

## References

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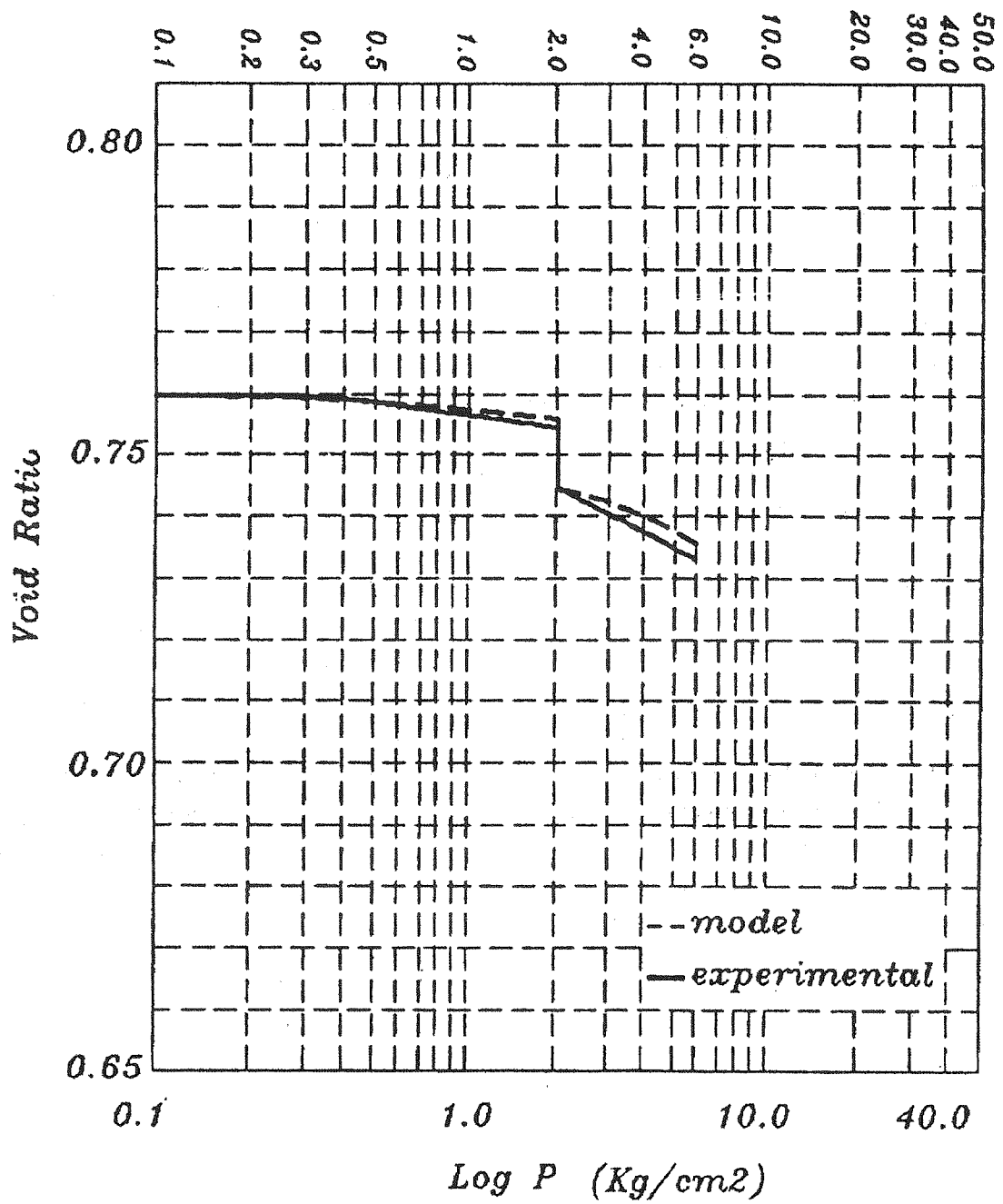


Figure (8) Collapsibility of collapsible soil by reducing suction

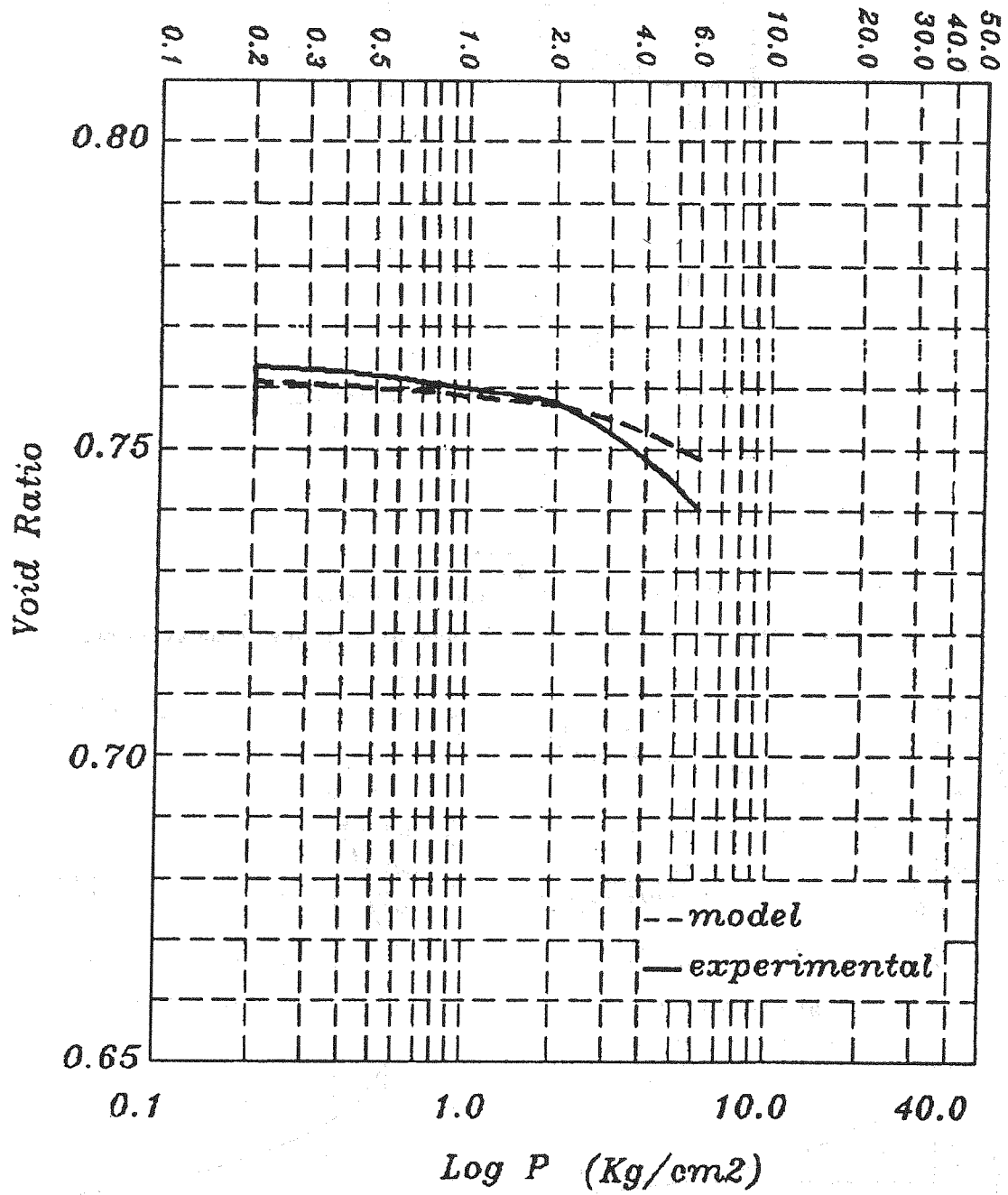
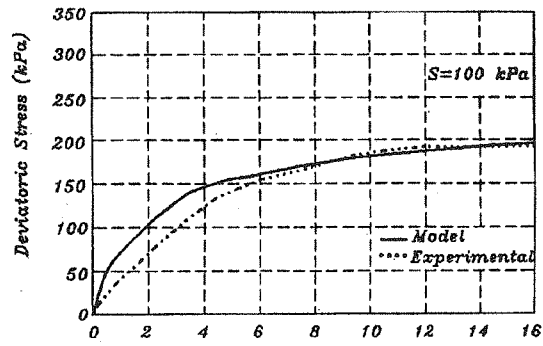
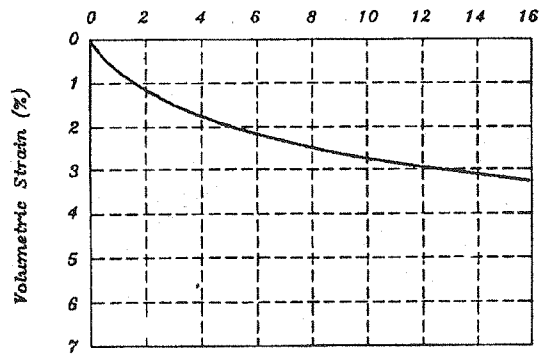


Figure (7) Swelling while reducing suction



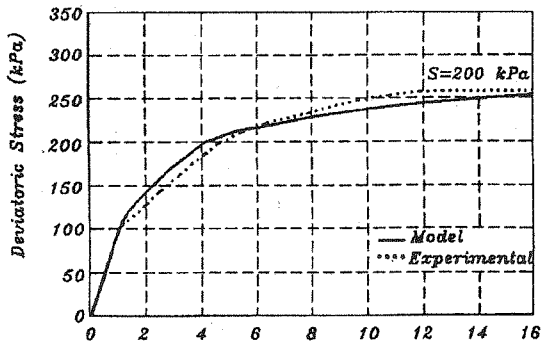
a) Axial Strain (%)



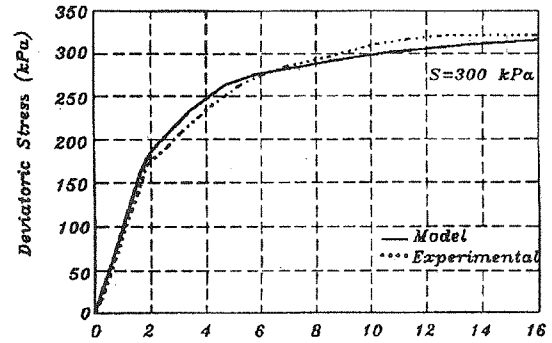
(b)

Figure (5) Experiment and model response

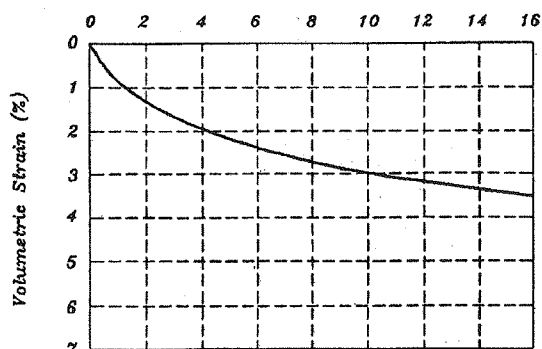
a) Stress deviator (Q) versus axial strain ( $\epsilon_z$ ) b) Volumetric strain ( $\epsilon_v$ ) versus axial strain ( $\epsilon_z$ )



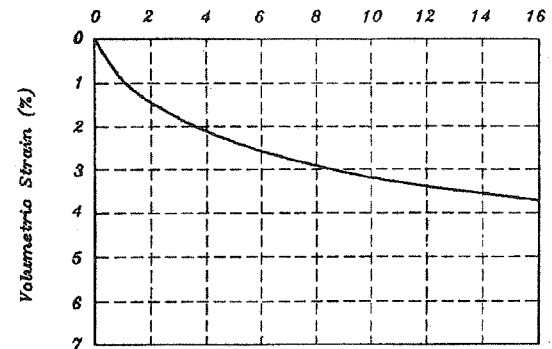
a) Axial Strain (%)



a) Axial Strain (%)



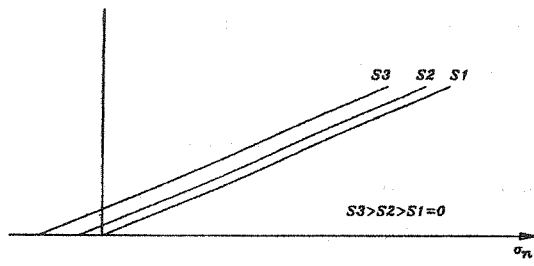
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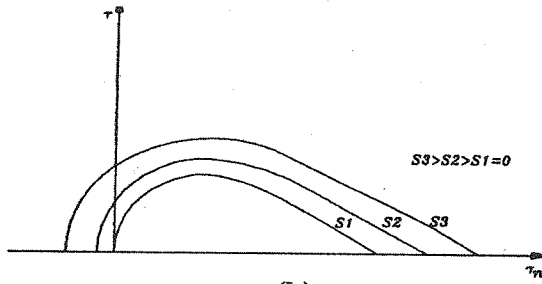
(b)

Figure (6) Experiment and model response a) Stress deviator (Q) versus axial strain ( $\epsilon_z$ )

b) Volumetric strain ( $\epsilon_v$ ) versus axial strain ( $\epsilon_z$ )

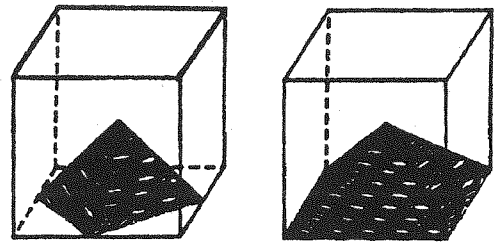


(a)

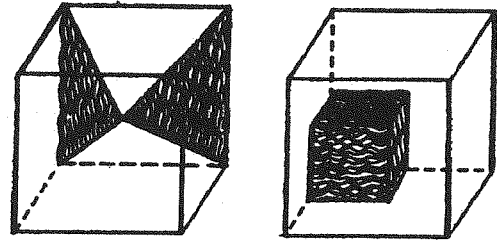


(b)

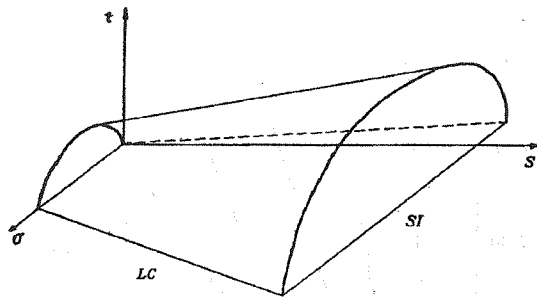
Figure (2) a) Yield criteria and b) Plastic potential functions



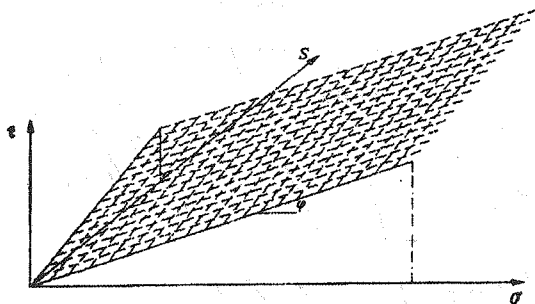
PLANES : 1,2,3&4 PLANES : 7,8,9&10



PLANES : 5 & 6 PLANES : 11,12&13



(b)



(a)

Figure (3) a) Yield and b) Plastic potential functions in  $t, \sigma_n, s$  space [2]

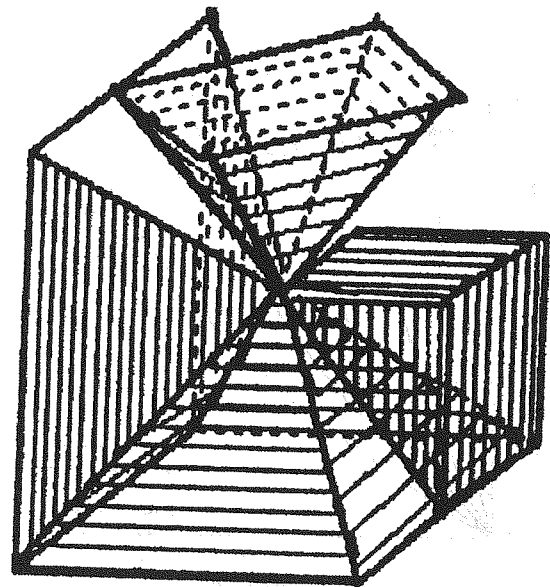


Figure (4) Demonstration of the orientation of 13 planes

which take place during plastic flow, are out of constitutive relations. Accordingly, the sampling plane constitutive formulations provide convenient means to classify loading events, generate history rules and formulate independent evolution rules for local variables.

The behaviour of unsaturated soil has also been modeled based on a semi-microscopic concept which is very close to the reality of particle movement in collapsible soils.

Kinematic and isotropic hardening based phenomenological features on sampling planes are contributed and appropriately summed up, therefore, the solution of any

complexities involved in change of strength because of change of moisture, can be obtained and presented.

In spite of producing the final results in macro scale, there is another significant feature that represents the ability of being informed of the semi-micro scales procedures during any complex stress path. This feature is very fruitful in clarifying the history and rate of all local average micro scales variations through the medium. The final thing which can be gained through this process is the information about failure and corresponding orientation through the medium.

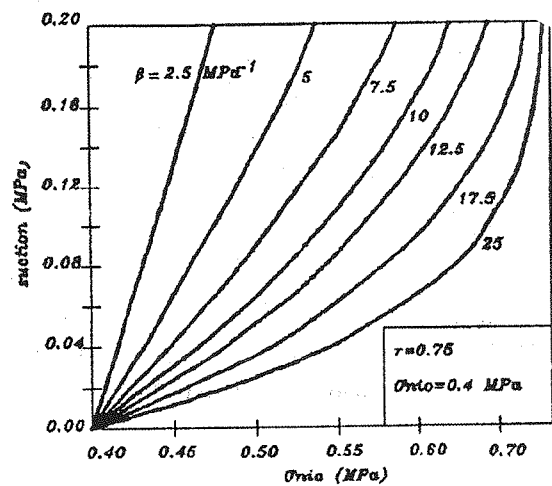
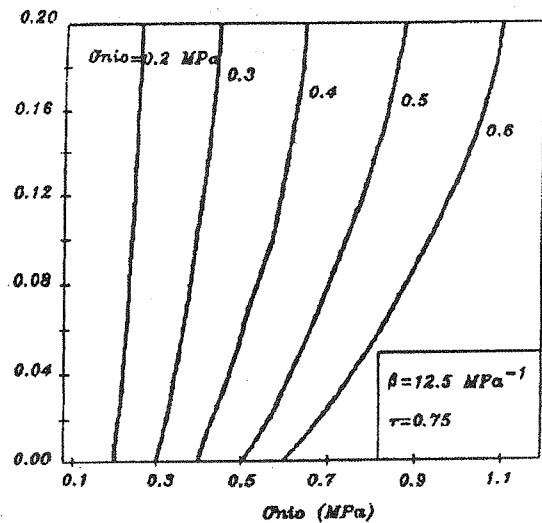
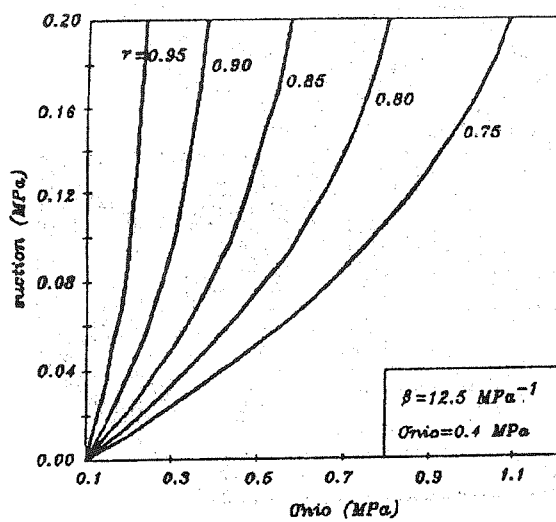


Figure (1) Different inter-relation of  $\sigma_{vio}$ ,  $r$ , and  $\beta$  [2]

uniaxial stress path is followed. Based on the numbering of planes shown in Figure 4, the planes number 1, 2, 3, 4 are predominant with respect to the plastic shear strain values and planes number 7, 8, 9, and 10 are active but have less plastic strains.

In modeling the behaviour of unsaturated soil, the existing suction increased the affecting normal stress on each plane. Accordingly, the strength of each plane increased due to existing suction. However, any reason causes reduction of suction such as saturating the soil by increasing moisture ratio, may causes reduction in strength and leads to collapsing the soil. Therefore, the model is capable of predicting the collapsibility of unsaturated collapsible soil.

Furthermore, lack of real isotropic and homogeneous response of sample material causes practical failure to be shown only in one orientation. The orientation of failure plane is approximate as stated earlier, these planes are sampling planes of an integration rule.

To present the capabilities of the proposed model, the response of the model under triaxial standard configuration is compared with the experimental results provided in reference [2].

The comparison of the model results with experiments, under complex and particular stress paths due to change of moisture predicted and compared with the experiments obtained in reference [2]. Figure 7 shows the comparison of model result and swelling test while decreasing the suction. The comparison of model prediction with experiment for a collapsible soil sample due to moisture change which leads to change of suction has been shown in Figure 8. Quite well and comparable model results with experiments show the capability of the

model in predicting the behaviour of partly saturated soil.

## Conclusions

From this study a model capable of predicting the behaviour of partly saturated soil on the basis of sliding mechanisms, elastic behaviour of particles and a higher effective hydrostatic pressure due to suction has been presented. The concept of multi-laminate framework was applied successfully for unsaturated soil. However, this is achieved by the use of a generally simplified, applicable, effective, and easily understandable relations between micro and macro scales. These relations demonstrate an easy way to handle any heterogeneous material property as well as mechanical behaviour of materials. Significantly, the stress-strain relations are primarily defined on the sampling planes, therefore, there is no need to handle tensorial invariance requirements which are a source of great difficulty in constitutive modeling. In this way, not only the tensorial invariance is subsequently ensured, but also some more effects, which in ordinary models are missed, are additionally included. This inclusion is achieved by combining the responses from sampling planes of all orientations within the material. Consequently, these results are one step closer to real plastic behaviour of soil.

This model is able to solve a three dimensional plasticity problem by a rather simple theory based on the phenomenological description of two dimensional plastic deformation and kinematic hardening of materials. This, actually, is achieved in such a way that the application of some difficult tasks such as induced anisotropy and rotation of principal stress and strain axes

sampling plane. These parameters are listed as follows:

- 1) Elastic modulus,  $E'$ .
- 2) Poisson ratio,  $\nu'$ .
- 3) Slope of critical state line,  $\eta_{co}$ .
- 4) Constant value in hardening function,  $A$ .
- 5) Peak angle of internal friction,  $\phi_f$ .
- 6) Parameter defining maximum soil stiffness due to suction,  $r$ .
- 7) The rate increment for  $r$  and  $\beta$ .
- 8) Suction ( $u_a - u_w$ ),  $s$ .
- 9) Specific volume,  $v$ .
- 10) Constant coefficient,  $k_s$  (only while using method in [2]).

$E'$  and  $\nu'$  are found in the usual way as for any other model. The other three parameters correspond to the plastic behaviour of one plane. In this research, these three parameters have been assumed to be the same for all 13 defined planes because of initial isotropic conditions.

$\phi_f$ ,  $\eta_{co}$ , and  $A$  are constants. Although,  $A$  is directly used as a constant parameter under monotonic loading. In the case of cyclic loading there are slight changes in the value of  $A$  which can be made without necessity of any more parameters [12]. The value of  $\phi_f$  is constant through the test.  $r$  and  $\beta$  can be obtained through drained triaxial test under the model, one set of test results concerning standard triaxial tests have been considered. In the case of accepting the validity of information in Figure 1 [2], model calibration is achieved with only stated tests.

The values of parameters used in computations are shown in Table 2.

**Table (2) - Parameter values.**

Parameter	Value	unit
$E$	8.0	MPa.
$\nu$	0.3	
$\eta_{co}$	0.738	
$A$	0.011	
$\phi_f$	0.535	rad.
$r$	0.3	
$\beta$	12.5	MPa <sup>-1</sup> .
$\sigma_{nc}$	0.1	MPa.
$k_s$	0.1	

The value of  $\eta_{co}$ ,  $\phi_f$ , and  $A$  are obtained by numerical trial and error method.  $\eta_{co}$  is provided through conforming the predicted variation of  $\epsilon_v$  versus axial strain with experiment and the two others by following similar procedure on variation of stress deviator ( $Q$ ) versus axial strain. Figure 5, and 7 show the variations of  $Q_{\epsilon_1}$  and  $\epsilon_v$  versus  $\epsilon_1$  for different suctions.

It must be noted that there are two other parameters  $\sigma_{nc}$  and  $\phi_e$  which define the reference stress and elastic domain. The value of the first has been taken equal to 0.1 MPa, and the other parameter is actually, small and thus can some times be ignored; also it is ineffective to make any change in the overall model results. Therefore the value of this parameter for all tests even for different sands is equal to 0.57°.

Regarding the concept of multilaminate framework and the definition of 13 planes passing through the corresponding point, the number of active planes in the above stated tests are the same. The reason of this feature is the fact that in all tests the same



Table (1) - Direction cosines and weight coefficients of integration points.

Direction cosines of integration points									Weights $w_i$
$l_i^1$	$m_i^1$	$n_i^1$	$l_i^2$	$m_i^2$	$n_i^2$	$l_i^3$	$m_i^3$	$n_i^3$	
$+\sqrt{\frac{1}{3}}$	$+\sqrt{\frac{1}{3}}$	$+\sqrt{\frac{1}{3}}$	$+\sqrt{\frac{1}{6}}$	$+\sqrt{\frac{1}{6}}$	$-\sqrt{\frac{2}{3}}$	$-\sqrt{\frac{1}{2}}$	$+\sqrt{\frac{1}{2}}$	0	27/840
$+\sqrt{\frac{1}{3}}$	$-\sqrt{\frac{1}{3}}$	$+\sqrt{\frac{1}{3}}$	$+\sqrt{\frac{1}{6}}$	$-\sqrt{\frac{1}{6}}$	$-\sqrt{\frac{2}{3}}$	$+\sqrt{\frac{1}{2}}$	$+\sqrt{\frac{1}{2}}$	0	27/840
$-\sqrt{\frac{1}{3}}$	$+\sqrt{\frac{1}{3}}$	$+\sqrt{\frac{1}{3}}$	$-\sqrt{\frac{1}{6}}$	$+\sqrt{\frac{1}{6}}$	$-\sqrt{\frac{2}{3}}$	$+\sqrt{\frac{1}{2}}$	$+\sqrt{\frac{1}{2}}$	0	27/840
$+\sqrt{\frac{1}{3}}$	$+\sqrt{\frac{1}{3}}$	$-\sqrt{\frac{1}{3}}$	$+\sqrt{\frac{1}{6}}$	$+\sqrt{\frac{1}{6}}$	$+\sqrt{\frac{2}{3}}$	$+\sqrt{\frac{1}{2}}$	$-\sqrt{\frac{1}{2}}$	0	27/840
$+\sqrt{\frac{1}{2}}$	$+\sqrt{\frac{1}{2}}$	0	$-\sqrt{\frac{1}{2}}$	$+\sqrt{\frac{1}{2}}$	0	0	0	1	32/840
$-\sqrt{\frac{1}{2}}$	$+\sqrt{\frac{1}{2}}$	0	$+\sqrt{\frac{1}{2}}$	$+\sqrt{\frac{1}{2}}$	0	0	0	1	32/840
$+\sqrt{\frac{1}{2}}$	0	$+\sqrt{\frac{1}{2}}$	$-\sqrt{\frac{1}{2}}$	0	$+\sqrt{\frac{1}{2}}$	0	1	0	32/840
$-\sqrt{\frac{1}{2}}$	0	$+\sqrt{\frac{1}{2}}$	$+\sqrt{\frac{1}{2}}$	0	$+\sqrt{\frac{1}{2}}$	0	1	0	32/840
0	$-\sqrt{\frac{1}{2}}$	$+\sqrt{\frac{1}{2}}$	0	$+\sqrt{\frac{1}{2}}$	$+\sqrt{\frac{1}{2}}$	1	0	0	32/840
0	$+\sqrt{\frac{1}{2}}$	$+\sqrt{\frac{1}{2}}$	0	$-\sqrt{\frac{1}{2}}$	$+\sqrt{\frac{1}{2}}$	1	0	0	32/840
1	0	0	0	0	1	0	1	0	40/840
0	1	0	1	0	0	0	0	1	40/840
0	0	1	0	1	0	1	0	0	40/840

proposed model to obtain the behaviour of an isotropic unsaturated homogeneous sand are as follows:

**a) initial condition**

The initial condition consist of evaluating initial values of  $\tau$ ,  $\sigma$ , and  $s$ . Starting from an isotropic form,  $\tau$  for all planes are equal to zero and the value of  $\sigma_{ni}$  is equal to  $(\sigma_{ni} - ks)$  for all planes. According to [2], the value of  $s$  can be found knowing the initial specific volume of soil. However, the value of  $ks$  can be carried out by the methods

presented in references [10] and [11]. In the case of employing method presented in reference [2],  $ks$  can be obtained through the results of at least two shear tests on two samples from the same soil with different suctions, accepting linear effect of suction on shear strength.

**b) strength and dilation parameters**

These parameters are  $E'$ ,  $v'$ ,  $\phi_r$ ,  $\eta_{co}$ ,  $A$ ,  $r$ , and  $\beta$ . The first two of these parameters correspond to elastic behaviour of soil skeleton and  $\phi_r$ ,  $\eta_{co}$ , and  $A$  to plastic flow on each

tion simulates the best variation of this property during the plastic flow which has been represented as a hyperbolic function as follows:

$$\eta_i = \frac{K_i \cdot \tan(\phi_f)}{A_i + K_i} \quad (18)$$

where,  $K_i = (\epsilon_{Ri}^{Pt} - \epsilon_{oi}^{Pt})$ ,  $\phi_f$  is peak internal frictional angle, and  $A_i$  is a soil parameter.  $\epsilon_{Ri}^{Pt}$  and  $\epsilon_{oi}^{Pt}$  are current and initial values of plastic shear strain on  $i_{th}$  plane. It must be noted that at first loading  $\epsilon_{oi}^{Pt}$  is equal to zero and its value is renewed at each change of load increment sign.

This approach makes a matched tally range of rotating the yield locus over its permissible zone.  $\eta_i$  starts from  $\phi_e$ , grows accompanied with the plastic shear strain and slowly approaches to failure line. However, as stated for dense soils, it has to slowly rotate back towards critical state line.

### Definition of Planes in Three Dimensional Media

As stated earlier, multilaminate framework makes a fair and reasonable relationship between micro and macro scale theories. This is simply achieved by a numerical integration over the surface of a sphere of unit radius in such a manner that any point on the surface is supposed to be a normal to the plane passing through the center of the sphere. Normally, the degree of preciseness depends on the number sampling planes considered in numerical summation.

To satisfy the conditions of applicability of the theory from the engineering viewpoint and also to reduce the extremely high computational costs, a limited number of necessary and sufficient sampling planes

are considered.

The choice of 13 independent planes for the solution of any three dimensional problem is a fair number. The orientation of the sampling planes as given by their direction cosines and the weight coefficients for numerical integration rule are given in Table 1.

One of the important features of multilaminate framework is that it enables identification of the active planes as a matter of routine. The application of any stress path is accompanied with the activities of some of the 13 defined planes in three dimensional media. The value of plastic strain on all the active planes are not necessarily the same. Some of these planes initiate plastic deformations earlier than the others. These priorities and certain active planes can change due to any change of direction of stress path, a number of active planes may stop activity and some inactive ones becomes active and some planes may take over others with respect to the value of plastic shear strain. Thus the framework is able to predict the mechanism of failure.

Figure 4 shows the orientation of all 13 planes in similar cubes. In order to clarify their positions, they have been presented in four cubes.

### Identification of Parameters

In a general case, for the most anisotropic, non - homogeneous material, 13 sets of material parameters corresponding to plastic sliding of each sampling planes are required. However, any knowledge about the similarity of the sliding behaviour of different sampling planes reduces the number of required parameters.

The parameters required to be used in

$\eta_c$  is not constant and changes by  $s$ . This variation is a hyperbolic function as follows:

$$\eta_c = \eta_{co} [\sigma'_{ni} / (\sigma'_{ni} + 2.8183ks)] \quad (8)$$

where,  $\eta_{co}$  is the slope of critical state line in saturated condition. Adopting the incremental procedure, at each calculating step, a new  $\eta_c$  is computed to be used in next step. Typical presentations of this function are shown in Figure 2. The obtained boundary surface of plastic potential function in three dimensional  $\tau$ ,  $\sigma$ , and  $s$  space is shown in Figure 3.

### Non - Associated Flow Rule and Consistency Condition

Substituting the suction effect as a hydrostatic stress in stress vector, flow rule is expressed as follows:

$$d\epsilon_i^P = \lambda_i \cdot \frac{\partial \psi_i}{\partial \sigma'_i} \quad (9)$$

where  $\lambda_i$  is proportionality scalar parameter and changes during plastic deformations.

In theory of plastic flow, consistency condition is a necessary condition which requires that a yield criterion be satisfied as far as the material is in a plastic state. Mathematically, this condition is stated as follows:

$$\{\partial F_i / \partial \sigma'_i\}^T \cdot d\sigma'_i + \partial F_i / \partial k_i \cdot dk_i = 0.0 \quad (10)$$

where in the first loading process  $K_i = \epsilon^{pt}_i$  and  $\epsilon^{pt}_i$  is plastic shear strain on the  $i^{th}$  plane. Substituting equation 10 in 11,  $d\lambda_i$  is obtained as follows:

$$d\lambda_i = - \frac{\{\partial F_i / \partial \sigma'_i\}^T \cdot d\sigma'_i}{\{\partial F_i / \partial K_i\} \cdot \{\partial \psi_i / \partial \tau_i\}} \quad (11)$$

or

$$d\epsilon_i^P = - \frac{\{\partial F_i / \partial \sigma'_i\}^T \cdot d\sigma'_i \cdot \{\partial \psi_i / \partial \sigma\}}{\{\partial F_i / \partial K_i\} \cdot \{\partial \psi_i / \partial K_i\}} \quad (12)$$

This relation can also be expressed in another form as:

$$d\epsilon_i^P = - \{1/Hp_i\} \cdot \{\partial F_i / \partial \sigma'_i\}^T \cdot \{\partial \psi_i / \partial \sigma'_i\} \cdot d\sigma'_i \quad (13)$$

where  $Hp_i$  is defined as hardening modulus of  $i^{th}$  plane and is obtained as follows:

$$Hp_i = - \{\partial F_i / \partial K_i\} - \{\partial \psi_i / \partial \tau_i\} \quad (14)$$

therefore,

$$d\epsilon_i^P = c_i^P \cdot d\sigma'_i \quad (15)$$

where:

$$C_i^P = \{1/Hp_i\} \cdot \{\partial \psi_i / \partial \sigma'_i\} \cdot \{\partial F_i / \partial \sigma'_i\}^T \quad (16)$$

$c_i^P$  as a whole, represent the plastic resistance corresponds to  $i^{th}$  plane and must be summed up as the contribution of this plane with the others. Accordingly, the conceptual numerical integration of multilaminate framework presents the following summation for computing  $c^P$ .

$$C^P = 4\pi \cdot \sum_{i=1}^n w_i \cdot \hat{C}_i^P \quad (17)$$

where  $w_i$  are weight coefficients and  $c^P$  is the global plastic complince matrix corresponding to a single point in the medium.

According to the proposed plastic shear hardening rule, hardening takes place on the  $i^{th}$  plane as a function of plastic shear-strain corresponding to that plane and is independent of other planes. A simple func-

criterion for all specified sampling planes are the same. Conceptually, after start of plastic deformation, the yield criterion for each plane may be different. Obviously, this is due to induced anisotropic behaviour of material after plastic deformation.

In the proposed model, the states of yield criterion on a sampling plane are considered as the possible zone in Mohr - Coulomb failure criterion. According to the Mohr - Coulomb criterion, failure involves the construction of an envelope to all possible circles of stress that can be drawn for a particular problem. These envelopes are generally curved but are usually replaced by a straight line. This is equivalent to assuming that the soil conforms to the Coulomb failure criterion which states that there is a linear relationship between the shear stress  $\tau$  at failure and the normal stress,  $\sigma_n$ . Therefore, the shear strength under drained condition or in dry soil increases with increasing normal stress on the failure locus:

$$\tau = c' + \sigma'_n \cdot \tan(\phi) \quad (5)$$

where,  $\tau$  is the shear stress on the failure plane,  $c'$  the cohesion of the material,  $\sigma'_n$  the normal effective stress on the failure surface, and  $\phi$  the angle of internal friction.

In this constitutive formulation, the yield criterion is defined by the ratio of the shear stress ( $\tau_i$ ) to the normal effective stress (affected by suction according to equation 4) ( $\sigma'_{ni}$ ) on  $i_{th}$  sampling plane. The simplest form of yield function i.e. a straight line on  $\tau$  versus  $\sigma'_n$  space is adopted. As the ratio  $\tau/\sigma_n$  increases, the yield surface represented by the straight line rotates anti-clock-wise due to hardening and approaches Mohr -

Coulomb's failure line and finally failure on corresponding plane takes place.

The equation of yield function is formulated as follows:

$$F_i(\tau_i, \sigma'_{ni}, n_i) = \tau_i - n_i(\sigma'_{ni} + ks) \quad (6)$$

where,  $\eta_i = \tan(\alpha_i)$  is a hardening parameter and assumed as a hyperbolic function of plastic shear strain on the  $i_{th}$  plane.  $\alpha_i$  is the slope of yield line. The effective normal stress on  $i_{th}$  plane for unsaturated soil is obtained as the summation of one part as what equilibrium presents and the second part which is equal to  $ks$ . The latter part is considered as the multiplication of a constant  $k$  by suction effect as normal stress on each plane. This hypothesis presents a linear increment on shear strength on each plane due to existing suction.

### Plastic Potential function

Significantly, the different forms of this function, with different convexities will effect on the obtained variation of volumetric plastic strain during plastic flow. Consequently, this change, mainly, is a change in transient levels of energy or in other words, the rate of entropy growth.

Feda [9], derived the plastic potential function which is used in this research. The modified form of this function for unsaturated soil is stated in terms of  $\tau_i$ ,  $\sigma_{ni}$ ,  $\sigma_{nio}$ , and  $s$  for the  $\tau$  versus  $\sigma'_n$  space as follows:

$$\Psi(\tau_i, \sigma'_{ni}, S) = \tau_i + n_c(\sigma'_{ni} + ks) \text{Ln}[\sigma'_{ni} / (\sigma'_{nio} + ks)] \quad (7)$$

where,  $\eta_c$  is the slope of critical state line and  $\sigma_{nio}$  is the initial value of effective normal stress on  $i_{th}$  plane. For unsaturated soil

In many instances, the scale of the microstructure is coarse enough to be out of the range of such specific considerations of slip theory, and the individual component blocks can be considered as a continuum with well - defined plastic resistances and hardening behaviour. In this research, the individual component blocks of the overall media deform collectively as a heterogeneous (but compatible in deformations with other blocks) assembly of continua, interacting with each other only through the boundary conditions applicable at their various interfaces. The deformations of such coarse heterogeneous assemblies are best considered in full detail, preserving the information of the internal variations of effective deformational resistances in individual component blocks and associated internal stresses. This can then be followed by averaging or "post - smoothing" approach permits the monitoring of the evaluation of internal deformations in addition to the over all deformation resistances.

As already defined, plastic strain is calculated from the study of the glide motion over an individual sampling plane. To start explaining the plasticity constitutive law for a sampling plane, the main features of plasticity law (i. e. yield criterion, plastic potential function, flow rule and hardening rule) must also be considered.

### Suction in Partly Saturated Soil

Normally, there are negative pore water pressures in unloaded unsaturated soil. The term negative pore water pressure is taken here to mean any pressure deficiency which occurs in situ while subjected to some form of externally applied stress system. However, the suction of any element of

soil,  $s$ , in the unloaded state, is modified by the effect of the overburden. This modified suction is, of course, the final pore water pressure, either positive or negative, that the element has reached. The equivalent hydrostatic pressure stands for suction can be obtained by either Cronley & Coleman [10] or Clisby [11].

In multi-laminate framework, the effect of a negative hydrostatic pressure is accounted as compressive normal pressure on all sampling planes. The variation of this normal pressure versus the degree of saturation can be accepted as follows [2]:

$$(\sigma_{nio} / \sigma_{nc}) = (\bar{\sigma}_{nio} / \sigma_{nc})^{(1-r) \exp(-\beta S) + r} \quad (4)$$

where,  $\sigma_{nio}$  is the effective normal stress for  $i_{th}$  plane in unsaturated soil,  $\sigma_{nc}$  may be for net normal stress in the case of saturated virgin state starting at a partially saturated condition through a wetting path,  $\bar{\sigma}_{nio}$  is normal stress for saturated condition,  $r$ ,  $\beta$ , and  $s$  are parameters defining maximum soil stiffness due to suction, the rate increment for  $r$ , and suction ( $u_a - u_w$ ), respectively. The values of parameters  $r$ ,  $\beta$ , and  $\bar{\sigma}_{nio}$  can be obtained from Figure 1. These informations are based on  $\sigma_{nc} = 0.1$  MPa. [2].

### Yield Criterion

In view of the complexities involved in the yielding of materials in two or three dimensional state of stress and hardening, it seems convenient to define a scalar function,  $F$ , as the yield criterion.

Under the concept of multilaminate framework, this function is defined in two dimensional stress space for each individual sampling plane. Starting from initially isotropic and homogeneous condition, yield

rial mass has an infinite number of yield functions, usually one for each of the planes in the physical space.

### Constitutive Equations

Generalizing the behaviour of partially saturated soil as elastic-plastic strain increments can be decomposed as elastic and plastic parts and schematically written as follows:

$$d\epsilon = d\epsilon^e + d\epsilon^p \quad (1)$$

The increment of elastic strain ( $d\epsilon^e$ ) is related to the increments of effective stress ( $d\sigma'$ ) by:

$$d\epsilon^e = C^e \cdot d\sigma' \quad (2)$$

where,  $C^e$  is elastic compliance matrix, usually assumed as linear. Conceptually, it is possible to compute  $C^e$  by using the multilaminate framework. However, if the single structural units are assumed to be elastically isotropic, using a common elasticity tensor, then trivially, the overall elastic response of the collective system will be isotropic, having the same elasticity tensor. Clearly, in this case, computing  $C^e$  by using multilaminate framework is not fruitful. When single structural unit constituents are anisotropic, then, whether or not the overall elastic response will be isotropic depends on the distribution of the single structural units. For a random distribution the overall response will be isotropic, whereas this response will be anisotropic if the distribution of particle orientations is biased by prior plastic deformation.

For the soil mass, the overall stress-strain increments relation, to obtain plastic

strain increments ( $d\epsilon^p$ ), is expressed as:

$$d\epsilon^p = C^p \cdot d\sigma' \quad (3)$$

where,  $C^p$  is plastic compliance matrix.

Clearly, it is expected that all the effects of plastic behaviour be included in  $C^p$ . To find out  $C^p$ , the constitutive equations for a typical slip plane must be considered in calculations. Consequently, the appropriate summation of all provided compliance matrices corresponding to considered slip planes yields overall  $C^p$ .

### Constitutive Equations for a Sampling Plane

A Sampling plane is defined as a boundary surface which is a contacting surface between two structural units of polyhedral blocks. These structural units are parts of an inhomogeneous continuum, for simplicity defined as a full homogeneous and isotropic material. Therefore, all inhomogeneities behaviour are supposed to appear in inelastic behaviour of corresponding slip planes. In many cases, however, the medium is known to be heterogeneous and the notion of continuum is used to describe it on a scale very much larger than the scale of the real particles. When this approach of pre-smoothing is taken a priori, without any knowledge of the distribution and aggregation of specific microstructure, information on all internal details, on the distribution of intergranular stresses, strains and many other real features is forfeited. Since in reality, this information is necessary to understand the overall deformation resistance of the soil, this aspect becomes too complicated. Therefore, the material which is contained inside a structural unit is treated a "black box".

of contact forces has been long the aim of numerous researchers [1], and [3].

In recent years, another class of models called multilaminate model, was developed by Zienkiewics and Pande (1977) [4] for jointed rock masses and Pands and Sharma (1980 , 1981, 1983) [5] for clays. Bazant and Oh (1982) [6] have developed a similar model for fracture analysis of concrete under the name "microplane model".

An elastoplastic model named, reflecting surface model' developed by Pande and Pietruszczak (1982) [7] and used to predict cyclic loading behaviour of normally consolidated and lightly over - consolidated clays. Shiomi et al (1982) [8] used this model for the prediction of liquefaction of sand layers.

This paper presents a multilaminate model capable of predicting the behavior of partly saturated soil under monotonic, cyclic loading and other respectively complex stress paths.

The concept of proposed model is natural, physically meaningful and extremely simple. According to this formulation which is based on a simple numerical integration, an appropriate connection between averaged micro and macro - mechanical behaviour of material has been presented. The inclusion of the rotation of principal stress and strain axes, induced anisotropy and the possibility of supervising and even controlling any variation through the medium are the significance of the model.

### **Basic Assumptions and Discussions**

In partially saturated soil, water in a moist particle occurs in the form of droplets or adsorbed water between the points of

contact of the individual grains and is therefore referred to as contact of the individual grains moisture. This water , retained by surface tension, holds the particles together and produces a resistance to applied stress resembling cohesion. To overcome the complexity of this potential of suction which is actually sets of micro - interaction between soil particles, a semi - micro model such as multilaminate is employed.

Multilaminate framework by defining the small continuum structural units as an assemblage of particles and voids which fill infinite spaces between the sampling planes, has appropriately justified the contribution of interconnection forces in overall macro - mechanics. Plastic deformations are assumed to occur due to sliding, separation/closing of the boundaries and elastic deformations are the overall responses of structural unit bodies. Therefore, the overall deformation of any small part of the medium is composed of total elastic response and an appropriate summation of sliding, separation / closing phenomenon under the current effective normal and shear stresses on sampling planes.

According to these assumptions, overall sliding, separation / closing of intergranular points of grains included in one structural unit, are summed up and contributed as the result of sliding, separation/closing surrounding boundary planes. This simply implies yielding / failure or even ill - conditioning and bifurcation response to be possible over any of the randomly oriented sampling planes. Consequently, plasticity control such as yielding should be checked at each of the planes and those of the planes which are sliding will contribute to plastic deformation. Therefore, the granular mate-

# *A Multilaminate Elastoplastic Model for Semi - Saturated Soil*

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## **Abstract**

*In this paper, a multilaminate based model capable of predicting the behaviour of semi-saturated soil on the basis of sliding has been presented. The capability of the model to predict the behaviour of soil under the condition of semi - saturation has been examined. The validity of presented model examined successfully by comparing the predicted numerical results of semi - saturated specimens and test results.*

## **Introduction**

The behaviour of partially saturated soils depends upon the amount of entrapped air and gas in the pore space. If the volume of gas or air is small, the soil may get saturated after application of additional stress. On the other hand, if the volume of air and gas is large, the soil may remain unsaturated throughout.

According to observations, the unsaturated soils undergo volume change even in undrained tests. Therefore, according to Mohr - Coulomb failure lines the strength envelope with respect to total stresses is straight line, the strength envelope with respect to total stresses is straight line but not horizontal. Furthermore, the air and gas occupy relatively large volume at small applied pressure. The volume of the gas reduces with increase in pressure because of compression. Also part of it may go into solution.

The pore water pressure and pore air pressure are different because of the surface

tension of water due to which air bubbles have a larger pressure than the surrounding liquid.

Due to the pressure of meniscus at the air - water interface, the water is under surface tension and the initial pore water pressure is negative.

For a typical soil that supports the overall applied loads through contact friction and cohesion, the overall mechanical response ideally may be described on the basis of micro - mechanical behaviour of grains interconnections. Naturally, this requires the description of overall stress, characterization of fabric, representation of kinematics, development of local rate constitutive relations and evaluation of the overall differential constitutive relations in terms of the local quantities.

The task of representing of the overall stress tensor in terms of micro level stresses and the condition, number and magnitude