Impacts of Different Deterioration Processes on Structural Time-dependent Reliability via Dynamic Bayesian Networks

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Abstract:

Engineering structures are typically subjected to time-dependent deterioration processes, such as corrosion, fatigue, and carbonation, which gradually reduce their service life and reliability. This study investigates the time-dependent reliability of structures under different deterioration mechanisms using Dynamic Bayesian Networks (DBNs). This analysis has the potential to significantly influence future decisions about the structure's usage. Three deterioration models: deterministic, stochastic, and Gamma process are implemented to represent distinct degradation behaviors. The methodology involves discretizing the resistance variable in DBN and comparing reliability indices obtained from DBN and Monte Carlo simulation (MCS) to validate the approach. The DBN results are validated against Monte-Carlo simulations, showing a maximum discrepancy of 3%, as well as providing standard deviation (0.0211) and root-mean-square error (0.023) of differences that demonstrates the DBN approach's validity and precision. This paper calculates the time-dependent reliability of a portal frame structure experiencing resistance deterioration, influenced by various deterioration models. Finally, it presents a comparison of the results from time-dependent reliability analysis utilizing various deterioration processes. Among the models, the Gamma process yields the highest reliability index over a 40-year period, while deterministic and stochastic models exhibit slightly lower reliability. Estimates derived from measurements are more realistic than those based on design values. The findings demonstrate the capability of DBN to incorporate measurement evidence, providing a robust basis for lifetime reliability assessment and maintenance planning of deteriorating structures also DBN effectively captures deterioration effects and probabilistic uncertainty over time, offering a computationally and time-efficient alternative to Monte-Carlo simulations.

Keywords:

Deterioration, Deterministic and stochastic process, Time-dependent reliability, Gamma distribution, Dynamic Bayesian Networks.

1. Introduction

Conventional structural analysis assumes that the parameters of structural models possess a fixed value. In numerous civil engineering applications, uncertainty in geometric dimensions, input forces, material specifications, and other parameters is unavoidable and cannot be disregarded as they significantly impact the final performance of the structure [1, 2]. Given uncertainties, it is preferable to assess structural safety using probabilistic methods. The reliability analysis method is a probabilistic framework used to calculate the structure's confidence margin which plays an essential role in the analysis and design of structures [3]. Structures and infrastructure systems, crucial for the usability of modern societies, experience a gradual decline in performance throughout their service lives due to environmental degradation [4, 5], a process to which natural disasters, deterioration mechanisms, and harmful environmental stressors collectively contribute, thereby degrading the structure’s lifetime performance [6-8]. Classical reliability methods for structures neglect the long-term impacts of environmental factors on the hardness and strength of concrete and steel, relying on simplifications, including the assumption of a constant loading rate.

One of the fundamental components of time-dependent reliability analysis in structures is the deterioration function, which is related to the structure's deterioration mechanism and is associated with uncertainty. Numerous empirical and analytical models have been suggested to address the structural resistance deterioration function while accounting for its uncertainties [9-11]. To facilitate an efficient examination of the structural time-dependent reliability of deterioration, numerous studies have been conducted. One of the initial models presented is the model by Ellingwood and Mori [9]. This applies to the reliability-based service-life assessment of aging concrete structures, as studied by Rodriguez et al. [12]

who investigated the extent of diameter reduction in rebars due to general and localized corrosion. Additionally, utilizing the variables of corrosion initiation time and corrosion propagation, Stewart and Rosowsky [13] proposed a formula for reducing the diameter of rebars. Enright and Frangopol [14] investigated the variables associated with corrosion initiation time and subsequently modeled the uncertainty stemming from this initiation using the lognormal distribution function. A process utilizing the outcrossing approach was presented [15], which facilitated the application of traditional reliability tools, including first-order reliability method and second-order reliability method. To forecast the likelihood and severity of cracking in reinforced concrete structures that are exposed to chloride ions, Stewart and Mullard [16] implement a spatial time-dependent reliability analysis. To track the progression of the corrosion process, models are implemented that predict the initiation and propagation of corrosion, as well as the initiation and growth of cracks. Numerous methodologies for time-dependent reliability were introduced by Melchers and Beck [17], such as importance sampling and the first-passage method. An analytical approach that integrates the stochastic processes of resistance degradation and variable load was developed by Van Noortwijk et al. [18]. This approach employs a Poisson process for load and a Gamma process for resistance. Much of the prior research in reliability primarily focuses on assessing the time-dependent bending strength affected by the corrosion of reinforced concrete elements, as well as investigating the shear resistance resulting from corrosion phenomena. In the past two decades, Bhargava et al. [19] and colleagues have conducted extensive research to estimate time-dependent corrosion models in reinforced concrete beams under various shear and bending modes. Classical reliability methods typically utilize only the statistical indices of parameters and rely on simplifying assumptions regarding their distribution, thereby increasing uncertainty. One method for reducing these uncertainties is to adopt a Bayesian perspective and employ Bayesian probabilistic networks.

Probabilistic Bayesian networks integrate Bayesian principles with graph theory. Bayesian networks are a graphical representation of the probability distribution of a collection of random variables. This network comprises nodes that signify random variables. The edges connecting the nodes represent the relationship between them, as well as the conditional probability distribution associated with each node [20]. For each node, a probability table is generated, which is then used in statistical inference. In the Bayesian approach, as opposed to the classical method, a comprehensive distribution is accounted for each parameter. In this case, all uncertainties can be accounted for in the parameter under discussion. A significant advantage of the Bayesian method is its capacity to integrate various types of information based on their uncertainty [21, 22]. Bayesian networks are typically constructed utilizing standard probabilistic models. Bayesian networks allow for the derivation of a collection of random variables' posterior distribution based on a given set of observations. This procedure is referred to as inference. These networks are highly appropriate for integrating the potential for deterioration, signifying a reduction of resistance over time [23].

Bayesian networks have gained attention in engineering risk analysis in recent years because of their intuitive characteristics and capacity to manage numerous dependent random variables [24]. Several researchers have employed Bayesian networks in modeling temporal functions, including deterioration [25]. Montes-Iturrizaga et al. [26] employed a Bayesian network to enhance inspection strategies for offshore structures vulnerable to multiple failure mechanisms. Attoh-Okine and Bowers [27] introduced an empirical model for bridge deterioration utilizing Bayesian networks. Sani et al. [28] introduced a novel model for the analysis of structural resistance and loads. A Bayesian network is advantageous in domains characterized by statistical states. In such instances, each variable possesses a unique and definitive value. This assumption is a static issue and is inadequate for numerous problems. A DBN is an augmented Bayesian network incorporating a time dimension, utilized for modeling dynamic systems [29]. DBNs address these limitations by augmenting traditional BN with temporal dependencies, facilitating the examination of the evolution and dynamic interaction of risks over time [30].The term 'temporal Bayesian network' is preferable to 'dynamic Bayesian network' because it implies that the model's structure remains constant while the temporal variables fluctuate over time, rather than being fixed and uniform. A DBN is a directed acyclic graphical model that is particularly suitable for modeling temporal deterioration and its accompanying uncertainties and can be readily combined with real-time data for state inference and model updating [31, 32].

This paper employs DBN to model the time-dependent reliability of a portal frame structure experiencing resistance deterioration, acknowledging the significant impact of deterioration on structural strength throughout its lifetime and the efficacy of DBN in probabilistic analysis and deterioration modeling. The DBN framework models the probabilistic evolution of structural resistance over time and update reliability based on available inspection or measurement data. Multiple approaches for modeling the degradation function have been proposed. In this study, instead of relying solely on classical models, recently proposed deterioration models have been employed to for the deterioration function, to ensure a consistent and comparable evaluation of structural degradation. Deterministic, stochastic, and Gamma process deterioration functions are selected to model the deterioration process. The main assumptions include: (1) structural loads are treated as random variables, (2) deterioration parameters are derived from experimental and literature-based data, and (3) the Gamma process is characterized by time-dependent shape and constant scale parameters. The DBN framework is employed as the core reliability assessment tool, enabling sequential updating of structural performance through its posterior inference capability when new evidence becomes available. To ensure the robustness and accuracy of the DBN predictions, an independent Monte Carlo simulation is conducted as an external validation benchmark. The integration of these modern deterioration models within a unified DBN–MCS framework, along with the posterior updating of structural reliability, represents the principal novelty of this research and addresses an existing gap in the literature. Ultimately, deterministic and stochastic deterioration functions, along with the Gamma process, have been utilized on a one-bay frame, and the results have been compared.

2. Models of structural deterioration

The strength and stiffness of structures in service may be altered as a result of aging, which exceeds the baseline conditions that were assumed during the structural design process. The service life of structural components and systems is influenced by the temporal alterations in material properties. Certain aging effects are harmless; others may lead to a deterioration of component or system integrity over time, potentially increasing the risk of structural failure. The prediction of deterioration is a crucial phase in the comprehensive lifetime management of buildings [33]. Figure 1 schematically illustrates the decline in the structure's resistance over time. The subsequent text delineates various perspectives on deterioration models, including deterministic, stochastic, and Gamma process deterioration functions.

2.1. Deterministic deterioration model

The initial approach to modeling structural degradation examined in this research is the structure's resistance based on a deterministic deterioration function. The degradation of structural resistance is typically represented by Equation 1 [9].

$$R(t) = R_0 \times G(t) \tag{1}$$

The stochastic deterioration function is represented by $G(t)$, the initial resistance is represented by R_0, and the structural resistance at time t is denoted by $R(t)$. Elementary polynomial models, specifically linear, exponential, and square root functions of time t, are used to evaluate the reliability of $G(t)$, a function that is typically represented by basic polynomial functions[9, 14, 34-36]. In recent years, various researchers have proposed a polynomial function to account for the effects of different deterioration mechanisms [28, 37].

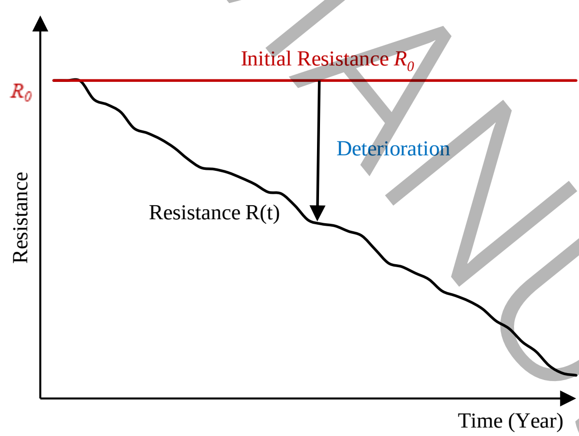

Fig. 1. Schematic representation of the resistance deterioration process over time

The parameters of this function are established through the analysis of physical and chemical processes and are contingent upon the type of deterioration mechanism, as well as the type and usage. This research employs a particular function for deterministic deterioration and examines the distinct impacts of each failure mechanism on reliability.

$$g(t) = 1 - m_F = 1 - \left(at + b\sqrt{t} + \exp\left(\frac{c}{t}\right) \right) = 1 - at - b\sqrt{t} - \exp\left(\frac{c}{t}\right) \tag{2}$$

Here, $m_F(\cdot)$ represents the deterministic time-dependent deterioration ratio, while $g(t)$ denotes a deterministic deterioration function, with terms chosen based on primary deterioration mechanisms. This polynomial function comprises three terms. The first term represents the linear model, the second term denotes the square root model, and the third term illustrates the exponential model, respectively. This formulation incorporates terms that explain the variations in resistance resulting from various degradation factors. The linear term denotes additional deterioration mechanisms, such as fatigue, while the exponential term represents corrosion and the square root term represents carbonation.

The deterioration function's parameters are typically estimated and can be adjusted through routine inspections or observations of the structure. Equation (2) estimates the parameters a, b, and c based on the assumption that each term is solely influenced by the designated deterioration mechanism. Fatigue affects a, carbonation affects b, and corrosion affects c. Consequently, the estimation of these parameters is facilitated by the quantification of the total deterioration that can be attributed to each mechanism. The structure's type, usage, and location determine the fractional attribution of deterioration among various damaging mechanisms. The deterioration of steel structures is primarily influenced by corrosion, while concrete degradation is largely affected by carbonation. Additionally, coastal structures are more susceptible to corrosion, whereas bridge decks are more vulnerable to fatigue.

2.2. Stochastic deterioration model

In time-dependent reliability analysis, the majority of structural deterioration models are empirical. The uncertainties associated with the most common degradation mechanisms may be substantial, as evidenced by experimental data. Studies have shown that the approximated mean degradation function is insufficient for accurately evaluating structural reliability when the coefficient of variation for the time-dependent degradation function $G(t)$ exceeds approximately 4%. Consequently, the uncertainty inherent in the degradation function must be considered [19]. Additionally, the resistances at two temporal points, $R(t_1)$ and $R(t_2)$, do not exhibit statistical independence or dependence when $G(t)$ is characterized as a non-stationary stochastic process. Without repair, the principles of deterioration physics are violated, and the derivative of $R(t)$ cannot be positive. As a result, any model of resistance deterioration must adequately account for the stochastic dependence inherent in the degradation process and the variability associated with structural deterioration. This deterioration modeling method is predicated on the formula of the deterministic deterioration model. Additionally, to incorporate uncertainties in the degradation process and more accurately represent the stochastic nature of structural deterioration, the stochastic deterioration model proposed by Li et al. [36]. may be utilized for $G(t)$, as expressed in the following equation:

$$G(t) = 1 - \sum_{i=1}^{n} \hat{d}(t_i).\varepsilon(t_i) \tag{3}$$

where, $\hat{d}(t_i)$ is the time-dependent mean deterioration during time points t_{i-1} and t_i,

$$\hat{d}(t_i) = q(t)\cdot\Delta(t) \tag{4}$$

$\varepsilon(t_i)$ constitutes a sequence of independent random variables that adhere to Gamma distributions, characterized by a mean value of one:

$$\varepsilon(t_i) \approx Gamma\left(\frac{\hat{d}(t_i)}{\xi}, \frac{\xi}{\hat{d}(t_i)}\right) \tag{5}$$

ξ is a time-invariant scaling factor. The mean value of $F(t)$, denoted as m_F, can be computed using Equation (2). Here, $F(t)$ represents the deterioration ratio of component resistance, which follows a Gamma distribution with a mean value of $\int_0^t q(t)\,dt$.

$$G(t) = 1 - F(t) \tag{6a}$$

$$F(t) \approx Gamma\left(\frac{\int_0^t q(t)dt}{\xi}, \xi\right) \tag{6b}$$

Furthermore, Equation (6b) remains applicable, with the parameter $\hat{d}(t_i)$ articulated as per Equation (7), thereby guaranteeing that resistance diminishes monotonically with independent increments while correlations retain among resistances at various temporal points.

$$\hat{d}(t_i) = m_F(t_i) - m_F(t_{i-1}) \tag{6}$$

where, $m_F(.)$ is the mean value of deterioration as a function of lifetime, which follows Eq. (2).

It should be noted that the formulation of the stochastic deterioration model is based on the same fundamental degradation expressions presented in Equation (1) for the deterministic model. To avoid unnecessary repetition of identical equations, these relations are not rewritten here.

2.3. Gamma process modeling of deterioration

A recent approach to modeling structural deterioration is the Gamma process. The Gamma process is an efficient mechanism for modeling the gradual and monotonic deterioration of building components. The Gamma process model possesses a stochastic characteristic in deterioration prediction and accounts for the temporal variability in the structural deterioration progression. The Gamma process model is characterized by independent, non-negative increments that follow Gamma distributions, which have a uniform scale parameter and a shape parameter that varies over time. Deterioration, characterized by uncertainty stemming from wear, corrosion, fatigue, and crack propagation, is non-decreasing and can be effectively modeled as a Gamma process [38], providing a suitable framework for random deterioration over time. The Gamma process models uncertainty in lifetime and/or deterioration rate. An appropriate model for the degradation of structural resistance is the Gamma process, which is distinguished by its independent non-negative increment properties [1, 39]. Various researchers have applied Gamma processes to data: Cinlar et al. [40] on concrete creep, Lawless and Crowder [41] on fatigue crack propagation, Kallen and Noortwijk [42] on corrosion-induced thinning, and Frangopol et al. [43] on corroded steel gates. Additionally, Samali et al. [44] employed the Gamma process to model the deterioration of bridges.

In mathematical terms, the continuous-time stochastic Gamma process [1] is defined as a random variable that represents the value of resistance deterioration for all $t \geq 0$. $Gamma(x|v(t), u)$ is the probability density function of the Gamma distribution with parameters $v(t)$ and u. Its probability density function is denoted as $f_{X(t)}(x)$. The shape parameter $v(t)$ is a non-decreasing, right-continuous, real-valued function of time. In this stochastic process, the initial deterioration value is set to zero to ensure a monotonic decline over time. The increase in deterioration from time t_1 to t_2 may be independent of the cumulative deterioration at time t_1 and is a nonnegative value. The Gamma distribution is characterized by a constant scale parameter and a time-dependent shape parameter, and the increment $X(t+\Delta t)$ - $X(t)$ follows this distribution, specifically $Gamma(x|v(t+\Delta t) - v(t), u)$. This distribution is applied to all increments over the time interval. The mean, variance, and coefficient of variation of the deterioration $X(t)$ are presented below:

$$E\left(X\left(t\right)\right)=\frac{v\left(t\right)}{u},\ \ \mathrm{var}\left(X\left(t\right)\right)=\frac{\sqrt{v\left(t\right)}}{u},\ \ \text{and}\ \ \nu_X\left(t\right)=\frac{1}{\sqrt{v\left(t\right)}} \tag{7}$$

The expectation is governed by a power law [15].

$$E\left(X\left(t\right)\right)=\frac{v\left(t\right)}{u}=\frac{\alpha t^{\beta}}{u} \tag{8}$$

Experimental studies can validate α and β, while the shape parameter $v(t)$ is a function of time in this context. The parameters α and β represent the rate of deterioration and the predominant deterioration mechanism, respectively.

Conversely, the structural resistance degrades over time as described by the equation $R_t = R_0\ (1 - \alpha\ t^{\beta})$, where R_t represents the resistance at time t, and α and β denote the deterioration rate and the predominant deterioration mechanism, respectively [9]. The shape parameter in the Gamma distribution within the deterioration model signifies the form of the structural resistance degradation and deterioration process. Deterioration models characterized by β = 1 (linear) for reinforcement corrosion, β = 2 (parabolic) for sulfate attack, and β = 0.5 (square root) for diffusion-controlled aging [9]. A noteworthy exception to the Gamma process is represented by the probability density function of $X(t)$ when β = 1.

$$f_{X(t)}(x)=Gamma\left(x\,|\,\alpha t,u\right) \tag{9}$$

3. Time-dependent reliability analysis using dynamic Bayesian network

A Bayesian network is a probabilistic model that integrates Bayesian principles with graph theory, represented as a directed acyclic graph [45]. It comprises a network architecture featuring nodes, edges, and conditional probability tables (CPTs). These networks offer a concise graphical representation of existing parameters and their interrelationships. The nodes of the network signify random parameters, while the edges denote the relationships among them. A CPT is generated for each node, utilized in statistical inference computations. The output of a Bayesian network is the joint probability mass function of the network's random variables.

DBNs are a distinct category of Bayesian networks that depict stochastic processes. They comprise a series of slices, each representing a basic Bayesian network at time t, containing one or more Bayesian network nodes [46]. The quantization of these links is represented by $P(X_i,0)$ for nodes in the initial slice and $P(X_i,t+1|\ X_i,t)$ for subsequent nodes, and the slices are interconnected by directed links from nodes in slice t to nodes in slice t+1. DBNs model systems that undergo dynamic changes or evolve over time. They allow users to manage the system and forecast future variable values (prediction). They also enable evaluation of unobserved variables from the past (smoothing), and assessment of unobserved variables at the present moment (filtering). In this context, the posterior distribution of the state at time t is computed by utilizing all available evidence up to that point. DBN can be regarded as a generalization of Markov process models, which have frequently been employed to simulate deterioration [23, 47, 48]. The model is further enhanced to accommodate broader scenarios involving resistance deterioration [23]. The DBN model comprises three components (Figure 2). One of the primary advantages of the defined model is that, when information is accessible, it can enhance the reliability updating process.

The initial component, the reliability model, comprises two elements: time-invariant model parameters, which remain constant, and time-variant model parameters, which change over time steps $t = 0$, It comprises the structural state E_t at time t, random variables $(x_1,x_2,\ldots,x_t)$, and structural resistance R_t, defined by the following mathematical expressions:

$$P\left(E_t=0\right)=\int_{\Omega_E} f_X\left(X\right) f_{R_t}\left(r_t\right) dX\, dr_t \tag{10}$$

where $\Omega_E = \{LS(X, r_t) \leq 0\}$and $LS\ (X,r_t)$ is the limit state function.

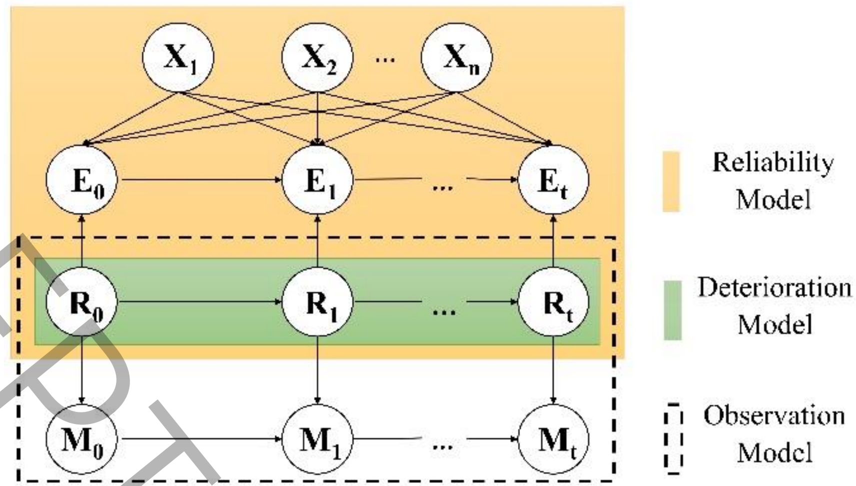

Fig. 2. Structural time-dependent reliability analysis using the general DBN model

The deterioration model at time step t and the preceding time step $t-1$ are defined in the second part. This section defines various deterioration models, specifically deterministic, stochastic, and Gamma process deterioration functions. Deterioration is represented as $\{R_0,\ldots,R_t,R_{t+1}\}$, where R_{t+1} is solely dependent on R_t. The change Δx between R_t and R_{t+1} adheres to deterioration models. The mathematical parameters of the model are provided by:

$$f\left(R_{t+1}\middle|R_0,R_1,\ldots,R_t\right)=f\left(R_{t+1}\middle|R_t\right) \tag{11}$$

where

$$R_{t+1}=R_t-\Delta x \tag{13a}$$

and

$$\begin{aligned} &\Delta x\sim Gamma\left(x\,|\,v(t+1)-v(t),u\right) && \text{for Gamma process model}\\ &\Delta x\sim R_0\times m_F && \text{for Deterministic approach}\\ &\Delta x\sim R_0\times F(t) && \text{for Stochastic approach} \end{aligned} \tag{13b}$$

In the final section, the observation model at any given time step can include data on the condition of a model parameter or its deterioration, which can be obtained from inspections, monitoring systems, environmental parameter recordings, or other pertinent measurements related to the model parameters. Indirect and error-prone, resistance measurements can be expressed as:

$$M_t=R_t+\varepsilon \qquad \varepsilon\sim \mathrm{N}\left(\mu,\sigma^2\right) \tag{14}$$

M_t represents the measurement of R_t, with the error corresponding to a normal distribution.

The measurement uncertainty is modeled using a Gaussian distribution, which is a widely accepted assumption in structural reliability and health monitoring applications. This probabilistic representation accounts for potential deviations between the true structural state and the measured values due to factors such as sensor noise, environmental variability, and modeling simplifications. The inclusion of measurement uncertainty plays a significant role in the updating process of the DBN. During Bayesian inference, the posterior probability of the structural state is obtained by combining the prior probability (predicted from the deterioration model) with the likelihood function derived from observations. As a result, higher measurement uncertainty leads to a broader posterior distribution and reduced confidence in the updated reliability, whereas lower measurement uncertainty results in a sharper posterior distribution and higher reliability accuracy.

To enhance the clarity of the proposed framework, the overall computational procedure adopted in this study is summarized in Figure 3. The flowchart integrates all methodological components, including the formulation of deterioration models, evaluation of time-variant structural resistance, discretization of state variables, construction of the DBN, sequential posterior updating using observational evidence, and the statistical validation of DBN-based reliability estimates through Monte Carlo simulation. This schematic representation provides a unified view of how deterministic, probabilistic, and Gamma process deterioration models are embedded within the DBN framework and subsequently compared under consistent conditions.

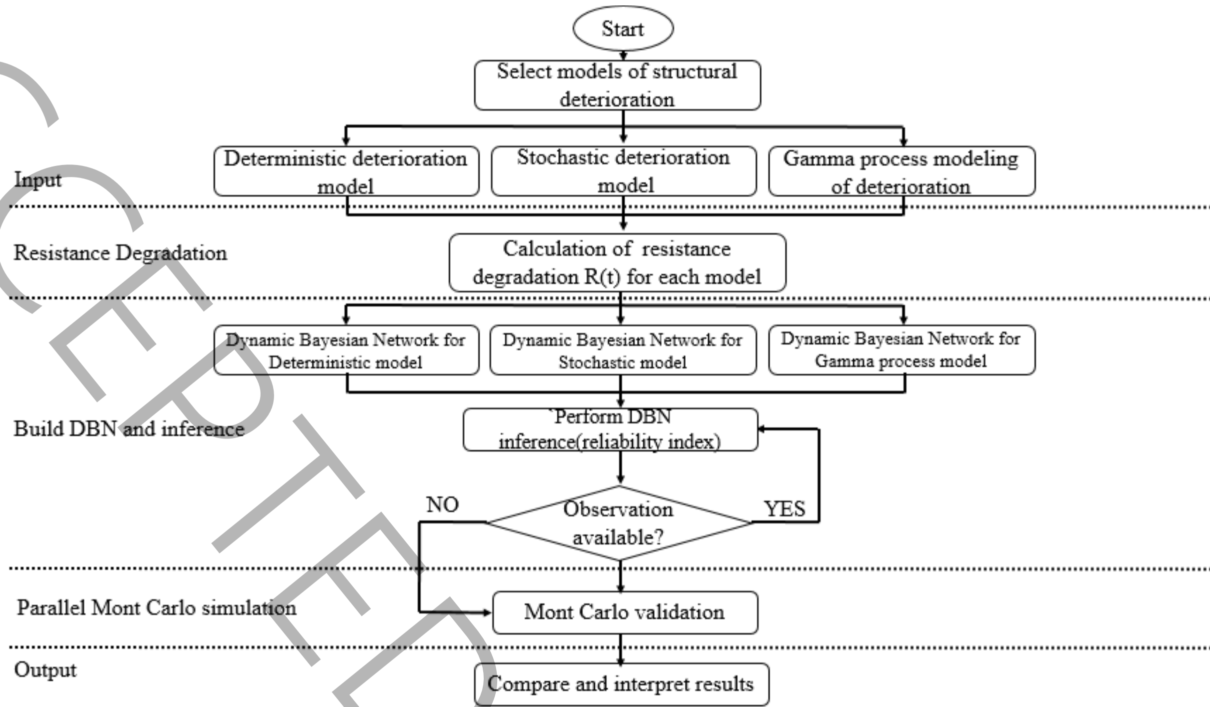

Fig. 3. Comprehensive flowchart of the proposed methodology, illustrating deterioration modeling, build DBN and inference, posterior updating, and Monte Carlo–based validation

4. The Impact of different deterioration processes

Figure 4 illustrates the portal frame structure [49], which is subject to concentrated forces in both the horizontal and vertical directions. The frame structure comprises five nodes and four elements. This figure illustrates the material properties of the elements.

The corrosion of steel reinforcement and the creep of concrete result in a decrease in the plastic moment capacities (M_1, M_2, and M_3). In an effort to avoid complexity, it is assumed that only M_3 is deteriorating, while the applied loads (P_1 and P_2) and moment capacities of M_1 and M_2 are considered statistically independent random variables. The parameters of the probabilistic distributions are delineated in Table 1. Deterioration processes are represented using deterministic, stochastic, and Gamma process deterioration functions as outlined in Section 2. In the Gamma process, the shape parameter $v(t)$ is defined as $\alpha.t$, while the scale parameter u is chosen. The parameter values are presented in Table 1.

The limit state is exclusively defined for moment capacity in this example. A component failure event occurs when the bending moment at a hinge attains its moment capacity. Given that a hinge may develop on either side of an element, a total of eight hinge locations are identified, labeled 1 through 8 in Figure 4. Prior research has utilized a plastic mechanism to determine the failure modes of these frame structures [50, 51].

Table 1. Statistical parameters and distribution types of random variables.

Random variables	Distribution	Mean	Coefficient of variation	References
P1	Gumbel	20 kN	0.30	[52]
P2	Gamma	40 kN	0.30	[52]
M1	Lognormal	75 kN.m	0.05	[52]
M2	Lognormal	75 kN.m	0.05	[52]
M3	Lognormal	101 kN.m	0.05	[52]
α	Deterministic	0.5	-	[53]
u	Deterministic	0.5	-	[53]
ξ	Deterministic	0.01	-	[37]
ε (kN.m)	Normal	0	0.15	[53]

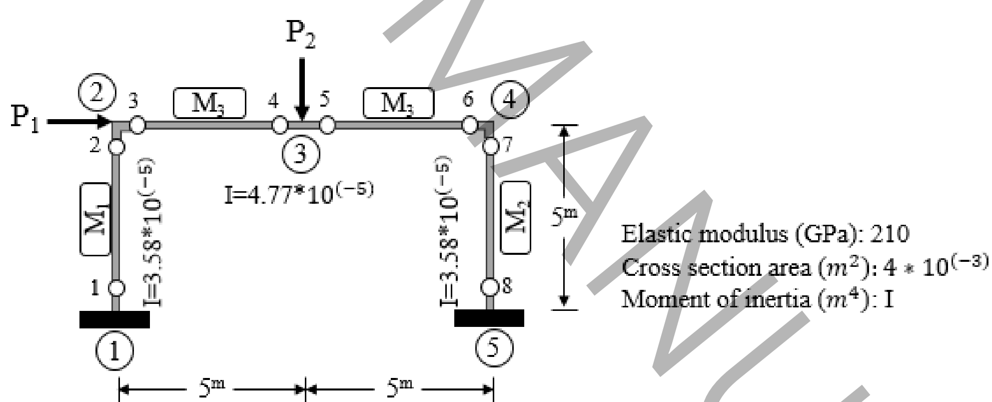

Fig. 4. A portal frame structure with 5 nodes and four members subjected to a horizontal and vertical concentrated forces

The plastic mechanisms method is applicable to simple structures, whereas identifying all failure modes in larger structures poses challenges. A systematic approach appropriate for such building frames is the Branch and Bound method [52]. This study identifies two predominant modes: the beam failure mode and the lateral failure mode of the frame structure, as illustrated in Figure 5. These two principal failure mechanisms have been employed in modeling the failure of elements within a DBN. Subsequently, the relevant limit state functions can be derived directly through plastic analysis or the virtual work theorem.

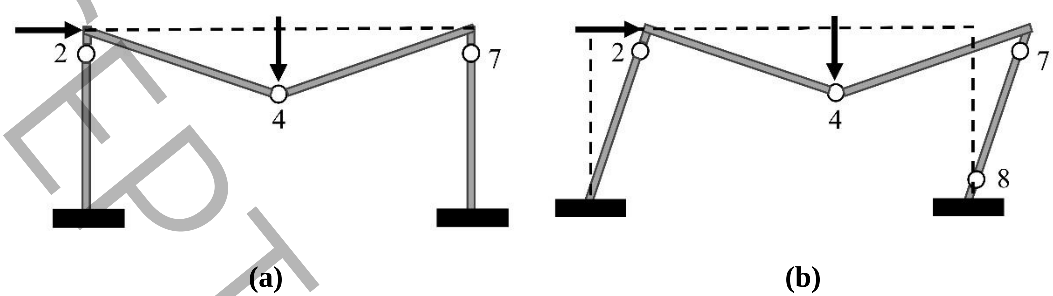

Fig. 5. Failure mechanisms of the five dominant failure modes identified by branch and bound method: (a) beam mode and (b) combined mode.

The DBN formulated for structural time-variant reliability in the context of deterioration is illustrated in Figure 6. Equivalent variables defined in finite domains are used to replace all random variables defined in continuous domains. Discretization is extensively employed for managing continuous random variables in Bayesian networks, as it represents the most straightforward method for exact inference [54]. Through the partition of its domain Ω_X into a finite collection of intervals $w_i = 1,...,m$, a discretization of the continuous variable X is obtained. This discretization occurs sequentially, with one variable addressed at a time and hierarchically within the Bayesian network, whereby parent variables are discretized prior to their children. Consequently, DBN comprises solely discrete variables. The network structure can be modeled with nodes, edges, CPTs, and connectivity via direct links between slices to determine the frame's reliability over time. The degradation process for R_3 is modeled using the three previously discussed methods. To make inferences in DBNs, three distinct types of CPTs need to be computed. The CPTs calculations are based on the mathematical parameters of the three models.

$$P\left(R_{t+1}=r_k \middle| R_t\right)=F_{R_k}\left(r_k^{+}\middle|R_{t+1}\right)-F_{R_k}\left(r_k^{-}\middle|R_{t+1}\right) \tag{15}$$

$$P\left(E_t=0\middle|R_t=r_k\right)=\int_{LS\leq 0} f_X\left(X\right)f_{R_t}\left(r_k\right)dX\,d r_k \tag{16}$$

$$P\left(M_t=m_k \middle| R_t\right)=F_{M_k}\left(m_k^{+}\middle|R_t\right)-F_{M_k}\left(m_k^{-}\middle|R_t\right) \tag{17}$$

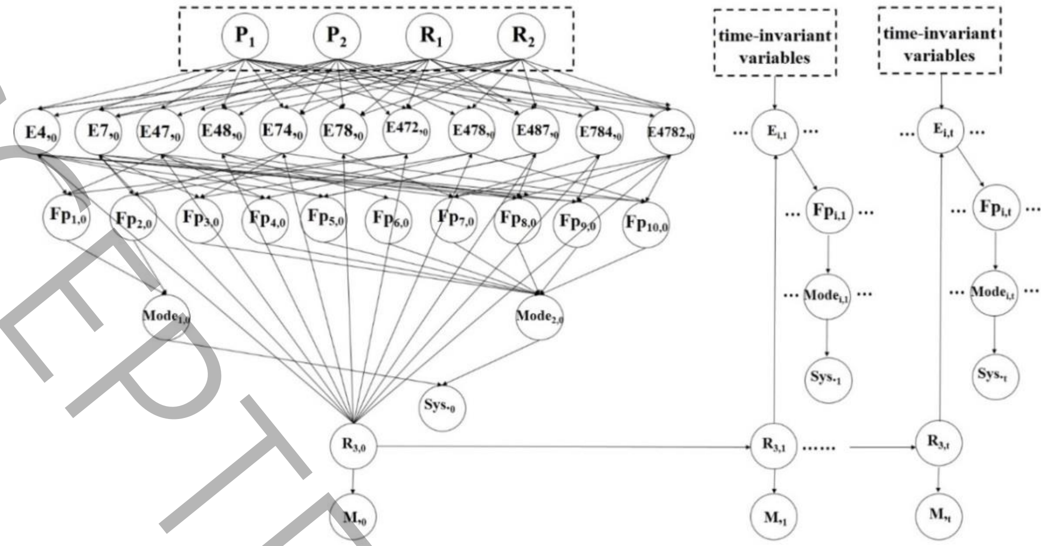

Fig. 6. Dynamic Bayesian network for time-dependent reliability.

where r_k^- and r_k^+ denote the lower and upper limits of the interval w_k and $f_{Rt}(r_k)$ represents the probability density function of uniform distribution on the interval $\{LS \le 0 | r_k^- < r_k < r_k^+\}$. Equation (16) can be evaluated using structural reliability techniques, including the first-order and second-order reliability methods.

This study proposes that, to estimate the parameters *a*, *b*, and *c* for the deterministic and stochastic deterioration models, the remaining resistance at the conclusion of the 40th year is 80% of the initial resistance, indicating 20% deterioration. The mechanisms of carbonation, corrosion, and fatigue contribute to the deterioration of the structure at rates of 10%, 45%, and 45%, respectively. These assumptions are more accurate for steel structures [37]. This assumption aligns with the findings of Li et al. [36], allowing us to compare our findings with those of the prior study. This research endeavors to evaluate the probability of failure for deterministic, stochastic, and Gamma process deterioration functions, recognizing that the reduction in structural strength will vary depending on the type of structure and its location. In deterministic deterioration, it is posited that $g(40) = 1 - a.40 - b\sqrt{40} - exp(\frac{c}{40})$. The three unknown parameters can thus be determined by equating the total decrease fraction to the individual terms. The values of *a*, *b*, and *c* are 0.09/40, 0.02/$\sqrt{40}$, and ln(0.09)×40, respectively.

Figure 7 illustrates the outcomes of a time-dependent reliability analysis employing a deterministic deterioration model through the DBN model and Monte-Carlo simulation method. The results of the DBN exhibit a maximum discrepancy of 3% compared to the Monte-Carlo results, with standard deviation(STDV) and root mean-square deviation (RMSD) values of 0.0211 and 0.023, respectively, thereby confirming the accuracy of the proposed DBN model. The stochastic deterioration model was computed using the stochastic deterioration function, incorporating the uncertainty of the deterioration process, with the deterioration ratio $F(t)$ characterized by a Gamma process.

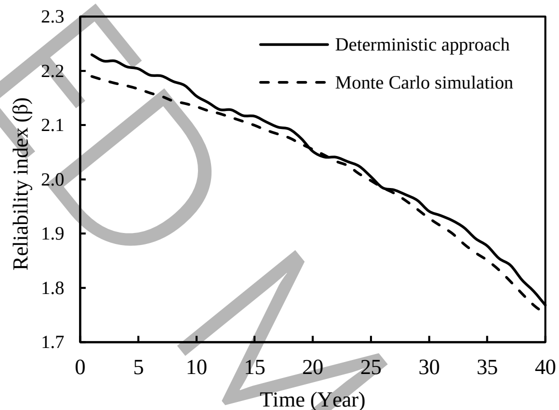

Fig. 7. Validation of dynamic Bayesian network results and Monte-Carlo simulation based on deterministic deterioration model

Figure 8 illustrates various time-dependent resistance functions derived from the combined and individual effects of degradation functions obtained through three probabilistic resistance degradation models: linear, square root, and exponential. These functions facilitate calculations of time-dependent reliability and joint probability. To investigate the influence of the degradation function on a stochastic model, the values of $G(t_i)$ are determined from independent Gamma distribution functions. The sample functions presented in Figure 8 illustrate the values of structural degradation for the statistically independent process. Figure 8 illustrates that the assumption of stochastic independence allows for strength reversals without any external influences.

In the stochastic method for determining failure probability in resistance degradation models over 40 years, the values of $G(t_i)$ are represented using various Gamma distribution functions, whereas in the deterministic method, the degradation function corresponds to the mean values of these different Gamma distribution functions.

Figure 9 illustrates the significance of quantitatively assessing the parameters that characterize the time-dependent increase in the deterioration of structural resistance due to aging, evaluated both in combination and separately, utilizing linear, square root, and exponential models within the deterministic and stochastic deterioration functions, respectively. Figure 9(a) presents results indicating the probability of failure under a deterministic approach, wherein uncertainties are disregarded; consequently, the value of $G(t_i)$ is numerically equivalent to the mean values of various Gamma distribution functions. The results of the stochastic deterioration function utilize samples of the deterioration function to compute the reliability index and, consequently, the probability of failure, as illustrated in Figure 9(b). This calculation incorporates probabilities and uncertainties derived from the Gamma distribution functions presented in Figure 8 for $G(t_i)$.

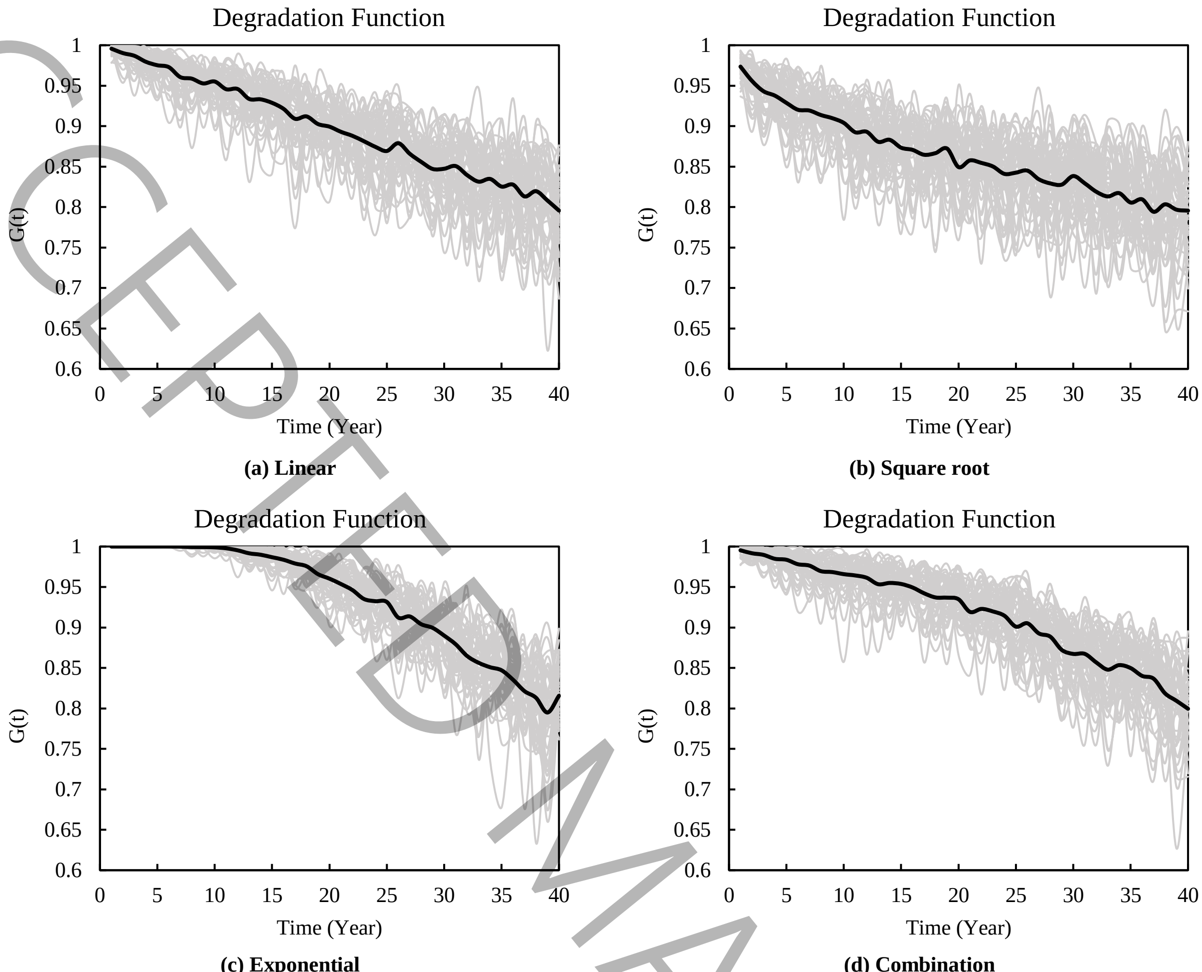

Fig. 8. Samples of degradation function for different mechanisms obtained by the stochastic deterioration function.

For deterministic and stochastic deterioration functions, the square root model's failure probability is at its highest for up to 40 years, while the exponential model's failure probability rises quickly. Ellingwood and Mori [9] and Li et al. [36] have also reported these observations. The configuration of the deterioration functions exerts a more significant impact as *G(40)* diminishes. The failure probability is highly sensitive to minor variations in strength when the residual strength at time t is low; this sensitivity compounds over time.

In the Gamma process, the linear shape parameter is defined as $v(t)=\alpha.t$, and the scale parameter is denoted as u. The parameter values are presented in Table 1. Figure 10 illustrates the comparison of results from the time-dependent reliability analysis using the Gamma process, alongside deterministic and stochastic deterioration models derived from DBN, with respect to the reliability index over a 40-year period.

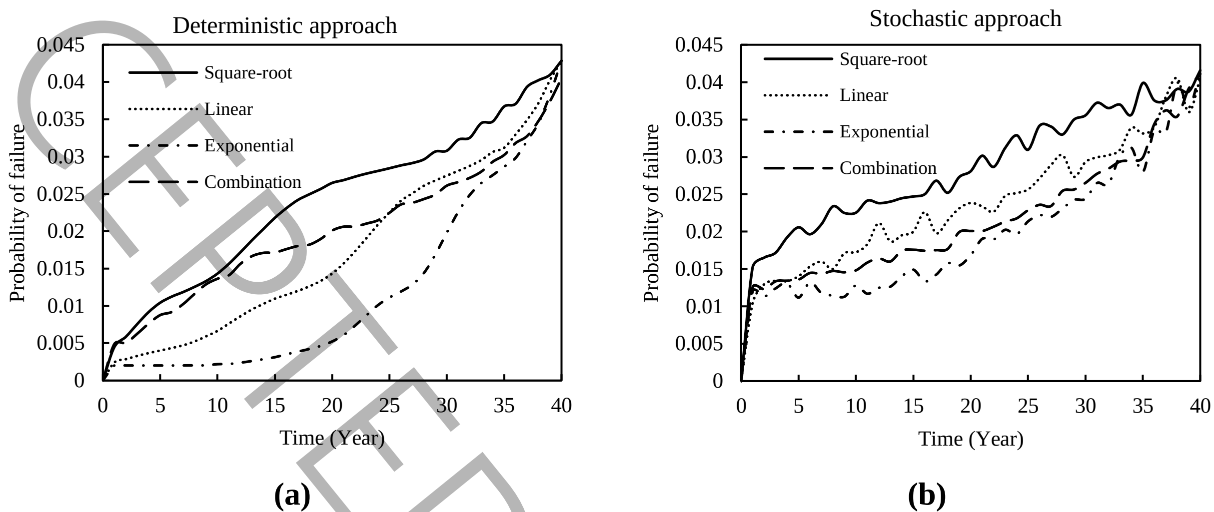

Fig. 9. Dependence of cumulative failure probability on combination and separate effect of three resistance degradation models (Linear, Exponential, and Square-root): (a) Deterministic degradation function and (b) Probabilistic degradation function.

The results derived from the DBN are in an unobserved state, as indicated in Section 3. Utilizing DBNs allows for the derivation of the posterior distribution of a collection of random variables based on a set of observations. In this article, the filtering model is employed to account for observations, representing the primary advantage of this tool. Figure 10's first line displays the results obtained by DBN regarding the reliability index over a 40-year period, excluding evidence. The resistance measurements of R_3 over 40 years are based on a deterministic deterioration function. Upon evaluating the evidence and adhering to the precise inference for dynamic Bayesian networks, the results depicted in Figure 10 and Table 2 indicate that the structural posterior reliability indices of filtering based on the Gamma process are inferior to those obtained without evidence, due to the selected Gamma process's overestimation of actual resistance deterioration, particularly after the seventh year.

Table2. Reliability index of the frame with evidence (Gamma process), without evidence (Gamma process), and deterministic and stochastic degradation models for different years

Models	**Years**						
	1	10	20	30	40	STDV	RMSD
Stochastic approach	2.24	2.05	1.93	1.74	2.00	0.0563	0.0591
Deterministic approach	2.25	2.11	2.05	1.99	2.08	0.0455	0.0477
Gamma approach without evidence	2.22	2.05	1.94	1.77	2.00	0.0329	0.0641
Gamma approach with evidence	2.26	2.05	1.97	1.90	2.01	-	-

The findings of deterministic and stochastic deterioration models indicate that these approaches align more closely with evidence-based filtering, and until the 30th year, their reliability indices are nearly identical; thereafter, both stochastic and deterministic models exhibit a decline in value.

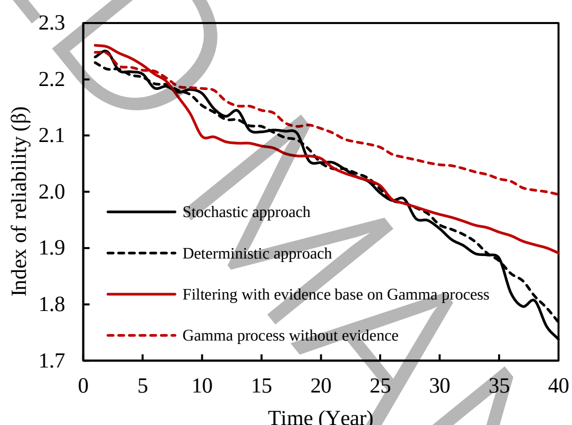

Fig. 10. Analysis of time-dependent reliability with evidence (Gamma process), without evidence (Gamma process), and deterministic and stochastic degradation models.

5. Conclusion

This study addressed the problem of time-dependent reliability assessment in the structure with resistance deterioration, where material degradation and load uncertainties reduce structural safety over time. The importance of this topic lies in its direct impact on lifetime performance, maintenance planning, and cost efficiency of infrastructure systems. The DBN approach was employed to model and infer the reliability evolution of a deteriorating structure, offering a probabilistic framework that integrates uncertainty and evidence updating. The study considered three deterioration models: deterministic and stochastic degradation models, as well as the Gamma process, with loads being treated as random variables. Their impact on the reliability index of a one-span frame with resistance deterioration was compared over a 40-year service life with the aid of DBN modeling in MATLAB software.

The adopted methodology provided a systematic and flexible framework capable of incorporating inspection data and updating structural reliability dynamically. The results of the frame structure reliability index show the importance of examining deterioration models and that the correct selection of deterioration models is vital in the design of structures and is very important from an economic point of view and that deterioration over time significantly reduces the reliability index of structures.The use of DBN allows for the explicit modeling of dependencies among parameters and enables posterior reliability estimation when new evidence becomes available. While the MCS was used as a reference for validation, the DBN demonstrated high computational and time-efficiency and accuracy, with only minor deviations.

The proposed DBN framework effectively captures time-varying deterioration mechanisms and provides accurate reliability predictions under different degradation models. The deterministic and stochastic models showed closer alignment with the evidence-based filtering approach compared to the Gamma process without evidence, especially during early service life. The Gamma process tended to overestimate deterioration after the seventh year, leading to lower posterior reliability indices. Quantitative comparison of deterioration functions demonstrated that the square-root model results in the highest failure probability, followed by the exponential and linear models. The DBN approach achieved less than 3% deviation from the MCS results, confirming its validity for structural reliability analysis. From an economic and maintenance perspective, the findings emphasize the importance of selecting suitable deterioration models for planning inspection intervals and maintenance strategies, ensuring cost-effective service life management. The study’s novelty lies in integrating deterministic, stochastic, and Gamma deterioration processes into a unified DBN framework that can incorporate evidence updating, providing a realistic and adaptive reliability assessment tool.

While the current study focused on the general deterioration behavior of structural systems, it is important to note that the same modeling framework can be directly extended to real-world structures such as bridges exposed to corrosive environments by appropriately calibrating the deterioration parameters. This extension will be considered in future research to evaluate the influence of corrosion on reliability and maintenance planning. Also integrate sensor-based inspection data for real-time updating. Additionally, linking the DBN model with optimization-based maintenance scheduling can provide more direct economic decision support.

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