Appendix

Nominal test motor specifications as provided by the manufacturer:

Armature Resistance (ohm) ± 12%	1.10
Max. Power Output (watt)	8.18
No Load Speed (RPM) ± 12%	5500
No Load Current (mA) ± 50%	210
Friction Torque (No Load Speed) (Oz-in)	0.30
Stall Torque (Oz - in)	7.44
Velocity constant (RPM/Volt)	953
Back EMF Constant (mV/RPM)	1.049
Torque Constant (Oz-in/Amp)	1.42
Armature Inductance (mH)	0.10
Mechanical Time Constant (mS)	30
Armature Inertia (x10-4 Oz- in - Sec ^{2})	3.32
Max. Efficiency (%)	64

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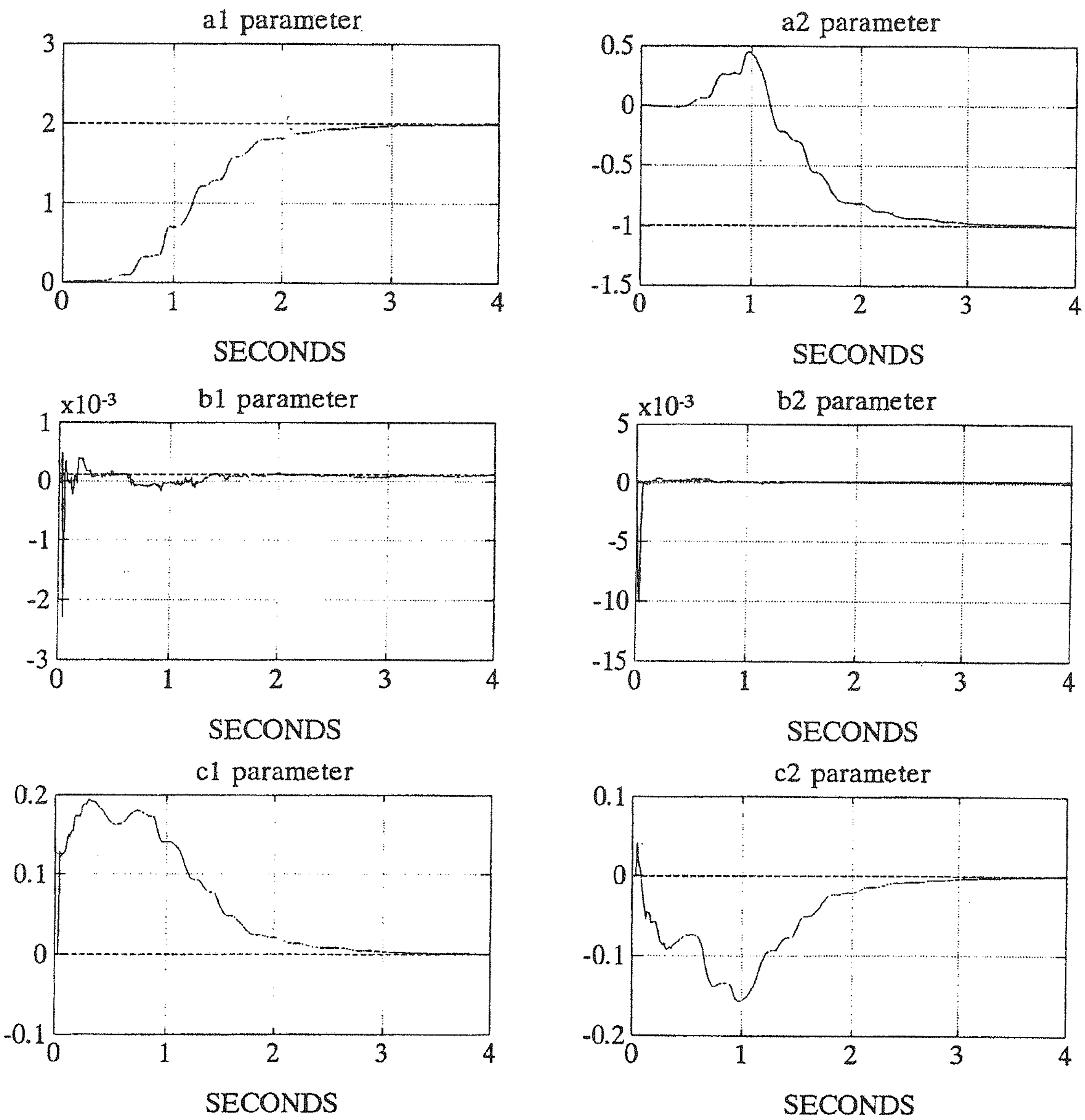

Fig (4) Convergence of the unknown parameter estimation

devices. The overall system was linearized by compensating the second order dynamic which contained all of the nonlinearities. Thus, the design of a linear controller to achieve the performance requirements became a relatively simple task. The experimental results presented in this paper, confirmed the effectiveness of the self-tuning controller in dealing with tracking problems. The proposed controller had two important advantages over the other algorithms:

- It was less sensitive to environmental noises.
- Its response was more rapid, therefore, this algorithm could be used in tracking problems with high frequency command signals.

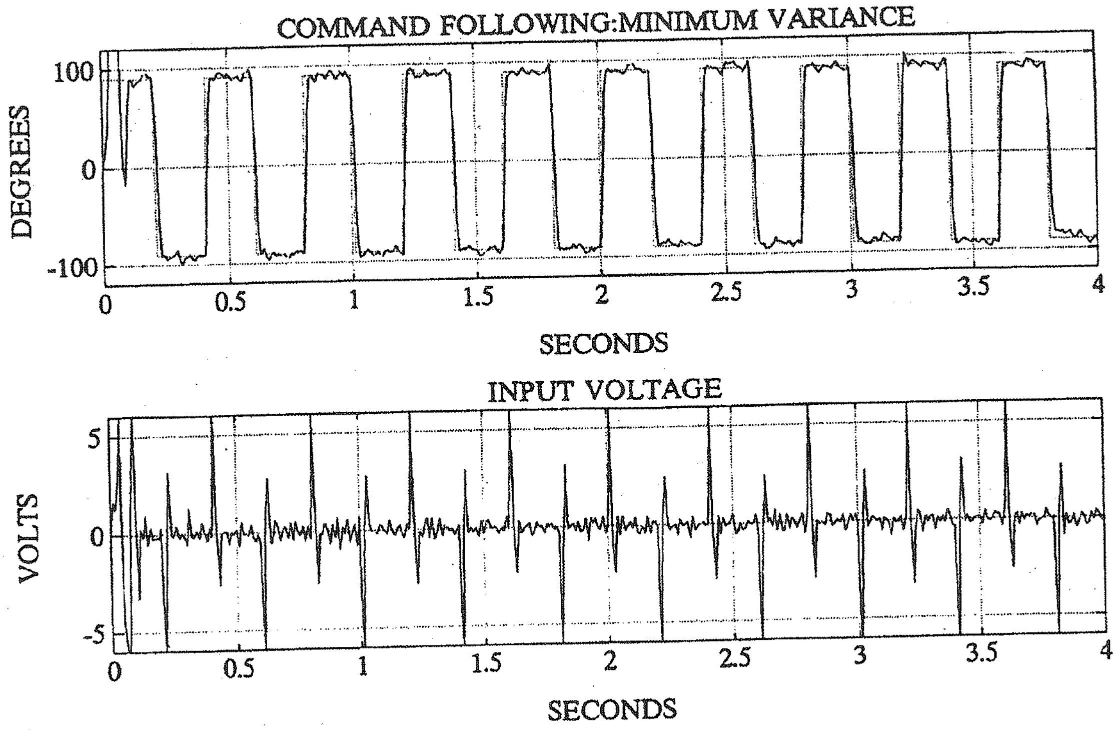

Fig (2) System tracking performance and control input for minimum variance criterion

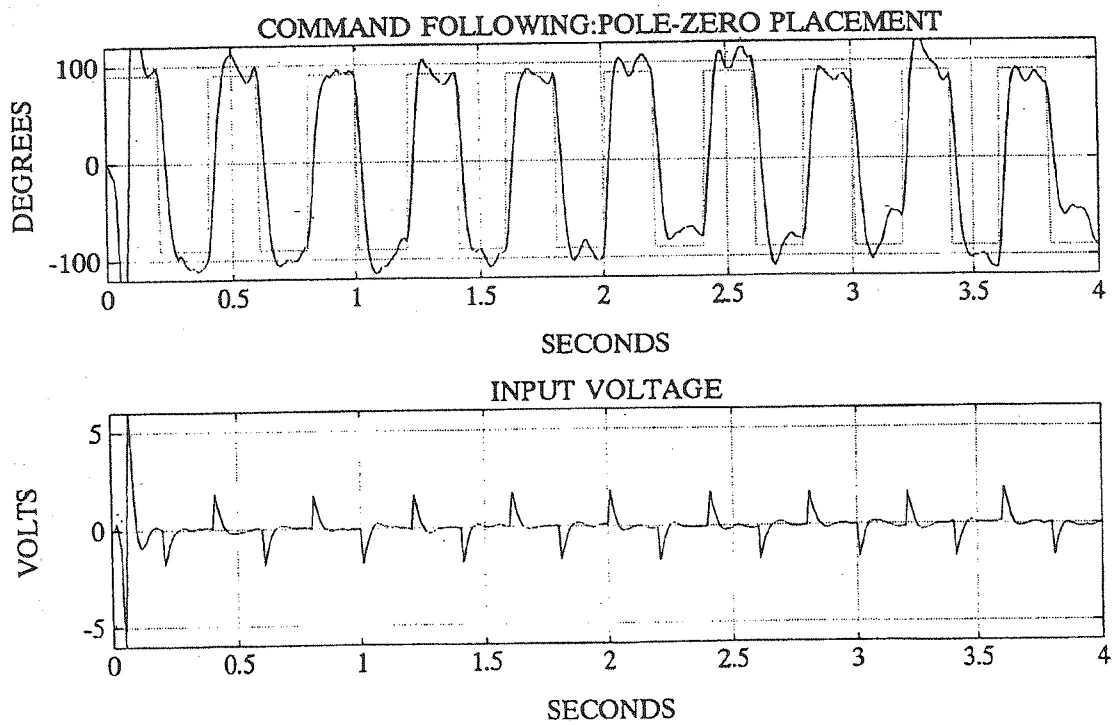

Fig (3) System tracking performance and control input for pole - placement criterion

ments. For comparison, zero and pole placement criterion as introduced in [9], is combined with the nonlinear compensation algorithm of this paper and then, the same signal is applied to the system. The results are shown in Figs. 3. a and 3.b. As it can be seen from the figures, the response of the system with the proposed controller is superior to the response of the system with previously introduced controllers existing in the literature of control system. Fig. 4 shows the convergence of the unknown parameters estimation.

7. Conclusion

In this paper, the third order model of a permanent magnet dc motor was derived and the self-tuning controller for the trajectory follow-up of the system with nonlinear load was proposed. The control system was successfully implemented on a digital computer using a discretized model with a discrete parameter estimation algorithm. The system was also tested on an experimental set-up consisting of a six-volt dc motor, carrying a point mass at the end of a bar which was vertically connected to the motor shaft.

The control scheme presented here, illustrates a practical method for applying this type of adaptive controller to industrial robotic manipulators and other positioning

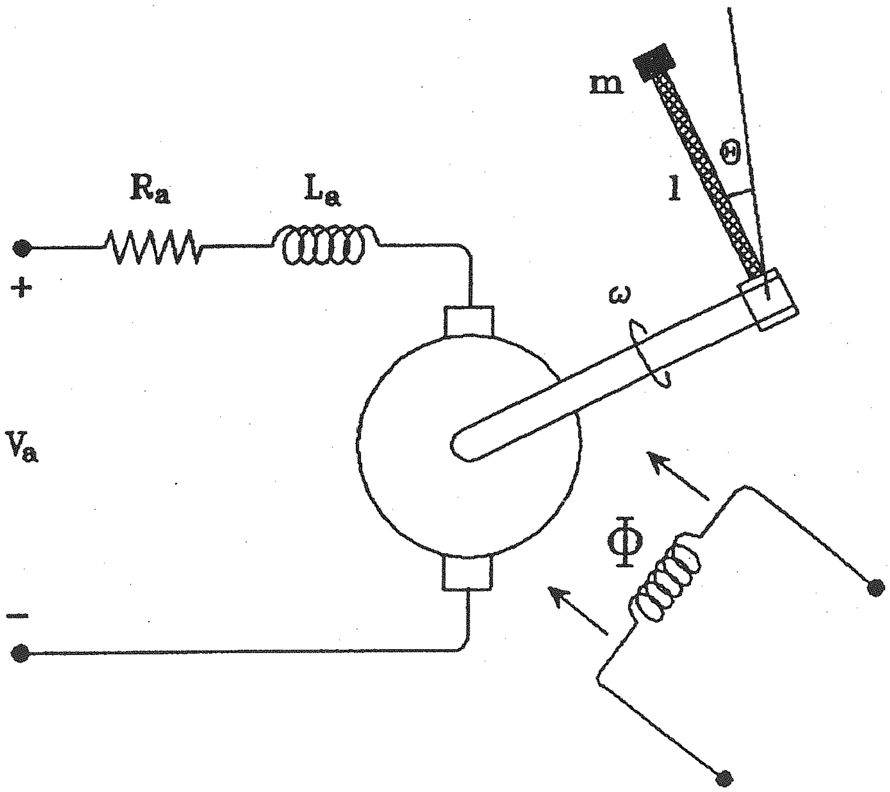

Fig (1) Physical system of a DC motor with nonlinear load

ed along with other system parameters through equations (29. a) and (29. b).

Now, considering the linearized dynamics of a dc motor, a self-tuning controller with minimum variance criterion is designed. Suppose that the input-output relation of the discrete system is written as:

$$y(t)+b_1y(t-1)+b_2y(t-2)=a_1u(t-1)+a_2u(t-2)+e(t)+c_1e(t-1)+c_2e(t-2) \quad (34)$$

where y(t), u(t) and e(t) are system output, system input and a white noise with unity variance, respectively. Defining the shift operator q, this relation can be written in the folowing form:

$$B(q)\,y(t) = A(q)\,u(t) + C(q)\,e(t) \quad (35)$$

A general controller for the system of equation (35) has been proposed by [9] as:

$$R(q)\,u(t) + S(q)\,y(t) = T(q)\,u_c(t) \quad (36)$$

Here, the aim is to find the polynomials R (q), S(q) and T(q) such that the closed loop system stays stable and output y(t) follows the command signal $u_c(t)$ with the minimum variance. In addition, since the system has an underdamped zero, the proposed algorithm should operate in such a way that none of the zero of the system is cancelled.

Considering the controller (36), the output of the closed loop system can be expressed as:

$$y = \frac{AT}{BR + AS}\,u_c + \frac{CR}{BR + AS}\,e \quad (37)$$

Therefore, the problem of designing the controller reduces to solving the following algebraic equations:

$$BR + AS = qC \quad (38)$$

where, the polynomials R(q) and S(q) are chosen as:

$$R(q) = q + r_0 \quad (39.\ a)$$

$$S(q) = s_1 q + s_2 \quad (39.\ b)$$

The polynomial T(q) should be chosen such that the steady-state error between the command input and the system output converges to zero. In other words, the coefficient of u_c in equation (37) converges to unity for the steady-state case. It can simply be demonstrated that, the design requirements are satisfied by choosing the following polynomial for T(q):

$$T(q) = t_1q^2 + t_2q + t_3 = \frac{C(q)}{a_1 + a_2} \quad (40)$$

6. Experimental Results

The proposed controller is applied to a six-volt 3540 MicroMo series DC motor [Appendix] carrying a load which is vertical to the shaft of the motor. The command signal is chosen to be a periodic pulse which changes between ± 90°. Note that the motor setup is in horizontal configuration so that, the system is affected by the gravitational force. The system output and the control input for the suggested controller are depicted in Figs. 2. a and 2. b. It is clear that, the output of the system follows the command signal after an initial transient process. This transient state occurs on the time that the nonlinear part of the system dynamics have not been compensated yet. In addition, due to the stochastic nature of the controller, it is also able to maintain the system performance in more noisy environ-

$$a_2 = -e^{\alpha\Delta},\ b_2 = \frac{\beta}{\alpha}\left(\Delta e^{\alpha\Delta} - \frac{e^{\alpha\Delta} - 1}{\alpha}\right),\ c_2 = -\frac{\gamma}{\alpha}\left(\Delta e^{\alpha\Delta} - \frac{e^{\alpha\Delta} - 1}{\alpha}\right) ml \tag{26. b}$$

Equation (25) expresses a special type of nonlinear model which is linear to its unknown parameters. Therefore, it could be written in matrix form as:

$$\theta_{k+2} = (F_{k+1})^T q_{k+1} \tag{27}$$

where,

$$F_{k+1} = [\theta_{k+1} \quad \theta_k \quad Va_{k+1} \quad Va_k \quad gSin\,\theta_{k+1} \quad gSin\,\theta_k])^T \tag{28. a}$$

$$q_{k+1} = [a_1 \quad a_2 \quad b_1 \quad b_2 \quad c_1 \quad c_2])^T \tag{28. b}$$

The elements of information vector F_{k+1} are nonlinear functions of the output angular position; but since the equation is linear with respect to its parameters, linear system identification methods can be used. In this paper, least-squares estimator is considered for system identification. Recursive least-squares algorithm for estimation of the parameter vector q from the system input-output data can be expressed as [6]:

$$\hat{q}_{k+1} = \hat{q}_k + \frac{P_k F_k}{(F_k)^T P_k F_k + 1} [\theta_{k+1} - (F_{k+1})^T \hat{q}_k] \tag{29.a}$$

$$P_{k+1} = P_k - \frac{(P_k F_k)(P_k F_k)^T}{(F_k)^T P_k F_k + 1} \tag{29. b}$$

5. Adaptive Controller Design Using Feedback Linearization

Feedback linearization methods for a broad class of electromechanical systems have been addressed in many papers [7, 8]. In general, the solution of the feedback linearization problem is quite difficult and needs nonlinear transformation of the state space bases. Here, the nonlinear system under consideration is a special case in which, the system dynamic is a combination of a linear part and a nonlinear part that can be separated as follows:

$$\dot{x}(t) = ax(t) + bu(t) + f(x, h, t) \tag{30}$$

where f (x, h, t) is the nonlinear part of the system dynamic and h defines the unknown parameter vector. In addition, if it is assumed that the nonlinear part of the system dynamic is linear with respect to its parameters, one can write:

$$f(x, h, t) = g(x, t)^T.h \tag{31}$$

where g (x, t) is a vector of nonlinear functions and also independent of h. Now, if the structures of nonlinear functions are known and h is also estimated, then the following nonlinear feedback will linearize the closed loop dynamics:

$$u(t) = r(t) - \frac{1}{b} g(x, t)^T.h \tag{32}$$

Similarly, if the design model of the dc motor and its nonlinear applied torque - equations (12) and (13) - are considered, one can linearize the system dynamics with the following feedback:

$$V_a = V_{ar} - \frac{\gamma}{\beta} ml\, gSin\theta \tag{33}$$

V_{ar} is the reference armature voltage proportional to the desired output of the motor. The coefficient of the term gSinθ is an unknown parameter which should be estimat-

$$X_{k+1} = e^{A\Delta} X_k + \left(\int e^{A\tau}\, \partial\tau\right) B u_k \tag{20}$$

Applying the above relation to equation (14. b) results in:

$$\omega_{k+1} = e^{\alpha\Delta}\, \omega_k + \frac{1}{\alpha}\left(e^{\alpha\Delta} - 1\right) u_k \tag{21}$$

Now, substituting equations (14. a) and (17) in equation (21), the following second order difference equation for θ is obtained:

$$\theta_{k+2} = \left(e^{\alpha\Delta} + 1\right) \theta_{k+1} - e^{\alpha\Delta}\, \theta_k + \frac{\Delta}{\alpha}\left(e^{\alpha\Delta} - 1\right) u_k \tag{22}$$

This model is called a "one - approximation model", since the approximation of the differential is only used once. It is clear that this model offers a higher degree of accuracy than equation (18). This model reduces to the former one by simply using the first two term in the Taylor expansion of $e^{\alpha\Delta}$.

3.3. Third Model

This model which is the first of the two exact models, is based on the direct discretization of equation (12). It results into a set of first order difference equations as in the following:

$$\omega_k = e^{\alpha\Delta}\, \omega_{k-1} + \frac{1}{\alpha}\left(e^{\alpha\Delta} - 1\right) u_{k-1} \tag{23. a}$$

$$\theta_{k+1} = \theta_k + \frac{1}{\alpha}\left(e^{\alpha\Delta} - 1\right) \omega_k - \frac{1}{\alpha}\left(\Delta - \frac{e^{\alpha\Delta} - 1}{\alpha}\right) u_k \tag{23. b}$$

The model expresses the relation between the input and the output of the system by recursive equations. At first, by utilizing the information at instance k - 1, angular velocity at instance k is computed and then it is used in finding angular position at instance k + 1. Here, it is seen that there are two time delays between the input and the output. Since, the computation of the output angle is indirect and it is done through another first order difference equation, the obtained model is an "exact indirect model".

3. 4. Fourth Model

The last discrete model that is introduced for describing the equation set (12), is a second order difference equation which accurately relates the output angle to the input with no approximation. This equation is obtained by simultaneously utilizing the equation set (23) as in the following:

$$\theta_{k+2} = \left(e^{\alpha\Delta} + 1\right)\theta_{k+1} - e^{\alpha\Delta}\theta_k - \frac{1}{\alpha}\left(\Delta - \frac{e^{\alpha\Delta} - 1}{\alpha}\right) u_{k+1} + \frac{1}{\alpha}\left(\Delta\, e^{\alpha\Delta} - \frac{e^{\alpha\Delta} - 1}{\alpha}\right) u_k \tag{24}$$

This model is referred to as an "exact direct model".

4. Estimation of the Discrete Model Parameters

Any of the four obtained discrete models should be transformed to a regression one which is more suitable for parameter estimation. For this purpose, equation (24) is combined with equation (13) to obtain:

$$\theta_{k+2} = a_1\theta_{k+1} + a_2\theta_k + b_1 Va_{k+1} + b_2 Va_k + c_1 g \operatorname{Sin}\theta_{k+1} + c_2 g \operatorname{Sin}\theta_k \tag{25}$$

where,

$$a_1 = e^{\alpha\Delta} + 1,\ b_1 = -\frac{\beta}{\alpha}\left(\Delta - \frac{e^{\alpha\Delta} - 1}{\alpha}\right),\ c_1 = -\frac{\gamma}{\alpha}\left(\Delta - \frac{e^{\alpha\Delta} - 1}{\alpha}\right) ml \tag{26. a}$$

where, $\alpha = -(B + K_a \cdot K_b/R_a) / J$, $\beta = K_a/JR_a$ and $\gamma = -1/J$. The disturbance torque on the motor shaft can be expressed with the following equation:

$$T_d(t) = mgl \, \mathrm{Sin}\, \theta(t) \quad (13)$$

where, g is the gravitational acceleration and the other parameters are as in Fig. 1.

3. Discretization of Nonlinear Model of DC Motors

Most of the physical systems in the world are continuous by nature. Nowadays due to the use of digital computers in system identification, discrete models are used more often. Discrete parameter estimation methods are utilized in discrete models of actual systems. Thus, as a first step the continuous models must be transformed to discrete ones. In this paper, four discrete equivalents are proposed for equations (12. a) and (12. b). Two of these are approximate models and the other two are accurate ones and only differ in the way they are expressed. The main aim of this section is to derive the second order difference equation of the angular position θ. It is clear that the first order difference equation of angular velocity ω, is obtained in the intermediate steps.

3. 1 . First model

By using first order approximation in equations (12. a) and (12. b) one obtains:

$$\frac{\theta_{k+1} - \theta_k}{\Delta} = \omega_k \quad (14.\ a)$$

$$\frac{\omega_{k+1} - \omega_k}{\Delta} = \alpha\omega_k + \beta V_{ak} + \gamma T_{dk} \quad (14.\ b)$$

where, Δ is the sampling period. For simplicity , the effects of external inputs including load torque and armature voltage are collected in one term as:

$$u_k = \beta V_{ak} + \gamma T_{dk} \quad (15)$$

Now, solve equation (14. b) for ω_{k+1}:

$$\omega_{k+1} = (1 + \alpha\Delta)\,\omega_k + \Delta u_k \quad (16)$$

Using equation (14. a), the following can be written:

$$\frac{\theta_{k+2} - \theta_{K+1}}{\Delta} = \omega_{k+1} \quad (17)$$

Equating equation (16) with (17) and utilizing (14. a) the second order difference equation describing the system behavior is obtained as:

$$\theta_{k+2} = (2 + \alpha\Delta)\,\theta_{K+1} - (1 + \alpha\Delta)\,\theta_k + \Delta^2 u_k \quad (18)$$

The accuracy of the model depends on Δ. This model is suitable for the cases where high resolution is not considered. Since the differential approximation has been used twice, this model is called the "two-approximation model",

3.2. Second model

Here, the exact discretization is used in one step. Consider the following set of simultaneous first order state equations:

$$\dot{x} = Ax + Bu \quad (19)$$

If this set is sampled with period Δ, the following first order discrete set is obtained [5]:

ical transients. This is achieved by equating the scalar μ with zero, which results in a second order design model.

Now, the effects of this simplification and the high frequency unmodeled dynamics on the winding voltage is considered. The relation of the output torque, T, with respect to the control input is written as in the following [4]:

$$T = K_a I \qquad (4.\ a)$$

$$\mu \frac{\partial J}{\partial t} = - I - \frac{K_b}{R_a}\dot{\theta} + \frac{1}{R_a} V_a \qquad (4.\ b)$$

Defining operator $p = \partial/\partial t$ and combining equations (4. a) and (4. b) result in:

$$T = \frac{K_a}{R_a}\left(\frac{V_a - K_b\dot{\theta}}{\mu P + 1}\right) = \frac{K_a}{R_a}\left(V_a - K_b\dot{\theta} + \mu\frac{(K_b\ddot{\theta} - \dot{V}_a)}{\mu P + 1}\right) \qquad (5)$$

The effect of the armature voltage on the output torque is seen by rewriting equation (5) as:

$$T = \frac{K_a}{R_a}(V_{as} + V_{ud}) = \frac{K_a}{R_a} V_{eff} \qquad (6)$$

where,

$$V_{as} = V_a - K_b\dot{\theta} \qquad (7.\ a)$$

$$V_{ud} = \frac{\mu(K_b\ddot{\theta} - \dot{V}_a)}{\mu P + 1} \qquad (7.\ b)$$

V_{as} is the armature winding voltage for the slow state of the system, i. e., the voltage which will be predicted in the case of low order design model. V_{ud} is a parasitic voltage to the armature winding and it reflects the effects of high frequency dynamics which have not been considered in the design model. The state space model of V_{ud} is written as:

$$\frac{\partial V_{ud}}{\partial t} = -\frac{1}{\mu} V_{ud} + (K_b\ddot{\theta} - \dot{V}_a) \qquad (8)$$

This equation shows that the unmodeled dynamics have a stable pole at $-1/\mu$. The derivatives of the control input V_a and the output angular velocity are the inputs to the parasitic system. It is clear that the control input with fast changes or numerous discontinuities will excite the unmodeled dynamics, and degrade the performance of the system. This matter is important specially to adaptive controllers which are based on variable structure theory and obtain the control law using sign function.

Here, the effect of the unmodeled dynamics on the combined dynamics of the actuator and the load, is considered by combining equations (1) and (5) as:

$$\frac{K_a}{R_a}\left(V_a - K_b\dot{\theta} + \frac{\mu(K_b\ddot{\theta} - \dot{V}_a)}{\mu P + 1}\right) = J\ddot{\theta} + B\dot{\theta} + T_d \qquad (9)$$

Simplifying this equation results in:

$$\frac{K_a}{R_a} V_a = J\ddot{\theta} + \left(B + \frac{K_a K_b}{R_a}\right)\dot{\theta} - \frac{K_a}{R_a}\frac{\mu(K_b\ddot{\theta} - \dot{V}_a)}{\mu P + 1} + T_d \qquad (10)$$

If the external input due to unmodeled dynamics in (10) is denoted by Z_{ud}, then the following relation can be written:

$$\mu\dot{Z}_{ud} = -Z_{ud} + \mu\frac{K_a(K_b\ddot{\theta} - \dot{V}_a)}{R_a} \qquad (11)$$

The second order model is obtained by substituting zero for μ, which results in the following design model of dc motors:

$$\dot{\theta} = \omega \qquad (12.\ a)$$

$$\dot{\omega} = \alpha\,\omega + \beta\,V_a + \gamma\,T_d \qquad (12.\ b)$$

systems with input voltage fluctuations.

Design of a self-tuning adaptive controller is outlined in the following. The proposed controller consists of two independent feedback loops. The first one utilizes a parameter estimator to compensate the undesired nonlinear dynamics of the load and the system. The second loop defines a suitable controller for the closed loop system by solving a minimum variance design problem. Special difficulties which exist in the control of a dc motor with a nonlinear load are listed in the following:

- The overall dynamic of the system is nonlinear.
- Discrete transfer function of a dc motor has a very underdamped zero.
- The system output should track different command signals.

This correspondence presents the combined nonlinear dynamic of motor and load. On the basis of this model, the effects of unmodeled dynamics using singular perturbation theory and two-time-scale model concept, are considered. Utilizing parameter estimator and a linearizing feedback, the nonlinear effects are compensated. Design of a self-tuning controller using minimum variance criterion is outlined for the linearized system. This controller guarantees the convergence of the output error to zero with desirable speed. Finally, the experimental results are investigated.

2 - Continuous Model of a Permanent Magnet DC Motor

The physical system of a permanent magnet dc motor with nonlinear load is shown in Fig. 1, where V_a, R_a, and L_a are the armature voltage, resistance, and inductance respectively. The armature winding of the system is subjected to a constant magnetic field represented by Φ. Nonlinear mechanical load is modeled as a point mass m at the end of an arm of length 1. ω is the angular speed of the motor and θ indicates the deviation of link with respect to a vertical line passing through the connection point in the trigonometric direction.

Using the electrical and mechanical balance laws, differential equations of a dc motor is written as in the following:

$$K_a I = J\dot{\omega} + B\omega + T_d \qquad (1.a)$$

$$L_a \frac{\partial I}{\partial t} + R_a I = V_a - K_b \omega \qquad (1.b)$$

where I, J, and B represent the armature current, moment of inertia, and speed proportional friction coefficient respectively. T_d indicates the nonlinear torque load on the motor shaft. These equations can be rewritten as:

$$\dot{\omega} = -\frac{B}{J}\omega + \frac{K_a}{J} I - \frac{1}{J} T_d \qquad (2.a)$$

$$\frac{L_a}{R_a}\frac{\partial I}{\partial t} = -I - \frac{K_b}{R_a}\omega + \frac{1}{R_a} V_a \qquad (2.b)$$

By defining $\mu = L_a/R_a$, $x_1=\theta$, $x_2 = \omega$ and $x_3 = I$, equations (2. a) and (2. b) are written in the state-space form as:

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \mu\dot{X}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -B/J & K_a/J \\ 0 & -K_b/R_a & -1 \end{bmatrix} \times \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1/R_a \end{bmatrix} V_a + \begin{bmatrix} 0 \\ -1/J \\ 0 \end{bmatrix} T_d \qquad (3)$$

This equation expresses a two-time-scale model. Its characteristic is expressed by the small positive scalar μ. Generally, the fast x_3 electrical transient is neglected in comparison with the slow x_1 and x_2 mechan-

Modeling and Adaptive Control of a Permanent Magnet DC Motor with Nonlinear Load Using Feedback Linearization

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Abstract

In this paper, a two - time scale model of a permanent magnet dc motor is introduced and it is shown that the fast part of the model has a parasitic effect on the windings of the armature. This new framework allows one to predict the behavior of the system in the case of input voltage fluctuations due to controller output and improve the robustness of the closed loop system. Next, derivation of an adaptive control law for a dc motor with nonlinear mechanical load is outlined. This controller utilizes feedback linearization to compensate the full nonlinear dynamics and minimum variance criterion to enable the closed loop system to follow input command signal of desired angular position. Then, the suggested algorithm is applied to control a dc motor with nonlinear load. The simulation and the experimental results confirm the theoretical derivation.

Keywords

Nonlinear Modeling, Adaptive Control, DC Motors, Tracking Problem, Unmodeled Dynamics, Parameter Estimation

1 - Introduction

Today, dc servomotors are widely utilized for various electromechanical applications, particularly in accurate position and velocity control. In some cases, such as robot manipulators and cybernatic arms, the dc motor is subjected to nonlinear loads. Obviously, major difficulties arise in the accurate motion control of these systems. The nonlinear effects can be suppressed by the use of high gear ratios. However, high gear ratios have their own disadvantages and trend is toward direct drive systems [1].

In order to improve the motion performance, efforts have been devoted to develop various control algorithms such as adaptive control [2, 3]. This attractive alternative approach to fixed gain controllers, allows the following of reference trajectories, especially, in the cases where servomotors are required to deal with variable loads. Generally a second order model representing the slow mechanical dynamics of a dc servomotor is used to design a controller.

In this note , a third order model of a dc motor is considered and the effect of neglecting the fast electrical dynamic on the applied voltage to the winding is investigated. This model provides a new framework, which allows one to predict the behavior of