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Original Article

Quantile regression for capital asset pricing model

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ABSTRACT: In this paper, we examine the Capital Asset Pricing Model (CAPM) and demonstrate that when the log returns of an asset are subject to extreme risks or outliers or nonlinear relationship, the Linear Regression (LR) model may not perform well in predicting future returns. Instead, we propose using Quantile Regression, which is more robust to such data anomalies and provides better predictions.

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1. Introduction

The Capital Asset Pricing Model (CAPM) developed by William Sharpe in 1964 [8] and John Lintner in 1975 [7] is a fundamental concept in finance that has revolutionized the field of asset pricing. The CAPM provides a framework for understanding the relationship between risk and expected return, and it has had a lasting impact on financial theory and practice.

Four decades after its introduction, the CAPM remains widely used in various applications, including estimating the cost of capital for firms and evaluating the performance of managed portfolios. Its relevance and utility have stood the test of time, making it a cornerstone of investment education. In fact, the CAPM is often the sole asset pricing model taught in MBA investment courses, underscoring its importance and enduring influence.

Sharpe's work on the CAPM earned him the Nobel Memorial Prize in Economic Sciences in 1990, recognition of his significant contribution to the fields of economics and finance. The CAPM continues to be a vital tool for investors, financiers, and researchers alike, providing insights into the intricate relationship between risk and reward in financial markets.

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The longevity and widespread adoption of the CAPM are testimony to its timeless appeal and practical applicability. Despite the emergence of newer asset pricing models, the CAPM remains an indispensable part of finance theory and practice, continuing to shape our understanding of financial markets and inform investment decision-making.

CAPM has been a widely used and influential model in finance theory, offering appealing predictions about risk measurement and the relationship between expected return and risk. However, despite its popularity, the CAPM has faced criticism due to its poor empirical record. The model's shortcomings may stem from both theoretical flaws and difficulties in implementing valid tests. One of the primary concerns is the limitation of the market portfolio, which is typically defined as a basket of U.S. common stocks. This raises questions about the appropriateness of using a narrow definition of the market portfolio, when in reality, there are various types of financial assets, including bonds, and other assets, that could be included. Furthermore, the model assumes that the market portfolio is comprehensive, which may not be feasible in practice.

The CAPM's empirical issues suggest that caution should be exercised when applying the model in real-world scenarios. While the model's predictions may be intuitively appealing, its limitations must be acknowledged, and alternative approaches should be explored. Expanding the market portfolio to include a broader range of financial assets and considering global markets may provide a more accurate representation of the risks and returns associated with various investments. Ultimately, the failure of the CAPM in empirical tests highlights the need for continued refinement and development of asset pricing models to ensure that investment decisions are informed by robust and reliable theories[3].

2. How CAPM work?

The Capital Asset Pricing Model builds upon the model of portfolio selection developed by Harry Markowitz in 1959 [2]. Markowitz's model assumes that investors are risk-averse and focus on selecting a portfolio that maximizes expected return while minimizing variance. The model is often referred to as a "mean-variance model" since it prioritizes these two factors in portfolio selection.

In Markowitz's model, investors select a portfolio at time t-1 that generates a stochastic return at time t. The goal is to identify the optimal portfolio that balances expected return and risk. To achieve this, investors seek out "mean-variance-efficient" portfolios, which minimize variance in portfolio return while maintaining a certain level of expected return.

The CAPM extends Markowitz's model by introducing the concept of a risk-free rate and a market portfolio. The risk-free rate represents the return an investor can earn from a completely riskless investment, such as a U.S. Treasury bond. The market portfolio, on the other hand, represents a diversified portfolio of all available assets, which includes both risky and riskless assets.

By incorporating these additional elements, the CAPM enables investors to assess the expected return and risk of their portfolios relative to the overall market. This allows investors to make more informed decisions about their investments and helps to explain the relationship between risk and return in financial markets.

CAPM is built upon the foundation of the Markowitz model, which provides an algebraic condition on asset weights in mean-variance-efficient portfolios. The CAPM adds two key assumptions to identify a portfolio that must be mean-variance-efficient: complete agreement and the ability to borrow and lend at a risk-free rate. Complete agreement refers to the idea that investors agree on the joint distribution of asset returns from t-1 to t, given market clearing asset prices at t-1. This means that investors share the same beliefs about the future returns of assets, and this belief is reflected in the market prices of assets.

The second assumption, the ability to borrow and lend at a risk-free rate, allows investors to borrow and lend funds without incurring any risk. This means that investors can borrow money to invest in assets that are expected to generate higher returns than the risk-free rate, and they can also lend money to other investors who are willing to pay a premium for the use of their funds.

With these two assumptions in place, the CAPM can be used to test the relation between risk and the expected return. By comparing the expected return of a portfolio with its risk, investors can determine whether the portfolio is mean-variance-efficient. If the portfolio is not efficient, then it suggests that there is a mispricing of assets in the market, and investors can exploit this mispricing to earn abnormal returns. Overall, the CAPM provides a useful framework for analyzing the tradeoff between risk and return in financial markets. By assuming complete agreement and the ability to borrow and lend at a risk-free rate, the CAPM can be used to identify mean-variance-efficient portfolios and to test the relation between risk and expected return.

3. Why we use Quantile Regression

Quantile regression is a powerful tool in statistics and machine learning, and it has several advantages over traditional linear regression. These are several reasons why we might choose to use quantile regression instead of traditional linear regression [4, 5, 6]. Recently some authors used quantile factor models (QFM) represent a new class of factor models for high-dimensional panel data [1].

1. Robustness to outliers: Traditional linear regression is sensitive to outliers, which can greatly affect the model's fit and predictions. Quantile regression is more robust to outliers since it focuses on the conditional quantiles rather than the mean. This makes it particularly useful when there are extreme values or outliers in the data.
2. Handling non-normal distributions: Traditional linear regression assumes that the residuals follow a normal distribution, but this assumption may not always hold true in practice. Quantile regression does not make any assumptions about the distribution of the residuals, making it a good choice for data with non-normal distributions.
3. Modeling heteroscedasticity: Heteroscedasticity refers to the phenomenon where the variance of the residuals changes across different levels of the predictor variables. Quantile regression can handle heteroscedasticity by estimating separate quantile regressions for each level of the predictor variable.
4. Identifying non-linear relationships: Quantile regression can identify non-linear relationships between the dependent and independent variables, whereas traditional linear regression assumes a linear relationship. By estimating separate quantile regressions for different quantiles, we can capture non-linear patterns in the data.
5. Estimating uncertainty: Quantile regression provides a way to estimate the uncertainty associated with the regression estimates. The quantile regression coefficients come with an associated standard error, which allows us to assess the precision of the estimates.
6. Incorporating covariates: Quantile regression can easily incorporate multiple covariates into the model, allowing us to examine their effects on the conditional quantiles. This is useful when we want to study how different factors influence the distribution of the response variable.
7. Interpretability: Quantile regression provides interpretable results, as the coefficients represent the change in the conditional quantile per unit change in the predictor variable, holding all other variables constant. This makes it easier to understand the effects of the predictors on the response variable.
8. Flexibility: Quantile regression models can be extended to accommodate various complexities, such as time-varying covariates, interaction terms, and non-linear relationships.
9. Computational efficiency: Quantile regression algorithms have become increasingly efficient in recent years, making them comparable to traditional linear regression methods in terms of computational speed. Alternative hypothesis testing: Quantile regression can be used for alternative hypothesis testing, such as testing whether a coefficient is equal to zero at a specific quantile. This can provide insights into the significance of certain predictors at different parts of the distribution.

4. CAPM and QCAMP

The general formula of CAPM is as follows:

$$r_i = r_f + \beta(R_M - r_f) + e_i, \tag{1}$$

where

1. r_i: return of i^{th} asset
2. r_f: rate of risk free asset
3. R_M: return of market
4. e_i: is withe noise with mean 0 and variance σ^2

The $(R_M - r_f)$ is the risk premium and ordinary least squares (OLS) estimation of beta is $\beta = \frac{cov(r_i, R_M)}{var(R_M)}$. Therefore if we take mathematical expectation both sides (1) we have;

$$E(r_i) = r_f + \beta(E(R_M) - r_f).$$

Fama and French [3] mentioned that the original version of the Capital Asset Pricing Model developed by [8] and [7] has not been successful in empirical testing.

We utilized a contemporary adaptation of the CAPM known as the Quantile Capital Asset Pricing Model (QCAPM) to evaluate the performance of our investments. The QCAPM model takes into consideration the quantile within a business or industry, such as production, marketing, and finance, and assesses the unique risks and characteristics associated with each one. By doing so, it enables a more precise estimation of expected returns and helps organizations make well-informed investment decisions. The general formula is as follows:

$$r_{i_q} = \alpha + \beta_q(R_{M_q}) + e_i, \tag{2}$$

that

$$\hat{\beta} = \text{argmin}_{\beta} \sum_{i=1}^{n} \rho\left(\frac{r_i - R_M \beta}{q}\right), \tag{3}$$

where $\rho(u)$ is the quantile loss function, which depends on the quantile level q and the residual $u = \frac{r_i - R_M\beta}{q}$.
The most common choice for the quantile loss function is the check function, which is defined as:

$$\rho(u) = \begin{cases} 0 & u < 0 \\ u & u \geq 0. \end{cases}$$

This function has two desirable properties:

- It is minimized when $u = 0$, which means that the estimated coefficient should be zero when the residual is zero.
- It is monotonically increasing, which means that larger residuals result in larger penalties.

5. Comparison of CAPM and QCAPM with real data

To analyze of real data, we consider high frequency daily historical data from 01/01/2010 to 01/01/2024 of IBM's log-returns for asset returns and S&P500's log-returns for market returns ($r_t = (\log(P_{t+1}) - \log(P_t))$ that r_t is the log-return and P_t is the price in t^{th} day). We assume that there aren no risk free assets.

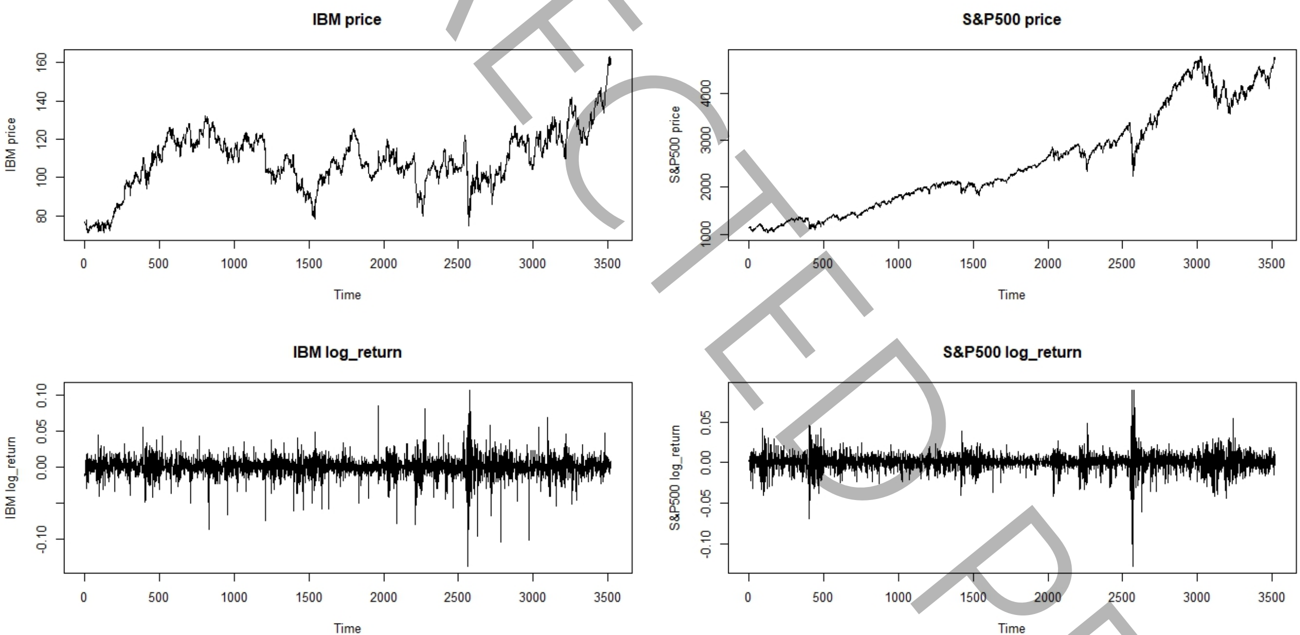

Figure 1: Historical data graphs of IBM and $S\&P500$

As depicted in Figure 1, the historical price and logarithm of return are displayed.

5.1. CAPM

For CAPM we used simple linear regression the results of which one as follows;

$$r_i = \beta_0 + \beta_1 R_M + e_i,$$

Note: Throughout this paper, a significance level of 0.05 is used, and all data analyses were performed using R version 4.4.0.
The result for CAPM is

Table 1: Regression coefficient estimation

Coefficients	Estimate	p-value
Intercept	0.000058	0.9
return S&P500	0.781	0.00

On the basis of the output in Table 1, at a significance level of 0.05, the null hypothesis for the intercept term is accepted therefore, the best model is as follows

$$\hat{r}_i = 0.781 R_M.$$

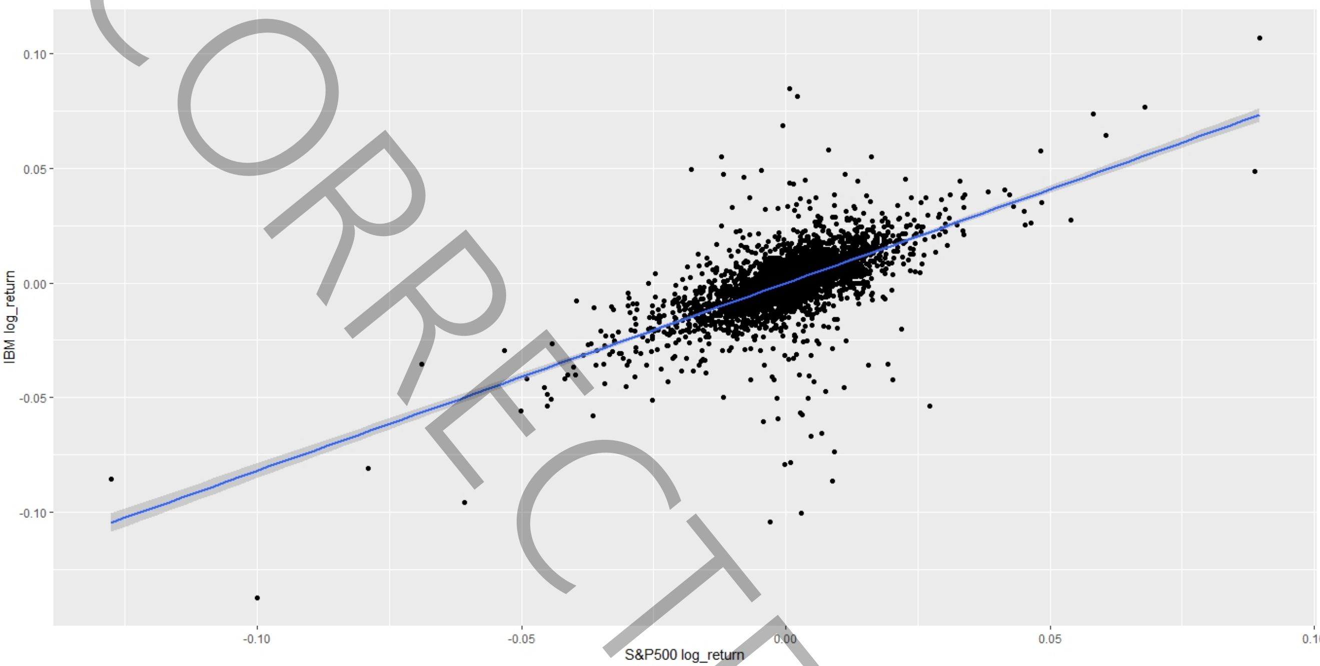

Figure 2: Graph log-returns of the IBM and $S\&P500$ with linear regression f.t

The points that are farthest away from the regression line in Figure 2 have the highest residuals or errors, which can result in a higher mean squared error (MSE) value. This indicates that these points are not well-explained by the regression model.

5.2. QCAPM

For the QCAPM we used quantile regression, the results are as follows;

$$r_{i_q} = \alpha_q + \beta_q(R_{M_q}) + e_i.$$

The results are;

Table 2: Quantile regression coefficient estimation

Coefficients	Estimate	lower bound	upper bound
Intercept	0.00007	-0.00065	0.00084
return S&P500	0.713	0.65	0.77

Based on the output in Table 2, we can see that confidence interval contain 0, then the null hypothesis for the intercept term is accepted, so best model is

$$\hat{r}_{i_q} = 0.713 R_{M_q}.$$

5.3. Comparison of the mean square errors of the CAPM and the QCAPM

Using k-fold cross validation, the Mean Squared Errors (MSE) of both models was calculated and presented in Table 3. As shown in the table, the MSE of the quantile regression model is lower compared to that of the classical linear regression model. This suggests that the quantile regression model outperforms the classical linear regression model in terms of predictive accuracy.

Table 3: Average of MSE of k-fold cross validation

Model	MSE
Linear Regression	0.00218
Quantile Regression	0.000117

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