



# *Latency Compensation in Multi Chaotic Systems Using the Extended OGY Control Method*

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## **ABSTRACT**

The problem discussed in this paper is the effect of latency time on the OGY chaos control methodology in multi chaotic systems. The Smith predictor, rhythmic and memory strategies are embedded in the OGY chaos control method to encounter loop latency. A comparison study is provided and the advantages of the Smith predictor approach are clearly evident from the closed loop responses. The complex plants considered are coupled chaotic maps controlled by the extended OGY scheme. Simulation results are used to show the effectiveness of the applied Smith predictor scheme structure in multi chaotic systems.

## **KEYWORDS**

Chaos, Latency in multi chaotic systems, OGY, Smith Predictor.

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## 1. INTRODUCTION

Delay is a generic problem in the control of chaotic systems. The effective delay time in any feedback loop is the sum of present delay times, e.g. the measurement duration, control design computations, and the system response time to the applied control signal [1, 2, 3].

There are two major chaos control strategies: one is the OGY method (based on linearization of the Poincaré map [4]) and the other is the Pyragas method (based on a time-delay feedback [5]).

There is another delay- named latency- which is introduced in time-delay feedback methodology. As defined in [6], latency times are the times required to compare the current signal with its time-delayed counterpart and apply the feedback into the system. In an experimental setup, there is always latency involved. Control loop latency is associated with finite propagation speed of the control signal, measurement delays of the system to generate the control signal, or processing times for feedback calculation. In optical or opto-electronic systems, for instance, the length of an optical fiber used to transmit the control signal can become of crucial importance [7]. The same holds for electronic systems such as a fast diode resonator, where a propagation delay acts as a limiting factor [8]. Latency has adverse effects on control systems, which are analysed in [9] and [10]. It is to be noted that delay and latency concepts are interchangeably used in many papers.

It has been shown in [11, 12] that longer latency times reduce the control abilities of the time-delayed feedback control. Similar results were found for the extended time-delayed feedback [13]. Latency times on the OGY controllers is analysed in [1, 2, 3]. It was also shown that the measurement delay problem can be solved systematically for the OGY control and difference control by rhythmic control and memory methods.

One approach to avoid the failure of chaos control due to latency is to simplify the controller as much as possible in order to minimize the control loop latency. An example of this approach includes methods that apply perturbations of a predetermined strength when the system crosses some threshold in the phase space [14, 15]. Using these methods, successful control of fast but low-dimensional chaos was demonstrated. A drawback of these methods is that they rely on the ability of easily defining proper windows and walls in the phase space. This task is conceptually much more difficult in the case of delayed systems where the phase space is infinite dimensional.

These inherent difficulties prevented the applications of such methods to practical delay systems [16].

The primary objective of this paper is to derive by means of a simple example the latency adverse effects on feedback systems and to provide an approach for latency tackling in the face of substantial control-loop latency. The paper is focused on the OGY chaos control methodology and employs the Henon map as a two dimensional chaotic system. Then, Rhythmic control, Memory control, and Smith predictor will be applied in the presence of loop latency and study if these methodologies can overcome the problems caused by the loop latency.

Then, upon the application of such compensating control methods in a single chaotic map, the proposed methods will be applied to complex multi-chaotic interconnected plants. In [17], complex structures containing chaotic subsystems with interactions was considered and an extended OGY method was presented to tackle the interaction effects. In this paper, the Smith predictor structure is appropriately inserted in the proposed structure and the simulation results show the efficiency of the final design methodology.

The paper is organized as follows: A brief review of the OGY control system in its original form and the extended OGY control methodology for interconnected chaotic maps are presented in section 2. In section 3, latency structure is introduced and the stability regions are calculated for the Henon chaotic map, with and without loop latency. In section 4, Rhythmic control method, Memory method, and Smith predictor structure are presented as compensation methods of latency in chaos control systems. In section 5, the presented methods of section 4 are implemented on the Henon chaotic map with latency and simulation results are provided to show the effectiveness of the proposed methodologies. Section 6 contains the implementation results for an interconnected multi- chaotic maps using the extended OGY control. Finally, section 6 concludes the paper.

## 2. PRILIMINARY THEORIES

OGY is a chaos control technique that does not require the exact dynamical equations governing the system. Moreover, its simple structure is valuable for practical implementations. Let a nonlinear plant be described as

$$\mathbf{z}_{i+1} = f(\mathbf{z}_i, p), \quad (1)$$

where  $f$  is a nonlinear function,  $\mathbf{z}_i \in \mathbb{R}^2$  and  $p$  is the accessible parameter. It can be shown that the OGY controller gain is [4]

$$K = \frac{\lambda_u}{\lambda_u - 1} \frac{\mathbf{f}_u^T}{\mathbf{f}_u^T \mathbf{g}}, \quad (2)$$

where  $\lambda_u$  is the unstable eigenvalue of the linearized system,  $\mathbf{f}_u^T$  is a related vector to the unstable eigenvalue such as

$$\begin{aligned} \mathbf{f}_u^T \mathbf{e}_u &= 1, \\ \mathbf{f}_u^T \mathbf{e}_s &= 0. \end{aligned} \quad (3)$$

$\mathbf{e}_s$  and  $\mathbf{e}_u$  are the stable and unstable right eigenvectors of the linearized system at system's fixed point ( $\mathbf{z}_F$ ), and  $\mathbf{g}$  is defined as

$$\mathbf{g} = \left. \frac{\partial \mathbf{z}_F(p)}{\partial p} \right|_{p=p_n}. \quad (4)$$

As presented in [17], for control of interconnected chaotic networks, a multivariable control design strategy is needed. The extended OGY, is a methodology to tackle the interconnection effects.

Consider the following chaotic systems that are initially decoupled

$$\begin{cases} x_{n+1} = f_1(x_n, y_n, p_1) \\ y_{n+1} = g_1(x_n, y_n, p_1), \\ c^1_n = y_n \end{cases} \quad (5)$$

and

$$\begin{cases} z_{n+1} = f_2(z_n, w_n, p_2) \\ w_{n+1} = g_2(z_n, w_n, p_2), \\ c^2_n = w_n \end{cases} \quad (6)$$

where  $x$ ,  $y$ ,  $z$  and  $w$  are the states,  $c^1$  and  $c^2$  are the outputs,  $p_1$  and  $p_2$  are the control parameters.

Now consider the following interconnected chaotic system:

$$\begin{cases} x_{n+1} = f_1(x_n, y_n, p_1, \alpha z_n) \\ y_{n+1} = g_1(x_n, y_n, p_1, \alpha z_n) \\ z_{n+1} = f_2(z_n, w_n, p_2, \beta x_n) \\ w_{n+1} = g_2(z_n, w_n, p_2, \beta x_n) \end{cases} \quad (7)$$

where  $\alpha$  and  $\beta$  are the scalar interaction factors. The system described by (7) is called a multi-chaotic system. Parameters  $\alpha$  and  $\beta$  affect the controller performance, since they change the actual dynamical equations of each primary system. The extended matrix form for the OGY controller with two manipulated parameters ( $\delta p_1, \delta p_2$ ) is

$$\begin{bmatrix} \delta p_1 \\ \delta p_2 \end{bmatrix} = 2 \left( (\lambda_U - I) F_U^T G \right)^{-1} \lambda_U F_U^T \delta \zeta_n, \quad (8)$$

where  $\lambda_U$  is a matrix whose diagonal terms are the unstable eigenvalues and

$$[F_U \ F_S]^T [E_U \ E_S] = \begin{bmatrix} F_U^T \\ F_S^T \end{bmatrix} [E_U \ E_S] = I, \quad (9)$$

where  $E_S$  and  $E_U$  are the stable and unstable eigenvectors and  $F_S$  and  $F_U$  are the related basis vectors, respectively.

The extended OGY controller gain for the case of n-manipulated parameters ( $\delta p_1, \dots, \delta p_n$ ) is

$$\begin{bmatrix} \delta p_1 \\ \vdots \\ \delta p_n \end{bmatrix} = n \left( (\lambda_U - I) F_U^T G \right)^{-1} \lambda_U F_U^T \delta \zeta_n. \quad (10)$$

It was shown in [17] that the proposed procedure can satisfactorily control Multi Chaotic Maps. The matrix  $(\lambda_U - I) F_U^T G$  must be non-singular, which is the controllability condition of the coupled chaotic system.

### 3. LATENCY STRUCTURE

In a closed loop chaotic system using the OGY control strategy, all latency times of different origins can be summed up to a new parameter  $\delta$ . An analytical expression will be derived for the upper bound of  $\delta$ , i.e., a maximum latency time, such that the design of an OGY controller is still possible.

To assess the latency effects, the OGY control for the Henon map with one control parameter was considered in [18]. The Henon map can be described by the following equations

$$\begin{cases} x_{n+1} = p - x_n^2 + b y_n, \\ y_{n+1} = x_n \end{cases} \quad (11)$$

where  $p_0 = 1.29$ ,  $b = 0.3$ . The linearized model around the system's fixed point ( $x_F, y_F$ ) is a  $2 \times 2$  system with two eigenvalues ( $\lambda_u = -1.8398, \lambda_s = 0.1630$ ). The feedback law and updating parameter formula are:

$$\begin{aligned} \delta p_n &= K \delta \mathbf{z}_n, \\ p(n) &= p_0 + 1.8401(x(n) - x_F) \\ &\quad - 0.3001(y(n) - y_F). \end{aligned} \quad (12)$$

Therefore, the closed loop feedback law with loop latency, can be written as:

$$\begin{aligned} p_n &= K \delta \mathbf{z}_{n-\delta}, \\ p(n) &= p_0 + 1.8401(x(n - \delta) - x_F) \\ &\quad - 0.3001(y(n - \delta) - y_F). \end{aligned} \quad (13)$$

It can be shown that the closed loop system is unstable with  $\delta = 1$ . Since the greater amount of latency will cause system instability, it is necessary to use compensation methods in control systems.

Suppose that the row vector  $K = [k_1 \ k_2]$ , is the control vector that varies according to the map parameters. With the control algorithm in place, the stability of the

fixed point is determined by the eigenvalues of the matrix  $A - BK$ .  $A$  is the Jacobian of the uncontrolled Henon map and  $B$  is the sensitivity of the map to some accessible system parameter  $p$  that is displaced by  $p_0$  from its normal value. The stability of the map depends on the matrix  $A - BK$ , which is the Jacobian of the controlled map [18].

To find the region in which the Henon Map presented in (11) may be stabilized with a control vector derived from the OGY method, we have

$$\begin{cases} x_{n+1} = (p_0 + k_1 x_n + k_2 y_n) - x_n^2 + b y_n \\ y_{n+1} = x_n \end{cases} \quad (14)$$

The linearized closed loop equations of the system around the fixed point is:

$$\begin{bmatrix} x_{n+1} \\ y_{n+1} \end{bmatrix} = \begin{bmatrix} -2x^* + k_1 & b + k_2 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_n \\ y_n \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} p_0. \quad (15)$$

The characteristic equation of the closed loop system is then equal to

$$\lambda^2 + (2x^* + k_1)\lambda + (k_2 - b) = 0. \quad (16)$$

Fig. 1 shows the boundary on the  $k_1 - k_2$  plane that can stabilize the chaotic orbit defined in the above equation. If the control parameters are chosen within the region bounded by the triangle and the control algorithm is applied, the system will be stabilized around the fixed point. The small filled circle on the figure shows the calculated state feedback using the OGY control method for the Henon map [17].

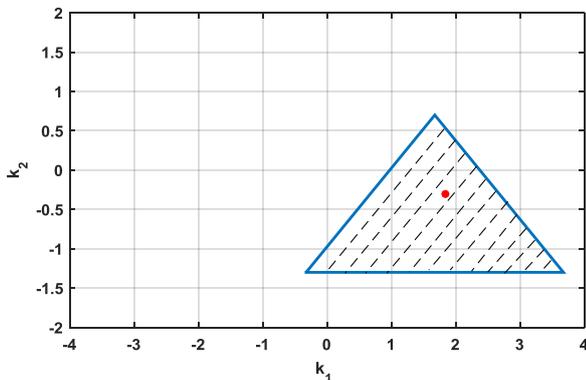


Fig. 1. Stability region for the Henon map using state feedback. The borders form the possible selection of  $k_1$  and  $k_2$  to stabilize the system with state feedback.

Now, let us recalculate the above region for  $\delta = 1$ . In this case, the equations of closed loop system are as:

$$\begin{cases} x_{n+1} = (p_0 + k_1 x_{n-1} + k_2 y_{n-1}) - x_n^2 + b y_n \\ y_{n+1} = x_n \end{cases} \quad (17)$$

The linearized model is:

$$\begin{bmatrix} x_{n+1} \\ y_{n+1} \\ y_n \end{bmatrix} = \begin{bmatrix} -2x^* & k_1 + b & k_2 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_n \\ y_n \\ y_{n-1} \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} p_0 \quad (18)$$

The characteristic equation of the closed loop system is then equal to

$$\lambda^3 + 2x^* \lambda^2 - (k_1 + b)\lambda - k_2 = 0 \quad (19)$$

Using the Jury Criterion, the boundary on the  $k_1 - k_2$  plane in this case will be as shown in Fig. 2:

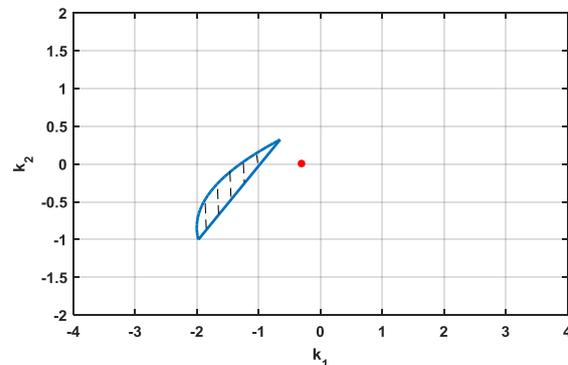


Fig. 2. Stability region for the Henon map using state feedback with  $\delta = 1$ .

It is clear that the OGY calculated feedback does not lie in the region of possible state feedback elements.

Finally, the  $k_1 - k_2$  plane is calculated for  $\delta = 2$  in which the system may be controlled by state feedback. In this case, the linearized closed loop system is presented as:

$$\begin{bmatrix} x_{n+1} \\ y_{n+1} \\ y_n \\ y_{n-1} \end{bmatrix} = \begin{bmatrix} -2x^* & b & k_1 & k_2 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_n \\ y_n \\ y_{n-1} \\ y_{n-2} \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} p_0 \quad (20)$$

It is easily shown that the closed loop system is unstable for all values of  $k_1$  and  $k_2$ .

#### 4. LATENCY COMPENSATION METHODS

##### A. The Rhythmic OGY Control

As explained in [1, 3], for a  $\delta$  time step delay, one can restrict control application rhythmically only every  $\delta + 1$  time-steps, and then leave the system uncontrolled for the remaining time-steps. For the following system with one state ( $x$ ) and one control parameter ( $r$ )

$$x_{t+1} = f(x_t, r) \quad (21)$$

the updating formula for system parameter with feedback gain ( $k$ ) is computed as

$$r_t = r_0 + k(x_t - x^*), \quad (22)$$

where  $x^*$  is the system fixed point. In the rhythmic control method,  $k$  is time dependant:

$$k = k(t). \quad (23)$$

In this method, the calculated control gain will be applied in every  $(\delta + 1)$ -fold iteration of the original system

( $k = k_0$ ) and the gain is zero ( $k = 0$ ) in remaining iterations. The closed loop characteristic equation is then calculated as

$$x_{t+(\delta+1)} = \lambda^{\delta+1}x_t + k_0\mu x_t, \quad (24)$$

where

$$\lambda = \left(\frac{\partial f}{\partial x}\right)_{x^*, r_0}, \quad \mu = \left(\frac{\partial f}{\partial r}\right)_{x^*, r_0}. \quad (25)$$

The advantage of this method is that system dimension is not increased and the stability analysis is simpler due to the compact form of (24).

$$|\lambda^{\delta+1} + k_0\mu| < 1. \quad (26)$$

### B. Memory OGY Control

In this method, the control parameter ( $r_t$ ) is a linear function of previous states. For the case of  $\delta = 1$  we have:

$$r_t = k_1x_{t-1} + k_2x_{t-2} + \dots + k_{n+1}x_{t-n-1}. \quad (27)$$

For  $\delta \geq 2$ ,  $k_1 = \dots = k_{\delta-1} = 0$ . If the state vector is  $[x_t \dots x_{t-n-1}]^T$ , for  $n$  steps memory and one step delay, the characteristic polynomial is

$$(\alpha - \lambda)\alpha^{n+1} + \sum_{i=1}^n k_i\alpha^{n-i} = 0. \quad (28)$$

This method allows control up to  $\lambda_{max} = 2 + n$ . In [3], the optimal values for  $k_i$  was presented. For more than one step delay, the maximal controllable  $\lambda$  is reduced and there is no general scheme for optimal selection of  $k_i$ . It means that for any arbitrary  $\lambda$  to be controlled, a memory length of  $n > \lambda - 2$  is needed.

There is another form for the memory method. If it is possible to use the previously applied control amplitudes  $r_{t-1}, r_{t-2}, \dots$ , the control parameter will be

$$r_t = k_1x_{t-1} + k_2x_{t-2} + \dots + k_\delta x_{t-\delta} + \eta_1 r_{t-1} + \dots + \eta_\delta r_{t-\delta}, \quad (29)$$

in which  $k_1 = \dots = k_{\delta-1} = 0$  for  $\delta \geq 2$  [3].

### C. Smith Predictor Structure

The presence of a large input delay  $k$  implies that the control action will be equally delayed [19]. Suppose that  $G(z^{-1})$  is the system transfer function with delay  $z^{-k}$ . A practical and yet well established approach to delay control is the Smith predictor method.

Fig. 3 represents a schematic structure for the Smith predictor.  $\tilde{G}(z^{-1})$  is delay free plant transfer function and  $C(z^{-1})$  is a designed controller that is considered satisfactory.

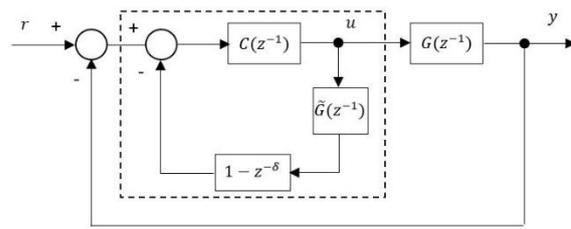


Fig. 3. Block diagram of the Smith predictor algorithm.

Using this structure, the controller generates the correct control action.

## 5. CONTROLLING LATENCY IN HENON MAP

In this section, the compensation control methods described in section 4 are applied to the Henon map.

### A. Rhythmic Control for Henon Map

Rhythmic control equation for the Henon map can be written as:

$$\delta p_n = K_n \delta z_{n-\delta}, \quad \delta \geq 1, \quad (30)$$

$$\begin{cases} K_n = K, & n = i \times (\delta + 1), i = 0, 1, 2, \dots \\ K_n = 0, & O.W. \end{cases}, \quad (31)$$

$$\begin{cases} x_{n+\delta+1} = p_0 + K_1(x_n - x^*) + K_2(y_n - y^*) - x_n^2 + by_n \\ y_{n+1} = x_n \end{cases} \quad (32)$$

Let  $Z_n = [x_n \ y_n]^T$  be the state vector, then  $Z_{n+1} = Z_n = Z^*$  at the system fixed point. Using (32), the following equation will be obtained:

$$Z_{n+(\delta+1)} = Z^* + (A^{\delta+1} + BK)(Z_n - Z^*), \quad (33)$$

where

$$A = \left(\frac{\partial f}{\partial Z}\right)_{Z^*, p_0} = \begin{bmatrix} -2x^* & b \\ 1 & 0 \end{bmatrix}, \quad (34)$$

$$B = \left(\frac{\partial f}{\partial p}\right)_{Z^*, p_0} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

For the Henon map with  $\delta = 1$ , we have

$$A^2 + BK = \begin{bmatrix} 4x^{*2} + b + k_1 & -2x^*b + k_2 \\ -2x^* & b \end{bmatrix}, \quad (35)$$

The closed loop eigenvalues are

$$\lambda_1 = 5.2252, \quad \lambda_2 = 0.0266, \quad (36)$$

which means that the closed loop with loop latency is not stable. In addition, simulation results show that this compensation method is not suitable for a Henon map with  $\delta = 1$ .

### B. Memory Method for Henon Map

Consider the Henon Map with  $\delta = 1$ . For this case, there is no state feedback controller to place the unstable eigenvalues to zero and stable eigenvalues unchanged

(like the OGY strategy), hence an additional feedback term is used:

$$p_n = k_1 x_{n-1} + k_2 y_{n-1} + k_3 p_{n-1}. \quad (37)$$

The closed loop linearized system with  $[x_n \ y_n \ y_{n-1} \ p_{n-1}]^T$  as the state vector, gives

$$\lambda^4 + (2x^* - k_3)\lambda^3 + (-k_1 - 2x^*k_3 - b)\lambda^2 + (bk_3 - k_2)\lambda = 0. \quad (38)$$

In this case, there is a solution to have the closed loop eigenvalues as  $\lambda_1 = \lambda_2 = 0$  and  $\lambda_3 = \lambda_4$ , by the following design:

$$k_1 = -3.38498, k_2 = 0.55194, k_3 = 1.8398.$$

Simulation results of the states and control parameter are shown in Fig. 4.

For  $\delta = 2$ , there is no solution with the following control law:

$$p_n = k_1 x_{n-2} + k_2 y_{n-2} + k_3 p_{n-1}. \quad (39)$$

But using additional terms of the previously applied control parameters as:

$$p_n = k_1 x_{n-2} + k_2 y_{n-2} + k_3 p_{n-1} + k_4 p_{n-2}, \quad (40)$$

there is a controller with satisfactory performance. In this case, the characteristic polynomial of the closed loop system is

$$\lambda^6 + (2x^* - k_3)\lambda^5 + (-k_4 - 2k_3x^* - b)\lambda^4 + (bk_3 - k_1 - 2k_4x^*)\lambda^3 + (bk_4 - k_2) = 0. \quad (41)$$

Using a controller with the following parameters

$$k_1 = 6.2279, k_2 = -1.0155, k_3 = 1.8398, k_4 = -3.3850, \quad (42)$$

the stable eigenvalues of the open loop system remain unchanged and the unstable ones will be zero in the closed loop system. Simulation results of the states and control parameter are shown in Fig. 5.

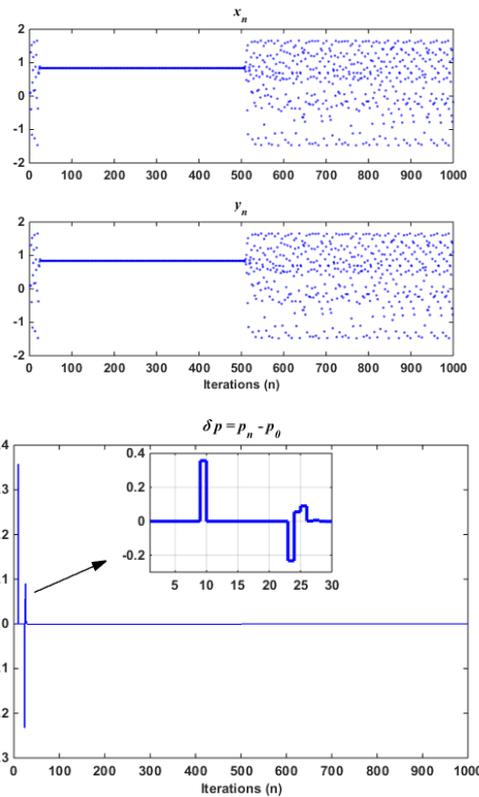


Fig. 4. Simulation of the Henon map with delay  $\delta = 1$  by the memory method.

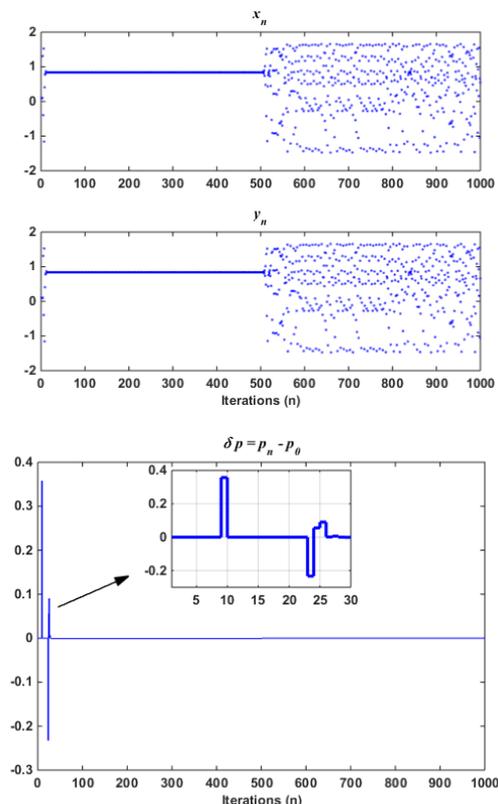


Fig. 5. Simulation of the Henon map with delay  $\delta = 2$  by the memory method.

### C. Smith Predictor for Henon Map

Fig. 6 presents the block diagram form of the Henon Map with loop latency.

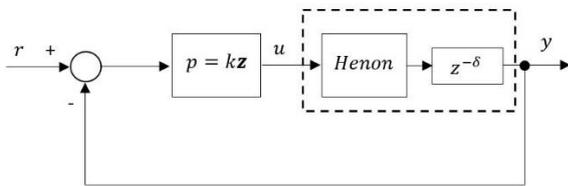


Fig. 6. Block diagram form of the Henon map with delay.

Closed loop instability is inevitable even with  $\delta = 1$ . The Smith predictor proposed to compensate the Henon system with latency is applied and simulation results indicate the success of this methodology. Fig. 7 demonstrates the system states and control parameter deviations for  $\delta = 2$ .

The results show that the Smith predictor compensates for the system's latency efficiently.

### 6. COMPENSATION IN MULTI CHATIC MAPS

For coupled Henon and Ushiki maps, the system equations will be as:

$$\begin{cases} x_{n+1} = p - x_n^2 + 0.3y_n \\ y_{n+1} = x_n + \alpha z_n \\ z_{n+1} = (a - z_n - 0.06w_n)z_n \\ w_{n+1} = (2.5 - 0.4z_n - w_n)w_n + \beta x_n \end{cases}, \quad (43)$$

where  $x, y, z$  and  $w$  are the states,  $p$  and  $a$  are the control parameters, and  $\alpha$  and  $\beta$  are the state interaction parameters between the two chaotic map dynamics [17].

As was shown in [17], the basic OGY method is not suitable for complex chaotic maps with interconnections and so the extended OGY methodology must be used. Simulation results show that the latency time can easily destabilize the controlled coupled Henon and Ushiki maps using the extended OGY methodology.

For  $\delta = 1$ , the states diverge and therefore a compensation method is needed. Hence, the proposed methods of section 4 will be applied to the system described in (43).

#### A. Rhythmic Control in Coupled Multi Chaotic Maps

For the linearized system described in (43) around the fixed point, with  $\alpha = \beta = 0.05$ ,

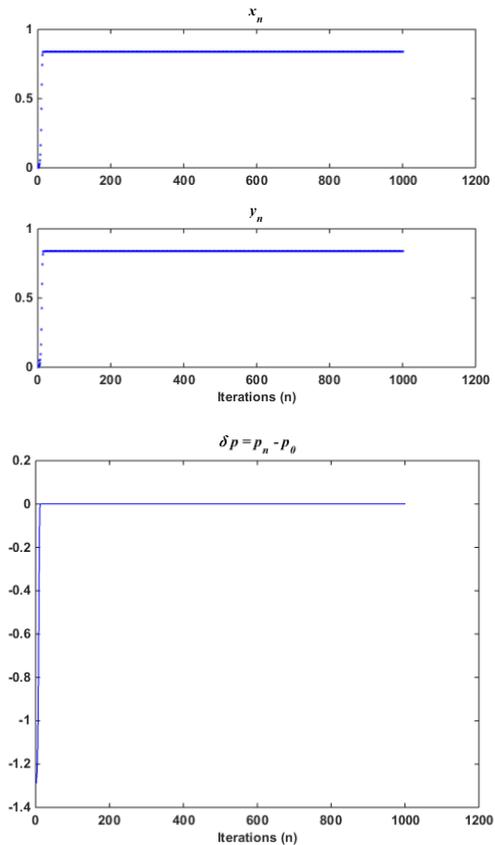


Fig. 7. Smith predictor applied to the Henon map with latency  $\delta = 2$ .

$$A = \begin{bmatrix} -1.7142 & 0.3 & 0 & 0 \\ 1 & 0 & 0.05 & 0 \\ 0 & 0 & -1.9752 & -0.1785 \\ 0.0500 & 0 & -0.1654 & 0.4827 \end{bmatrix}, \quad (44)$$

$$B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 2.9752 \\ 0 & 0 \end{bmatrix},$$

$$K = \begin{bmatrix} 1.2915 & -0.6919 & 0.1240 & -0.0189 \\ -0.0161 & -0.0124 & 0.8979 & 0.0606 \end{bmatrix}.$$

The control feedback law in this case is:

$$\begin{bmatrix} \delta p_n \\ \delta a_n \end{bmatrix} = K_n \begin{bmatrix} \delta x_{n-\delta} \\ \delta y_{n-\delta} \\ \delta z_{n-\delta} \\ \delta w_{n-\delta} \end{bmatrix}, \quad \delta \geq 1, \quad (45)$$

$$\begin{cases} K_n = K, & n = i \times (\delta + 1), i = 0, 1, 2, \dots \\ K_n = 0, & O.W. \end{cases} \quad (46)$$

The eigenvalues of  $A^2 + BK$  are 4.9772, 6.6158,  $-0.1428$ , and 0.2448 and the closed loop system is unstable. It means that the Rhythmic methodology cannot solve the loop latency problem in this multi-chaotic case.

### B. Memory Method in Coupled Multi Chaotic Maps

In this case, the control feedback law is

$$\begin{bmatrix} \delta p_n \\ \delta a_n \end{bmatrix} = K_n \begin{bmatrix} \delta x_{n-\delta} \\ \delta y_{n-\delta} \\ \delta z_{n-\delta} \\ \delta w_{n-\delta} \end{bmatrix}, \delta \geq 1, \quad (47)$$

$$\begin{bmatrix} p_n \\ a_n \end{bmatrix} = \begin{bmatrix} k_1 & k_2 & k_3 & k_4 \\ k_5 & k_6 & k_7 & k_8 \end{bmatrix} \begin{bmatrix} x_{n-1} \\ y_{n-1} \\ z_{n-1} \\ w_{n-1} \end{bmatrix} + \begin{bmatrix} q_1 & q_2 \\ q_3 & q_4 \end{bmatrix} \begin{bmatrix} p_{n-1} \\ a_{n-1} \end{bmatrix}. \quad (48)$$

If the state vector of the original system is defined as  $Z_n = [x_n \ y_n \ z_n \ w_n]^T$  and the input control parameter vector is  $P_n = [p_n \ a_n]^T$ , then the state space vector for applying memory method for system described in (43) with latency time  $\delta = 1$  via memory method will be:

$$\begin{bmatrix} Z_n \\ Z_{n-1} \\ P_{n-1} \end{bmatrix}$$

This choice will form a system matrix with  $10 \times 10$  dimension which leads to difficulties in calculating a suitable state feedback to set all closed loop eigenvalues to zero (except the stable open loop ones which remain unchanged). As it is illustrated in this example, calculation burden is a limiting feature of this methodology.

### C. Smith Predictor in Coupled Multi Chaotic Maps

In this section, Smith predictor is used for complex multi-chaotic systems. In order to examine the effect of this compensator, Smith predictor is applied to the system described in (43) as presented in Fig. 8.

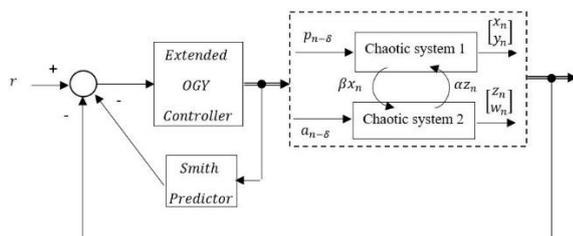


Fig. 8. Block diagram for Smith predictor applied in multi chaotic system.

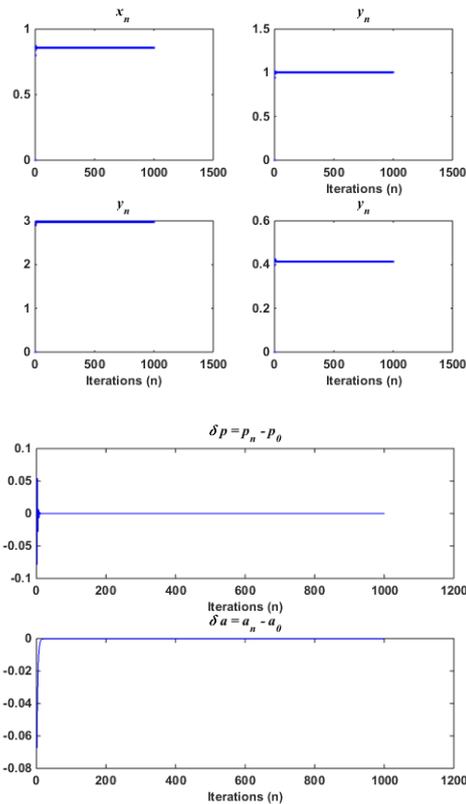


Fig. 9. States and two control parameters of interconnected maps described in (43) using extended OGY method as the controller and Smith predictor as latency time compensator.

$x, y, z$  and  $w$  are the states,  $p$  and  $a$  are the control parameters,  $n$  is the current iteration, and  $\delta$  is the latency of the multi chaotic system.

Simulation results in Fig. 9 indicate that Smith predictor can be an appropriate compensator for latency in multi chaotic systems.

### 7. CONCLUSIONS

In this paper, the problem of controlling chaotic systems in presence of loop latency was considered. The latency compensation methods were reviewed and they have been applied to both simple Henon map using OGY chaos control and a complex chaotic structure. For complex chaotic maps, the extended OGY control methodology must be used. The compensation methods analysed in this paper are Rhythmic control, Memory Method and Smith predictor. In Rhythmic control, the control signal was injected in every  $(\delta + 1)$ -fold rhythmically. It was a simple method, but the control was kept inactive for some time steps. The results show that Rhythmic control is not a suitable solution even for simple chaotic maps. In Memory method, previous applied control signals or states of the systems were involved in the control signal. Besides this method's advantages,

there were some problems in application such as instability and increasing the closed loop systems dimensions. It should be noted that in the Memory method, there is no rule for selecting the number of previous states/parameters to be involved in the control signal. Also, it is not a suitable solution for high dimensional systems. As it is shown in multi chaotic example, the dimension of added items (parameters or states) in the state vector of the system causes difficulties in calculations for finding an OGY controller. The Smith predictor is a structure in which the designed controller is not changed and the loop latency can be compensated. In both simple and complex chaotic maps, Smith predictor has a closed loop satisfactory performance in conjunction with the OGY controllers.

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