

The Upper-Bound of the Average Achievable Rate of Non Orthogonal Multiple Access

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Abstract:

The Non Orthogonal Multiple Access (NOMA) is a popular candidate for the next generation of wireless networks. Two advantages of NOMA are that it can achieve a higher rate and support more users compared to orthogonal multiple access (OMA). By increasing the number of users occupied a subchannel in NOMA, the achievable rate and the number of supported users of the system have been increased. Thus, we can assume there are infinite users in the system wanting to share one subchannel to derive the upper bound of the achievable rate of NOMA. In this article, the optimal power allocation function of infinite users in NOMA is derived, which maximizes the average achievable rate of the system under the maximum power constraint. The performances of the proposed power allocation strategy are compared with the simple case with only two users in NOMA. The simulation results show the gap between the average achievable rate and the outage probability of infinite users and two users in the NOMA system.

Keywords:

Resource allocation, NOMA, Successive interference cancellation, Convex optimization.

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1. Introduction

Increasing demand for wireless communication systems leads to the need to support more users. Non-Orthogonal Multiple Access (NOMA) is an important approach to handle this requirement. In fact, it brings factors such as being able to transmit data from multiple users at the same time, frequency, and code but with different power levels[1]. As it is previously known, the number of supported users is limited in OMA due to the fact that they will have to be orthogonal to each other in one of the aforementioned dimensions[2]. Whereas one of the advantages of NOMA is the absence of this restriction, which is one of the main factors that makes it a suitable candidate for next generation wireless networks [3]. NOMA enables us to allocate less power to the user with superior channel conditions; therefore, we will be able to transmit the signal to the user with the unfavorable channel conditions with higher power levels. In [4], the comparison between NOMA and OMA in the system level performance is investigated, where the superiority of NOMA compared to OMA in terms of the sum rate is proved theoretically. To support K users in OMA, the bandwidth (B) should be divided between them, which leads to a lower total rate compared to NOMA, where all users share the whole bandwidth (B). As a result, by increasing the number of users sharing the subcarrier, the total achievable rate of NOMA increases. Therefore, in this paper, it is assumed that infinite users share one subcarrier to derive the upper bound on the average achievable rate of NOMA. Note that considering infinite users for superposition coding is applied in the broadcast strategy to calculate the achievable rate of the channel when CSI is not available at the transmitter[5, 6].

The strategy of power allocation between users, who share one subcarrier in NOMA, is one of the important problems studied in the literatures. In previous works, there are many approaches for resource allocation in NOMA system that optimizes the desired objective functions[7]. For example, in [8] the optimal power allocation policy that maximizes the energy efficiency of the system is derived. Moreover, the optimum power allocation of the users to optimize the sum rate of them is proposed in different system models as[9-11]. The survey on this topic can be found in[12]. Due to the importance of NOMA

in the next generation of wireless communication networks, sum rate maximization is studied in recent papers [13-17] in more complex system models. The achievable sum rate of NOMA users in cell-free massive multiple-input multiple-output is maximized by considering the strict quality of service requirements for ultra-reliable low-latency communication [13]. The power allocation algorithm is proposed by the successive convex approximation method. The two user clustering algorithms are suggested based on graph theory. In [14], the considered system model is equipped with cash and device-to-device communication. The optimization problem is solved by a convex optimization method. In [15], the sum rate of full duplex NOMA is maximized by considering an active reconfigurable intelligent surface (RIS) in the system model. The semi-definite relaxation (SDR) and successive convex approximation (SCA) methods are used for solving the resource allocation problem. The cooperative simultaneously transmitting and reflecting reconfigurable intelligent surface (STAR-RIS) assisted non-orthogonal multiple access is investigated in [16], where the phase shifts, amplitude coefficients, and power allocation are optimized by the geometry programming (GP) method. The power allocation of users by considering a hybrid active-passive STAR-RIS is studied in [17] with the goal of sum rate maximization. Also, recently, a machine learning method has been proposed for resource allocation in NOMA [18]. In all these works, it is assumed that K users (finite users) share one subcarrier. Many papers even consider that only two users are allocated to each subcarrier [19]. By this assumption, the NOMA's ability to support numerous users in wireless communication is limited.

As mentioned earlier, in this paper, it is assumed that infinite users can share one subcarrier to derive the upper bound on the average achievable rate of NOMA. As a result, it is necessary to find the power allocation strategy of these infinite users. The channel gains of users are based on a single continuous probability density function, as a Rayleigh distribution. As the number of users tends to infinity, all possible values of channel gains are examined by users. Thus, the problem is obtaining the optimal power allocation function for continuous channel gain according to its probability in the fading channel. The optimal power allocation function is derived to maximize the average achievable rate of the system. The

Euler equation is used to derive the closed-form expression for the power allocation function of the users. Due to the maximum power constraint, the power is allocated to a limited range of the channel gain. Note that by assuming infinite users in the system, the complexity of the successive interference cancellation (SIC) algorithm in the receivers increases extremely, and it cannot be implemented in a practical system. In fact, the average achievable rate by the proposed power allocation function is the upper bound of the average achievable rate of NOMA. The derived upper bound of the rate of NOMA is compared with the practical NOMA system in which only two users share one subcarrier. The simulation result shows that NOMA with infinite users has considerably higher average achievable rate compared to NOMA with two users. As a result, the contribution of this paper is summarized as follows,

- Formulate the average achievable rate of NOMA with infinite users
- Derived a closed-form function for the power allocation of infinite users
- Comparison of the average achievable rate of the derived power allocation and two users NOMA and OMA

The rest of this paper is organized as follows: Section 2 presents the system model and the average achievable rate maximization problem by considering infinite users in a NOMA system. The optimum power allocation function of users is derived in Section 3. In section 4, the simulation result shows the gap between the average achievable rate of the derived power allocation function for infinite users and simple two-user NOMA systems. Finally, Section 5 summarizes findings.

2. Problem Formulation

Consider a NOMA downlink communication scenario in which a base station (BS) is communicating with K users. The received signal of the k 'th user ($y_k(t)$) is as follows

$$y_k(t) = g_k x(t) + w_k(t), \quad (1)$$

where $x(t)$, g_k and $w_k(t)$ are, respectively, the transmitted signal, the gain of the BS to the k 'th user, and the additive Gaussian noise of 1 variance of the k 'th user. It is assumed that the channel's gain for each block transmission is constant and will change randomly in the next block transmission. Without loss of generality, it is assumed that the users are sorted according to their channel gains as $s_1 < s_2 < \dots < s_K$ where $s_i = |g_i|^2$.

By applying superposition coding at the transmitter and SIC at the receivers, each user can decode the signal of weaker users, subtract from the received signal, and decode its own signal. As a result, only the signals of better users compared to the given users are considered as an interference [7]. Thus, the Signal to Noise plus Interference Ratio (SINR) of the k 'th user is as follows

$$SINR_k = \frac{P_k s_k}{1 + s_k \sum_{i=k+1}^K P_i}, \quad (2)$$

where, P_i $i \in 1, \dots, k$ is allocated power to the i 'th user. Thus, the rate of the k 'th user is as follows

$$R_k = \log\left(1 + \frac{P_k s_k}{1 + s_k \sum_{i=k+1}^K P_i}\right). \quad (3)$$

Since in this article the case of infinite users is investigated, obviously, K approaches infinity. When the number of users goes to infinity, we can assume that s_k it is continuous, which is represented by a continuous variable s . Therefore, the power allocation strategy of users converts to a continuous function $p(s)$. As a result, the allocated power to the user with channel gain s is derived by discretizing the continuous function $p(s)$ as $p(s)ds$. Therefore, the P_k allocated power of the k 'th user should be replaced by $p(s)ds$. Also, $\sum_{i=k+1}^K P_i$ it is replaced by $I(s) = \int_s^\infty p(u)du$ which is the total interference of users with channel gain s .

As a result, to rewrite Eq. (3), by assuming a continuous variable, we have

$$R(s) = \log\left(1 + \frac{sp(s)ds}{1 + sI(s)}\right), \quad (4)$$

since $ds \ll 1$ Eq. (4) can be approximated as $R(s) = \frac{sp(s)ds}{1 + sI(s)}$.

In this article, we want to find the upper bound of the average achievable rate of NOMA by considering infinite users. For this, we should find the optimum power allocation function of the users ($p(s)$) which maximizes the average achievable rate of the total NOMA users. Assume that the probability of the existence of a user with channel gain s is $f(s)$ which is the probability density function of channel gain. Thus, the total average achievable rate of all users is

$$R_{avg} = \int_0^\infty f(s)R(s) = \int_0^\infty f(s) \frac{sp(s)ds}{1 + sI(s)}. \quad (5)$$

As mentioned, our problem is to find the power allocation function $p(s)$ that maximizes R_{avg} . According

to the $I(s)$ definition, we have $p(s) = \frac{-dI(s)}{ds} = -I'(s)$. As a result, finding $p(s) I(s)$ and is

equivalent. Therefore, Eq. (5) can be written as $\int_0^\infty f(s) \frac{-sI'(s)}{1 + sI(s)} ds$. Also, assume that the maximum

power limitation of the BS transmitter is P_T which leads to a constraint on the $p(s)$ as $\int_0^\infty p(s)ds \leq P_T$.

Another constraint on $p(s)$ is that it should be nonnegative function. As a result, the optimization problem is as follows

$$\begin{aligned} \max_{I(\cdot)} R_{avg} &= \max_{I(\cdot)} \int_0^\infty f(s) \frac{-sI'(s)}{1 + sI(s)} ds \\ \text{s.t.} \quad &\int_0^\infty -I'(s)ds \leq P_T \\ &-I'(s) \geq 0. \end{aligned} \quad (6)$$

3-Power Allocation

To solve our optimization problem, we can apply Lagrange multipliers as

$$\mathcal{L} = \int_0^\infty (f(s) \frac{-sI'(s)}{1+sI(s)} + I'(s)(\lambda - v(s)))ds + \lambda P_T, \quad (7)$$

where λ and $v(s)$ are considered as, respectively, an arbitrary coefficient and an arbitrary function, and both are non-negative with regard to the constraints of Eq. (6). The integrand of Eq. (7) can be thought of as a function extracted as

$$G(s, I, I') = (f(s) \frac{-sI'(s)}{1+sI(s)} + I'(s)(\lambda - v(s))). \quad (8)$$

Subsequently, the Euler equation [20] is a good candidate for solving this problem. Hence, there will be

$$G_I - \frac{dG_{I'}}{ds} = 0 \text{ where}$$

$$\begin{aligned} G_I &= \frac{\partial G}{\partial I} = \frac{s^2 I'(s) f(s)}{(1+sI(s))^2} \\ G_{I'} &= \frac{\partial G}{\partial I'} = \frac{-sf(s)}{1+sI(s)} + (\lambda - v(s)), \\ \frac{dG_{I'}}{ds} &= -\frac{f(s) + sf'(s) + s^2(f'(s)I(s) - f(s)I'(s))}{(1+sI(s))^2} - v'(s). \end{aligned} \quad (9)$$

Thus we have

$$G_I - \frac{dG_{I'}}{ds} = \frac{f(s) + sf'(s) + s^2 f'(s)I(s)}{(1+sI(s))^2} + v'(s) = 0. \quad (10)$$

In addition, it is known from the complementary slackness conditions:

$$\lambda(P_T + \int_0^\infty I'(s)ds) = 0 \text{ and } v(s)I'(s) = 0, \quad (11)$$

when $I'(s) = -p(s) < 0$, $v(s) = 0$ is deducted and $f(s) + sf'(s) + s^2 f''(s)I(s) = 0$. As a result $I(s)$ can be calculated as

$$I(s) = \frac{-f(s) - sf'(s)}{s^2 f''(s)}. \quad (12)$$

In the following, we are going to depict that for a total of M disjoint subintervals in which $I'(s) = -p(s) < 0$, there is a maximum of one disjoint subinterval with a positive power allocation in each subinterval. To prove this claim, we can suppose otherwise. Assume $s_{m_1} < a_1 < a_2 < b_1 < b_2 < s_{m_u}$

in which $I'(s) = -p(s) = \frac{d((-f(s) - sf'(s)) / (s^2 f''(s)))}{ds} < 0$ holds for $s \in [s_{m_1}, s_{m_u}]$ and there are two positive power intervals, i.e., $[a_1, a_2]$ and $[b_1, b_2]$, with zero power intervals in $[a_2, b_1]$. As stated in [20], for a piece-wise continuous external solution, the Weierstrass-Erdmann (corner) conditions, as stated below, are applicable at any corner point, for instance s_a ,

$$\begin{aligned} G_{I'}|_{s=s_a^+} &= G_{I'}|_{s=s_a^-}, \\ G - I'G_{I'}|_{s=s_a^+} &= G - I'G_{I'}|_{s=s_a^-}. \end{aligned} \quad (13)$$

Therefore, considering the first equation in Eq. (13)

$$f(s_a) \frac{s_a}{1 + s_a I(s_a^-)} + \lambda - v(s_a^-) = f(s_a) \frac{s_a}{1 + s_a I(s_a^+)} + \lambda - v(s_a^+). \quad (14)$$

Based on the slackness conditions in Eq. (11), $v(a_2^-) = 0$ because $a_2^- \in [a_1, a_2]$. According to the fact that

$I(s)$ is a continuous smooth function ($I(a_2^+) = I(a_2^-)$) and referring to Eq. (13), we have $v(a_2^+) = 0$.

Similarly, it can be shown that $v(b_1^-) = 0$. Also, in the interval (a_2, b_1) , we have $p(s) = -I'(s) = 0$, as a result,

$$v'(s) = \frac{s^2 f'(s)(-I(a_2) - (f(s) + sf'(s)) / (s^2 f'(s)))}{1 + s^2 I(a_2)}. \quad (15)$$

Since $\frac{d((-f(s) - sf'(s)) / (s^2 f'(s)))}{ds} < 0$ in the interval $[s_{m_l}, s_{m_u}]$, $v'(s)$ is a negative and $v(s)$ is

decreasing function in the interval (a_2, b_1) which is contradicts with $v(a_2^+) = v(b_1^-) = 0$. Thus, there is at most one positive power allocation interval in $[s_{m_l}, s_{m_u}]$.

3-2- Rayleigh Fading Channel

In a Rayleigh fading channel, the probability density function of s is exponential with parameter λ where for simplicity we assume $\lambda = 1$. Thus, we have $f(s) = e^{-s}$. By substituting $f(s)$ in Eq. (12), it can be stated that

$$I(s) = \frac{-e^{-s} - s \times (-e^{-s})}{s^2 \times (-e^{-s})} = \frac{1-s}{s^2}. \quad (16)$$

Therefore, $I'(s) = \frac{s-2}{s^3}$ which lead to the power allocation function as $p(s) = \frac{2-s}{s^3}$.

Subsequently, for having $p(s) > 0$, i.e., having non-zero power for users, we will have to abide by the following condition

$$s - 2 < 0, \text{ hence } s < 2. \quad (17)$$

Also, due to the maximum power limitation, the positive power allocation interval has a lower bound

which is $I(s_L) = P_T$. Therefore, $s_L = \frac{-1 + \sqrt{1 + 4P_T}}{2P_T}$. On the other hand, the upper bound of positive

power allocation interval is derived as $I(s_U) = 0$ which leads to the $s_U = 1$. As a result, in the Rayleigh

fading channel the power is allocated to the users which have channel gain in the interval

$\left[\frac{-1 + \sqrt{1 + 4P_T}}{2P_T} \quad 1 \right]$. Note that $\frac{-1 + \sqrt{1 + 4P_T}}{2P_T}$ is always lower than 1 for any value of P_T . As a result,

the power allocation interval never becomes zero for a Rayleigh fading channel.

Finally, the maximum average achievable rate of NOMA with infinite users in a rayleigh fading channel can be calculated as

$$R_{avg}^* = \int_0^\infty f(s) \frac{-sI'(s)}{1 + sI(s)} ds = \int_{s_L}^1 e^{-s} \frac{s \frac{2-s}{s^3}}{1 + s(\frac{1-s}{s^2})} ds = \int_{s_L}^1 e^{-s} \frac{2-s}{s} ds. \quad (18)$$

3-2- Nakagami-m Fading Channel

By considering Nakagami-m fading channel with fading parameter m and $E(|g|^2) = \Omega$, the probability of s is a gamma distribution with probability density function as

$$f(s) = \frac{1}{\theta^m \Gamma(m)} s^{m-1} e^{-s/\theta}, \quad (19)$$

where $\theta = \frac{\Omega}{m}$ and $\Gamma(.)$ is the Gamma function. By substituting $f(s)$ in Eq. (12), we have

$$I(s) = \frac{-(s - m\theta)}{s(s + \theta - m\theta)}. \quad (20)$$

Thus, for $I(s_U) = 0$, we have $s_U = m\theta = \Omega$. Based on Eq. (19), Eq.(20) and Eq. (12), the power allocation function is as follows

$$p(s) = \frac{-s^2 + 2m\theta s - m^2\theta^2 + m\theta^2}{s^2(s + \theta - m\theta)^2}. \quad (21)$$

For $p(s) > 0$, we have $s < \theta(m + \sqrt{m})$. Based on the Eq. (21), the s_L is calculated as

$$s_L = \frac{-(mP_T + 1 - m\theta P_T) + \sqrt{(mP_T + 1 - m\theta P_T)^2 + 4m\theta P_T}}{2P_T}. \quad (22)$$

Finally, R_{avg}^* is calculated as follows

$$R_{avg}^* = \int_0^\infty f(s) \frac{-sI'(s)}{1 + sI(s)} ds = \int_{s_L}^{s_U} \frac{As^{m-2} e^{-s/\theta} (-s^2 + 2m\theta s - m^2\theta^2 + m\theta^2)}{\theta(s + \theta - m\theta)} ds. \quad (23)$$

where $A = \frac{1}{\theta^m \Gamma(m)}$.

4-Simulation Result

In this section, numerical results of the proposed power allocation strategy are provided. By considering $\Omega = 1$, the average achievable rate in the Nakagami-m fading channel for different values of m is shown in Fig. 1. Note that for $m = 1$, the Nakagami-m fading includes the Rayleigh distribution. For $m < 1$, the fading is more severe than Rayleigh fading, and for $m > 1$ closely approximates the Rician distribution with parameter K where $m = \frac{(1 + K)^2}{1 + 2K}$.

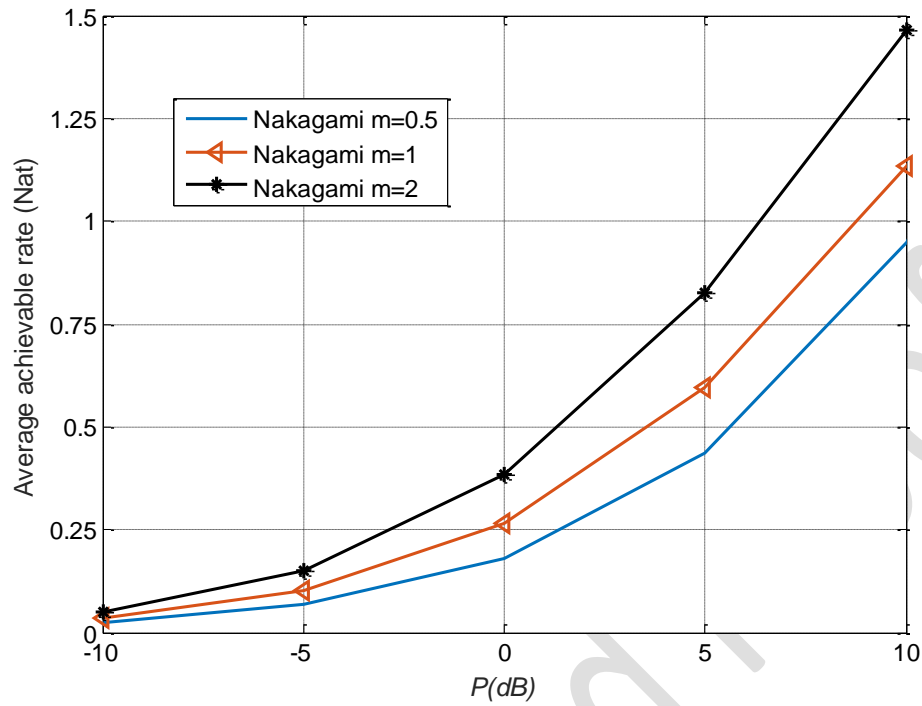


Fig.1.Average achievable rate of the infinite users in Nakagami-m fading channel for different value of m

Assume a Rayleigh fading channel with average one. The maximum average achievable rate in this channel is calculated based on Eq. (18) shown in Fig.2.

For comparison, two different strategies for user access are considered. The first one is NOMA, in which only two users share the subcarrier. Assume that there are two users with channel gains s_1 and s_2 where

$$s_1 < s_2.$$

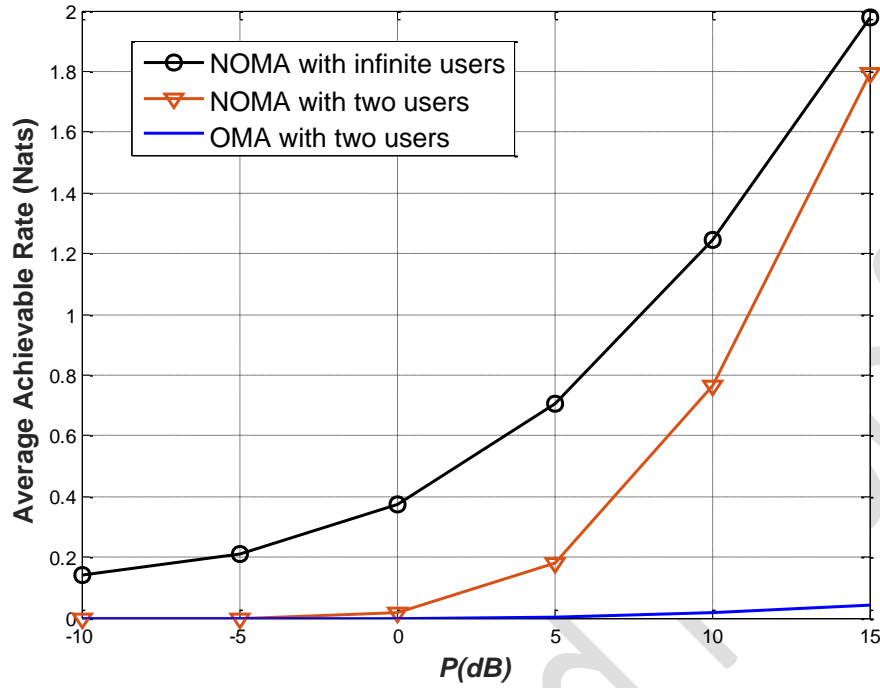


Fig.2. Average achievable rate of the infinite and two users NOMA and two users OMA versus different transmit powers in Rayleigh fading channel.

As we know, the optimal power allocation of NOMA with two users by goal of rate maximization without any constraint on the rate of users is $p(s_1) = 0$ and $p(s_2) = P_T$. For supporting two users in NOMA, it is necessary to assume minimum rate for the worse user (s_1) denoted by R_{th} . Note that the user with channel gain s_L has the minimum rate in the proposed approach because has maximum interference. As a result, for fair comparison, R_{th} it should be set to $R(s_L)$. This value can be calculated by substituting s_L in Eq. 4 as follows

$$R_{th} = R(s_L) = \log\left(1 + \frac{s_L P(s_L)}{1 + s_L I(s_L)}\right) = \log\left(1 + \frac{(2 - s_L) / s_L^2}{1 + s_L P_T}\right). \quad (24)$$

For the sake of comparison, the rate for two users must be higher than or equal to R_{th} . Thus we have

$$\log(1 + \frac{s_2(1-a)P_T}{s_2aP_T + 1}) \geq R_{th}, \quad (25)$$

where a is a power allocation ratio between two users and is obtained as $a = \frac{1 + s_2P(s) - e^{R_{th}}}{s_2P(s) - e^{R_{th}}}$. Note that

in the case where derived a is not in the interval $[0, 1]$, R_{th} can not be reached for the worse user due to the maximum power limitation P_T . Thus, the resource allocation is infeasible, and the rates of these cases are set to zero. Now, the value of the rate for two users can be calculated as $R_{total} = R_{th} + \log(1 + as_1P_T)$ in the case where $a \in [0, 1]$.

The second strategy is OMA. In this strategy, it is forced to support two users, the same as NOMA with two users. As a result, the bandwidth is divided between two users. The rate at this case can be calculated as

$$R_{OMA} = \frac{1}{2}(\log(1 + s_2P_T) + \log(1 + s_1P_T)), \quad (26)$$

where the capacity is calculated per hertz and $B=1$. For fair comparison, when the rate of each user is below R_{th} , its rate is set to zero. Note that if we assume that infinite users are supported by OMA, the bandwidth should be divided between them which leads to the $\lim_{k \rightarrow \infty} \frac{B}{k}$. As a result, the bandwidth of each user tends to zero.

The simulation results are derived by mont carlo simulation for NOMA and OMA with two users with 10000 runs and are shown in Fig.2. As can be seen, the proposed power allocation function achieves 70% a higher rate compared to the rate of two NOMA users. Also, by increasing the total power, the gap between these strategies decreases. This is due to the fact that R_{th} can be achieved by a weaker user with high power in the most cases. Note that the complexity of the SIC for infinite users is extremely high and can not be implemented in practical wireless communication systems. Thus, the average achievable rate

of the proposed power allocation can be considered as an upper bound on the average achievable rate of the NOMA. As expected, the rate of OMA with two users is much lower than that of other strategies. Note that the gap between OMA and NOMA with two users is very bigger than the gap between NOMA with two users and NOMA with infinite users. Therefore, one can conclude that the NOMA with two users can achieve suitable average rate by reasonable complexity of SIC.

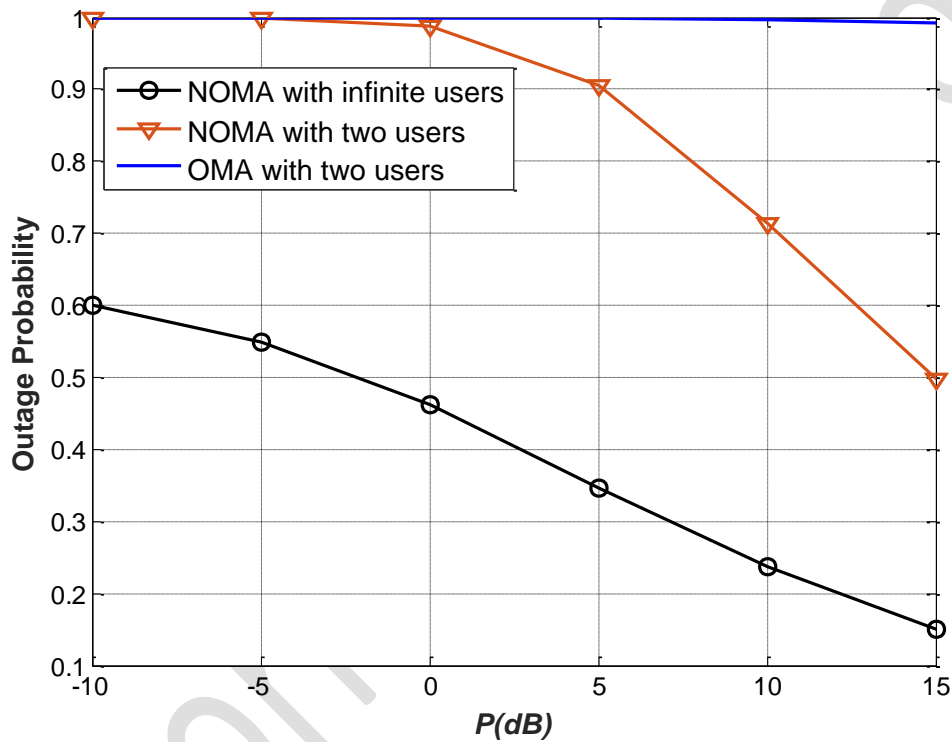


Fig.3.Outage probability of the infinite and two users NOMA and two users OMA versus different transmit powers

In Fig.3, the outage performance of the proposed power allocation function for infinite users is compared with NOMA and OMA with two users NOMA.

In infinite users NOMA system, the outage probability is the probability of users who have a lower gain than s_L . Because their rate are zero and the rate of users who have gain higher than s_L is higher than R_{th} .

By assuming a Rayleigh fading channel, this probability is $1 - e^{-s_L}$. In two users NOMA system, the outage accrues when the R_{th} can not be achieved by the worse users. Similar to NOMA with two users, in

OMA the case where the rate of a user is lower than R_{th} , is considered an outage. According to the simulation result, the outage performance of the proposed power allocation function of infinite users outperforms than OMA and NOMA with two users.

5-Conclusion

In this paper, the upper bound on the average achievable rate of NOMA system is derived. For this purpose, it is assumed that the system has infinite users, and the optimum power allocation function for these users is derived by solving the average rate maximization problem. The Euler equation is used for solving this problem. According to the simulation results, the average achievable rate of infinite users is average 70% higher than two users in NOMA systems. The gap between the average achievable rate of two users OMA and NOMA is much bigger than the gap between infinite and two users NOMA. Also, the outage probability of the proposed power allocation function for infinite users is lower than that of two-user NOMA and OMA.

6- References

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