2T_0\omega_{r0}\Delta\omega_r$$

$$\Delta T_e - \Delta T_L = 2H\,\omega_b\frac{d}{dt}\Delta\omega_r$$

$$\frac{L_M}{D}\left(\psi_{dr0}\Delta\psi_{qs} + \psi_{qs0}\Delta\psi_{dr} - \psi_{qr0}\Delta\psi_{ds} - \psi_{ds0}\Delta\psi_{qr}\right) - 2T_0\omega_{r0}\Delta\omega_r = 2H\,\omega_b\frac{d}{dt}\Delta\omega_r \quad \text{(A.8)}$$

Linearized voltage equations and the torque-speed dynamic relationship given by (A.7) and (A.8), respectively, describe the small signal model of induction machine and can be used to show dynamic response of motor for small deviations from operating point. Any set of linearly independent system variables which satisfy the $\dot{x} = Ax + Bu$ may be used to describe the state of an induction machine. Let:

$$\dot{x} = \begin{bmatrix}\Delta\dot{\psi}_{ds} & \Delta\dot{\psi}_{qs} & \Delta\dot{\psi}_{dr} & \Delta\dot{\psi}_{qr} & \Delta\dot{\omega}_r\end{bmatrix}^T$$

$$u = \begin{bmatrix}\Delta v_{ds} & \Delta v_{qs} & \Delta v_{dr} & \Delta v_{qr}\end{bmatrix}^T \quad \text{(A.9)}$$

And write:

$$A = \begin{bmatrix}
-\frac{\omega_b R_s L_{rr}}{D} & \omega_b & \frac{\omega_b R_s L_m}{D} & 0 & 0 \\
-\omega_b & -\frac{\omega_b R_s L_{rr}}{D} & 0 & \frac{\omega_b R_s L_m}{D} & 0 \\
\frac{\omega_b R_r L_m}{D} & 0 & -\frac{\omega_b R_r L_{ss}}{D} & s_0\omega_b & -\omega_b\psi_{qro} \\
0 & \frac{\omega_b R_r L_m}{D} & -s_0\omega_b & -\frac{\omega_b R_r L_{ss}}{D} & \omega_b\psi_{dro} \\
\frac{-L_m\psi_{qro}}{2HD} & \frac{L_m\psi_{dro}}{2HD} & \frac{L_m\psi_{qso}}{2HD} & -\frac{L_m\psi_{dso}}{2HD} & -\frac{T_0\omega_{ro}}{H}
\end{bmatrix}$$

$$B = \omega_b\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & 0 & 0\end{bmatrix} \quad \text{(A.10)}$$

These equations form a minimal set of dynamics variables along with the inputs to the system provide a complete description of the induction motor behavior. The main advantage of small signal modeling of induction machine is to give a complete overview of the machine behavior when the machine experiences voltage fluctuations.

APPENDIX B:

Ratings and parameters of 500 hp induction motor.

Rated voltage [V]	2300
Rated power [hp]	500
Rated current [A]	93.6
Rated speed [rpm]	1773
$R_s[pu]$	0.018464
$L_{ls}[pu]$	0.08499
$L_{lr}[pu]$	0.08499
$L_m[pu]$	3.807
$R_r[pu]$	0.08499
$j[kg.m^2]$	11.06

Ratings and parameters of 2250 hp induction motor.

Rated voltage [V]	2300
Rated power [hp]	2250
Rated current [A]	472
Rated speed [rpm]	1786
$R_s[pu]$	0.0103079
$L_{ls}[pu]$	0.0803
$L_{lr}[pu]$	0.0803
$L_m[pu]$	4.635
$R_r[pu]$	0.0078198
$j[kg.m^2]$	63.87

8. NOMENCLATURE

v_d, v_q	dq axes voltages
i_d, i_q	dq axes currents
ψ_d, ψ_q	dq axes flux linkages
R_s	Stator resistance
L_{ls}	Stator leakage inductance
R_r	Rotor resistance
L_{lr}	Rotor leakage inductance
L_m	Mutual inductance
H	Inertia constant
j	Moment of inertia
ω_b	Base angular frequency
ω_r	Rotor angular speed
f_m	Modulation (flicker) frequency
k	Modulation depth
T_L	Load torque

Subscripts

s	Stator variables
r	Rotor variables
o	Steady state values

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