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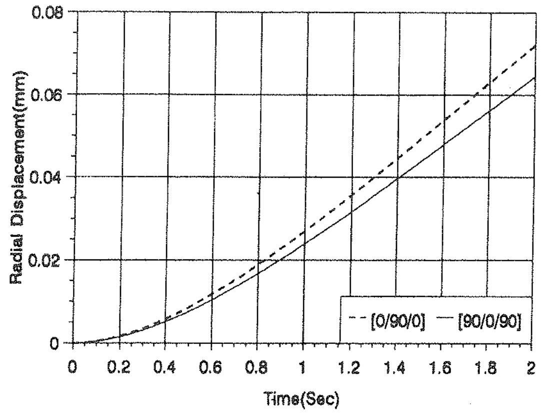

Figure (17) Transient response of radial displacement of central layer for two different layer stacking

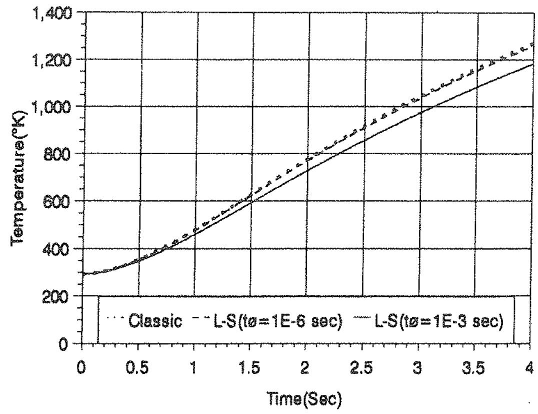

Figure (18) Effect of relaxation time on temperature time history

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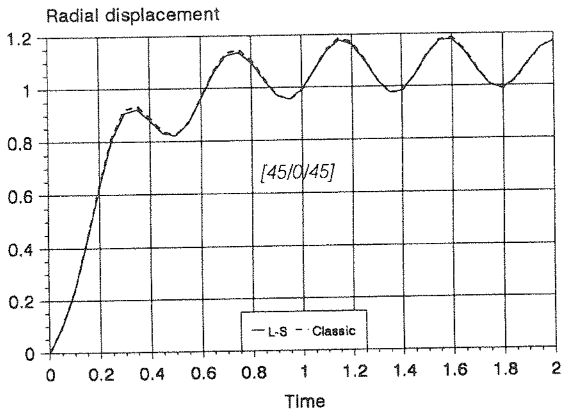

Figure (11) Transient non-dimensional radial displacement for the Classical and L-S theories

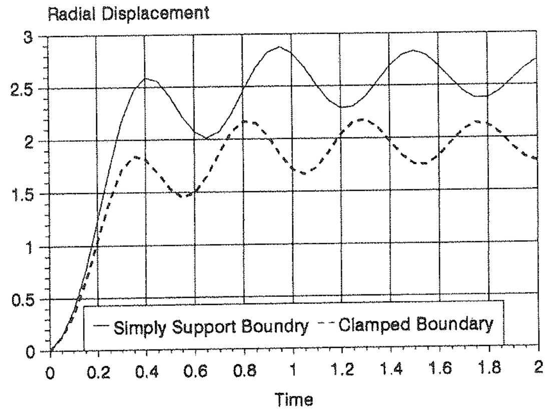

Figure (14) Effect of boundary condition on coupled response of the shell

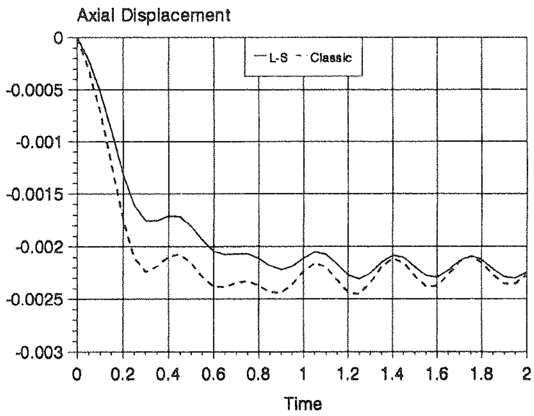

Figure (12) Transient non-dimensional longitudinal displacement for the Classical and L-S theories

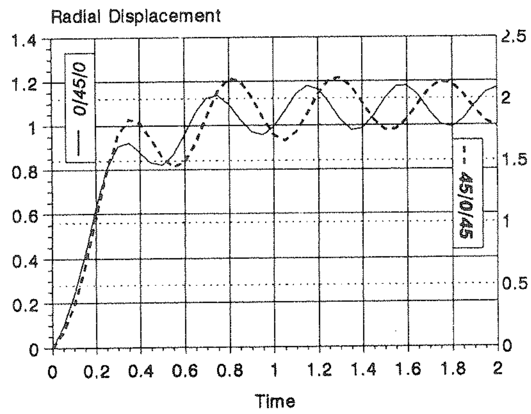

Figure (15) Effect of layer stacking on the response of heated shell

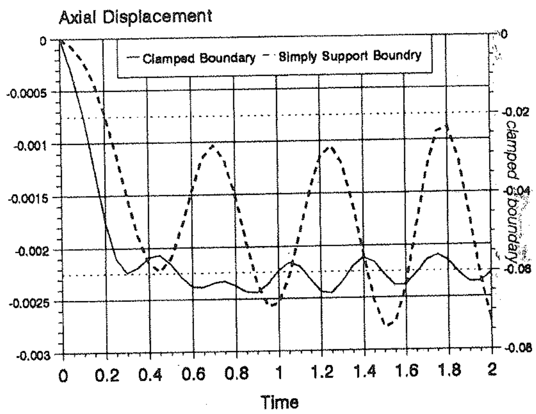

Figure (13) Effect of boundary condition on coupled response of the shell

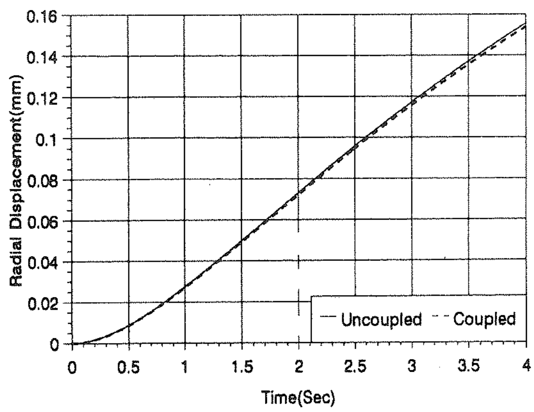

Figure (16) The time history of radial displacement of central layer at crown of shell

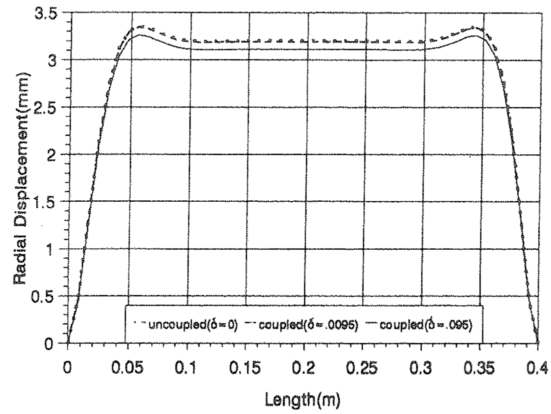

Figure (5) The variation of radial displacement versus the shell length

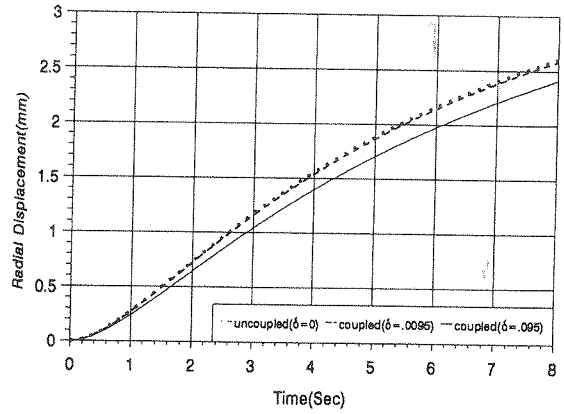

Figure (8) The time history of radial deflection at the crown of shell

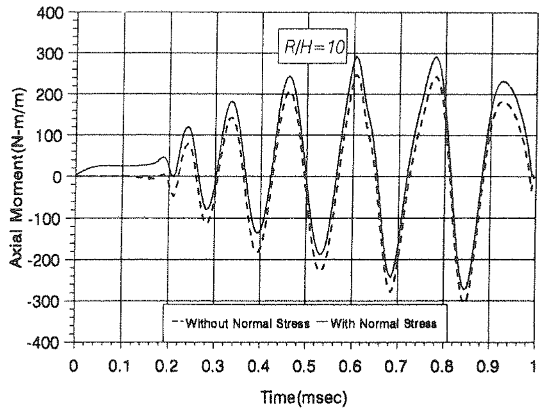

Figure (6) Time history of axial moment of inside shell surface at middle length for R/h = 10

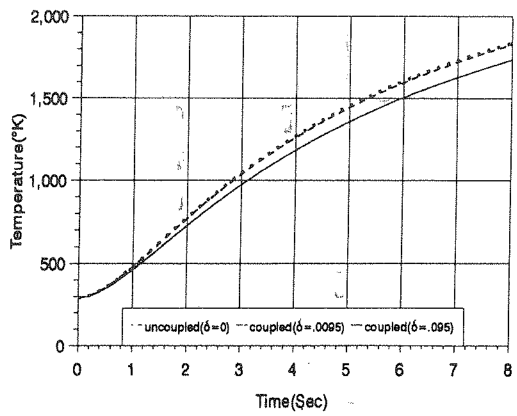

Figure (9) The time history of inside temperature at the crown of shell

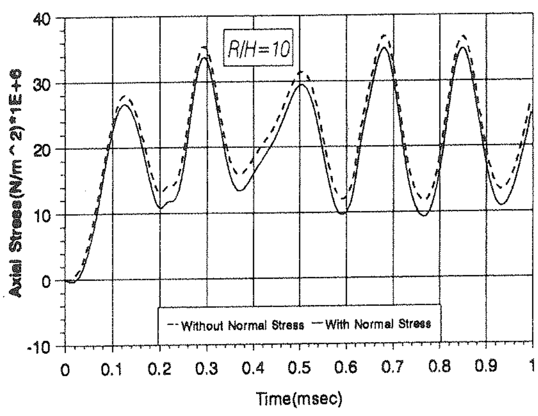

Figure (7) Axial Stress versus time at middle length for R/h = 10

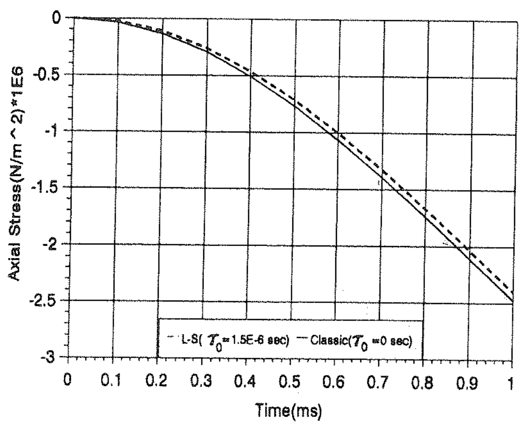

Figure (10) Effect of τ_0 on axial stress derived under L-S theory in cylindrical shell

As expected, ply angle will have an important effect on the responses of the heated shells. The results are presented in Fig. (15).

As the last example , an open spherical shell with clamped edges made of three layer laminates is presented. The sphere is considered to be exposed to the inside thermal shock given by Eq. (41). The thermal conditions at the ends of shell is assumed isolated. Fig. (16) shows radial displacement of central layer at crown of shell. As it indicates, coupling effect is important and cannot be neglected. Fig. (17) presents the variation of transient response of radial displacement of central layer for two different cross - ply (0/90/0) and (90/0/90). As expected, ply angle will have an important effect on the response of shell. In Fig. (18) time history of temperature is shown. The effect of relaxation time can be seen.

In all of numerical results presented herein, zero initial conditions for displacements and velocities have been assumed.

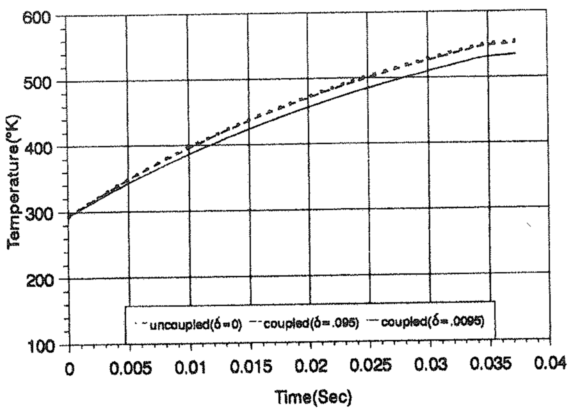

Figure (1) The inside temperature versus time

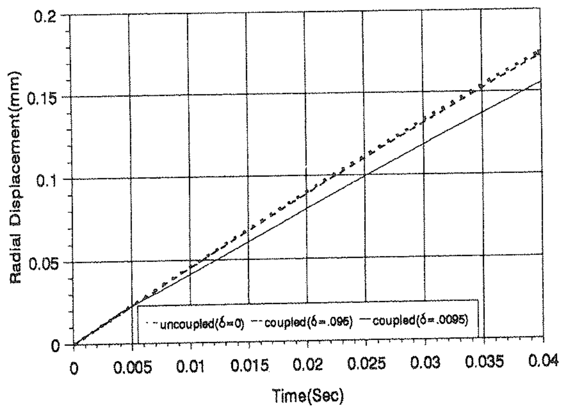

Figure (2) The middle plane lateral deflection

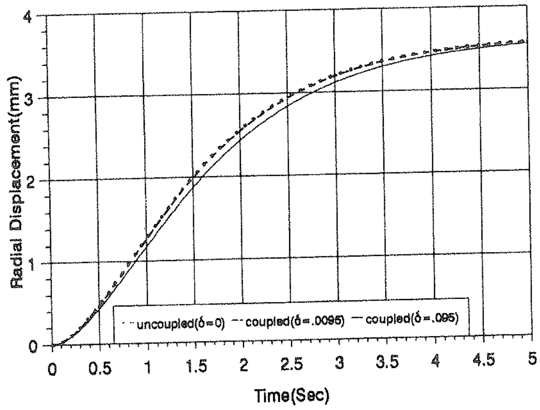

Figure (3) Time history of radial displacement

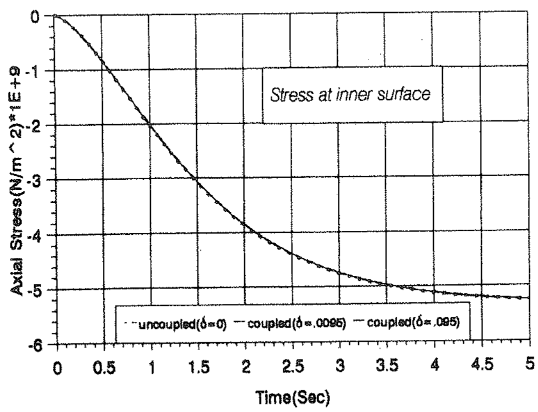

Figure (4) Time history of axial stress at inner surface

$$- 2 \frac{\beta_{\phi\theta}}{R + z} \cot g\phi \dot{V} + \frac{\beta_{\theta\theta} - \beta_{\phi\phi}}{R + z} \dot{W}_{,\phi}]$$

$$- K_{zz} T_{,zz} - \frac{2}{R + z} K_{zz} T_{,z} - \frac{1}{(R + z)^2} K_{\phi\phi} T_{,\phi\phi}$$

$$- \frac{\cot g\phi}{(R + z)^2} K_{\phi\phi} T_{,\phi\phi} = 0 \qquad (46)$$

For Cylindrical Shells

$$R = (1 + t_0 \frac{\partial}{\partial t}) [\rho c \dot{T}] + T_a [\beta_{xx} (\dot{U}_{,x} + t_0 \ddot{U}_{,x})$$

$$+ \frac{\beta_{\theta\theta}}{R + z} (\dot{W} + t_0 \ddot{W}) + \beta_{zz} (\dot{W}_z + t_0 \ddot{W}_z) + \beta_{\chi\theta} (\dot{V}_{,\chi} + t_0 \ddot{V}_{,\chi})]$$

$$- K_{xx} \frac{\partial^2 T}{\partial x^2} - K_{zz} (\frac{\partial^2 T}{\partial z^2} + \frac{1}{R + z} \frac{\partial T}{\partial z}) = 0 \qquad (47)$$

Substituting equation (44) in either of equations (46) or (47), the following integrals provide two set of independent equations for T_0 and T_1

$$\int_z (R) * (1) * dz = 0 \qquad (48)$$

$$\int_z (R) * (z) * dz = 0 \qquad (49)$$

The finite element model of shell using Galerkin method is constructed and the α method is used to solve the time dependent finite element equation.

In the following a multilayer composite cylindrical shell exposed to the inside thermal and pressure shock given by equations (40) and (42) is considered.

Table (7) Geometry and material properties of multilayer cylindrical and spherical shells

E_{11} 196 GN /m²	E_{22} 4.83 GN /m²
E_{1n} 4.83 GN /m²	E_{12} 3.44 GN /m²
G_{2n} 3.44 GN /E_{11}	ν_{12} 0.05
ν_{2n} 0.3	K_1 180 W / mk
K_2 67 W/mk	K_3 67 W / mk
α_1 $1.3 * 10^{-6}$ 1/k	α_2 $1.5 * 10^{-6}$ 1/k
α_3 $15 * 10^{-6}$ 1/k	h_i 10000 W/m² k
h_0 200W/m² k	R 0.2 m

The thermal conditions at the ends of shell is assumed isolated. Figs. (11) and (12) compare the transient non - dimensional radial and longitudinal displacements for the Classical and L - S models in a three - layer stacking [45/0/45] cylindrical shell. Owing to the anisotropy in the thermoelastic moduli, the effects of boundary conditions in the present laminated shell are expected to be more complicated than those in isotropic shells. Different edge conditions (together with the effects of the anisotropic thermoelastic moduli, specially the coefficients of thermal expansion) may cause different mode of vibration. Figs. (13) - (14) summarize the effects of various boundary conditions on coupled response of the shell.

7 - Coupled Thermoelasticity of Composite Shells

The composite structures are more frequently used in industry due to their many advantages. Many shell structures are made of composite materials to take advantage of their strength and light weight.

Dynamic thermoelasticity of orthotropic cylindrical shells and multilayer shells are discussed by Wu et al. (1991) and Wang et al. (1991). Eslami, Shakeri and Shiari discussed in a series of papers the coupled thermoelasticity of composite laminated shells of revolution (1996, 1997). The discussion includes the effect of coupling terms, normal and shear stresses, the relaxation time, and the stacking sequence of the laminated layers. The classical and Lord - Shulman models are basically used in the analysis. The finite element model is also made on the assumption of equivalent single layer model.

The formulation of the composite shells of revolution are the same as those given for isotropic shell (here we adopt the assumption of second order theory and Flugge model). However , the constitutive law of composite material should be considered instead of the simple Hooke's law. For a layered composite the relations must be considered for each single layer. The stress - strain relation for the k - th orthotropic layer bounded by surfaces at $z = h_k$ and $z = h_{k-1}$ are given by:

$$\begin{bmatrix} \sigma_\chi \\ \sigma_\theta \\ \sigma_z \\ \tau_{\theta z} \\ \tau_{\chi z} \\ \tau_{\chi\theta} \end{bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{13} & 0 & 0 & \bar{Q}_{16} \\ \bar{Q}_{21} & \bar{Q}_{22} & \bar{Q}_{23} & 0 & 0 & \bar{Q}_{26} \\ \bar{Q}_{31} & \bar{Q}_{32} & \bar{Q}_{33} & 0 & 0 & \bar{Q}_{36} \\ 0 & 0 & 0 & \bar{Q}_{44} & \bar{Q}_{45} & 0 \\ 0 & 0 & 0 & \bar{Q}_{54} & \bar{Q}_{55} & 0 \\ \bar{Q}_{61} & \bar{Q}_{62} & \bar{Q}_{63} & 0 & 0 & \bar{Q}_{16} \end{bmatrix}$$

$$\begin{bmatrix} \varepsilon_\chi \\ \varepsilon_\theta \\ \varepsilon_z \\ \gamma_{\theta z} \\ \gamma_{\chi z} \\ \gamma_{x\theta} \end{bmatrix} - \begin{bmatrix} \bar{\beta}_\chi \\ \bar{\beta}_\theta \\ \bar{\beta}_z \\ 0 \\ 0 \\ \bar{\beta}_{x\theta} \end{bmatrix} [T] \qquad (43)$$

where $[\bar{Q}_{ij}]_k$ and $[\bar{\beta}_i]_k$ are stiffiness and thermoelastic matrices, respectively. The temperature distribution across the shell thickness is assumed as:

$$T(\alpha_1, \alpha_2, z, t) = T_0(\alpha_1, \alpha_2, t) + zT_1(\alpha_1, \alpha_2, t) \qquad (44)$$

Substituting for the strains in equation (43) from equation (34) and using the expressions for the stresses in equation (43) and integrating, the relations between stress and moment resultants and the middle plane strains and temperatures field are found.

The energy equation on the basis of Lord - Shulman assumption is:

$$K_{ij} T_{,ij} - (1 + \tau_o \frac{\partial}{\partial t}) [c_v \rho \dot{T} + T_\alpha \beta_{ij} \dot{\varepsilon}_{ij}] = 0 \qquad (45)$$

For a multilayer composite spherical (ϕ, θ, z) and cylindrical (x , θ, z) shells the above equation yields:

For Spherical Shell

$$R = \rho c t_0 \ddot{T} + t_0 T_a [\frac{\beta_{\phi\phi}}{R+z} \ddot{U}_{,\phi} + \frac{\beta_{\phi\theta}}{R+z} \ddot{V}_{,\phi} + \beta_{zz} \ddot{W}_{,z}$$

$$+ \frac{\beta_{\theta\theta}}{R+z} \operatorname{cotg}\phi \ddot{U} - 2 \frac{\beta_{\phi\theta}}{R+z} \operatorname{cotg}\phi \ddot{V} + \frac{\beta_{\theta\theta} - \beta_{\phi\phi}}{R+z} \ddot{W}_{,\phi}]$$

$$\rho c T + T_a [\frac{\beta_{\phi\phi}}{R+z} \dot{U}_{,\phi} + \frac{\beta_{\phi\theta}}{R+z} \dot{V}_{,\phi} + \beta_{zz} \dot{W}_{,z} + \frac{\beta_{\theta\theta}}{R+z} \operatorname{cotg}\phi \dot{U}$$

Table (4) Effect of normal stress (cylindrical shells)

Theory	W (m) * 10^{-8} (R/h = 30)	W (m) * 10^{-9} (R/h = 10)
Including σ_n	20976	67291
Excluding σ_n	20852	65881
Difference	0.7%	2%

Fig. (6) shows the time history of axial moment of inside shell surface at middle length for R/h = 10. Fig. (7) is the plot of axial stress versus time at the same location for R/h = 10. The axial stress is the sum of $N_x/h + 12z\, M_x/h^3$. Since the axial force is dominant, the axial stress follows its pattern.

Spherical Shells

The next example is a hemispherical shell with the following data R = 0.3 m, h = 0.01 m, E = 200 Gpa, ρ = 8000kg/m^3, ν = 0.3. Sphere is clamped at edges. The pressure pulse of equation (42) is applied and the lateral deflection at the crown of the shell is given in Table (3). It is noted that the consideration of normal stress improves the results up to 1.4 percent for R/h = 30 and 4.6 percent for R/h = 10.

Table (5) Effect of normal stress (spherical shells)

Theory	W (m) * 10^{-7} (R/h = 30)	W (m) * 10^{-9} (R/h = 10)
Including σ_n	1731	53871
Excluding σ_n	1736	56347
Difference	0.2%	4.4%

The final example is a hemispherical shell with clamped edges under thermal shock given by equation (40) and the following data;

Table (6) Geometry and material properties of spherical shell

E 200 GN/m^2	k 50W/m^0 k
α 11.8 * 10^{-6} 1/k	G 76.9GN/m^2
R 0.15 m	ν 0.3
ρ 7904 kg/m^3	h 0.15m
c_v 500J/kg^{0}k	h_ι 10000W /m^{2}k
h_0 200$_w$/m^{2}k	T_a 293^{0}k

In Figs. (8) and (9) the time history of the radial deflection and inside temperature at the crown of shell is given. It is noted that the mechanical coupling δ = 0.095 has considerable effect on shell response and cause the reduction of w and T. For a suitable representation of relaxation time effect, consider a cylindrical shell with clamped edges under thermal shock given by equation (40) and geometry and material properties as Table (1). Fig . (10) illustrates the difference between the values calculated under two theories, the classical coupled theory (τ_0 = 0) and L - S theory (τ_0 = 1.5*10^{-6}). The axial stresses curve for L-S theory become smaller than the classical theory. Maximum value of difference is about 38 percent which occur at t = 0.4*10^{-3} second at outer surface of shell.

shell is assumed isolated, and the shell is considered to be exposed to inside thermal shock given by the following equations:

$$T_i(t) = 2207\,(1 - e^{-13100t}) + 293°\,k \qquad (40)$$

The temperature of the inside surface raise from 293° K to 2500° K in 0.45 msec, and the shell behavior is studied up to 0.04 sec which is about 90 times the time period required for the thermal shock to reach its steady state condition. Shell is divided into 50 elements along its length and the time increment is 1E-6 sec. In Fig. (1) the inside temperature versus time is shown. The effect of mechanical coupling $\delta = \frac{(1-\nu)\alpha^2 T_a E}{(1-\nu)(1-2\nu)\rho c_v}$ is shown in this figure. For $\delta = 0$, the mechanical coupling term is ignored from the energy equation and the problem is decoupled. For the given shell $\delta = 0.0095$ and it is noticed that the effect of damping is negligible. For larger α or smaller c_υ the value of δ is larger. For $\delta = 0.095$ the mechanical coupling has noticeable effect. In Fig. (2) this comparison is shown for the middle plane lateral deflection. It is noticed that while at t = 0.04 sec the thermal shock is reached to its steady state condition, but the lateral deflection is still increasing. The reason is that the characteristic time of heat transfer is much larger than the mechanical characteristic time for stress wave . This behavior is different when the shell is under pressure shock.

Now consider the same shell under low rate thermal shock. The equation of teperature shock applied to the cylindrical shell is:

$$T_i(t) = 2207\,(1 - e^{-1.3t}) + 293°\,k \qquad (41)$$

The rate of temperature variation with respect to time is slower compared to equation (40). Temperature reaches to its maximum value within 35 sec. Time increment is selected $\Delta t = 0.01$ sec and the shell behavior is studied up to 5 sec. In Fig. (3) time history of radial displacement is shown. Similar to Figs. (1) and (2), the values of temperature and displacement for coupled condition ($\delta = 0.095$) is less than the values for semi - coupled condition ($\delta = 0$). This means that the coupled effect act like a damper and thus it could be regarded as "thermoelastic damping". At the begining of the shock, due to lower values of strains, the difference between coupled ($\delta = 0.095$) and semi-coupled ($\delta = 0$) is negligible and as time increases this difference also increases. When temperature reaches its steady state condition the strains reach their maximum values while their time rate is decreased and the effect of mechanical coupling will also increase. In Fig. (4) time history of axial stress at inner surface is shown. The variation of radial displacement versus the shell length are shown in Fig.(5).

The effect of normal stress is studied in the next example. A simply supported cylindrical shell of L = 1.0 m, R = 0.15 m, h = 0.005 m, E = 196 Gpa, $\rho = 8000\ kg/m^3$, and $\nu = 0.3$ under inside uniform pressure shock of

$$P(t) = 8 * 10^6\,(1 - e^{-13100t}) \qquad (42)$$

is considered . Pressure reaches its maximum value at 0.45 msec. Table (2) gives the radial displacement for middle length at t = 5E - 4 sec. The difference between two cases are about 0.7 percent for R/h = 30 and 2 percent for R/h = 10.

$$\frac{\partial (A_2 Q_1)}{\partial \alpha_1} + \frac{\partial (A_1 Q_2)}{\partial \alpha_2} + A_1 A_2 (\frac{N_1}{R_1} + \frac{N_2}{R_2})$$

$$+ A_1 A_2 [q_n - I_1 \ddot{w} - I_2 \ddot{w}' - \frac{I_3}{2} \ddot{w}''] = 0$$

$$\frac{\partial (A_2 M_1)}{\partial \alpha_1} + \frac{\partial (A_1 M_{21})}{\partial \alpha_2} + M_{12} \frac{\partial A_1}{\partial \alpha_2} - M_2 \frac{\partial A_2}{\partial \alpha_1}$$

$$- Q_1 A_1 A_2 + A_1 A_2 [m_1 - I_2 \ddot{u} - I_3 \ddot{\beta}_1] = 0$$

$$\frac{\partial (A_1 M_z)}{\partial \alpha_2} + \frac{\partial (A_2 M_{12})}{\partial \alpha_1} + M_{21} \frac{\partial A_2}{\partial \alpha_1} - M_1 \frac{\partial A_1}{\partial \alpha_2}$$

$$- Q_2 A_1 A_2 + A_1 A_2 [m_2 - I_2 \ddot{\upsilon} - I_3 \ddot{\beta}_2] = 0$$

$$\frac{\partial (A_2 S_1)}{\partial \alpha_1} + \frac{\partial (A_1 S_2)}{\partial \alpha_2} + A_1 A_2 (\frac{M_1}{R_1} + \frac{M_2}{R_2}) - A A_1 A_2$$

$$+ A_1 A_2 [m_n - I_2 \ddot{w} - I_3 \ddot{w}' - \frac{I_4}{2} \ddot{w}''] = 0$$

$$\frac{\partial (A_2 T_1)}{\partial \alpha_1} - A_1 A_2 (\frac{P_1}{R_1} + \frac{P_2}{R_2}) - B A_1 A_2$$

$$+ A_1 A_2 [-q_n - \frac{I_3}{2} \ddot{w} - \frac{I_4}{2} \ddot{w}' - \frac{I_5}{4} \ddot{w}''] = 0 \qquad (37)$$

where

$$I_n = \int_{-h}^{h} \sum_{k=1}^{N} \rho_k z^{(i-1)} (1 + \frac{z}{R_1})(1 + \frac{z}{R_2}) dz \qquad (i = 1, 2, ..., 5)$$

(38)

q_z and m_z are the components of external forces and moments acting on the middle plane of shell. These forces and moments are related to the external applied forces q^+_i and q^-_i as:

$$q_i = q^+_i (1 + \frac{h}{2R_1})(1 + \frac{h}{2R_2}) + q^-_i (1 - \frac{h}{2R_1})(1 + \frac{h}{2R_2})$$

$$m_i = \frac{h}{2} [q^+_i (1 + \frac{h}{2R_1})(1 + \frac{h}{2R_2}) - q^-_i (1 - \frac{h}{2R_1})(1 + \frac{h}{2R_2})]$$

(39)

where q^+_i acts on outer surface and q_i acts on inner surface of shell. q_n is selected positive in the opposite direction of normal to the middle plane.

The energy equation based on Lord and Shulman model is considered and the Galerkin finite element technique is used to derive the equilibrium equation of shell.

Cylindrical Shells

Consider a thin cylindrical shell of clamped edges and following geometrical and material properties.

Table (3) Geometry and material properties of cylindrical shell

E 200 GN/m^2	K 50W//m^0k
α 15 * 10^{-6} 1/k	L 0.40m
R 0.1085 m	ν 0.3
ρ 7904 kg/m^3	h 0.002m
c_v 500JkgOk	h_i 10000W /m^2k
h_0 200w/m^2k	T_a 293^0k

The thermal conditions at the ends of

$$\varepsilon^0_1 = \frac{1}{A_1}\frac{\partial u}{\partial \alpha_1} + \frac{\upsilon}{A_1 A_2}\frac{\partial A_1}{\partial \alpha_2} + \frac{w}{R_1}$$

$$\varepsilon^0_2 = \frac{1}{A_2}\frac{\partial u}{\partial \alpha_2} + \frac{\upsilon}{A_1 A_2}\frac{\partial A_2}{\partial \alpha_1} + \frac{w}{R_2}$$

$$k_1 = \frac{1}{A_1}\frac{\partial \beta_1}{\partial \alpha_1} + \frac{\beta_2}{A_1 A_2}\frac{\partial A_1}{\partial \alpha_2}$$

$$k_2 = \frac{1}{A_2}\frac{\partial \beta_2}{\partial \alpha_2} + \frac{\beta_2}{A_1 A_2}\frac{\partial A_2}{\partial \alpha_1}$$

$$\beta^0_1 = \frac{1}{A_1}\frac{\partial \upsilon}{\partial \alpha_1} - \frac{\upsilon}{A_1 A_2}\frac{\partial A_1}{\partial \alpha_2}$$

$$\beta^0_2 = \frac{1}{A_2}\frac{\partial \upsilon}{\partial \alpha_2} - \frac{\upsilon}{A_1 A_2}\frac{\partial A_2}{\partial \alpha_1}$$

$$\beta'_1 = \frac{1}{A_1}\frac{\partial \beta_2}{\partial \alpha_1} - \frac{\beta_1}{A_1 A_2}\frac{\partial A_1}{\partial \alpha_2}$$

$$\beta'_2 = \frac{1}{A_2}\frac{\partial \beta_1}{\partial \alpha_2} - \frac{\beta_2}{A_1 A_2}\frac{\partial A_2}{\partial \alpha_1}$$

$$\mu^0_1 = \frac{1}{A_1}\frac{\partial w}{\partial \alpha_1} - \frac{\upsilon}{R_1} + \beta_1$$

$$\mu'_1 = \frac{1}{A_1}\frac{\partial w'}{\partial \alpha_1} \quad , \quad \mu''_1 = \frac{1}{A_1}\frac{\partial w''}{\partial \alpha_1}$$

$$\mu^0_2 = \frac{1}{A_2}\frac{\partial w}{\partial \alpha_2} - \frac{\upsilon}{R_2} = \beta_1$$

$$\mu'_2 = \frac{1}{A_2}\frac{\partial w'}{\partial \alpha_2} \quad , \quad \mu''_1 = \frac{1}{A_2}\frac{\partial w''}{\partial \alpha_2}$$

$$\varepsilon'_1 = k_1 + \frac{w'}{R_1} \quad , \varepsilon'_2 = k_2 + \frac{w'}{R_2}$$

$$\varepsilon''_1 = \frac{w''}{R_1} \quad , \varepsilon'_2 = k_2 + \frac{w''}{R_2} \qquad (35)$$

These relations are obtained based on Flugge second order shell theory where the term z/R is retained in the equations compared to the unity.

The forces and moments resultants based on the second order shell theory are defined as:

$$< N_1 , N_{12} , Q_1 > = \int_{-h}^{+h} (\sigma_1 , \tau_{12} , \tau_{1n}) \left(1 + \frac{z}{R_2}\right) dz$$

$$< N_2 , N_{21} , Q_2 > = \int_{-h}^{+h} (\sigma_2 , \tau_{12} , \tau_{2n}) \left(1 + \frac{z}{R_1}\right) dz$$

$$< M_1 , M_{12} > = \int_{-h}^{+h} (\sigma_1 , \tau_{12}) \left(1 + \frac{z}{R_2}\right) zdz$$

$$< M_2 , M_{21} > = \int_{-h}^{+h} (\sigma_2 , \tau_{21}) \left(1 + \frac{z}{R_1}\right) zdz$$

$$< S_i , P_i , T_i > = \frac{1}{2}\int_{-h}^{+h} (\tau_{in} , \sigma_n z^2 / 2 , \tau_{in} z^2 / 2) \left(1 + \frac{z}{R_i}\right) dz$$

$$< A, B > = \int_{-h}^{+h} (\sigma_n , \sigma_n z) \left(1 + \frac{z}{R_1}\right) \left(1 + \frac{z}{R_2}\right) dz \qquad i, j = 1, 2$$

(36)

The equations of motion can be obtained using the Hamilton's variational principle. For this general case where the normal stress and strain are included in the governing equations the Hamilton's principle yields the following equilibrium equations

$$\frac{\partial (A_2 N_1)}{\partial \alpha_1} + \frac{\partial (A_1 N_{21})}{\partial \alpha_2} + N_{12}\frac{\partial A_1}{\partial \alpha_2} - N_2\frac{\partial A_2}{\partial \alpha_1} + \frac{Q_1 A_1 A_2}{R_1}$$

$$+ A_1 A_2 [q_1 - I_1 \ddot{u} - I_2 \ddot{\beta}_1] = 0$$

$$\frac{\partial (A_1 N_2)}{\partial \alpha_2} + \frac{\partial (A_2 N_{12})}{\partial \alpha_1} + N_{21}\frac{\partial A_2}{\partial \alpha_1} - N_1\frac{\partial A_1}{\partial \alpha_2}$$

$$+ \frac{Q_1 A_1 A_2}{R_2} + A_1 A_2 [q_2 - I_1 \ddot{\upsilon} - I_2 \ddot{\beta}_2] = 0$$

under axisymmetric loading is discussed by Eslami and et all (1994). In this paper, due to the assumption of long cylindrical shells, the axial displacement is ignored. The coupled thermoelasticity of shells of revolution based on the second order shell theory and the Flugge assumption is given by Eslami et. al. (1995, 1999). The papers referred so far deal with homogeneous isotropic shell materials and consider the classical coupled thermoelasticity assumption. The latter paper (Eslami et. al. 1999) considers the Lord - Shulman assumption in addition to the classical theory. Also, the effect of normal and transverse shear stresses are considered in this paper.

The basic assumption to consider the normal stress and strain in the shell equations requires to relate the displacement components along the principle orthogonal curvilinear coordinate of shell to the displacement components on middle plane as given by the following relations;

$$U(\alpha_1, \alpha_2, z) = u(\alpha_1, \alpha_2) + z\beta_1(\alpha_1, \alpha_2)$$

$$V(\alpha_1, \alpha_2, z) = \upsilon(\alpha_1, \alpha_2) + z\beta_2(\alpha_1, \alpha_2)$$

$$W(\alpha_1, \alpha_2, z) = w(\alpha_1, \alpha_2) + zw'(\alpha_1, \alpha_2) + \frac{z^2}{2} w''(\alpha_1, \alpha_2) \tag{32}$$

where (α_1, α_2, z) are the principle orthogonal curvilinear coordinates of shell and u, υ and w are the middle plane displacements, β_1 and β_2 are rotations of the tangent line to the middle plane along α_1 and α_2 axes, respectively, and w' and w'' represent the non-zero transverse normal strains. Consideration of these two terms violate the third Love's assumption (which states γ_n = 0) and part of the Love's forth assumption (which states ε_n = 0). Furthermore, if the transverse shear strains γ_{1n} and γ_{2n} are not made zero , the rotations β_1 and β_2 are no longer simply described in terms of the middle plane displacements, and the restrictions imposed by the other part of the Love's forth assumption (which state γ_{1n} = γ_{2n} = 0) is removed.

From the general strain - displacement relations in curvilinear coordinate in terms of the covarient derivative:

$$\varepsilon_{ij} = \frac{1}{2}(U_{i}|_{j} + U_{j}|_{i}) \tag{33}$$

The strain - displacement relations for the second order shell theory follows to be:

$$\varepsilon_1 = \frac{1}{1+\frac{z}{R_1}}\left(\varepsilon^0{}_1 + z\varepsilon'_1 + \frac{z^2}{2}\varepsilon''_1\right)$$

$$\varepsilon_2 = \frac{1}{1+\frac{z}{R_2}}\left(\varepsilon^0{}_2 + z\varepsilon'_2 + \frac{z^2}{2}\varepsilon''_2\right)$$

$$\varepsilon_n = w' + zw''$$

$$\gamma_{1n} = \frac{1}{1+\frac{z}{R_1}}\left(\mu^0{}_1 + z\mu'_1 + \frac{z^2}{2}\mu''_1\right)$$

$$\gamma_{2n} = \frac{1}{1+\frac{z}{R_2}}\left(\mu^0{}_2 + z\mu'_2 + \frac{z^2}{2}\mu''_2\right)$$

$$\gamma_{1z} = \frac{1}{1+\frac{z}{R_1}}(\beta^0{}_1 + z\beta'_1) + \frac{1}{1+\frac{z}{R_2}}(\beta^0{}_2 + z\beta'_1) \tag{34}$$

Where the definition of terms given in equations (34) are involves five new material constants that are not present in classical theory.

c) The heat conduction law of G-L theory does not involve flux - rate term. For a material having a center of symmetry at each of its points, this law reduces to the clssical Fourier's law in classical theory. Accordingly, G-L theory can admit second sound even without violating the classical Furier's law. But the formulation of L - S theory itself is based on a modified Fourier's law.

d) The symmetry of k_{ij} is an integral part of the structure of G-L theory. This is not the case with L -S theory.

5 - Coupled Thermoelasticity in Composite Structures

Structural components made of composite materials are frequently expected to operate at elevated or low temperatures. Whilst it is generally accepted that in most thermal stress applications involving metals, the thermoelastic coupling term can be neglected without introducing significant errors, the same may not be true for composite materials. A survey of the literature, however, indicates that very little work has been done in the area of transient thermal stress analysis in composite structures (Woo et. al., 1980; Wang et. al., 1991; Wu 1991). This is partly due to the fact that these problems are usually analytically intactable and recourse is often sought in numerical procedures such as the finite difference and finite element methods (Zukas, 1974; Eslami, 1996, 1997; Shirakawa et.al., 1982; Minagawa, 1988; Change and Shyong, 1994). However, most commercially available finite element packages handle only uncoupled transient heat conduction problems. Furthermore, it is well known that moisture and temperature arising from adverse environmental conditions can significantly affect the mechanical properties of certain composite materials such as carbon fibre - reinforced plastics (Padovan, 1980). The temperature dependence of these properties should be taken into account in a proper analysis of the coupled transient problems.

6 - Coupled Thermoelasticity of Shells

Owing to the mathematical difficulties encountered in the analytical treatment of coupled thermoelasticity problems, mainly due to presence of coupling terms in governing equations, the close form solution of this class of problems are scarce. For complicated structures, such as shells, numerical methods of solution are inevitable. McQuillen and Brull in 1970 studied coupled thermoelasticity of cylindrical shells by using the traditional Galerkin method to obtain the approximate solution. He considered the first order shell theory based on Love assumptions and essentially ignored the normal stress, transverse shear stress and rotary inertia, but assumed a nonlinear temperature distribution across the shell thickness. He concluded that the difference between the coupled and uncoupled solutions are about one percent. Li et. al. (1983), Ghoneim (1986), Sabbaghian (1980), and Eslami and Vahedi in 1992 used the analytical and Galerkin finite element methods and applied to the coupled thermoelasticity of thick cylinders and spheres. The coupled thermoelasticity of cylindrical shells based on first order shell theory for long circular cylindrical shells rate is included among the constitutive variables. A remarkable feature of this theory is that it does not violate the clssical Fourier's law, if the material had a center of symmetry at each point. Moreover, even in the general anisotropic case, the heat conduction equation of the theory does not include the flux - rate term (Ignaczak, 1986). Suhubi (1975), who has formulated this theory independently, has referred to it as the "temperature - rate dependent thermoelasticity". In order to deduce the governing equations of the linear (G - L) theory following relations must be considered (Green and Lindsay, 1972) :

$$\sigma_{ij} = C_{ijkl}\ \varepsilon_{kl} - \beta_{ij}\ (\theta + t_1\ \dot{\theta}) \tag{26}$$

$$\eta = \eta_o + \frac{c_v}{T_o}\ \theta + \frac{c_v\ t_2}{T_o}\ \dot{\theta} + \frac{1}{\rho}\ \beta_{ij}\ u_{i,j} \tag{27}$$

where t_1 and t_2 are the associated time lags. Consideration of the proposed Constitutive laws results into the following governing equations for general anisotropic material.

$$C_{ijkl}\ u_{k,lj} - \beta_{ij}\ \theta_{,j} - t_1\ \beta_{ij}\ \dot{\theta}_{,j} + \rho X_i = \rho \ddot{u}_i \tag{28}$$

$$k_{ij}\ \theta_{,ij} = \rho c_v\ (\dot{\theta} + t_2\ \ddot{\theta}) + \theta_o\ \beta_{ij}\ \dot{u}_{i,j} \tag{29}$$

For isotropic materials these equations reduce to the following system of equations

$$\mu \nabla^2\ u_i + (\lambda + \mu)\ u_{k,ki} - (3\lambda + 2\mu)\alpha\theta_{,i} - t_1\ (3\lambda + 2\mu)\ \alpha\theta_{,i} = \rho u_i \tag{30}$$

$$k\ \nabla^2\ \theta + \rho R = \rho c_v\ \dot{\theta} + \rho c_v\ t_2\ \ddot{\theta} + (3\lambda + 2\mu)\ \alpha\theta_o\ \dot{e}_{kk} \tag{31}$$

During the two past decades much attention has been made toward this theory (Gurtin and Papkin, 1968; Bem, 1982, 1983; Chandrasekhariah and Srikantiah, 1986 (a), 1986 (b) ; Sherief 1992 ; Eslami et . al., 1997).

4 - Fundamental Differences Between the Theories

The two theories, L - S and G - L theories, are thus structurally different from another, and one cannot be obtained as a particular case of the other. While it is the flux - rate term that incorporates the second sound phenomenon into L - S theory, it is the temperature - rate that plays the pivotal role in G - L theory. We have seen that if we drop q_i from the constitutive equation (24), L -S theory reduces to classical theory. If we drop θ from the constitutive equations, G-L theory reduces to classical theroy. In other words, classical theory may be recovered from G - L theory, by setting t_1 and t_2 equal to zero. We can classify the fundamental differences between the theories as:

a) While L - S modifies only the parabolic - type heat transport equation of classical theory to account for second sound, in the process of its formulation G-L theory modifies all the constitutive equations of classical theory. As such , the second sound phenomenon forms an integral part of the infrastructure of G-L theory rather than the one imposed from outside as in L-S theory.

b) Because of the ad hoc approach employed in its formulation, L-S theory does not involve any new material constant other than the thermal relaxation parameter τ. On the other hand, G -L theory, which is formulated on firm thermodynamical grounds, 1973; Mengi and Turhun, 1978; Provest and Tao, 1983; Sadd and Didlake, 1977; Sadd and Cha, 1982) have employed the modified Fourier equation to study some practically relevant problems and have found that in heat transfer problems involving very short time intervals and very high heat flux, the parabolic heat equation (Classical Fourier Law) gives significantly different results than hyperbolic equation (Modified Foruier Law). On the basis of these studies it may generally be inferred that the last term in equation (24) should not be neglected practically when the elapsed time during a transient is less than, say, about 10^{-5} (s) or when the heal flux involved is greater than , say about 10^{-5} W / cm^2 or when the heal flux involved is greater than, say, about 10^{-5} W/cm^2 (Chandrasekharaiah, 1986 (b)). Materials encountered in practice, except for pure liquids, gases, and homogeneous solids, can be described as complex systems made up of solid, liquid, and gas, e.g., porous -capillary bodies, cellular systems suspensions, pastes, etc. The cumulative effect of different transfer mechanisms, for instance, heat conduction, heat transfer by particle - to - particle contact, free convection in a closed space, radiation, etc., is often described by the heat conduction transfer equation. The presence of moisture and its method of binding with a material play an important role in heat transfer. The thermal conductivity determined experimentally is an average value. It should be also noticed that in order to obtain a better fitting of experimental and calculated results thermal conductivity is expressed as a function of temperature, moisture content, etc. We will call these non homogeneous inner structure materials. Luikov (1966) suggested that depending on process intensity, τ can range from 10^{-3} to 10^{+3} seconds. Brazhnikov (1975) gives τ=20 to 30 seconds for meat products. Michalowski (1988) claimed that for the falling drying rate period, the average value of τ is of the order of several thousand seconds. Similar data have been published elsewhere (Antonishyn, 1974; Raspopov, 1967; Todos, 1970 ; Kaminski, 1990). These seem unnaturally high.

3 - 2 - Thermoelasticity with two relaxation times (G - L theory)

The idea of formulating a thermoelasticity theory with second sound without making any assumptions in regard to the form of the heat conduction law was first advanced by Muller (1967, 1971). By considering general constitutive relations for the entropy flux and entropy source, and by making use of a generalized entropy inequality, Muller developed a rigorous nonlinear theory of thermoelasticity, which included temperature-rate among the constitutive variables and consequently adapted second sound. But this theory is so complex and implicit that it is not easily manageable in regard to its applications. McCarthy (1972), has made use of this theory to study a specific application., viz., the propagation of acceleration waves. In 1972, Green and Lindsay formulated a theory of thermoelasticity with second sound, which is closely related to Muller's theory. This theory is simpler and more explicit than Muller's, and is based on an entropy in equality, proposed by Green and Laws (1972). In this theory the classical form of the entropy flux and source are preserved and as in Muller's theory, the temperature -

$$q_i + \tau \frac{\partial q_i}{\partial t} = - k_{ij} \theta_{,j} \quad (24)$$

The relaxation time τ is the short time required to establish a steady - state heat conduction when a temperature gradient is suddenly applied to a solid. With the view of illustrating how L - S theory yields results which are qualitatively different from those of classical thermoelasticity theory in specific problems, Chandrasekharaiah (1986) discussed the Danilovskaysa's problem from the solution of field equations, which is valid for small values of time. It is evident that there occur two waves propagating with different but finite speeds, υ_1 and υ_2. Accordingly, the wave propagating with speed υ_1 is predominantly "elastic" (E wave) and the other is predominantly thermal (T wave) in nature. We verify that $\upsilon_1 < \upsilon_2$ and that as $\tau \to 0$, $\upsilon_1 \to 0$, and $\upsilon_2 \to \infty$. The constant τ, has a definite physical interpretation. It represents the time lag needed to establish the steady state heat conduction in an element of volume when a temperature gradient is suddenly imposed on that element. Chester (1963) has explained a clear physical meaning and has estimated that:

$$\tau = \frac{3K}{\rho c_v \upsilon_p^2} \quad (25)$$

where υ_p is the speed of ordinary sound (first sound). Various authors have determined τ for different types of materials and have found it to range 10^{-10} (s) for gases to 10^{-14} (s) for metals, with the values of τ for liquids and insulators falling within this range (Nettleton, 1960; Chester, 1963; Maurer, 1969). Francis (1972) and Bargman (1974) have given a table of values of τ for some materials. (See Table II)

Table (2)

Material	Acoustic velocity (cm/s)	Thermal relaxation time, τ (s)
Uranium dioxide	$4.06 * 10^5$	$6.6 * 10^{-14}$
Uranium silicate	$3.6 * 10^5$	$1.5 * 10^{-12}$
Liquid He II	$2.52 * 10^4$	$2.0 * 10^{-9}$
Aluminum alloys	$5.07 * 10^5$	$8.0 * 10^{-12}$
Carbon Steels	$5.07 * 10^5$	$1.6 * 10^{-12}$

An experimental procedure for determining the actual value of τ for a given material has been proposed by Mengi and Turhan (1978). The general problem of measuring short - time thermal transport effects has been discussed by Chester (1966). He provides some justification to the fact that the so - called second sound must exist in any solid since all solid continua exhibit phonon - type excitations. In an idealized solid, for example, the thermal energy can be transported by different mechanisms: by quantized electronic excitations, which are called free electrons, and by the quanta of lattice vibrations, which are called phonons. These quanta undergo collisions of a dissipative nature, causing a thermal resistance in the medium. The relaxation time τ is associated with the average communication time between these collision for the commencement of resistive flow. Since τ is found to be very small, many authors (e.g. Boley, 1964; Nowinski, 1978) have argued that the last term in the right hand side of equation (24) may be ignored in many practical problems. But some researchers (eg. Vernotte, 1958 (a), 1958 (b), 1961 ; Baumister and Hamill, 1969, 1971; Jackson and Walker, 1970, 1971; Maurer and Thompson

$$\delta = \frac{(3\lambda + 2\mu)^2 \alpha^2 T_o}{(\lambda + 2\mu) \rho c_v} \quad (23)$$

It is well known that, as long as $\delta << 1$, the coupling term in equation (22) may be neglected whenever inertia effects are small. (See Table I)

Table (1)

Material	Coupling constant(δ)
Aluminium	0.0356
Copper	0.0168
Iron	0.0297
Lead	0.0733

However, coupling does have some interesting effects on problems of wave propagation. Since both the specific heat c_V and the coefficient of thermal expansion α approach zero as the temperature approaches zero, while the ratio c_V / α approaches a constant for each material, the coupling constant approaches zero for very low temperatures (Bargman , 1974).

3 - Second Sound

The classical theory of thermoelasticity allows thermal disturbances to propagate with infinite velocity. To remedy this physically unacceptable situation, modified dynamic thermoelasticity theories have been proposed to allow for so - called "Second Sound" effects, the first sound being the usual sound (wave) and the second sound as the thermal wave propagation. Non classical theories predicting the occurrence of such disturbances are known as theories with finite wave speed or theories with second sound. The concept of the so - called "hyperbolic nature " involving finite speed of thermal disturbance dates as far back as Maxwell (1867). Thermal disturbances of a hyperbolic nature have also been derived using various approaches (Landau, 1941 ; Vernotee, 1958). Most of these approaches are based on the general notation of relaxing the heat flux in classical Fourier heat conduction equation, thereby, introducing a non - Fourier effect. There is also some contradiction to these non - classical propositions in thermoelasticity, with arguments questioning the applicability of finite speeds of propagation in gases to that occurring in solid continua. The aim of this section is to present two models of a linear thermoelastic body in which disturbances propagate with finite wave speed: the L - S model proposed, among other, by Lord and Shulman (1967), and G - L model introduced into the technical literature by Green and Lindsay (1972).

3 - 1 - Thermoelasticity with one relaxation time (L - S Theory)

The L - S theory is obtained as a result of modification of the classical thermoelasticity through a generalization of the heat conduction law. Such generalization was obtained for gases by Maxwell (1867), Maurer (1949) and was proposed for rigid bodies by Cattaneo (1948, 1958). Taking this analytical model into account in the description of a thermoelastic process in a deformable body, by Lord and Shulman (1967), leads to the following system of field equations of L - S theory. When the specific free energy does not depend on the temperature and temperature gradient, the modified Foruier's law, can be proposed as follow:

energy in terms of the temperature change and the principal invariants of strain tensor, it is found that a linear stress-strain-temperature relation (Duhamel-Neuman relation) can be obtained (Nowacki, 1975). For an anisotropic body the field equations using the constitutive equations, reads:

$$k_{ij}\,\theta_{,ij} + \rho R = \rho c_v \dot{\theta} + \theta_o\, \beta_{ij}\, \dot{u}_{i,j} \tag{15}$$

$$C_{ijkl}\; u_{k,lj} - \beta_{ij}\,\theta_{,j} + \rho X_i = \rho \ddot{u}_i \tag{16}$$

Where β_{ij} is the thermal expansion tensor, k_{ij} is the thermal conductivity tensor and $\theta = T - T_0$ is the temperature excess over a reference absolute temperature T_0. Evidently, (15) is the equation of heat transport and (16) is the equation of motion. These equations are coupled for linear classical thermoelasticity theory of homogeneous and anisotropic solids. We see that of the four field equations governing u_i and θ, the three (scalar) equations of motion, given by (16), are of hyperbolic type and the fourth one, the heat transport equation (15), is of parabolic type. Accordingly, the theory predicts a finite speed for elastic disturbances but an infinite speed for thermal disturbances.

2 - 1 - Thermoelastic Coupling

An elastic material corresponds to the postulate that the specific free energy, hence entropy η, stress σ_{ij} and heat flux q_i, depend at most on strain e_{ij}, temperature T and temperature gradient $T_{,i}$. With these assumptions, and the restrictions imposed on them by the second law, one obtains from equation (15) the heat conduction equation. For an isotropic material, linear stress - strain - temperature relation must be read (Parkus, 1968; Nowaki, 1969):

$$\sigma_{ij} = 2\mu\varepsilon_{ij} + \delta_{ij}\,\lambda\varepsilon_{kk} - (3\lambda + 2\mu)\,\alpha\theta \tag{17}$$

Where λ, μ are the Lame' elastic constants and α is the coefficient of linear thermal expansion. Moreover, with the particular assumption of Fourier's law of heat conduction.

$$q_i = -\,k_{ij}\,\theta_{,j} \tag{18}$$

by the additional assumption $|\theta| \leq T_0$ the temperature is governed by:

$$\rho R + k_{ij}\,\theta_{,ij} = \rho c_v\,\dot{\theta} + (3\lambda + 2\mu)\alpha T_o\,\dot{e}_{ij} \tag{19}$$

Where c_V is the specific heat at constant deformation. Equations (18) and (19) are supplemented by the equations of motion (16) and appropriate boundary and initial conditions. Upon substitution, the equation of motion reads

$$(\lambda + \mu)\, u_{j,ij} + \mu u_{i,jj} - \rho \ddot{u}_i = (3\lambda + 2\mu)\,\alpha\theta_{,i} - \rho X_i \tag{20}$$

In particular, in the one - dimensional geometry for the half - space $x \geq 0$, the coupled thermoelastic equations reduce to (body force is absent)

$$\frac{\partial^2 u}{\partial x^2} - \frac{1}{C_L^2}\frac{\partial^2 u}{\partial t^2} = \frac{(3\lambda + 2\mu)\,\alpha}{(\lambda + \mu)}\frac{\partial \theta}{\partial x} \tag{21}$$

$$\kappa_V \frac{\partial^2 \theta}{\partial x^2} - \frac{\partial \theta}{\partial t} = \delta\,\frac{(\lambda + 2\mu)}{(3\lambda + 2\mu)}\frac{\partial^2 u}{\partial x\,\partial t} - \frac{R}{c_V} \tag{22}$$

Where $\kappa_V = k / (\rho c_V)$ is the thermal diffusivity at constant deformation, $C^2_L = (\lambda + 2\mu)/\rho$ is the wave speed, and the coupling constant

1 - 3- Linearized theory

The system of equations (1) - (4) and (14), is highly non - linear, even for an elastic body. Experience shows however, that many wave propagation effects in elastic solids can be adequately described by a linearized theory. For small displacements and displacement gradients, it is not necessary to distinguish between their values at the position of a particle before and after deformation. Then, Green's strain tensor is given by $2E_{ij} = 2\varepsilon_{ij} = u_{i,j} + u_{j,i}$.

2 - Thermoelasticity

The conventional quasi - static approach to thermoelastic problems in the presence of time - dependent temperature fields rests on the assumption that the inertia terms may be neglected in the governing field equations. This hypothesis, which reaches to Duhamel (1837), is known to yield useful results in a wide variety of applications. It is evident, however, that the quality of the approximation must depend both on the size of the relevant intrinsic inertia parameters and on the nature of the time variations inherent in the temperature distribution. If, in particular, the temperature field exhibits sufficiently steep time - gradients, the dynamic effects disregarded in the traditional treatment of the problem may be expected to become significant. Moreover, when the inertia terms are taken into account, the entire character of the problem is altered; the process of transmission of the thermal stresses is then no longer purely diffusive but involves the propagation of elastic waves. The first attempt to examine inertia effects in a transient thermoelastic problem is apparently due for Danilovskaya (1950). Danilovskaya (1952) generalized her previous solution to accommodate convective boundary conditions. A particularly important contribution to the subject under discussion is due to Nowacki (1957 , 1959) who obtained several closed exact solutions to the (uncoupled) three - dimensional thermoelastic equations of motion. The conventional coupled thermoelasticity theory is now proved to be an elegant model for studying coupled effects of elastic and thermal fields and the learned works, for example of Biot (1956), Sternbery and Chakravortry (1959), Chadwick (1960), Boley and Wiener (1960), Boley and Tolins (1962) Francis (1966), Achenbach (1967, 1968), Boley and Hetnarski (1968), Nickell and Sackman (1968), McQuilan and Brull (1970), Carlson (1972), Oden (1975), Dost and Suhubi (1975 (a), 1975 (b)), Bahar and Hetnarski (1968), Dragos (1979), Dhaliwal and Sharief (1980 , 1981), Dhaliwal and Singh (1980), Taningawa and Takeuti (1982), Li et. al. (1983), Takeuti et. al. (1983), Inan (1983), Liu (1984, 1985), Li and Chen (1984), Chang and Wang (1986), Tamma and Namburu (1989), Carter and Brooker (1989), Lee and Sim (1992), Geng (1994), Change et . al. (1994), Eslami et. al. (1992 , 1994, 1995) contain comprehensive accounts of the theory and applications thereof. However, because of its dependence on the classical Fourier's law, this theory, with all merits to its credit, also suffers from the deficiency of allowing infinite heat propagation speed. During the last three decades, attempts have been made to remove this deficiency on various grounds, and generalized versions of the theory have come into existence. We consider these development in following sections. In particular, by employing an expansion of the free

$$\sigma_{ji,j} + \rho X_i = \rho \ddot{u}_i \quad (2)$$

$$\sigma_{ij} = \sigma_{ji} \quad (3)$$

iii) Law of conservation of mass:

$$\rho_d = \frac{\rho}{\sqrt{g}}, \quad g = \det(\sigma_{ij} + 2e_{ij}) \quad (4)$$

iv) Law of conservation of energy (the first law of thermodynamics)

$$\rho(\dot{U} - R) = \sigma_{ij}\dot{e}_{ij} - q_{i,i} \quad (5)$$

In these equations, u_i is the displacement vector, e_{ij} is the Green strain tensor, σ_{ij} is the (Second) Piola -Kirchhoff stress tensor, and X_i, U and R are the body force, the internal energy, and the strength of the internal heat source, respectively, referred to the unit mass. In an arbitrary material element the mass density is ρ (> 0) in the initial (undeformed) state and ρ_d in the deformed state. Further, q_i is the heat flux within the material, referred to the unit area in the initial state. A superposed dot ($\dot{}$) denotes material differentiation with respect to time t and $( )_{,i}$ denotes partial differentiation with respect to the initial rectangular coordinate x_i. The second law of thermodynamic (Clausius - Duhemel inequality), expressing in non - negative entropy production, reads:

$$\rho(T\eta - R) + q_{i,i} - (q_i / T) T_{,i} \geq 0 \quad (6)$$

where η is the specific entropy, and $T \geq 0$ is the absolute temperature. Eliminating R between eqs (5) and (6) and introducing the specific free energy

$$\psi = U - T\eta \quad (7)$$

yields:

$$\rho(\dot{\psi} + \dot{T}\eta) - \sigma_{ij}\dot{e}_{ij} + (q_i / T) T_{,i} \geq 0 \quad (8)$$

It is assumed that ψ, η, q_i and σ_{ij} depend only on e_{ij}, T and $T_{,i}$. Equation (6) and inequality (8) then yield the following relations. (Parkus, 1976; Nowinski, 1978)

$$\sigma_{ij} = \rho\left[\frac{\partial\psi}{\partial e_{ij}}\right] \quad (9)$$

$$\eta = -\frac{\partial\psi}{\partial T} \quad (10)$$

$$\frac{\partial\psi}{\partial T_{,i}} = 0 \quad (11)$$

$$q_{i,i} = \rho(R - T\dot{\eta}) \quad (12)$$

$$q_i T_{,i} \geq 0 \quad (13)$$

1 - 2 - The Constitutive equations

The character of the material is expressed through a set of independent constitutive equations.

$$G_\alpha[\sigma_{ij}, q, \psi, \eta, T, e_{ij}, x_i, t] = 0, \quad (\alpha = 1, \ldots, 11) \quad (14)$$

The equations (9) - (13) and the Constitutive equations (14) must hold for every history of deformation and temperature in the body. Equations (1) - (4) and (14) are 19 equations for 19 unknowns ρ, x_i, σ_{ij}, q_i, e_{ij} (or ψ) , η and T. The inequality (8) places restrictions on the possible constitutive functionals, in particular by the principle of frame - in difference (Bargman, 1974).

Coupled Thermoelasticity With Special Reference to Shells

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Abstract

The paper presents a review of the work done on the dynamic interaction between thermal fields and solid bodies. This reflects the intense interest which has been shown recently in this field, owing to the great practical importance of dynamic effects in modern aeronautics and astronautics, nuclear reactors, high - energy particle accelerators, and its potential importance in cryogenic and laser applications. First, a brief summery of the general theory of continuum thermo - mechanics is given. Second, problems of thermoelasticity, i.e. thermoelastic coupling and second sound are reviewed. Recent works in coupled thermoelasticity in composite structures are then reviewed. An extensive bibliography is included.

1 - Introduction

When the materials are subjected to an external load disturbance, due to their resistance to deformation as well as motion, they transmit mechanical waves. In fact, the velocity of propagation of the disturbance is the square root of the resistance to deformation and inertia. Frequently, the external disturbance may be of thermal nature. For example, sudden heat deposition in a body, giving rise to thermal expansion, will create mechanical waves. The dynamic effect depends on the ratio of two significant times the time t_D characterizing the external (thermal) disturbance, and the mechanical time t_M characterizing the propagation of a disturbance across the body. If t_D and t_M are of the same order of magnitude, dynamic effects are important. They may be neglected if $t_D >> t_M$, where the problem is quasi - static. The dynamic intraction between thermal fields and solid elastic bodies is the topic of this review paper. Intense interest has been shown recently in this field, owing to the great practical importance of dynamic effects in modern aeronautics and astronautics and nuclear reactors (Bargman 1974). For convenient and better reference, the basic equations of thermo - mechanics are presented (Truesdell, 1960, 1965; Carlson, 1972).

1 - 1 - The Principals of Conservation and Irreversibility

The thermoelasticity theory is based on the following fundamental equations:

i) Kinematic relations:

$$2e_{ij} = u_{i,j} + u_{j,i} + u_{m,i} u_{m,j} \tag{1}$$

ii) Equations of balance of momenta: