

Performance Analysis of Local Processors-Assisted Cell-Free massive Multiple Input Multiple Output Systems

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Abstract:

This paper proposes a local processors-assisted structure (LPs-AS) for cell-free massive multiple-input multiple-output (CF-mMIMO) systems, consisting of a central processor (CP), several access points (APs), and some local processors (LPs). Each LP is connected to the CP and a subset of APs and is used as a precoding unit in downlink (DL). This proposed LPs-AS enables us to implement DL precoders with a semi-distributed approach. In our proposed semi-distributed implementation (SDI), we design precoders at the LPs. This approach differs from centralized implementation (CI) where the precoders are implemented at the CP and distributed implementation (DI) where the precoders are designed at APs. We evaluate LPs-AS in terms of spectral efficiency (SE) and analytically derive its achievable SE. Furthermore, we propose a power control algorithm to maximize its sum SE, compute the computational complexity (CC) of its minimum mean square error (MMSE) precoders, and compare this CC with its counterpart in CI and DI. Numerical results demonstrate that employing an optimal number of LPs (between 2 and 4) in our proposed LPs-AS, not only enables us to design DL precoders with significantly low CC but also results in an efficient SDI that effectively addresses the problem of low SE in DI.

Keywords:

Local processors-assisted structure, Semi-distributed implementation, Local Processor, Computational Complexity, Spectral Efficiency

1. Introduction

Cell-free massive multiple input multiple output (CF-mMIMO) system is a promising technology for 6G and beyond wireless networks, potentially providing high and relatively uniform data rates for all user equipment (UEs). This system comprises a substantial number of access points (APs) and a considerably smaller number of UEs in which the APs collaborate to serve the UEs coherently, using the same time-frequency resource. Data transmission between APs and UEs is performed in the time division duplex (TDD) mode and each AP acquires its channel state information (CSI) through uplink (UL) pilots, leveraging uplink-downlink (UL-DL) channel reciprocity. All APs are connected to a CP via front-haul links. This CP performs the majority of signal processing and resource allocation tasks[1-7].

Beamforming the UEs' signals is a key feature of the CF-mMIMO systems in the DL, achieved by transmitting a UE signal through multiple APs. Beamforming is performed using specific vectors known as precoders [8]. Designing precoders with low computational complexity (CC) that can provide high spectral efficiency (SE) for UEs is one of the fundamental challenges in CF-mMIMO systems [9]. These precoders can be implemented either centrally or in a distributed manner. In the centralized implementation (CI), precoders are designed at the CP, whereas in the distributed implementation (DI), they are designed at APs. Both centralized and distributed precoders are optimized or heuristically designed from UL combiners using the UL-DL duality theorem [7, 10-12]. The primary challenge of centralized precoders, both optimized and heuristic, is their high CC and the substantial communication overhead required for their design [9, 13, 14]. Additionally, optimized distributed precoders necessitate some level of cooperation among APs [15-17] or bidirectional training between UEs and APs rendering them unsuitable for many applications that require prompt responses [18-20]. The most straightforward and widely applicable precoders are heuristic distributed precoders, designed solely from the local CSI of each AP using the UL-DL duality theorem [8, 21]. These precoders require neither cooperation among APs nor training between APs and UEs. However, they are incapable of providing UEs with high SE.

Recent Works:

Precoding is a fundamental component of DL transmission in MIMO, mMIMO and CF-mMIMO systems that has generated an extensive literature. Here, we review some related works. Traditional centralized precoding schemes, such as zero-forcing (ZF) [7] and weighted minimum mean-square-error (WMMSE) [10], require APs to transmit their local CSI to the CP via front-haul links. The CP then computes optimized precoding vectors and distributes them back to the APs, resulting in substantial signaling overhead and imposing a heavy burden on the front-haul infrastructure. To mitigate these challenges, distributed precoding methods have been explored. For instance, [16] introduces a cooperative team minimum mean-square error (TMMSE) precoding scheme based on transmitter-specific CSI, extending traditional centralized MMSE precoding to distributed operations. Similarly, [22] proposes an iterative distributed approach where precoding vectors are optimized locally at each AP through bidirectional training between users and APs, coupled with periodic cross-term information sharing among APs. However, these distributed methods suffer from computational inefficiency due to their reliance on iterative optimization, making them impractical for latency-sensitive applications. Alternatively, non-cooperative precoding techniques—such as matched filtering (MF) and local MMSE operate solely on local CSI, eliminating the need for fronthaul-based CSI exchange. A notable example is maximum ratio transmission (MRT) [23], a decentralized linear precoding method designed to maximize received signal gain at the target user. However, unlike cellular massive MIMO systems, CF-mMIMO systems exhibit weaker channel hardening effects [24], limiting the performance of purely local CSI-based approaches. Furthermore, most existing precoding methods rely on convex optimization techniques [25], which struggle with non-convex problems and scale poorly as network size increases. While problem-specific algorithms can be developed, they demand considerable expertise and time-intensive customization, hindering their practicality for large-scale deployments. In CF-mMIMO systems, using artificial intelligence (AI) to design precoders and to achieve optimal resource allocation have been used in some recent works. As for example, in [26], a reinforcement learning based precoding scheme is proposed for CF-mMIMO systems and in [27], a deep learning-based

power control algorithm is proposed to achieve max-min, max-product and max-sum rate optimization in these systems.

Contributions:

In this paper, we propose a local processors-assisted structure (LPs-AS) for CF-mMIMO systems that enables the implementation of precoders in a semi-distributed manner. In our semi-distributed implementation (SDI), precoders are designed in local processors (LPs), each connected to the CP and a subset of APs. The performance of semi-distributed precoders, in terms of the SE they deliver to UEs, is comparable to that of centralized precoders. However, semi-distributed precoders achieve this performance with substantially lower CC. On the other hand, designing semi-distributed precoders heuristically resolves the issue of low achievable SE in their distributed counterparts and introduces an efficient alternative for this issue.

The contributions of this paper include:

- We introduce our proposed LPs-AS for CF-mMIMO systems and derive its achievable SE analytically.
- We propose semi-distributed maximum ratio (MR) and minimum mean square error (MMSE) precoders for our LPs-AS by modifying their centralized and distributed counterparts. Then we compute the CC of MMSE precoders for all CI, DI, and SDI.
- WE propose a power control algorithm for our proposed LPs-AS to maximize the sum SE.

Organization: In Section Two, we propose our LPs-AS for CF-mMIMO systems and analyze its DL operation. In "Section Three", we compute the heuristic semi-distributed MR and MMSE precoders by modifying their centralized and distributed counterparts. In "Section Four", we calculate the CC of semi-distributed MMSE precoders and compare it with that of their centralized and distributed counterparts. In Section Five, we derive an expression for the achievable SE of our proposed LPs-AS. In Section Six, we

propose a power control algorithm to maximize the sum SE for our proposed LPs-AS. Section Seven details the numerical results, and Section Eight concludes the paper.

Notations: x and $\mathbf{x}$ represent a scalar and vector, respectively. We denote by $(\cdot)^*$, $(\cdot)^T$, $(\cdot)^H$ and $\overline{(\cdot)} = \frac{(\cdot)}{\|\cdot\|}$, $E\{(\cdot)\}$, $\overline{(\cdot)} = (\cdot) - E\{(\cdot)\}$, $|\cdot|$ and $\|\cdot\|$ as the complex conjugate, the transpose, the Hermitian transpose, the normalization of $(\cdot)$, the expected value of a random variable, its deviation from the mean, the magnitude of a complex number and the L_2 norm, respectively. We denote the complex Gaussian distribution with mean μ and variance σ^2 by $\mathcal{N}_C(\mu, \sigma^2)$. Finally, $\mathbf{I}_N$ is the $N \times N$ identity matrix. Table 1 defines the variables used in this paper.

Table 1: Definitions of Variables Used in the Paper.

Variable	Definition
L_T	Total number of APs
L	APs number of each LP
K	UEs number
N	Antennas number of each AP
M	Number of LPs
$s_i \ i \in \{1, \dots, K\}$	Transmitted symbol of UEs
$\mathbf{x}_{l,m} \ l \in \{1, \dots, L\} \ m \in \{1, \dots, M\}$	Sum precoded symbols of all K UEs at (l,m)th AP
$\mathbf{w}_{il,m} \ i \in \{1, \dots, K\} \ l \in \{1, \dots, L\} \ m \in \{1, \dots, M\}$	Precoder of ith UE at (l,m)th AP
$\mathbf{h}_{il,m} \ i \in \{1, \dots, K\} \ l \in \{1, \dots, L\} \ m \in \{1, \dots, M\}$	Channel between ith UE and (l,m)th AP
$\beta_{il,m} \ i \in \{1, \dots, K\} \ l \in \{1, \dots, L\}$	Channel gain between ith UE and (l,m)th AP

$m \in \{1, \dots, M\}$	
$d_{il,m} \quad i \in \{1, \dots, K\} \quad l \in \{1, \dots, L\}$ $m \in \{1, \dots, M\}$	Distance between i th UE and (l,m) th AP
$\alpha_{il,m} \quad i \in \{1, \dots, K\} \quad l \in \{1, \dots, L\}$ $m \in \{1, \dots, M\}$	Shadowing between i th UE and (l,m) th AP
$y_i \quad i \in \{1, \dots, K\}$	Received signal at i th UE
$n_i \quad i \in \{1, \dots, K\}$	AWGN at i th UE
$\mathbf{H}_{i,m} \quad i \in \{1, \dots, K\} \quad m \in \{1, \dots, M\}$	Channel between i th UE and all L APs of m th LP
$\mathbf{W}_{i,m} \quad i \in \{1, \dots, K\} \quad m \in \{1, \dots, M\}$	Downlink precoder of i th UE at m th LP
$\mathbf{V}_{i,m} \quad i \in \{1, \dots, K\} \quad m \in \{1, \dots, M\}$	UL combiner of i th UE at m th LP
$\rho_{k,m}$	DL power of k th UE at m th LP
$\mu_{k,m}$	Square root of $\rho_{k,m}$
$p_i \quad i \in \{1, \dots, K\}$	UL transmitted power of i th UE
P_{Max}	Maximum DL power of each AP
δ	Accuracy Parameter of optimization problem

2. System Model

We consider DL transmission of a CF-mMIMO system consisting of L_T APs and K UEs where APs have N antennas and UEs have a single antenna. In our proposed LPs-AS, we introduce M LPs in the network that are connected to the CP as in Fig 2. Each LP serves $L = L_T/M$ neighboring APs via front-haul links.

A primary task of an LP in the DL is to compute the precoders for its APs. Figure 1 is a geometric illustration of the components of a CF-mMIMO system, with the introduced LPs in our proposed LPs-AS.

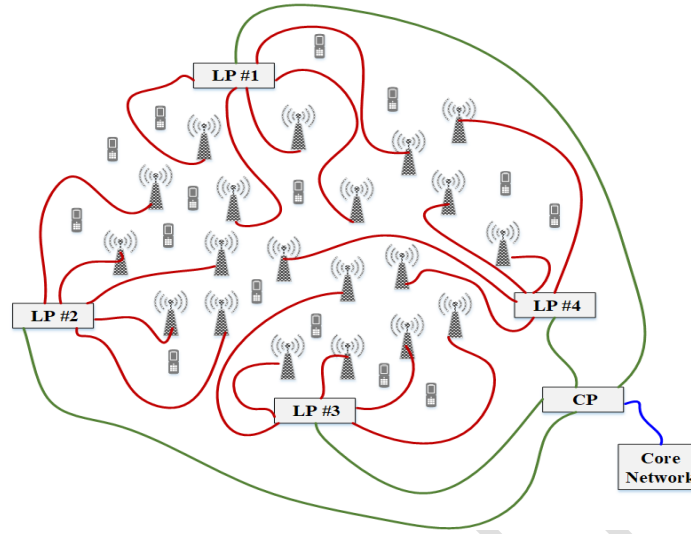


Fig. 1: A geometrical illustration of the introduced local processors in our proposed LPs-AS for CF-mMIMO systems.

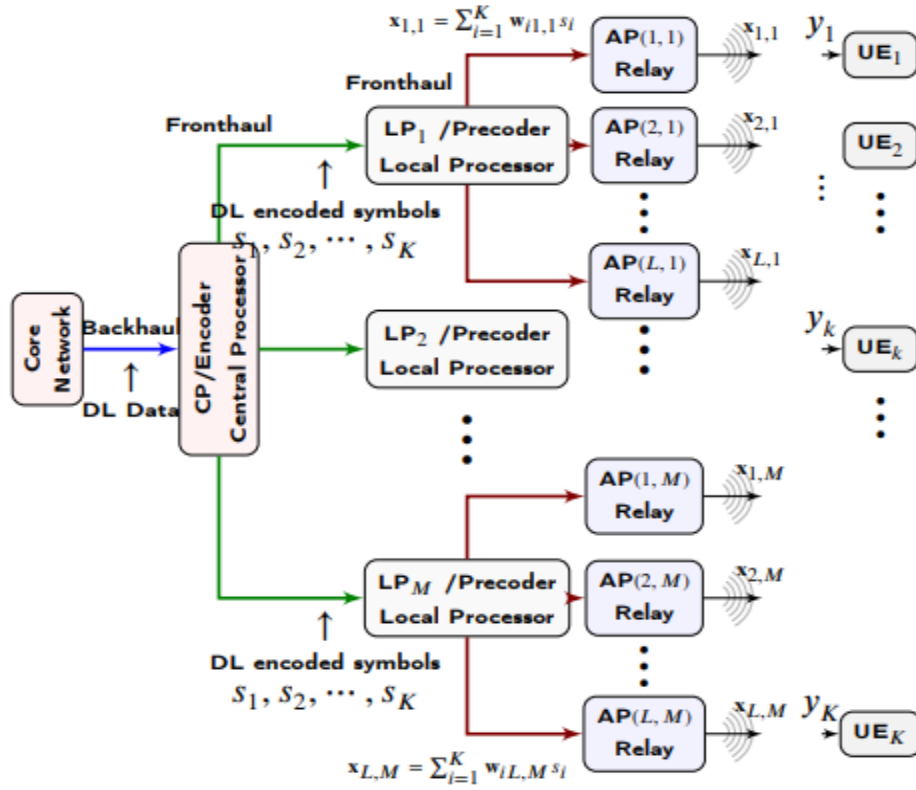


Fig. 2: DL signal processing layers for our proposed LPs-AS for CF-mMIMO systems.

2.1 Downlink Transmission in Our Proposed LPs-AS

Fig. 2 illustrates the DL signal generation in our proposed LPs-AS where all APs serve all K UEs at the same time-frequency slot. The CP encodes the UEs' DL data into symbols $s_1, s_2, \dots, s_K$ where $E\{|s_i|^2\} = 1$ and $E\{s_i\} = 0$. The UEs' encoded symbols $s_1, s_2, \dots, s_K$ are transmitted to M LPs where they are linearly precoded by m th LP into the following transmit signal and transmitted by (l, m) th AP:

$$\mathbf{x}_{l,m} = \sum_{i=1}^K \mathbf{w}_{il,m} s_i, \quad \forall m = 1, \dots, M, l = 1, \dots, L. \quad (1)$$

In our proposed LPs-AS, (l, m) th AP receives $\mathbf{x}_{l,m}$ from m th LP and broadcast it. The received signal at k th UE is expressed by

$$y_k = \sum_{m=1}^M \sum_{l=1}^L \mathbf{h}_{kl,m}^H \mathbf{x}_{l,m} + n_k, \quad (2)$$

where $n_k \sim \mathcal{N}_c(0, \sigma^2)$ is the independent additive white Gaussian noise and $\mathbf{h}_{kl,m}$ is the channel response between (l, m) th AP and k th UE. We substitute (1) in (2) as

$$y_k = \sum_{m=1}^M \sum_{i=1}^K \mathbf{H}_{k,m}^H \mathbf{W}_{i,m} s_i + n_k, \quad (3)$$

and respectively define $\mathbf{H}_{k,m}$ and $\mathbf{W}_{i,m}$ as the collective channel to k th UE from all APs of m th LP and their collective precoders by

$$\mathbf{H}_{k,m} = \begin{bmatrix} \mathbf{h}_{k1,m} \\ \vdots \\ \mathbf{h}_{kL,m} \end{bmatrix}, \quad \mathbf{W}_{i,m} = \begin{bmatrix} \mathbf{w}_{i1,m} \\ \vdots \\ \mathbf{w}_{iL,m} \end{bmatrix} \in \mathbb{C}^{LN}. \quad (4)$$

We assume that m th LP has access only to its local DL channels, i.e. $\mathbf{H}_{1,m}, \mathbf{H}_{2,m}, \dots, \mathbf{H}_{K,m}$. These CSIs are employed by m th LP to compute the required local precoders $\mathbf{W}_{1,m}, \mathbf{W}_{2,m}, \dots, \mathbf{W}_{K,m}$ for all network UEs.

In this LPs-AS, the network can be scaled up by adding new LPs (i.e., by increasing M) and APs. The additional APs are inexpensive as their role is simply to relay $\mathbf{x}_{l,m}$ to all network UEs.

3 Heuristic MR and MMSE Precoders for Our Proposed LPs-AS

The DL precoder optimization is known as an NP-hard problem in mMIMO and CF-mMIMO systems (this is in contrast to the optimal design of UL combiners) [8, 13, 28]. Due to its high CC, the DL precoder is typically designed heuristically, leveraging UL combiners based on the UL-DL duality theorem [8, 13]. Here, we propose semi-distributed MR and MMSE DL precoders by leveraging the UL-DL duality and modifying the UL combiners for CI and DI. From the UL-DL duality, the general form for semi-distributed DL precoders is:

$$\mathbf{W}_{k,m} = \sqrt{\rho_{k,m}} \bar{\mathbf{V}}_{k,m} \in \mathbb{C}^{LN \times 1}, \quad (5)$$

where $\rho_{k,m}$ is the total transmit power assigned to k th UE by m th LP, $\bar{\mathbf{V}}_{k,m} = \frac{\mathbf{V}_{k,m}}{\|\mathbf{V}_{k,m}\|}$ is the normalized

UL combiner that m th LP uses to extract and detect the signal of k th UE. The MR and MMSE UL combiners are proposed in [8] for CI and DI. We modify them to obtain the semi-distributed MMSE and MR DL precoders as follows.

MMSE precoding is computed by substituting

$$\mathbf{V}_{k,m}^{\text{MMSE}} = p_k \left(\sum_{i=1}^K p_i \mathbf{H}_{i,m} \mathbf{H}_{i,m}^H + \sigma^2 \mathbf{I}_{NL} \right)^{-1} \mathbf{H}_{k,m}, \quad (6)$$

in (5) where p_k is k th UE uplink power. Intuitively, the MMSE precoders aim to transmit a specific strong signal to k th desired UE while limiting the interferences caused to unintended UEs.

MR precoding is computed by substituting

$$\mathbf{V}_{k,m}^{\text{MR}} = \mathbf{H}_{k,m}. \quad (7)$$

in (5). This scheme maximizes the numerator of the effective SINR (i.e. the fraction of the transmit power that is received at the desired UE). In contrast to the MMSE Precoder, the computation of the MR precoder is very simple as the MR decoder ignores the interference in the network.

4 Computational Complexity

Table 2: Computational complexities (number of complex multiplications) for computing the MMSE precoders.

Scheme	number of Complex Multiplications	CC Factor
CI	$NL_T F_{CI}$	$F_{CI} = K \frac{NL_T + 1}{2} + NL_T + \frac{N^2 L_T^2 - 1}{3}$
DI	$NL_T F_{DI}$	$F_{DI} = K \frac{N + 1}{2} + N + \frac{N^2 - 1}{3}$
SDI	$NL_T F_{SDI}$	$F_{SDI} = K \frac{NL + 1}{2} + NL + \frac{N^2 L^2 - 1}{3}$

Here, we follow the method in ([13], Appendix B.1.1) to determine the required CC in our proposed LPs-AS for computing the semi-distributed MMSE precoder and compare it with the corresponding CC in centralized and distributed precoders presented in [8]. The semi-distributed MMSE precoder can be calculated by $\mathbf{W}_{k,m} = \mathbf{A}^{-1}\mathbf{B}$, where

$$\mathbf{C} = \sum_{i=1}^K p_i \mathbf{H}_{i,m} \mathbf{H}_{i,m}^H \in \mathbb{C}^{NL \times NL}, \quad (8)$$

$$\mathbf{A} = \mathbf{C} + \sigma^2 \mathbf{I}_{LN}, \quad \mathbf{B} = p_k \bar{\mathbf{H}}_{k,m} \in \mathbb{C}^{NL}. \quad (9)$$

The number of complex multiplications to compute $\mathbf{C}$, $\mathbf{A}^{-1}$ and $\mathbf{A}^{-1}\mathbf{B}$ in each LP are $\frac{(NL)^2+NL}{2}K$, $\frac{(NL)^3-NL}{3}$ and $(NL)^2$, respectively. Thus, the required multiplications for computing semi-distributed MMSE precoder in our proposed LPs-AS is $NL_T F_{SDI}$ for $ML = L_T$ where we define

$$F_{SDI} = \frac{K}{2} + NL \left(1 + \frac{K}{2} \right) + \frac{N^2 L^2 - 1}{3}. \quad (10)$$

as a CC factor simplifies our comparisons. Table 2 presents a summary of the CC for SDI, CI, and DI, expressed as the required number of complex multiplications necessary for computing MMSE precoders.

4 Spectral Efficiency of Our Proposed LPs-AS

We now derive an achievable SE expression for our proposed LPs-AS. To this end, we define $\mathbf{b}_k \in R_+^M$, $\boldsymbol{\mu}_k \in R_+^M$ and $\mathbf{C}_{ki} \in \mathbb{C}^{M \times M}$ with following entries $\forall m, m' = 1, \dots, M$:

$$\mathbf{b}_k(m) = E \left\{ \mathbf{H}_{k,m} \mathbf{V}_{k,m} \right\}, \quad (11)$$

$$\boldsymbol{\mu}_k(m) = \sqrt{\rho_{k,m}}, \quad (12)$$

$$\mathbf{C}_{ki}(m, m') = E \left\{ \mathbf{H}_{k,m} \bar{\mathbf{V}}_{i,m} \bar{\mathbf{V}}_{i,m'}^H \mathbf{H}_{k,m'} \right\}. \quad (13)$$

Lemma 1 Using (11), (12), and (13), the LPs-AS DL achievable SE of the k th UE is lower bounded by $SE_k = \log_2(1 + SINR_k)$ where $SINR_k = \frac{S_k}{IN_k}$ is the signal-to-interference plus noise ratio and

$$S_k = \left| \mathbf{b}_k^T \boldsymbol{\mu}_k \right|^2, \quad (14)$$

$$IN_k = \sum_{i=1}^K \boldsymbol{\mu}_i^T \mathbf{C}_{ki} \boldsymbol{\mu}_i - \left| \mathbf{b}_k^T \boldsymbol{\mu}_k \right|^2 + \sigma^2. \quad (15)$$

Proof. We substitute (5) in (3) and rewrite it as follows

$$y_k = \sum_{m=1}^M \sqrt{\rho_{k,m}} E \left\{ \mathbf{H}_{k,m} \bar{\mathbf{V}}_{k,m} \right\} s_k + I_k + n_k, \quad (16)$$

where a total interference of

$$I_k = \sum_{m=1}^M \sqrt{\rho_{k,m}} \mathbf{H}_{k,m} \bar{\mathbf{V}}_{k,m} s_k + \sum_{i=1, i \neq k}^K \sum_{m=1}^M \sqrt{\rho_{i,m}} \mathbf{H}_{k,m} \bar{\mathbf{V}}_{i,m} s_i \quad (17)$$

is received by the k th UE. We have $E\{I_k\} = 0$ since $E\{s_i\} = 0$ and I_k is uncorrelated with s_k since $s_1, \dots, s_K$ are independent. Therefore, we can easily show that $E\{s_k^* I_k\} = 0$ and

$$\begin{aligned} E\{|I_k|^2\} &= \sum_{i=1}^K E \left\{ \left| \sum_{m=1}^M \sqrt{\rho_{i,m}} \mathbf{H}_{k,m} \bar{\mathbf{V}}_{i,m} \right|^2 \right\} \\ &\quad - \left| E \left\{ \sum_{m=1}^M \sqrt{\rho_{k,m}} \mathbf{H}_{k,m} \bar{\mathbf{V}}_{k,m} \right\} \right|^2. \end{aligned} \quad (18)$$

The proof is completed by using ([13], Lemma 3.3, page 244) and applying the expressions defined in (11), (12), and (13).

6 Power allocation via Sum SE maximization

We propose a power control algorithm by using the Weighted Minimum Mean Square Error (WMMSE) approach to maximize the sum SE in our proposed LPs-AS as follows:

$$\begin{aligned} &\underset{\mu_{k,m} \geq 0, \forall k,m}{\text{maximize}} \sum_{k=1}^K \log_2 (1 + \text{SINR}_k) \\ &\text{subject to} \sum_{k=1}^K \mu_{k,m}^2 \|\bar{\mathbf{w}}_{kl,m}\|^2 \leq P_{\max}, \quad l = 1, \dots, L \\ &\quad \quad \quad m = 1, \dots, M \end{aligned} \quad (19)$$

where $\bar{\mathbf{w}}_{kl,m} = \frac{\mathbf{w}_{kl,m}}{\|\mathbf{w}_{kl,m}\|}$ is the weighted precoder assigned to the (l,m) th AP by the m th LP, which determines the portion of $\rho_{k,m}$ that the (l,m) th AP allocates to k th UE, and $P_{\max}$ is the maximum DL power of APs. This problem is non-convex, is an extension of the NP-hard problem in [29], and finding its global optimum entails an exponential complexity through the monotonic optimization method [30]. Thus, we focus on finding a local optimum using the WMMSE [10, 31, 32] approach

6.1 WMMSE Approach

We define the mean square error (MSE) of data detection by $e_k = E\{|\hat{s}_k - s_k|^2\}$. We have

$$e_k = E\{|\hat{s}_k|^2\} + E\{|s_k|^2\} - E\{\hat{s}_k^* s_k\} - E\{\hat{s}_k s_k^*\}, \quad (20)$$

where $\hat{s}_k$ is an estimate of s_k and is computed by applying the receiver weight u_k to y_k at the k th UE:

$$\hat{s}_k = u_k y_k = u_k \sum_{m=1}^M h_{k,m}^H \sum_{i=1}^K \sqrt{\rho_{i,m}} \bar{\mathbf{w}}_{k,m} s_i + u_k n_k \quad (21)$$

From (20) and (21), we obtain e_k :

$$e_k = u_k^2 \left(\sum_{i=1}^K \boldsymbol{\mu}_i^T \mathbf{C}_{ki} \boldsymbol{\mu}_i + \sigma^2 \right) - 2u_k \mathbf{b}_k^T \boldsymbol{\mu}_k + 1, \quad (22)$$

which depends on $(\{\boldsymbol{\mu}_i\}, u_k)$. Using this approach, the problem (19) is converted to

$$\begin{aligned} & \underset{\substack{\mu_{k,m} \geq 0, \forall k,m \\ d_k > 0, u_k, \forall k}}{\text{minimize}} \sum_{k=1}^K d_k e_k - \ln(d_k) \\ & \text{subject to } \sum_{k=1}^K \mu_{k,m}^2 \|\bar{\mathbf{w}}_{kl,m}\|^2 \leq P_{\max}, \quad \begin{matrix} l = 1, \dots, L \\ m = 1, \dots, M \end{matrix} \end{aligned} \quad (23)$$

where the non-negative weights d_k are introduced to equate (23) with (19). The problem in (23) is convex with respect to either u_k , d_k or $\mu_{k,m}$. Thus, we employ alternating optimization, wherein we alternatively

optimize each of the variables u_k , d_k and $\mu_{k,m}$ individually while keeping the other two variables fixed. This iterative algorithm is guaranteed to converge quickly, where the optimal values of the variables are updated in each iteration until a stopping criterion is met. The optimization problems for determining the optimal values of u_k and d_k are unconstrained. Therefore, we optimize them by setting their first-order derivatives to zero, i.e., $u_k(\sum_{i=1}^K \mu_i^T C_{ki} \mu_i + \sigma^2) - b_k^T \mu_k = 0$. and $e_k - \frac{1}{d_k} = 0$. Thus, the optimal values of u_k are given by

$$u_k^{\text{opt}} \leftarrow \frac{b_k^T \mu_k}{\sum_{i=1}^K \mu_i^T C_{ki} \mu_i + \sigma^2}. \quad (24)$$

By substituting u_k^{opt} in (22), we obtain:

$$e_k = 1 - \frac{(b_k^T \mu_k)^2}{\sum_{i=1}^K \mu_i^T C_{ki} \mu_i + \sigma^2} = \frac{1}{1 + \text{SINR}_k}. \quad (25)$$

From $d_k = 1/e_k$, we obtain

$$d_k^{\text{opt}} \leftarrow \frac{1}{u_k^2 \left(\sum_{i=1}^K \mu_i^T C_{ki} \mu_i + \sigma^2 \right) - 2u_k b_k^T \mu_k + 1}. \quad (26)$$

Substituting (22) in (23) leads to the following optimization problem for $\mu_{k,m}$

$$\begin{aligned} & \underset{\substack{\mu_{k,m} \geq 0, \forall k, m \\ d_k > 0, u_k, \forall k}}{\text{minimize}} \sum_{k=1}^K d_k u_k^2 \sum_{i=1}^K \mu_i^T C_{ki} \mu_i + \sigma^2 - 2u_k b_k^T \mu_k + 1 \\ & \text{subject to } \sum_{k=1}^K \mu_{k,m}^2 \left\| \bar{\mathbf{w}}_{kl,m} \right\|^2 \leq P_{\max}, \quad l = 1, \dots, L \\ & \quad \quad \quad m = 1, \dots, M \end{aligned} \quad (27)$$

The matrices C_{ki} are positive semi-definite. Consequently, for fixed u_k and d_k , the problem in (27) is a convex Quadratically Constrained Quadratic Program (QCQP), which can be easily solved, e.g., by using CVX. Algorithm 6.1 gives a concise summary of the iterative updating procedures to solve (23).

Substituting (24) and (26) into the objective function articulated in (23), we obtain $K - \sum_{k=1}^K \ln(1 + \text{SINR}_k)$. The minimization of this function is equivalent to the maximization of the sum SE function, i.e., (19) is equivalent to (23). Table 3 summarizes the proposed iterative algorithm to solve the optimization problem (23).

Table 3: Iterative Sum SE Maximization for SDI.

• Initialization: – Set the solution accuracy $\delta > 0$ – Set arbitrary feasible initial powers $\{\mu_k\}$ • while $\sum_{k=1}^K (d_k e_k(\{\mu_i\}, u_k) - \ln(d_k))$ is either improved more than δ or not improved at all do – Update u_k for all $k = 1, \dots, K$ using (24). – Update d_k for all $k = 1, \dots, K$ using (26). – Solve (27) to update $\mu_{k,m}$ for all $k = 1, \dots, K$ and all $m = 1, \dots, M$ given the current values of u_k and d_k. • end while • Output: Optimal square roots of the transmit powers

7 Numerical Results

In this section, we numerically evaluate our proposed LPs-AS for CF-mMIMO systems. To achieve this, we focus on the CC of MMSE precoders designed by SDI and the achievable SE that SDI provides for UEs and compare these results with their counterparts in the CI and DI. In our simulation, we consider $L_T = 100$ APs and $K = 40$ UEs, uniformly and independently distributed over a square area of $1000\text{m} \times 1000\text{m}$. We employ the urban microcell model in [8, 33, 34] to generate the large-scale fading coefficients by assuming that the APs are 10 meters above the UEs:

$$\beta_{kl,m} = -30.5 - 37.6 \log_{10}(d_{kl,m}) + \alpha_{kl,m} \text{ in dB}, \quad (28)$$

where the shadow fading $\alpha_{kl,m}$ is generated as a normal distribution $\alpha_{kl,m} \sim \mathcal{N}_c(0, 4^2)$ and $d_{kl,m}$ is the distance in meters between k th UE and (l, m) th AP. The channels are generated as uncorrelated Rayleigh distribution

$$\mathbf{h}_{kl,m} \sim \mathbf{N}_C \left(\mathbf{0}, 10^{-10} \frac{\beta_{kl,m}}{10} \mathbf{I}_N \right). \quad (29)$$

We plotted Figs. 4 and 5 via Monte-Carlo simulations by averaging over 100 uniformly generated setups of APs and UEs locations, and 500 independent channel realizations (29) for each setup. The simulation parameters are presented in Table. 4.

Table 4: The Simulation Parameters of Page[28].

Simulation Parameter	Value
L_T	100
K	40
N	4
p_i	100 mW
P_{Max}	1000 mW
δ	0.0001
Area	1000m×1000m
Bandwidth	20 MHz
Path-Loss Exponent	3.76
Noise Power	-94 dBm
Standard Deviation of Shadowing	4 dB

7.1 Complexity Comparison

In Table. 5, we calculated the CC of MMSE precoders designed by the SDI and its corresponding CI. As illustrated, increasing the number of LPs in the LPs-AS results in a more significant decrease in CC. This table highlights the primary advantage of SDI over its corresponding CI, where utilizing only 2 LPs results in approximately an 80% reduction in CC. This substantial reduction in CC within the SDI is attributed to designing precoders using divided information in the LPs. Therefore, it is a general property of LPs-AS and is not limited to heuristic MMSE precoders. We will leverage this property for designing optimized precoders in our future works.

Table 5: The required complexity for computation of MMSE precoder in SDI compared to the CI versus the number of LPs.

Number of LPs:	1	2	4	8
CI	2.1×10^7			
SDI	2.1×10^7	6.2×10^6	1.9×10^6	6.8×10^5
CC Ratio	1	0.285	0.089	0.031

7.2 SE Comparison

In this section, we have studied two scenarios:

- First scenario that we allocate equal powers to $\rho_{k,m}$
- Second scenario that we utilize our proposed power control algorithm to allocate optimum powers to $\rho_{k,m}$

In each of the above scenarios, we evaluate the systems under MR and MMSE preodering. Regarding MR and MMSE precoders, it is worth expressing that MR precoders only maximize the desired power of each UE and do not have any effect on the interference, on the other hand MMSE precoders maximize the SINR of each UE, so they are capable of canceling the interference. In our simulations, we have considered $L_T =$

100 APs and compare the results of simulations for different values of M . We know that the APs number of each LP equals $L = \frac{L_T}{M}$, so smaller number of M results in larger number of L . Moreover, large value of L means that each LP accesses the larger amount of local CSI of APs. This leads to designing MR precoders with higher beamforming power and MMSE precoders with higher interference cancelation. After this introduction, now we propose a comprehensive explanation for the simulation results of the paper. All the results that we observe in the paper can be summarized as follows:

- MMSE precoders provide higher SE for UEs than MR precoders with both equal power and optimum power.
- Using proposed power control algorithm to allocate optimum power to UEs improve the SE of UEs with both MR and MMSE precoders.
- Increasing the number of LPs M results in decreasing the SE of UEs with both equal and optimum power and MR and MMSE precoders. The only exception of this result is related to allocating optimum power to UEs under MR precoders where increasing M leads to higher SE of UEs.

To justify the first result, we focus on the advantage of MMSE precoders over MR precoders in canceling the interference. Due to this capability, MMSE precoders can provide higher SE for UEs compared to MR precoders. To justify the second result, we express that our proposed power control algorithm in this paper aims to maximize the sum SE of UEs. So, applying this power control algorithm results in allocating optimum powers to UEs that achieve higher SE for them compared to the case with equal power. The third result is the most important finding of this paper. To justify it, we remind that we utilize $L_T = 100$ APs in our simulation and emphasize that small the number of LPs M results in large number of APs in each LP L , because we have $L = \frac{L_T}{M}$. So, small number of LPs leads to high amount of CSI sharing in each LP. Designing MR precoders from high amount of CSI improves the beamforming power of these precoders. Moreover, designing MMSE precoders from high amount of CSI enhances the capability of interference cancelation in these precoders. Based on this explanation, using $M = 2$ results in the best performance with

both equal and optimum power and under MR and MMSE precoders. The only exception of this general result is the case of applying power control algorithm with MR precoders where we achieve the best performance with $M = 8$ instead of $M = 2$. In this case, the proposed power control algorithm can not efficiently cancel the interference to achieve the maximum sum SE because the normalized beamforming vector of MR precoders and the allocated power to this beamforming vector with our proposed power control are determined to satisfy two different criteria. It is worth emphasizing that the normalized beamforming vector of MR precoder is determined so that each UE achieves the maximum power. On the other hand, our proposed power control algorithm allocates the power to this normalized beamforming vector so that the sum SE of all UEs is maximized. Mathematically, this justification can be summarized as follows:

In our proposed LPs-AS, we obtained the modified MR precoder of k th UE to all APs of the m th LP as follows:

$$\mathbf{W}_{k,m} = \sqrt{\rho_{k,m}} \frac{\mathbf{H}_{k,m}}{\|\mathbf{H}_{k,m}\|} \in \mathbb{C}^{NL \times 1}, \quad (30)$$

where

$$\mathbf{H}_{k,m} = [\mathbf{h}_{k1,m}, \dots, \mathbf{h}_{kL,m}] \in \mathbb{C}^{NL \times 1}. \quad (31)$$

In (30), the $\frac{\mathbf{H}_{k,m}}{\|\mathbf{H}_{k,m}\|}$ is normalized beamforming vector and $\rho_{k,m}$ is the allocated power to this

normalized beamforming vector. According to the definition of MR precoder [13], the normalized

beamforming vector $\frac{\mathbf{H}_{k,m}}{\|\mathbf{H}_{k,m}\|}$ is determined so that each UE receives the maximum power. On the other

hand, the aim of our power control algorithm is to determine the optimum values of $\rho_{k,m}$ so that the sum

SE of all UEs is maximized. So, in our power control algorithm, we are only allocating the powers $\rho_{k,m}$ to the fixed directions $\frac{\mathbf{H}_{k,m}}{\|\mathbf{H}_{k,m}\|}$. In this case, utilizing $M = 2$ results in lacking the efficiency of power control

algorithm to cancel the interference because, the beamforming vectors $\frac{\mathbf{H}_{k,m}}{\|\mathbf{H}_{k,m}\|}$ are designed so that each UE receives the maximum power and most of allocated powers $\rho_{k,m}$ are used to increase the received power of each UE not to cancel the interference. After this general discussion and justification, now, we focus on the simulation results and explain them.

In Fig. 3, we examine the impact of three main factors—the number of LPs, the type of precoder, and the method of power allocation—on the average sum SE of UEs in the LPs-AS. As illustrated in this figure, MMSE precoders along with utilizing a small number of LPs provide higher SE for the UEs in the LPs-AS. Furthermore, the proposed power control algorithm results in the improvement of sum SE, particularly with MR precoders.

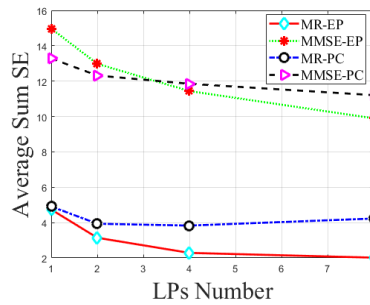
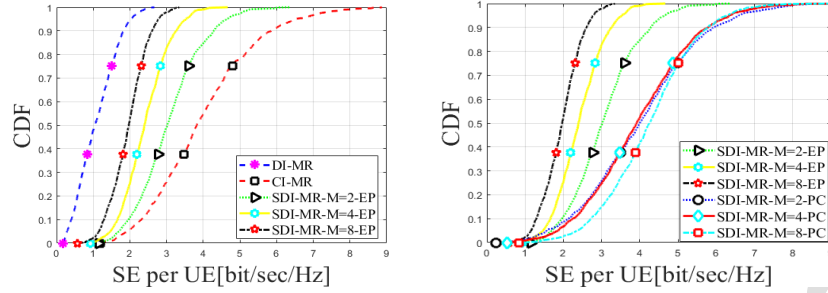


Fig. 3: The effect of LPs number on the average sum SE of LPs-AS.

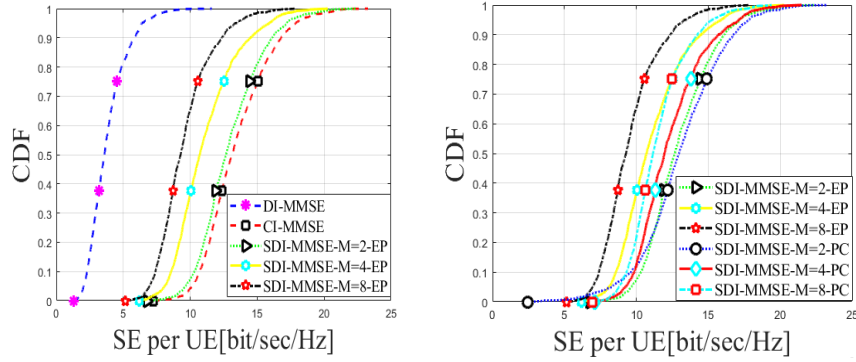


a) Equal power allocation.

b) Proposed power allocation algorithm.

Fig. 4: CDF of DL SE per UE of CF-mMIMO system using MR precoder.

In Figs. 4a and 4b, we evaluate the UEs' SE with MR precoders designed by SDI. As illustrated in Figure 4a, designing MR precoders using our proposed SDI can address the low SE of UEs in the DI, particularly when a small number of LPs are utilized in our LPs-AS. Furthermore, as shown in Fig. 4b, it is evident that the application of our proposed power control algorithm leads to an improvement in the SE of UEs and eliminates the dependency of SE on the number of LPs. In Figs. 5a and 5b, we employ MMSE precoders designed by our proposed SDI to evaluate the SE of UEs in the LPs-AS. The results are highly similar to those obtained with MR precoders; however, MMSE precoders demonstrate superior performance due to their interference cancellation capabilities. Based on our results for heuristic precoders from Table. 5 and Figs. 4 and 5, we present the following significant finding for our LPs-AS: Utilizing an optimal number of LPs in our proposed LPs-AS and designing precoders with our proposed SDI significantly reduces the CC of precoder design without substantially degrading the SE of UEs. We will investigate this property for optimized precoders in our future papers.



a) Equal power allocation.

b) Proposed power allocation algorithm.

Fig. 5: CDF of DL SE per UE of CF-mMIMO system using MMSE precoder.

8 Conclusion

In this paper, we proposed a novel LPs-AS for CF-mMIMO systems, which facilitates an SDI for precoder design in these systems. Our proposed LPs-AS utilizes several LPs to design precoders, with each LP connected to the CP and multiple APs, functioning as a precoding unit for its respective APs. We evaluated the LPs-AS in terms of SE and CC and proposed a power control algorithm to maximize the sum SE in this structure. Our numerical results demonstrated that by using a small number of LPs in the LPs-AS, our proposed SDI significantly outperformed its corresponding CI in terms of CC of precoder design. Furthermore, it effectively addressed the low SE of UEs in its corresponding DI.

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