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Original Article

The character table of a subgroup $2^7{:}G_2(2)$ of $Sp_8(2)$

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ABSTRACT: In this paper, the ordinary character table of a finite extension of structure $\overline{G} = 2^7{:}G_2(2)$ is computed via the Fischer-Clifford matrices technique. The group $\overline{G}$ sits maximally in the affine subgroup $2^7{:}Sp_6(2)$ of the symplectic group $Sp_8(2)$.

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1. Introduction

The Chevalley group $G_2(2)$ of type 2 has an absolutely irreducible representation of dimension 6 over $GF(2)$ and hence the split extension $\overline{S} = 2^6{:}G_2(2)$ exists [22]. In fact, $\overline{S}$ sits maximally in the maximal subgroup $\overline{S}_1 = 2^6{:}Sp(6,2)$ of the sporadic simple group Fi_{22} [5]. Moreover, the Schur Multiplier $M(\overline{S})$ of $\overline{S}$ is isomorphic to the cyclic group $\mathbb{Z}_2$ of order 2 and we obtain the 2-cover group $2.\overline{S}$ of $\overline{S}$ which is isomorphic to a group of structure $\overline{G} = 2^7{:}G_2(2)$. Furthermore, $\overline{G}$ also sits maximally in the 2-cover $2.\overline{S}_1$ of $\overline{S}_1$ which in turn is a maximal subgroup of $2.Fi_{22}$. Note that $2.\overline{S}_1 \cong ASp_8(2) = 2^7{:}Sp_6(2)$, where $ASp_8(2)$ is the affine subgroup of the symplectic group $Sp_8(2)$. Therefore, $Sp_8(2)$ contains an isomorphic copy of $\overline{G}$. The information above is verified with GAP [20] and we state it as Theorem 1.1.

Theorem 1.1. *The cover groups $\overline{G}$ and $2.\overline{S}_1$ of $\overline{S}$ and $\overline{S}_1$, respectively, are subgroups of $Sp_8(2)$.*

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In the online ATLAS [22], it can be seen that $Sp_8(2)$ has an absolutely irreducible representation of dimension 16 over $GF(2)$ and hence the split extension $\overline{G}_1 = 2^{16}{:}Sp_8(2)$ can be constructed. Using the Fischer-Clifford matrices technique [6], the ordinary irreducible character table of $\overline{G}_1$ was constructed in [14]. The group $\overline{G}$ is one of the inertia factors of the action of $\overline{G}_1$ on the irreducible characters $\mathrm{Irr}(2^{16})$ of 2^{16}. Therefore, the set $\mathrm{Irr}(\overline{G})$ plays an important role in the construction of the ordinary character table of $\overline{G}_1$ and for this reason we consider $\overline{G}$ as a subgroup of $Sp_8(2)$ and compute its ordinary character table using the Fischer-Clifford matrices technique. A Fischer-Clifford matrix (defined in Subsection 3.1) of $\overline{G}$, denoted by $M(g)$, is a square matrix with complex entries and is constructed for each conjugacy class $[g]$ of $Sp_8(2)$. In practice, we do not use the formal definition of a Fischer-Clifford matrix $M(g)$ to compute the character table of a finite extension group, for example $\overline{G}$, but instead use the powerful arithmetical properties (see Subsection 3.1) associated with $M(g)$. These properties are naturally inherited from the row and column orthogonal relations of the ordinary character table of a finite extension group. Therefore, we can compute the ordinary character tables of complicated finite extensions of p-groups with great efficiency using the Fischer-Clifford matrices technique. Since $\overline{G}$ is a split extension with its kernel 2^7 a 7-dimensional $G_2(2)$-module, the Fischer-Clifford matrices technique is an appropriate choice to compute the ordinary character table of $\overline{G}$.

From now on, we let $\overline{G}$ be the split extension of the elementary abelian 2-group $P = 2^7$ by $G = G_2(2)$. The conjugacy classes of $\overline{G}$ will be computed from each coset $Pg \in \frac{G}{P}$, where g is a conjugacy class $[g]_G$ representative of G. The method that will be used for this purpose is called the coset analysis technique (see [10, 11]). MAGMA routines in [1], based on the coset analysis technique, are used to compute the classes of $\overline{G}$ and their p power maps. The advantage of this routines is that we only need the generators of G as matrices of degree 7 over $GF(2)$ and identify P with a 7-dimensional vector space $V_7(2)$ over $GF(2)$. We do not require a representation of the whole of $\overline{G}$. Having the classes of $\overline{G}$ in coset analysis format, we can proceed to construct the Fischer-Clifford matrices $M(g)$ of $\overline{G}$ corresponding to each class representative $g \in G$. The ordinary character table of $\overline{G}$ is then constructed using the matrices $M(g)$ and the ordinary character tables of the so-called inertia factors H_i which are subgroups of G. The character table of $\overline{G}$ will be partitioned into blocks according to each inertia group $\overline{H_i}$ of $\mathrm{Irr}(P)$ in $\overline{G}$. See Subsection 3.1 for the definitions of $\overline{H_i}$ and H_i.

The character table of a nonsplit extension $\overline{G_2} = 2^{7\cdot}G_2(2)$ was computed in [18] by the Fischer-Clifford matrices technique. Although, the ordinary character tables of $\overline{G}$ and $\overline{G_2}$ do not coincide, but they have the same number of irreducible characters. Computations are done in GAP and MAGMA [4]. ATLAS notation [5] is used, unless stated otherwise.

2. Conjugacy Classes of $\overline{G}$

2.1. Construction of G as a 7×7 Matrix Group over $GF(2)$

A finite group F is called a $n \times n$ matrix group over a finite field K if each element of F can be represented as an invertible $n \times n$ matrix with entries in K. In [1], the symplectic group $Sp_6(2) \leq ASp_8(2)$ was constructed as a 7×7 matrix group over $GF(2)$. Since $G \leq Sp_6(2)$, we construct G as a 7×7 matrix group over $GF(2)$ within $Sp_6(2)$ and its generators g_1 and g_2 of orders of 2 and 6 are listed in Figure 1.

$$g_1 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \qquad g_2 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

Figure 1: Generators of G

2.2. Actions of G on P and $Irr(P)$

The linear group $G = < g_1, g_2 >$ acts on P, where we regard P as a 7-dimension vector space $V_7(2)$ over $GF(2)$, and obtain 4 orbits of lengths 1, 1, 63 and 63 with corresponding stabilizers $P_1 = G$, $P_2 = G$, $P_3 = 4^2{:}D_{12}$ and $P_4 = 4^2{:}D_{12}$. The permutation character $\chi(G|P) = 1aaaa + 14aa + 21aa + 27aa$ of G on the classes of P is computed in terms of the ordinary irreducible characters of G (see character table of $G_2(2)$ either in ATLAS or in GAP library). Evaluating $\chi(G|P)(g)$ on a class representative $g \in G$ gives the number of elements of P fixed by an element g. Since G has 4 orbits on P it follows by Brauer's Theorem [7] that G also has 4 orbits on $\mathrm{Irr}(P)$. To act G on the dual space P^* of P can be identified with the action of G on $\mathrm{Irr}(P)$. This action resulted in four orbits of lengths 1, 28, 36 and 63 with corresponding stabilizers $H_1 = G$, $H_2 = 3_+^{1+2}{:}8{:}2$, $H_3 = L_2(7){:}2$ and $H_4 = 4^2{:}D_{12}$ which are the inertia factors of $\overline{G}$ on $\mathrm{Irr}(P)$. Table 1 contains a summary of the information pertaining to the actions of G on P and $\mathrm{Irr}(P)$. The fusion maps of the classes of the inertia factors H_i into the classes of G are found in Table 2. For example, the fusion map of H_4 into G indicates that the elements of order 2 contain in the conjugacy class $2D$ of H_4 belong to the elements of order 2 in the conjugacy class $2B$ of G.

Table 1: Actions of G on P and $\mathrm{Irr}(P)$

	Action of G on P	**Action of G on $\mathrm{Irr}(P)$**
Number and lengths of Orbits	$\lvert O_1\rvert = 1$	$\lvert O_1\rvert = 1$
	$\lvert O_2\rvert = 1$	$\lvert O_2\rvert = 28$
	$\lvert O_3\rvert = 63$	$\lvert O_3\rvert = 36$
	$\lvert O_4\rvert = 63$	$\lvert O_4\rvert = 63$
Structure of point stabilizers	$P_1 = G_2(2)$	$H_1 = G_2(2)$
	$P_2 = G_2(2)$	$H_2 = 3_+^{1+2}{:}8{:}2$
	$P_3 = 4^2{:}D_{12}$	$H_3 = L_2(7){:}2$
	$P_4 = 4^2{:}D_{12}$	$H_4 = 4^2{:}D_{12}$
Size of stabilizers	$\lvert P_1\rvert = 12096$	$\lvert H_1\rvert = 12096$
	$\lvert P_2\rvert = 12096$	$\lvert H_2\rvert = 432$
	$\lvert P_3\rvert = 192$	$\lvert H_3\rvert = 336$
	$\lvert P_4\rvert = 192$	$\lvert H_4\rvert = 192$
Number of conjugacy classes $[g]$ of P_i and H_i	$\lvert [g]_{P_1}\rvert = 16$	$\lvert [g]_{H_1}\rvert = 16$
	$\lvert [g]_{P_2}\rvert = 16$	$\lvert [g]_{H_2}\rvert = 14$
	$\lvert [g]_{P_3}\rvert = 14$	$\lvert [g]_{H_3}\rvert = 9$
	$\lvert [g]_{P_4}\rvert = 14$	$\lvert [g]_{H_4}\rvert = 14$

Table 2: The fusion maps of H_i into G

$\lvert C_{H_2}(h)\rvert$	$[h]_{H_2}$ $\longrightarrow$	$[g]_G$	$\lvert C_{H_2}(h)\rvert$	$[h]_{H_2}$ $\longrightarrow$	$[g]_G$
432	$1A$	$1A$	24	$6A$	$6A$
48	$2A$	$2A$	6	$6B$	$6B$
12	$2B$	$2B$	8	$8A$	$8A$
216	$3A$	$3A$	8	$8B$	$8A$
18	$3B$	$3B$	12	$12A$	$12C$
24	$4A$	$4A$	12	$12B$	$12A$
12	$4B$	$4B$	12	$12C$	$12B$
$\lvert C_{H_3}(h)\rvert$	$[h]_{H_3}$ $\longrightarrow$	$[g]_G$	$\lvert C_{H_3}(h)\rvert$	$[h]_{H_3}$ $\longrightarrow$	$[g]_G$
336	$1A$	$1A$	6	$6A$	$6B$
16	$2A$	$2A$	7	$7A$	$7A$
12	$2B$	$2B$	8	$8A$	$8B$
6	$3A$	$3B$	8	$8B$	$8B$
8	$4A$	$4A$	12		
$\lvert C_{H_4}(h)\rvert$	$[h]_{H_4}$ $\longrightarrow$	$[g]_G$	$\lvert C_{H_4}(h)\rvert$	$[h]_{H_4}$ $\longrightarrow$	$[g]_G$
192	$1A$	$1A$	32	$4A$	$4A$
64	$2A$	$2A$	32	$4B$	$4C$
48	$2B$	$2B$	16	$4C$	$4B$
16	$2C$	$2A$	16	$4D$	$4C$
16	$2D$	$2B$	6	$6A$	$6B$
16	$2E$	$2B$	8	$8A$	$8A$
6	$3A$	$3B$	8	$8B$	$8B$

2.3. Coset Analysis Technique

The conjugacy classes of $\overline{G}$ is computed by the method of coset analysis (see [10, 11, 12]). In this subsection, we give a brief description of the coset analysis method for a split extension $SE = EA{:}Q$ of an elementary abelian p-group EA of order p^n by a linear matrix group Q of degree n over the field $GF(p)$. The group EA is regarded as a vector space $V_n(p)$ of dimension n over the finite field $GF(p)$ and is a Q-module over $GF(2)$, where upon the matrix group Q acts naturally. A coset $(EA)q$ is considered for each conjugacy class $[q]$ representative q in Q and then we consider the action of the stabilizer $C_q = EA{:}C_Q(q) = \{x \in SE | x((EA)q)x^{-1} = (EA)q\}$ of the coset EAq in SE by conjugation on the elements of $(EA)q$. Since C_q is split extension we will first act EA on $(EA)q$ to form k orbits $Q_1, Q_2, ..., Q_k$, with each orbit Q_i containing $|EA|/k$ elements. Under the action of the centralizer $C_Q(q)$ of $q \in Q$, f_j of the k orbits Q_i fuse together to form an orbit O_j. The orbit O_j contains the elements from the coset $(EA)q$ which belong to a conjugacy class $[x_j]$ of SE with class representative x_j. Note that $\sum f_j = k$. The order of the centralizer $C_{SE}(x_j)$ of the class representative x_j is computed by $|C_{SE}(x_j)| = \frac{k|C_Q(q)|}{f_j}$. In this manner, from a coset $(EA)q$, we obtain a set of conjugacy classes $\bigcup_{j=1}^{c(q)}[x_j]$ of SE, with class representatives $X(q) = \{x_1, x_2, ..., x_{c(q)}\}$.

We use a MAGMA routine, labelled as Programme A in [1], to compute the parameter f_j for our group $\overline{G}$. The value k is obtained by evaluate the permutation character $\chi(G|P)$, computed in Section 2.2, on a class representative $g \in G$. The MAGMA routine, Programme B in [1], is used to compute the orders of the elements of $\overline{G}$ as well as their p-power maps (see column 9 of Table 3). All the information pertaining to the elements of $\overline{G}$ is found in Table 3.

3. The Fischer-Clifford Matrices of $\overline{G}$

In this section, the Fischer-Clifford matrices technique will be applied to $\overline{G}$. A more general and detailed treatment of the Fischer-Clifford matrices technique is found in [1, 3, 6, 9, 12, 21]. For recent publications on the applications of the Fischer-Clifford matrices technique, see for example, [2, 13, 15, 16, 17].

3.1. General Construction of a Fischer-Clifford Matrix $M(g)$ of $\overline{G}$

From Section 2.2, $\overline{G}$ has four orbits O_i on $\mathrm{Irr}(P)$ with corresponding inertia groups $\overline{H}_i = P{:}H_i = \{x \in \overline{G} | \theta_i^x = \theta_i\}$, $i = 1, 2, 3, 4$, where $\theta_i \in \mathrm{O}_i$ are representatives of the orbits O_i and $H_i \cong \frac{\overline{H_i}}{P}$ the inertia factor groups (see Table 1 above). Since P is elementary abelian, each θ_i extends to a $\psi_i \in \mathrm{Irr}(\overline{H_i})$, i.e. $\psi_i \downarrow_P = \theta_i$, by Mackey's Theorem (see Theorem 5.1.15 in [12]). Furthermore, by Theorem 5.1.7, Remark 5.1.8 and Theorem 5.1.19 in [12], an ordinary irreducible character $\chi = (\psi_i\overline{\beta})^{\overline{G}}$ of $\overline{G}$ is obtained by induction of $\psi_i\overline{\beta} \in \mathrm{Irr}(\overline{H_i})$ to $\overline{G}$, where $P \subseteq \ker(\overline{\beta})$ of $\overline{\beta} \in \mathrm{Irr}(\overline{H}_i)$. Note that $\overline{\beta}$ is a lifting of $\beta \in \mathrm{Irr}(H_i)$ to $\overline{H}_i$. Hence, by Gallagher [8] we have Theorem 3.1 below for our group $\overline{G}$.

Theorem 3.1. $\mathrm{Irr}(\overline{G}) = \bigcup_{i=1}^{4} \left\{ (\psi_i\overline{\beta})^{\overline{G}} | \overline{\beta} \in \mathrm{Irr}(\overline{H_i}), P \subseteq \ker(\overline{\beta}) \right\} = \bigcup_{i=1}^{4} \left\{ (\psi_i\overline{\beta})^{\overline{G}} | \beta \in Irr(H_i) \right\}$.

Therefore, the set $\mathrm{Irr}(\overline{G})$ is partitioned into 4 blocks B_i with each block B_i corresponding to an inertia group $\overline{H_i}$ and $|\mathrm{Irr}(\overline{G})| = |\mathrm{Irr}(H_1)| + |\mathrm{Irr}(H_3)| + |\mathrm{Irr}(H_3)| = 16 + 14 + 9 + 14 = 53$.
We define the set

$$R(g) = \{(i, y_k) \,|\, 1 \leq i \leq 4, H_i \cap [g] \neq \emptyset, 1 \leq k \leq r\},$$

where y_k, $k = 1, 2, \ldots, r$, are representatives of conjugacy classes $[y_k]$ of H_i that fuse into a class $[g]$ of $H_1 = G$. Also, from the coset Pg, we have the set $X(g) = \{x_1, x_2, \ldots, x_{c(g)}\}$ of class representatives of $\overline{G}$. Furthermore, we have the set of pre-images $\eta^{-1}(y_k) = \bigcup_{s=1}^{c(k)}\{y_{k_s}\}$ of a class representative $y_k \in H_i$ under the natural homomorphism $\eta{:}\overline{H_i} \longrightarrow H_i$ such that y_{k_s} is conjugate to $x_j \in X(g)$. Then we evaluate $(\psi_i\overline{\beta})^{\overline{G}} \in \mathrm{Irr}(\overline{G})$ on $x_j \in X(g)$ as in Theorem 3.2 below.

Theorem 3.2. $(\psi_i\overline{\beta})^{\overline{G}}(x_j) = \sum_{y_k:(i,y_k)\in R(g)} \left[\sum_{s=1}^{c(k)} \frac{|C_{\overline{G}}(x_j)|}{|C_{\overline{H_i}}(y_{k_s})|}\psi_i(y_{k_s}) \right] \beta(y_k).$

Proof. See [21] □

For a class representative $g \in G$, a Fischer-Clifford matrix $M(g) = \left(a^j_{(i,y_k)}\right)$ of $\overline{G}$ is defined in Equation (1) below,

$$\left(a^j_{(i,y_k)}\right) = \left(\sum_{s=1}^{ck} \frac{|C_{\overline{G}}(x_j)|}{|C_{\overline{H_i}}(y_{k_s})|}\psi_i(y_{k_s}) \right). \tag{1}$$

Whereas, Figure 2 represents $M(g)$ in matrix form.

Table 3: The Conjugacy Classes of $\overline{G}$

$[x]_G$	k	f_j	d	w	$[x]_{\overline{G}}$	$C_G(g)$	$\lvert C_{\overline{G}}(x)\rvert$	**2P**	**3P**	**5P**	**7P**	$\to Sp_8(2)$
1A	128	1	(0,0,0,0,0,0,0)	(0,0,0,0,0,0,0)	1A	12096	1548288	1A	1A	1A	1A	1A
		1	(1,1,1,1,0,0,1)	(1,1,1,1,0,0,1)	2A		1548288	1A	2A	2A	2A	2A
		63	(0,1,0,1,0,1,1)	(0,0,0,0,1,0,0)	2B		24576	1A	2B	2B	2B	2B
		63	(1,0,0,0,0,0,0)	(1,0,0,0,0,0,0)	2C		24576	1A	2C	2C	2C	2C
2A	32	1	(0,0,0,0,0,0,0)	(0,0,0,0,0,0,0)	2D	192	6144	1A	2D	2D	2D	2H
		1	(1,1,1,1,0,0,1)	(0,0,0,0,0,0,0)	2E		6144	1A	2E	2E	2E	2I
		3	(0,1.0,0,0,0,0)	(0,0,0,0,0,0,0)	2F		2048	1A	2F	2F	2F	2G
		3	(1,0,0,0,0,0,1)	(0,0,0,0,0,0,0)	2G		2048	1A	2G	2G	2G	2J
		24	(0,0,1,1,1,0,1)	(1,0,0,0,0,1,0)	4A		256	2B	4A	4A	4A	4C
2B	16	1	(0,0,0,0,0,0,0)	(0,0,0,0,0,0,0)	2H	48	768	1A	2H	2H	2H	2N
		1	(1,1,1,0,1,0,0)	(0,0,0,0,0,0,0)	2I		768	2C	4B	4B	4B	4I
		1	(0,0,0,1,1,0,1)	(0,1,1,1,0,0,1)	4B		768	1A	2I	2I	2I	2O
		1	(1,1,1,1,0,0,1)	(1,0,1,0,0,0,0)	4C		768	2C	4C	4C	4C	4H
		6	(1,0,0,0,0,0,0,)	(1,0,1,0,0,0,0)	4D		128	2C	4D	4D	4D	4J
		6	(1,1,0,0,1,1,0)	(0,1,1,1,0,0,1)	4E		128	2B	4E	4E	4E	4G
3A	2	1	(0,0,0,0,0,0,0)	(0,0,0,0,0,0,0)	3A	216	432	3A	1A	3A	3A	3B
		1	(1,1,1,1,0,0,1)	(1,1,1,1,0,0,1)	6A		432	3A	2A	6A	6A	6D
3B	8	1	(0,0,0,0,0,0,0)	(0,0,0,0,0,0,0)	3B	18	144	3B	1A	3B	3B	3C
		1	(1,1,1,1,0,0,1)	(0,1,1,1,0,0,1)	6B		144	3B	2A	6B	6B	6G
		3	(1,1,0,1,1,1,0)	(1,0,1,0,1,1.0)	6C		48	3B	2C	6C	6C	6E
		3	(1,0,0,0,0,0,0)	(0,1,1,1,1,0,0)	6D		48	3B	2B	6D	6D	6F
4A	8	1	(0,0,0,0,0,0,0)	(0,0,0,0,0,0,0)	4F	96	768	2D	4F	4F	4F	4L
		1	(0,1,0,0,0,1,0)	(0,0,0,0,0,0,0)	4G		768	2D	4G	4G	4G	4M
		6	(1,1,0,0,0,0,0)	(0,0,0,0,0,0,0)	4H		128	2F	4H	4H	4H	4K
4B	8	1	(0,0,0,0,0,0,0)	(0,0,0,0,0,0,0)	4I	48	384	2D	4I	4I	4I	4L
		1	(0,0,0,0,0,1,0)	(0,0,0,0,0,0,0)	4J		384	2D	4J	4J	4J	4M
		6	(0,0,0,0,0,0,1)	(0,0,0,0,0,0,0)	4K		64	2F	4K	4K	4K	4K
4C	8	1	(0,0,0,0,0,0,0)	(0,0,0,0,0,0,0)	4L	32	256	2D	4L	4L	4L	4X
		1	(0,1,0,0,0,1,0)	(0,0,0,0,0,0,0)	4M		256	2D	4M	4M	4M	4W
		2	(1,0,1,0,0,1,1)	(0,0,0,0,0,0,0)	4N		128	2F	4N	4N	4N	4V
		4	(1,1,1,0,0,0,1)	(0,0,0,0,0,0,0)	4O		64	2G	4O	4O	4O	4Y
6A	2	1	(0,0,0,0,0,0,0)	(0,0,0,0,0,0,0)	6E	24	48	3A	2D	6E	6E	6M
		1	(0,1,1,1,0,0,1)	(0,0,0,0,0,0,0)	6F		48	3A	2E	6F	6F	6N
6B	4	1	(0,0,0,0,0,0,0)	(0,0,0,0,0,0,0)	6G	6	24	3B	2H	6G	6G	6X
		1	(1,0,0,0,0,0.0)	(0,0,1,1,0,0,0)	12A		24	3B	2I	6H	6H	6W
		1	(1,1,1,0,0,0,1)	(0,0,0,0,0,0,0)	6H		24	6C	4B	12A	12A	12I
		1	(1,0,1,1,0,1,1)	(0,0,1,1,0,0,0)	12B		24	6C	4C	12B	12B	12J
7A	2	1	(0,0,0,0,0,0,0)	(0,0,0,0,0,0,0)	7A	7	14	7A	7A	7A	1A	7A
		1	(1,1,1,1,0,0,1)	(1,1,1,1,0,0,1)	14A		14	7A	14A	14A	2A	14A
8A	4	1	(0,0,0,0,0,0,0)	(0,0,0,0,0,0,0)	8A	8	32	4L	8A	8A	8A	8I
		1	(0,1,0,0,0,1,0)	(0,0,0,0,0,0,0)	8B		32	4L	8B	8B	8B	8J
		1	(1,1,1,0,0,0,1)	(0,0,0,0,0,0,0)	8C		32	4N	8D	8D	8C	8H
		1	(1,0,1,1,0,1,1)	(0,0,0,0,0,0,0)	8D		32	4N	8C	8C	8D	8H
8B	4	1	(0,0,0,0,0,0,0)	(0,0,0,0,0,0,0)	8E	8	32	4F	8E	8E	8E	8F
		1	(1,1,0,0,0,0,1)	(0,0,0,0,0,0,0)	8F		32	4F	8F	8F	8F	8G
		1	(0,0,1,1,0,0,0)	(0,0,0,0,0,0,0)	8G		32	4H	8G	8H	8H	8E
		1	(0,0,1,1,0,0,0)	(0,0,0,0,0,0,0)	8H		32	4H	8H	8G	8G	8E
12A	2	1	(0,0,0,0,0,0,0)	(0,0,0,0,0,0,0)	12C	12	24	6E	4F	12C	12C	12O
		1	(1,1,1,1,0,0,0)	(0,0,0,0,0,0,0)	12D		24	6E	4G	12D	12D	12P
12B	2	1	(0,0,0,0,0,0,0)	(0,0,0,0,0,0,0)	12E	12	24	6E	4I	12G	12E	12O
		1	(1,1,1,1,0,0,0)	(0,0,0,0,0,0,0)	12F		24	6E	4J	12H	12F	12P
12C	2	1	(0,0,0,0,0,0,0)	(0,0,0,0,0,0,0)	12G	12	24	6E	4I	12E	12G	12O
		1	(0,0,0,0,0,0,0)	(0,0,0,0,0,0,0)	12H		24	6E	4J	12F	12H	12P

The Fischer-Clifford matrix $M(g)$ (see Figure 2) is partitioned row-wise into blocks $M_i(g)$, where each block corresponds to an inertia group $\overline{H}_i$. We write $|C_{\overline{G}}(x_j)|$, for each $x_j \in X(g)$, at the top of the columns of $M(g)$ and at the bottom we write $m_j = [C_g{:}C_{\overline{G}}(\overline{x_j})] = |P|\frac{|C_G(g)|}{|C_{\overline{G}}(\overline{x_j})|} = \frac{f_j|P|}{k}$. On the left of each row we write $|C_{H_i}(y_k)|$, where $[y_k]$, $k = 1, 2, \dots, r$, are the classes of an inertia factor H_i that fuse into the class $[g]$ of G. Since $|X(g)| = |R(g)|$ it follows that $M(g)$ is a square matrix of size $c(g)$. When there is no class fusion of an inertia factor H_i into a class $[g]$, the block $M_i(g)$ is omitted from $M(g)$.

$$
M(g) = \begin{array}{c} \\ |C_G(g)| \\ |C_{H_2}(y_1)| \\ |C_{H_2}(y_2)| \\ \vdots \\ |C_{H_3}(y_1)| \\ |C_{H_3}(y_3)| \\ \vdots \\ |C_{H_4}(y_1)| \\ |C_{H_4}(y_2)| \\ \vdots \\ \\ \end{array}
\begin{array}{cccc}
|C_{\overline{G}}(x_1)| & |C_{\overline{G}}(x_2)| & \cdots & |C_{\overline{G}}(x_{c(g)})| \\
\hline
1 & 1 & \cdots & 1 \\
\hline
\frac{|C_G(g)|}{|C_{H_2}(y_1)|} & a^2_{(2,y_1)} & \cdots & a^{c(g)}_{(2,y_1)} \\
\frac{|C_G(g)|}{|C_{H_2}(y_2)|} & a^2_{(2,y_2)} & \cdots & a^{c(g)}_{(2,y_2)} \\
\vdots & \vdots & \vdots & \vdots \\
\hline
\frac{|C_G(g)|}{|C_{H_3}(y_1)|} & a^2_{(3,y_1)} & \cdots & a^{c(g)}_{(3,y_1)} \\
\frac{|C_G(g)|}{|C_{H_3}(y_2)|} & a^2_{(3,y_2)} & \cdots & a^{c(g)}_{(3,y_2)} \\
\vdots & \vdots & \vdots & \vdots \\
\hline
\frac{|C_G(g)|}{|C_{H_4}(y_1)|} & a^2_{(4,y_1)} & \cdots & a^{c(g)}_{(4,y_1)} \\
\frac{|C_G(g)|}{|C_{H_4}(y_2)|} & a^2_{(4,y_2)} & \cdots & a^{c(g)}_{(4,y_2)} \\
\vdots & \vdots & \vdots & \vdots \\
\hline
m_1 & m_2 & \cdots & m_{c(g)}
\end{array}
$$

Figure 2: The Fischer-Clifford Matrix $M(g)$

Instead of using the above formal definition, i.e. Equation 1, we use the arithmetical properties 1-8 of $M(g)$ below (see [12]) to compute the entries $a^j_{(i,y_k)}$. Since $\overline{G}$ is a split extension of an elementary abelian group P of order 128, we use relation 6 to compute the values for column 1 of Figure 2. The relations 3 and 4 are inherited from the row and orthogonality relations of an ordinary character table of a finite group.

1. $a^j_{(1,g)} = 1$ for all $j = \{1, 2, \dots, c(g)\}$.
2. $|X(g)| = |R(g)|$.
3. $\sum_{j=1}^{c(g)} m_j \, a^j_{(i,y_k)} \, \overline{a^j_{(i',y'_k)}} = \delta_{(i,y_k),(i',y'_k)} \, \frac{|C_G(g)|}{|C_{H_i}(y_k)|} \, |P|$.
4. $\sum_{(i,y_k)\in R(g)} a^j_{(i,y_k)} \, \overline{a^{j'}_{(i,y_k)}} \, |C_{H_i}(y_k)| = \delta_{jj'} \, |C_{\overline{G}}(x_j)|$.
5. $M(g)$ is square and nonsingular.

Since P is an elementary abelian 2-group, then we obtain the additional properties 6-8 of $M(g)$ below,

6. $a^1_{(i,y_k)} = \frac{|C_G(g)|}{|C_{H_i}(y_k)|}$.
7. $|a^1_{(i,y_k)}| \geq |a^j_{(i,y_k)}|$.
8. $a^j_{(i,y_k)} \equiv a^1_{(i,y_k)} (\text{mod } 2)$.

3.2. Computing the Fischer Clifford Matrices of $\overline{G}$

Using Table 2 and the construction process of a matrix $M(g)$ given in Section 3.1, we compute all the Fischer-Clifford matrices of $\overline{G}$ and they are contained in Table 4.

Table 4: The Fischer-Clifford Matrices of $\overline{G}$

$M(g)$	$M(g)$
$M(1A) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 28 & -28 & 4 & -4 \\ 36 & -36 & -4 & 4 \\ 63 & 63 & -1 & -1 \end{pmatrix}$	$M(2A) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 4 & -4 & 4 & -4 & 0 \\ 12 & -12 & -4 & 4 & 0 \\ 3 & 3 & 3 & 3 & -1 \\ 12 & 12 & -4 & -4 & 0 \end{pmatrix}$
$M(2B) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 4 & -4 & -4 & 4 & 0 & 0 \\ 4 & -4 & 4 & -4 & 0 & 0 \\ 1 & 1 & -1 & -1 & -1 & 1 \\ 3 & 3 & -3 & -3 & 1 & -1 \\ 3 & 3 & 3 & 3 & -1 & -1 \end{pmatrix}$	$M(3A) = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$
$M(3B) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 3 & -3 & -1 & 1 \\ 3 & 3 & -1 & -1 \end{pmatrix}$	$M(4A) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 4 & -4 & 0 & 0 \\ 1 & 1 & 1 & -1 \\ 2 & 2 & -2 & 0 \end{pmatrix}$
$M(4B) = \begin{pmatrix} 1 & 1 & 1 \\ 4 & -4 & 0 \\ 3 & 3 & -1 \end{pmatrix}$	$M(4C) = \begin{pmatrix} 1 & 1 & 1 \\ 4 & -4 & 0 \\ 3 & 3 & -1 \end{pmatrix}$
$M(6A) = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$	$M(6B) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \end{pmatrix}$
$M(7A) = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$	$M(8A) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}$
$M(8B) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}$	$M(12A) \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$
$M(12B) = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$	$M(12C) = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$

4. The Character Table of $\overline{G}$

Since we obtained the fusion maps of classes of the inertia factors H_i into G, the character tables of the inertia factors H_i (stored in the GAP library) and the Fischer-Clifford matrices of $\overline{G}$, we proceed to construct the ordinary character table of $\overline{G}$. A partial character table (see Figure 3) of $\overline{G}$ is constructed by multiplying the columns $C_i(g)$ for $i \in \{1, 2, 3, 4\}$ of the ordinary character tables of the inertia factors H_i associated with the classes y_k of H_i, that are conjugate to $[g]_G$, by the rows of the Fischer-Clifford matrices in a block $M_i(g)$.

$$\begin{bmatrix} C_1(g)\, M_1(g) \\ C_2(g)\, M_2(g) \\ C_3(g)\, M_3(g) \\ C_4(g)\, M_4(g) \end{bmatrix}.$$

Figure 3: Partial Character Table of $\overline{G}$

To construct the set $\mathrm{Irr}(G)$, we append the partial character tables coming from each class $[g_i]$, $i = 1, 2, \ldots, 16$, of G as depicted in Figure 4.

$$\left[\begin{array}{c|c|c|c} C_1(g_1)\, M_1(g_1) & C_1(g_2)\, M_1(g_2) & \cdots & C_1(g_{16})\, M_1(g_{16}) \\ C_2(g_1)\, M_2(g_1) & C_2(g_2)\, M_2(g_2) & \cdots & C_2(g_{16})\, M_2(g_{16}) \\ C_3(g_1)\, M_3(g_1) & C_3(g_2)\, M_3(g_2) & \cdots & C_3(g_{16})\, M_3(g_{16}) \\ C_4(g_1)\, M_4(g_1) & C_4(g_2)\, M_4(g_2) & \cdots & C_4(g_{16})\, M_4(g_{16}) \end{array}\right].$$

Figure 4: Structure of $\mathrm{Irr}(\overline{G})$

The character table of $\overline{G}$ (see Table 5) obtained in this manner is a 53×53–complex valued square matrix with the irreducible characters partitioned into blocks Δ_i, for $i \in \{1, 2, 3, 4\}$, such that $\Delta_1 = \{\chi_i | 1 \le i \le 16\}$, $\Delta_2 = \{\chi_i | 17 \le i \le 30\}$, $\Delta_3 = \{\chi_i | 31 \le i \le 39\}$ and $\Delta_4 = \{\chi_i | 40 \le i \le 53\}$, where $\chi_i \in \mathrm{Irr}(\overline{G})$. The fusion map of the classes of $\overline{G}$ into $Sp_8(2)$ is obtained with the aid of the GAP function "PossibleClassFusions" and the ordinary characters tables of $\overline{G}$ and $Sp_(8)$ and is captured in the last column of Table 3. A GAP routine in [19] was implemented for checking the consistency and accuracy of the character table of $\overline{G}$. To reconstruct the character table in GAP, that is Table 5, interested readers can use the link below. The file contains the class orders, centralizer orders and the irreducible characters of $\overline{G}$ which were computed in this paper.

https://drive.google.com/file/d/1pv7JhGzdiGJ1eBOh39WWEvmd_LOh5pte/view?usp=drive_link

Table 5: The Character Table of $\overline{G}$

$[g]_G$	1A				2A					2B						3A	
$[x]_{\overline{G}}$	1A	2A	2B	2C	2D	2E	2F	2G	4A	2H	4B	2I	4C	4D	4E	3A	6A
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	1	1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	1	1
χ_3	6	6	6	6	-2	-2	-2	-2	-2	0	0	0	0	0	0	-3	-3
χ_4	6	6	6	6	-2	-2	-2	-2	-2	0	0	0	0	0	0	-3	-3
χ_5	7	7	7	7	-1	-1	-1	-1	-1	1	1	1	1	1	1	-2	-2
χ_6	7	7	7	7	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-2	-2
χ_7	14	14	14	14	-2	-2	-2	-2	-2	2	2	2	2	2	2	5	5
χ_8	14	14	14	14	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	5	5
χ_9	14	14	14	14	6	6	6	6	6	0	0	0	0	0	0	-4	-4
χ_{10}	21	21	21	21	5	5	5	5	5	-3	-3	-3	-3	-3	-3	3	3
χ_{11}	21	21	21	21	5	5	5	5	5	3	3	3	3	3	3	3	3
χ_{12}	27	27	27	27	3	3	3	3	3	3	3	3	3	3	3	0	0
χ_{13}	27	27	27	27	3	3	3	3	3	-3	-3	-3	-3	-3	-3	0	0
χ_{14}	42	42	42	42	2	2	2	2	2	0	0	0	0	0	0	6	6
χ_{15}	56	56	56	56	-8	-8	-8	-8	-8	0	0	0	0	0	0	2	2
χ_{16}	64	64	64	64	0	0	0	0	0	0	0	0	0	0	0	-8	-8
χ_{17}	28	-28	-4	4	4	-4	4	-4	0	4	-4	-4	4	0	0	1	-1
χ_{18}	28	-28	-4	4	4	-4	4	-4	0	-4	4	4	-4	0	0	1	-1
χ_{19}	28	-28	-4	4	4	-4	4	-4	0	-4	4	4	-4	0	0	1	-1
χ_{20}	28	-28	-4	4	4	-4	4	-4	0	4	-4	-4	4	0	0	1	-1
χ_{21}	56	-56	-8	8	8	-8	8	-8	0	0	0	0	0	0	0	2	-2
χ_{22}	56	-56	-8	8	-8	8	-8	8	0	0	0	0	0	0	0	2	-2
χ_{23}	56	-56	-8	8	-8	8	-8	8	0	0	0	0	0	0	0	2	-2
χ_{24}	168	-168	-24	24	-8	8	-8	8	0	0	0	0	0	0	0	-3	3
χ_{25}	168	-168	-24	24	-8	8	-8	8	0	0	0	0	0	0	0	-3	3
χ_{26}	168	-168	-24	24	-8	8	-8	8	0	0	0	0	0	0	0	-3	3
χ_{27}	168	-168	-24	24	-8	8	-8	8	0	0	0	0	0	0	0	-3	3
χ_{28}	224	-224	-32	32	0	0	0	0	0	-8	8	8	-8	0	0	8	-8
χ_{29}	224	-224	-32	32	0	0	0	0	0	8	-8	-8	8	0	0	8	-8
χ_{30}	336	-336	-48	48	16	-16	16	-16	0	0	0	0	0	0	0	-6	6
χ_{31}	36	-36	4	-4	12	-12	-4	4	0	4	4	-4	-4	0	0	0	0
χ_{32}	36	-36	4	-4	12	-12	-4	4	0	-4	-4	4	4	0	0	0	0
χ_{33}	216	-216	24	-24	-24	24	8	-8	0	0	0	0	0	0	0	0	0
χ_{34}	216	-216	24	-24	24	-24	-8	8	0	0	0	0	0	0	0	0	0
χ_{35}	216	-216	24	-24	24	-24	-8	8	0	0	0	0	0	0	0	0	0
χ_{36}	252	-252	28	-28	-12	12	4	-4	0	-4	-4	4	4	0	0	0	0
χ_{37}	252	-252	28	-28	-12	12	4	-4	0	4	4	-4	-4	0	0	0	0
χ_{38}	288	-288	32	-32	0	0	0	0	0	-8	-8	8	8	0	0	0	0
χ_{39}	288	-288	32	-32	0	0	0	0	0	8	8	-8	-8	0	0	0	0
χ_{40}	63	63	-1	-1	15	15	-1	-1	-1	7	-1	7	-1	-1	-1	0	0
χ_{41}	63	63	-1	-1	-9	-9	7	7	-1	1	5	1	5	-3	1	0	0
χ_{42}	63	63	-1	-1	-9	-9	7	7	-1	-1	-5	-1	-5	3	-1	0	0
χ_{43}	63	63	-1	-1	15	15	-1	-1	-1	-7	1	-7	1	1	1	0	0
χ_{44}	126	126	-2	-2	6	6	6	6	-2	8	4	8	4	-4	0	0	0
χ_{45}	126	126	-2	-2	6	6	6	6	-2	-8	-4	-8	-4	4	0	0	0
χ_{46}	189	189	-3	-3	-3	-3	13	13	-3	-3	-3	-3	-3	-3	5	0	0
χ_{47}	189	189	-3	-3	-3	-3	13	13	-3	3	3	3	3	3	-5	0	0
χ_{48}	189	189	-3	-3	21	21	5	5	-3	-3	9	-3	9	1	-3	0	0
χ_{49}	189	189	-3	-3	21	21	5	5	-3	3	-9	3	-9	-1	3	0	0
χ_{50}	378	378	-6	-6	-30	-30	2	2	2	0	0	0	0	0	0	0	0
χ_{51}	378	378	-6	-6	-6	-6	-6	-6	2	-6	6	-6	6	-2	2	0	0
χ_{52}	378	378	-6	-6	-6	-6	-6	-6	2	6	-6	6	-6	2	-2	0	0
χ_{53}	378	378	-6	-6	18	18	-14	-14	2	0	0	0	0	0	0	0	0

$[g]_G$	3B				4A				4B			4C			6A	
$[x]_{\overline{G}}$	3B	6B	6c	6D	4F	4G	4H	4I	4J	4K	4L	4M	4N	4O	6E	6F
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	1	1	1	1	1	1	-1	-1	-1	1	1	1	1	1	1
χ_3	0	0	0	0	-2	-2	-2	0	0	0	2	2	2	2	1	1
χ_4	0	0	0	0	-2	-2	-2	0	0	0	2	2	2	2	1	1
χ_5	1	1	1	1	3	3	3	-3	-3	-3	-1	-1	-1	-1	2	2
χ_6	1	1	1	1	3	3	3	3	3	3	-1	-1	-1	-1	2	2
χ_7	-1	-1	-1	-1	2	2	2	-2	-2	-2	2	2	2	2	1	1
χ_8	-1	-1	-1	-1	2	2	2	2	2	2	2	2	2	2	1	1
χ_9	2	2	2	2	-2	-2	-2	0	0	0	2	2	2	2	0	0
χ_{10}	0	0	0	0	1	1	1	1	1	1	1	1	1	1	-1	-1
χ_{11}	0	0	0	0	1	1	1	-1	-1	-1	1	1	1	1	-1	-1
χ_{12}	0	0	0	0	3	3	3	3	3	3	-1	-1	-1	-1	0	0
χ_{13}	0	0	0	0	3	3	3	-3	-3	-3	-1	-1	-1	-1	0	0
χ_{14}	0	0	0	0	-6	-6	-6	0	0	0	-2	-2	-2	-2	2	2
χ_{15}	2	2	2	2	0	0	0	0	0	0	0	0	0	0	-2	-2
χ_{16}	-2	-2	-2	-2	0	0	0	0	0	0	0	0	0	0	0	0
χ_{17}	1	-1	1	-1	4	-4	0	4	-4	0	0	0	0	0	1	-1
χ_{18}	1	-1	1	-1	4	-4	0	-4	4	0	0	0	0	0	1	-1
χ_{19}	1	-1	1	-1	4	-4	0	4	-4	0	0	0	0	0	1	-1
χ_{20}	1	-1	1	-1	4	-4	0	-4	4	0	0	0	0	0	1	-1
χ_{21}	2	-2	2	-2	-8	8	0	0	0	0	0	0	0	0	2	-2
χ_{22}	2	-2	2	-2	0	0	0	0	0	0	0	0	0	0	-2	2
χ_{23}	2	-2	2	-2	0	0	0	0	0	0	0	0	0	0	-2	2
χ_{24}	0	0	0	0	8	-8	0	8	-8	0	0	0	0	0	1	-1
χ_{25}	0	0	0	0	8	-8	0	-8	8	0	0	0	0	0	1	-1
χ_{26}	0	0	0	0	-8	8	0	0	0	0	0	0	0	0	1	-1
χ_{27}	0	0	0	0	-8	8	0	0	0	0	0	0	0	0	1	-1
χ_{28}	-1	1	-1	1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{29}	-1	1	-1	1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{30}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	2
χ_{31}	3	-3	-1	1	0	0	0	0	0	0	4	-4	0	0	0	0
χ_{32}	3	-3	-1	1	0	0	0	0	0	0	4	-4	0	0	0	0
χ_{33}	0	0	0	0	0	0	0	0	0	0	8	-8	0	0	0	0
χ_{34}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{35}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{36}	3	-3	-1	1	0	0	0	0	0	0	-4	4	0	0	0	0
χ_{37}	3	-3	-1	1	0	0	0	0	0	0	-4	4	0	0	0	0
χ_{38}	-3	3	1	-1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{39}	-3	3	1	-1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{40}	3	3	-1	-1	3	3	-1	3	3	-1	3	3	-1	-1	0	0
χ_{41}	3	3	-1	-1	3	3	-1	-3	-3	1	-1	-1	3	-1	0	0
χ_{42}	3	3	-1	-1	3	3	-1	3	3	-1	-1	-1	3	-1	0	0
χ_{43}	3	3	-1	-1	3	3	-1	-3	-3	1	3	3	-1	-1	0	0
χ_{44}	-3	-3	1	1	6	6	-2	0	0	0	2	2	2	-2	0	0
χ_{45}	-3	-3	1	1	6	6	-2	0	0	0	2	2	2	-2	0	0
χ_{46}	0	0	0	0	-3	-3	1	-3	-3	1	-3	-3	1	1	0	0
χ_{47}	0	0	0	0	-3	-3	1	3	3	-1	-3	-3	1	1	0	0
χ_{48}	0	0	0	0	-3	-3	1	-3	-3	1	1	1	-3	1	0	0
χ_{49}	0	0	0	0	-3	-3	1	3	3	-1	1	1	-3	1	0	0
χ_{50}	0	0	0	0	-6	-6	2	0	0	0	6	6	-2	-2	0	0
χ_{51}	0	0	0	0	6	6	-2	6	6	-2	-2	-2	-2	2	0	0
χ_{52}	0	0	0	0	6	6	-2	-6	-6	2	-2	-2	-2	2	0	0
χ_{53}	0	0	0	0	-6	-6	2	0	0	0	-2	-2	6	-2	0	0

$[g]_G$	6B				7A		8A				8B				12A		12B		12C	
$[x]_{\overline{G}}$	6G	6H	12A	12B	7A	14A	8A	8B	8C	8D	8E	8F	8G	8H	12C	12D	12E	12F	12G	12h
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	-1	-1	-1	-1	1	1	-1	-1	-1	-1	1	1	1	1	1	1	-1	-1	-1	-1
χ_3	0	0	0	0	-1	-1	0	0	0	0	0	0	0	0	1	1	C	C	-C	-C
χ_4	0	0	0	0	-1	-1	0	0	0	0	0	0	0	0	1	1	-C	-C	C	C
χ_5	1	1	1	1	0	0	-1	-1	-1	-1	-1	-1	-1	-1	0	0	0	0	0	0
χ_6	-1	-1	-1	-1	0	0	1	1	1	1	-1	-1	-1	-1	0	0	0	0	0	0
χ_7	-1	-1	-1	-1	0	0	0	0	0	0	0	0	0	0	-1	-1	1	1	1	1
χ_8	1	1	1	1	0	0	0	0	0	0	0	0	0	0	-1	-1	-1	-1	-1	-1
χ_9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	-2	0	0	0	0
χ_{10}	0	0	0	0	0	0	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1
χ_{11}	0	0	0	0	0	0	1	1	1	1	-1	-1	-1	-1	1	1	-1	-1	-1	-1
χ_{12}	0	0	0	0	-1	-1	-1	-1	-1	-1	1	1	1	1	0	0	0	0	0	0
χ_{13}	0	0	0	0	-1	-1	1	1	1	1	1	1	1	1	0	0	0	0	0	0
χ_{14}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{15}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{16}	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{17}	1	-1	-1	1	0	0	0	0	0	0	2	-2	0	0	1	-1	1	-1	1	-1
χ_{18}	-1	1	1	-1	0	0	0	0	0	0	2	-2	0	0	1	-1	-1	1	-1	1
χ_{19}	-1	1	1	-1	0	0	0	0	0	0	-2	2	0	0	1	-1	1	-1	1	-1
χ_{20}	1	-1	-1	1	0	0	0	0	0	0	-2	2	0	0	1	-1	-1	1	-1	1
χ_{21}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	2	0	0	0	0
χ_{22}	0	0	0	0	0	0	0	0	0	0	0	0	B	-B	0	0	0	0	0	0
χ_{23}	0	0	0	0	0	0	0	0	0	0	0	0	-B	B	0	0	0	0	0	0
χ_{24}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	1	-1	1	-1	1
χ_{25}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	1	1	-1	1	-1
χ_{26}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-1	C	-C	-C	C
χ_{27}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-1	-C	C	C	-C
χ_{28}	1	-1	-1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{29}	-1	1	1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{30}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{31}	1	-1	1	-1	1	-1	2	-2	0	0	0	0	0	0	0	0	0	0	0	0
χ_{32}	-1	1	-1	1	1	-1	-2	2	0	0	0	0	0	0	0	0	0	0	0	0
χ_{33}	0	0	0	0	-1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{34}	0	0	0	0	-1	1	0	0	A	-A	0	0	0	0	0	0	0	0	0	0
χ_{35}	0	0	0	0	-1	1	0	0	-A	A	0	0	0	0	0	0	0	0	0	0
χ_{36}	-1	1	-1	1	0	0	2	-2	0	0	0	0	0	0	0	0	0	0	0	0
χ_{37}	1	-1	1	-1	0	0	-2	2	0	0	0	0	0	0	0	0	0	0	0	0
χ_{38}	1	-1	1	-1	1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{39}	-1	1	-1	1	1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{40}	1	1	-1	-1	0	0	1	1	-1	-1	1	1	-1	-1	0	0	0	0	0	0
χ_{41}	1	1	-1	-1	0	0	-1	-1	1	1	-1	-1	1	1	0	0	0	0	0	0
χ_{42}	-1	-1	1	1	0	0	1	1	-1	-1	-1	-1	1	1	0	0	0	0	0	0
χ_{43}	-1	-1	1	1	0	0	-1	-1	1	1	1	1	-1	-1	0	0	0	0	0	0
χ_{44}	-1	-1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{45}	1	1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{46}	0	0	0	0	0	0	1	1	-1	-1	1	1	-1	-1	0	0	0	0	0	0
χ_{47}	0	0	0	0	0	0	-1	-1	1	1	1	1	-1	-1	0	0	0	0	0	0
χ_{48}	0	0	0	0	0	0	1	1	-1	-1	-1	-1	1	1	0	0	0	0	0	0
χ_{49}	0	0	0	0	0	0	-1	-1	1	1	-1	-1	1	1	0	0	0	0	0	0
χ_{50}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{51}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{52}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{53}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

A $= -2\sqrt{2}i$, B $= -2\sqrt{2}$, C $= -\sqrt{3}i$

Data availability statement

The authors confirm that the data supporting the findings of this study are available within the article.

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