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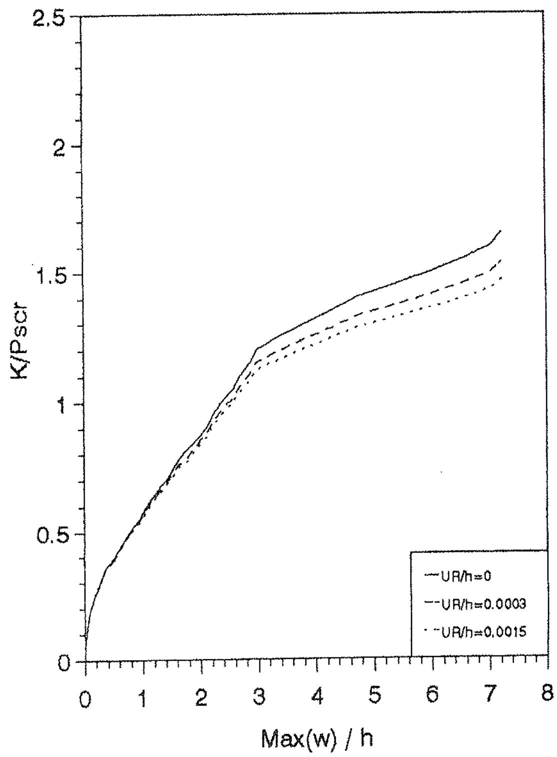

Fig. (4) Effect of initial imperfections on the dynamic buckling stress ratio forA =4.

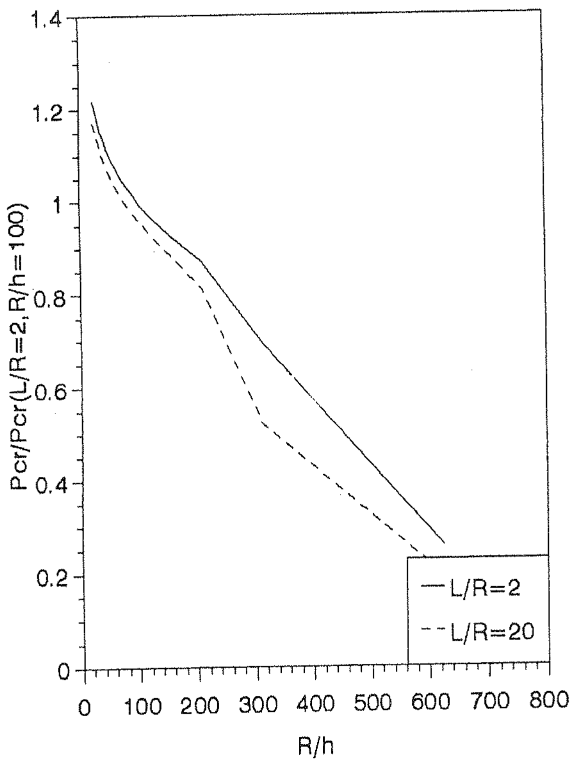

Fig. (5) Effect of shell length, radius and thickness on the dynamic buckling load forA = 4.

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The duration of pulse is chosen as $\Delta t = A.l/[E/(1-\nu^2)]^{0.5}$ and ruled by A. Fig. (2) illustrates the results for two different values of A. It can be seen that as the pulse duration decreases, the critical value of K increases. Figs (3) and (4) illustrate the influence of initial imperfections on the buckling state for $A = 2.5$ and $A = 4$, respectively. The initial imperfections of the shell are chosen to be:

$$w_0 = a_0 \cdot h \cdot \cos\frac{2\pi x}{l_x}\cos\frac{2\pi\theta}{l_\theta} \cdot \qquad (44)$$

where, according to reference [15], the following form can be adopted for a_0:

$$a_0 = \frac{U}{\pi^2 h^2} l_x^{1.5}\, l_\theta^{1.5} \qquad (45)$$

where, l_x and l_θ are the axial and circumferential half - wave lengths. Experimental results show that the most amplified mode is an axisymmetric mode with [6]:

$$l_x = \sqrt{2 l_0} \qquad l_\theta = 3 l_x \qquad (46)$$

and

$$l_\theta = \pi\sqrt{Rh}\,/\,[12(1-\nu^2)]^{\frac{1}{4}} \qquad (47)$$

is the axisymmetric classical static buckling half-wavelength. In Figs (3) and (4), the initial deviations are presented by the unevenness factor *UR/h*, which is a dimensionless parameter. The influence of the initial imperfections in decreasing the buckling load is readily observed. In Fig. (5), effect of shell length, radius and thickness on the buckling load are investigated simultaneously. For this purpose two curves, one represents a shallow shell with $(l/R = 2)$ and the other represents a medium length (or relatively long) shell with $(l/R = 20)$ are illustrated. In this figure, dynamic buckling load ratio Pcr/Pcr $(1/R = 2,\ R/h = 100)$ is plotted against the dimensionless ratio R/h, for $A = 4$.

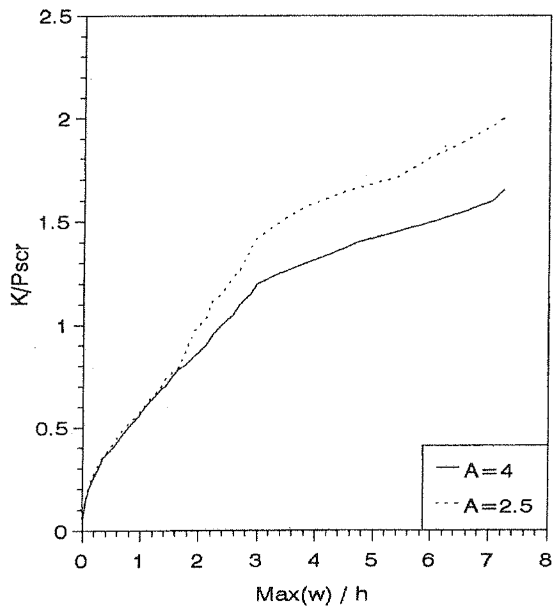

Fig. (2) Dynamic to static buckling stress ratio for different.

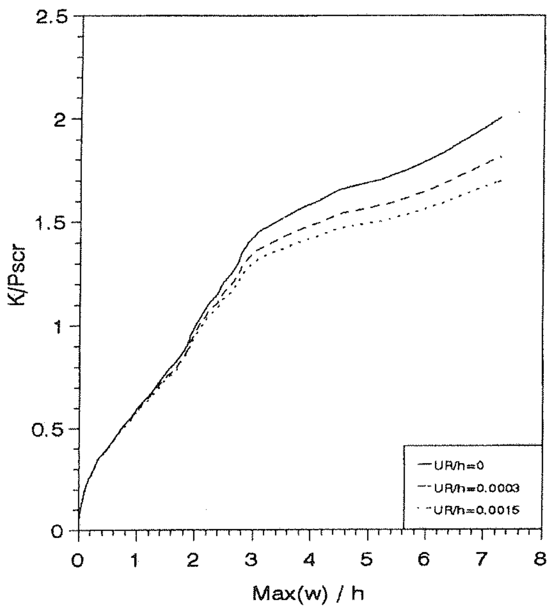

Fig. (3) Effect of initial imperfections on the dynamic buckling stress ratio forA = 2.5.

11-The increments of displacements and strains are added to the displacements and strains of the end of the preceding time interval to obtain their values at the end of the present time interval.

12-Based on the effective strains obtained, the effective stresses are determined from eqn (33).

13-The foregoing results are substituted in eqn (29) to yield the increments of stress resultants. Adding these increments to the values obtained in the preceding time interval, gives their values at the end of the present time interval.

14-The above procedure is continued until the final time of the loading is reached.

Buckling Criterion

The generalized concept of dynamic buckling proposed by Budiansky [2] is used in this paper. This concept is associated with the dynamic buckling of a structure where small changes in the magnitude of loading lead to large changes in the structure response. For this purpose, the response of the shell to impact loads of various amplitudes is studied. Since the response of the shell, subjected to an impact load of amplitude K, depends both on time and space, it is necessary to characterize this response by a specific value. To this end, using the solution procedure presented in the previous section, the extremum value of w at each time increment ($|w_{max}|$) is determined. Then, among the results obtained for $|w_{max}|$ in the loading time duration, the maximum value is specified ($\max_{t,c,\theta} |\omega|$). In this way, the maximum value of $|\omega|$ during application of loading is determined. Based on these results, the dimensionless ratio ($\max_{t,x,q} |w|$) /h is plotted versus K/P_{scr} that is another dimensionless ratio. Here, K is the amplitude of the applied load and P_{scr} is the classical static buckling load. According to the buckling criterion stated above, abrupt reduction in slope of the mentioned curve (minimum slope) indicates a dynamic buckling state.

Results and Discussions

The presented analysis is applied to investigate the dynamic response of a simply supported cylindrical shell whose geometrical specifications are:

$$L/R = 2 \qquad R/h = 100 \qquad h = 3[\mathrm{mm}] \tag{40}$$

The material properties are considered to be:

$$E = 2 \times 10^5\ [\mathrm{MPa}] \qquad \nu = 0.3 \qquad \sigma_{yp} = 250\ [\mathrm{MPa}]$$
$$m_0 = 4 \qquad \alpha_0 = 1.31479 \times 10^{-12} \qquad \rho = 7830\ [\mathrm{kg/m^3}] \tag{41}$$

The step load represents the worst type of load - time function. For this reason , the dynamic stability of the shell is investigated for the step loading. To have a continuous increased load, the starting and ending regions of the load-time curve must be very steep lines. Also, to improve the accuracy, the time duration of load is divided into a large number of load steps. For this special case, the classical static buckling stress can be determined from the following equation [21]:

$$\sigma_{st.} = \frac{E_s}{\sqrt{3(1-\nu_p^2)}} \cdot \left(\frac{h}{R}\right) \cdot \left(\frac{E_T}{E_s}\right)^{\frac{1}{2}} \tag{42}$$

so that:

$$\frac{K}{P_{scr}} = \frac{\sigma_{Dyn.}}{\sigma_{st.}} = \frac{\sigma_{Dyn.}}{\frac{E_s}{\sqrt{3(1-\nu_p^2)}} \cdot \left(\frac{h}{R}\right) \cdot \left(\frac{E_T}{E_s}\right)^{\frac{1}{2}}} \tag{43}$$

, 2 , ... This is accomplished by employing simultaneously the finite difference method with respect to spatial coordinates x and θ , and the fourth order Runge- Kutta method with respect to time. The results are the calculated increment values at the end of each time interval. To employ the finite difference method, the shell must be discretized by introducing a grid of nodes in the axial and circumferential directions. To insure stability of the results, the distances between nodes must be very small compared with the axial and tangential buckling wave lengths. This can be checked after the results for w and its variations in the axial and circumferential directions are determined. The initial and boundary conditions necessary to accomplish the numerical solution of eqn (23) are as follows: Initial conditions:

$$\begin{aligned} &\delta U_k^{t_i} = 0 \qquad (0 \le x \le l\ ,\ 0 \le \theta \le 2\pi) \\ &U_{k,t}^{t_i}(t_{i-1} \le t \le t_i) = U_{k,t}^{t_i}(t_i \le t \le t_{i+1}) \end{aligned} \tag{36}$$

where

$$U_1 \equiv u \qquad U_2 \equiv \upsilon \qquad U_3 \equiv w \tag{37}$$

The governing boundary conditions for simply supported edges are:

$$\begin{aligned} &U_k = \delta U_k = 0 \qquad M_x = \delta M_x = 0 \qquad \delta N_x = -[\bar{N}(t) - \bar{N}(t_i)] \\ &N_x = -\bar{N}(t) \qquad (0 \le x \le l,\ t_i \le t \le t_{i+1}) \end{aligned} \tag{38}$$

in which, $\bar{N}$ is the external axial load. For clamped edges we have:

$$\delta U_i = \delta U_{3,\chi} = 0 \qquad N_\chi = -\bar{N}(t) \qquad \delta N_\chi = -[\bar{N}(t) - \bar{N}(t_i)] \tag{39}$$

Therefore, the solution algorithm can be summarized in the following steps:

1- The distances between the nodes in the axial and tangential directions are chosen.

2-The initial imperfections distribution is specified.

3-Time duration of loading is divided into a number of loading steps.

4-The external load distribution is defined. In general, the loading may be asymmetric.

5-After application of the load increment , the strain and rotation expressions are calculated from eqns (7) and (35), using the finite difference method with respect to χ and θ. Then, using eqn (30), the effective strain increment is determined.

6-E_T and E_s modulii corresponding to present effective stress are found from eqn (34) and substituted in eqns (21) and (29).

7-After the foregoing evaluations have been accomplished, the remaining terms of eqn (23) are substituted by finite difference expressions. In this case, all terms are determined for the beginning of the present time interval.

8-The above substitutions, reduce eqn (23) to three second order ordinary differential equation accompanied by the initial and boundary equations [eqns (36), (37) and (38)] that must be solved by the fourth order Runge-Kutta method to obtain the increment values at the end of the time interval.

9-If necessary , to obtain higher accuracy, the preceding steps can be repeated from step (6) until the successive results of the same time interval are sufficiently close.

10-After the displacement increments are determined, effective strain increments as well as strain components are calculated from eqns (7), (35) and (30).

$$\delta N_{x\theta} = \frac{E_s h}{2(1+\nu_p)}\,\delta\gamma_{x\theta m} + \frac{1}{2(1+\nu_p)}\,.\,N_{x\theta}\,(E_T - E_s)\,.\,\frac{\delta\epsilon_e}{\epsilon_e}$$

$$\delta M_x = \frac{E_s h^3}{12(1-\nu_p^2)}(\delta\kappa_x + \nu_p\delta\kappa_\theta) + \frac{M_x}{\epsilon_e}\left(\frac{E_T}{E_s} - 1\right).\,\delta\epsilon_e$$

$$\delta M_\theta = \frac{E_s h^3}{12(1-\nu_p^2)}(\delta\kappa_\theta + \nu_p\delta\kappa_x) + \frac{M_\theta}{\epsilon_e}\left(\frac{E_T}{E_s} - 1\right).\,\delta\epsilon_e$$

$$\delta M_{x\theta} = \frac{E_s h^3}{12(1+\nu_p)}\,\delta\kappa_{x\theta} + \frac{1}{2(1+\nu_p)}\,.\,M_{x\theta}\,(E_T - E_s).\,\frac{\delta\epsilon_e}{\epsilon_e} \tag{29}$$

The effective strain increment, $\delta\epsilon e$, can be calculated from the following relation:

$$\delta\epsilon_e = \frac{2}{3}[(\delta\epsilon_x)^2 + (\delta\epsilon_\theta)^2 - (\delta\epsilon_x\,.\,\delta\epsilon_\theta) + 3\,(\delta\gamma_{x\theta})^2]^{\frac{1}{2}} \tag{30}$$

Determination of E_T and E_S Modulii

The total strain can be considered to be the sum of elastic, plastic and creep strains. If the time duration of the loading is short, the creep strain may be neglected and thus:

$$\epsilon = \epsilon_E + \epsilon_P \tag{31}$$

The following state equation may be considered because of its simplicity and good accuracy (specially for steels):

$$\epsilon = \frac{1}{E}[\sigma + \alpha_0(\sigma - \sigma_{yp})^{m_0}] \tag{32}$$

where, α_0 and m_0 are material constants and σ_{yp} is the yield stress. This equation can be extended to two and three dimensional state of stress, in the following form :

$$\epsilon_e = \frac{1}{E}[\sigma_e + \alpha_0(\sigma_e - \sigma_{yp})^{m_0}] = H(\sigma_e) \tag{33}$$

The function $\epsilon_e = H(\sigma_e)$ can be determined from the uniaxial stress-strain diagram of the material, conveniently. From eqns (25) and (33), the E_T and E_s modulii are readily found to be:

$$E_T = \frac{E}{[1 + \alpha_0\,.m_0(\sigma_e - \sigma_{yp})^{m_0 - 1}]}$$

$$E_s = \frac{E}{[1 + \alpha_0(\sigma_e - \sigma_{yp})^{m_0} / \sigma_e]} \tag{34}$$

Effect of Initial Imperfections

Examination of the strain- displacement equations (7) shows that only the κ and β expressions are changed in this case. The new rotation expressions are:

$$\beta_x = (w + w_0)_{,x}$$

$$\beta_\theta = \frac{\upsilon + (w + w_0)_{,\theta}}{R} \tag{35}$$

As the moment resultants are dependent on the curvature variations only, they are not affected by the initial geometrical imperfections. On the other hand, the right sides of the equations of motion include the displacement derivatives with respect to time. Thus, w_0 will not appear in these terms. The work of external pressure also depends on the value of w and not w_0. So that the general form of eqn (23) is valid in this case, too, provided that the values of β and M expressions are changed corresponding to eqns (35) and (7).

Solution Algorithm

Substituting eqns (7) and (35) into eqns (30), (29) and (25), three nonlinear partial differential equations in terms of displacement increments and time are obtained. These equations are solved successively for each time incerement $(t_i < t < t_{i+1})$, i = 0 , 1 the displacement components, u, v, and w will change and the new displacement components become:

$$\bar{u} = u + \delta u$$

$$\bar{\upsilon} = \upsilon + \delta\upsilon$$

$$\bar{w} = w + \delta w \qquad (22)$$

due to these changes in displacements, the stress and moment resultants will also change, so that the governing equations (16) will have the following form:

$$R\,.\,(N_x + \delta N_x)_{,x} + (N_{x\theta} + \delta N_{x\theta})_{,\theta} = R\rho h\,(u + \delta u)_{,tt}$$

$$R\,.\,(N_{x\theta} + \delta N_{x\theta})_{,x} + (N_\theta + \delta N_\theta)_{,\theta} + \frac{1}{R}(M_\theta + \delta M_\theta)_{,\theta}$$
$$+ (M_{x\theta} + \delta M_{x\theta})_{,x} - (N_\theta + \delta N_\theta)(\beta_\theta + \delta\beta_\theta) - (N_{x\theta} + \delta N_{x\theta})(\beta_x + \delta\beta_x)$$
$$- \bar{p}\,[\upsilon + \delta\upsilon) - (w + \delta w)_{,\theta}] = R\rho h\,(\upsilon + \delta\upsilon)_{,tt}$$

$$R\,.\,(M_x + \delta M_x)_{,xx} + 2(M_{x\theta} + \delta M_{x\theta})_{,x\theta} + \frac{1}{R}(M_\theta + \delta M_\theta)_{,\theta\theta}$$
$$- (N_\theta + \delta N_\theta) - \{R.\,(N_x + \delta N_x).\,(\beta_x + \delta\beta_x)_{,x} + (N_{x\theta} + \delta N_{x\theta})$$
$$[R\,.(\beta_\theta + \delta\beta_\theta)_{,x} + (\beta_x + \delta\beta_x)_{,\theta}] + (N_\theta + \delta N_\theta).\,(\beta_\theta + \delta\beta_\theta)_{,\theta}\}$$
$$- \bar{p}\,R[(\upsilon + \delta\upsilon)_{,\theta} + (w + \delta w)] = R\rho h\,(w + \delta w)_{,tt} \qquad (23)$$

eqn (23) is applicable to cylindrical shells under compression or external pressure or combination of these loadings, regardless of the shell length.

Establishment of the Stress-Strain Equations

The incremental changes in stress components due to incremental changes in loading, are found from eqn (19) and are:

$$\delta\sigma_x = \frac{E_s}{1 - \nu_p^2}(\delta\epsilon_x + \nu_p\,\delta\epsilon_\theta) + \frac{\delta E_s}{1 - \nu_p^2}(\epsilon_x + \nu_p\,\epsilon_\theta)$$

$$\delta\sigma_\theta = \frac{E_s}{1 - \nu_p^2}(\delta\epsilon_\theta + \nu_p\,\delta\epsilon_x) + \frac{\delta E_s}{1 - \nu_p^2}(\epsilon_\theta + \nu_p\,\epsilon_x)$$

$$\delta\tau_{x\theta} = G_s\,.\,\delta\gamma_{x\theta} + \delta G_s\,.\,\gamma_{x\theta} \qquad (24)$$

Assume that the uniaxial tensile stress - strain curve can be extended to two and three dimensional states of stress by means of the effective stress and strain concepts. According to the definition, the secant and tangent modulii are related to the effective stresses and strains as follows:

$$E_s = \sigma_e / \epsilon_e$$

$$E_T = \delta\sigma_e / \delta\epsilon_e \qquad (25)$$

Thus, from eqn (25), it is easily seen that:

$$\delta E_s = \delta\left(\frac{\sigma_e}{\epsilon_e}\right) = \frac{\delta\sigma_e}{\epsilon_e} - \frac{\sigma_e}{\epsilon_e^2}\,\delta\epsilon_e = \left(\frac{\delta\sigma_e}{\delta\epsilon_e} - \frac{\sigma_e}{\epsilon_e}\right).\,\frac{\delta\epsilon_e}{\epsilon_e} \qquad (26)$$

or

$$\delta E_s = (E_T - E_s)\,.\,\frac{\delta\epsilon_e}{\epsilon_e} \qquad (27):$$

and eqn (24) can be rewritten as:

$$\delta\sigma_x = \frac{E_s}{1 - \nu_p^2}(\delta\epsilon_x + \nu_p\,\delta\epsilon_\theta) + \frac{\sigma_x}{E_s\,\epsilon_e}(E_T - E_s)\,.\,\delta\epsilon_e$$

$$\delta\sigma_\theta = \frac{E_s}{1 - \nu_p^2}(\delta\epsilon_\theta + \nu_p\,\delta\epsilon_x) + \frac{\sigma_\theta}{E_s\,\epsilon_e}(E_T - E_s)\,.\,\delta\epsilon_e$$

$$\delta\tau_{x\theta} = \frac{E_s}{2\,(1 + \nu_p)}\,.\,\delta\gamma_{x\theta} + \frac{1}{2\,(1 + \nu_p)}\,.\,\gamma_{x\theta}\,(E_T - E_s)\,.\,\frac{\delta\epsilon_e}{\epsilon_e} \qquad (28)$$

Substitution of the above equations into eqn (20), gives:

$$\delta N_x = \frac{E_s h}{1 - \nu_p^2}(\delta\epsilon_{xm} + \nu_p\,\delta\epsilon_{\theta m}) + \frac{N_x}{\epsilon_e}\left(\frac{E_T}{E_s} - 1\right).\,\delta\epsilon_e$$

$$\delta N_\theta = \frac{E_s h}{1 - \nu_p^2}(\delta\epsilon_{\theta m} + \nu_p\,\delta\epsilon_{xm}) + \frac{N_\theta}{\epsilon_e}\left(\frac{E_T}{E_s} - 1\right).\,\delta\epsilon_e$$

curvature expressions from eqn (7) and employing Euler equations and eqn (9), the governing equations become:

$$R\,.\,N_{x,x} + N_{x\theta,\theta} = R\rho h u_{,tt}$$

$$R\,.\,N_{x\theta,x} + N_{\theta,\theta} + \frac{1}{R}M_{\theta,\theta} + M_{x\theta,x} - (N_\theta \beta_\theta + N_{x\theta}\beta_x) - p\,(\upsilon - w_{,\theta}) = R\rho h \upsilon_{,tt}$$

$$R\,.\,M_{x,xx} + 2M_{x\theta,x\theta} + \frac{1}{R}M_{\theta,\theta\theta} - N_\theta - [R\,.\,N_x\,.\,\beta_{x,x}$$

$$+ N_{x\theta}(R\,.\beta_{\theta,x} + \beta_{x,\theta}) + N_\theta\,.\,\beta_{\theta,\theta}] - p.\,R - p\,(\upsilon_{,\theta} + w) = R\rho h w_{,tt} \quad (16)$$

These are the equilibrium equations of a cylindrical shell under the action of external fluid pressure. In the case of dead-loading external pressure, the term $-\,p\,(\upsilon_{,\theta} - \omega)$ is omitted from the third of eqns (16) and the term $-\,p\,(\upsilon - \omega_{,\theta})$ is omitted from the second equation.

Stress-Strain Relations

According to secant modulus theory, beyond the elastic region, the elasticity modulus E can be replaced by the secant modulus E_s. Therefore, the modified Hook's relations can be generalized to the plastic state, as [21]:

$$\in_x = \frac{1}{E_s}(\sigma_x - \nu_p \sigma_\theta) + \alpha T$$

$$\in_\theta = \frac{1}{E_s}(\sigma_\theta - \nu_p \sigma_x) + \alpha T$$

$$\gamma_{x\theta} = \frac{\tau_{x\theta}}{G_s} = \frac{2(1+\nu_p)}{E_s}\tau_{x\theta} \quad (17)$$

in which, υ_p is the plastic Poisson ratio [21]:

$$\nu_p = 0.5 - \frac{E_s}{E}(0.5 - \nu) \quad (18)$$

Therefore, from eqn (17), we obtain:

$$\sigma_x = \frac{E_s}{1-\nu_p^2}(\in_x + \nu_p \in_\theta) - \frac{E_s\,.\,\alpha T}{1-\nu_p}$$

$$\sigma_\theta = \frac{E_s}{1-\nu_p^2}(\in_\theta + \nu_p \in_x) - \frac{E_s\,.\,\alpha T}{1-\nu_p}$$

$$\tau_{x\theta} = \frac{E_s}{2(1+\nu_p)}\gamma_{x\theta} \quad (19)$$

The stress and moment resultants, based on the first order shell theory, are given by:

$$N_{ij} = \int_{-h/2}^{h/2}\sigma_{ij}\,.\,dz$$

$$M_{ij} = \int_{-h/2}^{h/2}\sigma_{ij}\,.\,z\,.dz \quad (20)$$

thus, after substituting eqns (19) and (6) into eqn (20) and assuming that there is no temperature gradient in the shell, the resulting equations will be:

$$N_x = \frac{E_s h}{1-\nu_p^2}(\in_{xm} + \nu_p \in_{\theta m})$$

$$N_\theta = \frac{E_s h}{1-\nu_p^2}(\in_{\theta m} + \nu_p \in_{xm})$$

$$N_{x\theta} = \frac{E_s h}{2(1+\nu_p)}\gamma_{x\theta}$$

$$M_x = \frac{E_s h^3}{12(1-\nu^2_p)}(\kappa_x + \nu_p \kappa_\theta)$$

$$M_\theta = \frac{E_s h^3}{12(1-\nu^2_p)}(\kappa_\theta + \nu_p \kappa_x)$$

$$M_{x\theta} = \frac{E_s h^3}{12(1+\nu_p)}\kappa_{x\theta}$$

The Basic Equations

The general governing equations were presented in eqn (16). Now, the applied loads at time t, are incermented. As a result,

$$\beta_x = w_{,x}$$

$$\beta_\theta = \frac{\upsilon + w_{,\theta}}{R} \tag{8}$$

The forces and moments per unit length of the shell in normal and shear directions are related to the stress components through the following relations:

$$N_x = C(\epsilon_{xm} + \nu\epsilon_{\theta m})$$

$$M_x = D(\kappa_x + \nu\kappa_\theta)$$

$$N_\theta = C(\epsilon_{\theta m} + \nu\epsilon_{xm})$$

$$M_\theta = D(\kappa_\theta + \nu\kappa_x)$$

$$N_{x\theta} = \frac{1-\nu}{2} C\gamma_{x\theta m})$$

$$M_{x\theta} = (1-\nu) D\kappa_{x\theta} \tag{9}$$

where:

$$C = \frac{Eh}{1-\nu^2}$$

$$D = \frac{Eh^3}{12(1-\nu^2)}$$

Substituting eqns (6) and (9) into eqn (5) and dividing the strain energy into membrane energy U_m, and bending energy U_b, gives:

$$U = U_m + U_b$$

where:

$$U_m = \frac{RC}{2}\iint(\epsilon^2_{xm} + \epsilon^2_{\theta m} + 2\nu\epsilon_{xm}\epsilon_{\theta m} + \frac{1-\nu}{2}\gamma^2_{x\theta m})dx\,.\,d\theta$$

$$U_b = \frac{RD}{2}\iint(\kappa^2_x + \kappa^2_\theta + 2\nu\kappa_x\kappa_\theta + 2(1-\nu)\kappa^2_{x\theta})dx\,.\,d\theta \tag{10}$$

The potential energy due to external pressure can be written as:

$$\Omega = R\iint p\,.\,w dx\, d\theta \tag{11}$$

If the external pressure is provided by means of external fluid, 9 different expressions will be obtained [20]:

$$\Omega = pR\iint[w + \frac{1}{2R}(\upsilon^2 - \upsilon.w_{,\theta} + \upsilon_{,\theta}\,.\,w + w^2)]\,dx\,d\theta \tag{12}$$

The kinetic energy of the shell, neglecting the rotary inertia terms due to the thin shell assumption, has the form:

$$T = \frac{1}{2}\iint\rho Rh\,(u^2_{,t} + \upsilon^2_{,t} + w^2_{,t})\,dx\,d\theta \tag{13}$$

Thus, the Lagrangian function of the shell is defined as:

$$I = \int_{t_1}^{t_2}\int_0^{2\pi}\int_0^{l} L(x, \theta, u, u_{,x}, u_{,\theta}, u_{,t}, \upsilon, \upsilon_{,\chi}, \upsilon_{,\theta}, \upsilon_{,t}$$
$$, w, w_{,x}, w_{,\theta}, w_{,xx}, w_{,x\theta}, w_{,\theta\theta}, w_{,t})\,dx\,.\,d\theta\,.\,dt \tag{14}$$

or in the expanded form, is:

$$I = \int_{t_1}^{t_2}\int_0^{2\pi}\int_0^{l}\{\frac{RC}{2}(\epsilon^2_{xm} + \epsilon^2_{\theta m} + 2\nu\epsilon_{xm}\epsilon_{\theta m} + \frac{1-\nu}{2}\gamma^2_{x\theta m})$$
$$+ \frac{RD}{2}[\kappa^2_x + \kappa^2_\theta + 2\nu\kappa_x\kappa_\theta + 2(1-\nu)\kappa^2_{x\theta}]$$
$$+ R\,.\,p\,[w + \frac{1}{2R}(\upsilon^2 - \upsilon.\,w_{,\theta} + \upsilon_{,\theta}\,.w + w^2)]$$
$$- \frac{1}{2}\rho Rh\,(u^2_{,t} + \upsilon^2_{,t} + w^2_{,t})\}\,dx.d\theta.dt \tag{15}$$

According to Hamilton's principle, and upon application of the Euler equations, the stationary value of the functional appeared in eqn (14) leads to the equations of motion of shell. Thus, after substituting strain and

Hamilton's Principle - Nonlinear Equilibrium Equations

Hamilton's principle for the conservative systems (neglecting the internal dissipation of energy), can be written in the following form:

$$\delta I = \int_{t_1}^{t_2} \delta L . dt = 0 \tag{1}$$

The Lagrangian function L can be expressed in terms of the total potential energy V and the total kinetic energy of the system T as follows:

$$L = T - V \tag{2}$$

The total potential energy of the system is the sum of strain energy U and potential energy due to external loads , so that:

$$V = U + \Omega \tag{3}$$

The strain energy of an elastic shell whose geometric parameters are shown in Fig. (1) can be written as:

$$U = \frac{1}{2}\iiint (\sigma_x \epsilon_x + \sigma_\theta \epsilon_\theta + \tau_{x\theta} \gamma_{x\theta} + \tau_{\theta z} \gamma_{\theta z} + \tau_{xz} \gamma_{xz}) R . dx. d\theta . dz \tag{4}$$

and assuming plane -stress state for thin shells, leads to:

$$U = \frac{1}{2}\iiint (\sigma_x \epsilon_x + \sigma_\theta \epsilon_\theta + \tau_{x\theta} \gamma_{x\theta}) R. dx. d\theta . dz \tag{5}$$

Using Love's assumptions, the strain-displacement relations can be expressed as:

$$\epsilon_x = \epsilon_{xm} - z . \kappa_x$$

$$\epsilon_\theta = \epsilon_{\theta m} - z . \kappa_\theta$$

$$\gamma_{x\theta} = \gamma_{x\theta m} - 2z . \kappa_{x\theta} \tag{6}$$

where k is curvature of the middle surface and the subscript m refers to the strains on the middle surface of the shell. According to Sander's assumptions [19], the general strain -displacement relations can be simplified to give the following terms for the strains on the middle surface and the curvatures in terms of the displacements:

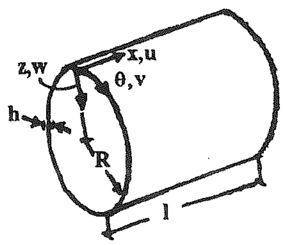

Figure (1) Coordinate axes and displacement components used in the present formulations.

$$\epsilon_{xm} = u_{,x} + \frac{1}{2}(w_{,x})^2 = u_{,x} + \frac{1}{2}\beta_x^2$$

$$\epsilon_{\theta m} = \frac{1}{R}(\upsilon_{,\theta} - w) + \frac{1}{2R^2}(\upsilon + w_{,\theta})^2 = \frac{1}{R}(\upsilon_{,\theta} - w)^2 + \frac{1}{2}\beta_\theta^2$$

$$\gamma_{x\theta m} = (\frac{1}{R}u_{,\theta} + \upsilon_{,x}) + \frac{1}{R} w_{,x}(\upsilon + w_{,\theta}) = (\frac{1}{R}u_{,\theta} + \upsilon_{,x}) + \beta_x \beta_\theta$$

$$\kappa_x = w_{,xx} = \beta_{x,x}$$

$$\kappa_\theta = = \frac{1}{R^2}(\upsilon_{,\theta} + w_{,\theta\theta}) = \frac{\beta_{\theta,\theta}}{R}$$

$$\kappa_{x\theta} = \frac{1}{2R}(\upsilon_{,x} + 2w_{,\theta x}) = \frac{1}{2}\beta_{\theta,x} \tag{7}$$

where β is the rotation of the cross section:

euation to determine the boundaries of both principal and combination parametric resonances for shell structures subjected to periodic excitation [9].

The foregoing studies were based on the analytical approach. In parallel, numerical methods were also employed. For example Gilat et al. [10], used the finite difference method with respect to the axial coordinate and the Runge-Kutta method with respect to time. Liaw and Yang [11, 12] used a doubly curved quadrilateral imperfect thin shell finite element in their analysis and in reference [13] the finite element method in connection with Liapanov's definition of stability is used. Simitses [14] summarized the concepts and methodologies used in estimating critical conditions for dynamic buckling of suddenly loaded structures and classified them in groups.

The often used equations of motion of cylindrical shells, are generally based on the works of Donnell [15] . In deriving these equations, it is comonly assumed that: the shell is short, no difference between the works done by dead loading external pressure and external fluid pressure is distinguished, terms containing product of rotations and shear forces in the moment equations are negligible compared to the remaining terms and some of the in - plane inertia forces and the rotary inertia can be ignored. Furthermore, in many researches it is assumed that the buckling occurs in axisymmetric manner. Therefore , using the above assumptions, the Donnell equations of motion are further simplified.

In the present paper, the general nonlinear equations of motion of a thin cylindrical shell subjected to axial compression or external pressure (deloading external pressure or fluid pressure) or combined loading are derived by application of Hamilton's principle. The simplifications commonly used in stability analysis of cylindrical shells are disregarded in this study. The presented equations are valid for large deflections and moderate strains, so that the analysis can be applied slightly beyond the buckling state.

Elasto-plastic analysis is usually accomplished by incremental methods. The proposed method is initialized by establishement of the state equation. To this end, a state equation with a good accuracy is introduced. The procedure of analysis is based on previous works of the authors [16 , 17, 18]. According to the established state equation, the stress and moment resultants are determined and substituted into the perturbed shell equations. These equations are then modified to take into account the influence of the initial deviations.

The solution to the problem is obtained progressively by employing the finite difference method with respect to the circumferential and longitudinal coordinates and the fourth order Runge-Kutta method with respect to time.For determination of the dynamic buckling state, the buckling criterion established by Budiansky [2] , is adopted.

In addition to the accuracy of the basic equations, the present method is rapidly convergent. The results of the present paper are more accurate compared to the described references. The advantage and generality of the present method is easily noticed when the nonlinearity of the equations of motion, nonlinearity of the material properties , nonlinearity of the strain-displacement equations and the possibility of asymmetric or local buckling are considered.

Elastic - Plastic Dynamic Buckling Analysis of Imperfect Cylindrical Shells

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Abstract

Based on the Hamilton's principle, the nonlinear dynamic equilibrium equations of a cylindrical shell subjected to axial loading or external pressure (dead loading or external fluid pressure), considering the effect of the initial imperfections, are derived. Using the resulted equations, dynamic elastic and plastic buckling of imperfect cylindrical shells are discussed. The nonaxisymmetric and local bucklings are predicted using the proposed numerical method. Since the equilibrium equations are based on large deflection assumption, the results can be used slightly beyond the buckling state (postbuckling). Finally, calculations are carried out for typical cases of a cylindrical shell subjected to a step dynamic loading in the axial direction.

Introduction

It is of both theoretical interest and practical importance to investigate the stability behavior of circular cylindrical shells subjected to dynamic loads. Shells under time dependent loading may be exposed to different kinds of dynamic instabilities, such as parametric resonance or dynamic buckling. The present study is restricted to dynamic buckling phenomena. The stability behavior of thin shells is usually very sensitive to the initial imperfections that are often induced due to the difficulty in achieving manufacturing accuracy. A collection of efforts to investigate the effect of initial deviations on the stability behavior of thin shells can be found for example, in reference [1].

An often employed method to solve dynamic buckling problems is to monitor the structural response under increasing load levels applying the criterion of Budiansky [2, 3]. Budiansky and Hutchinson [4] developed a theory to relate critical dynamic loads to static buckling loads of imperfect shells.

Lindberg [5, 6] proposed the critical amplification criterion which is more appropriate for pulse loads, while the threshold divergence criterion used by Budiansky is appropriate for step loads [6]. Another method of analysis is to substitute an approximate solution for the displacement components in the Hamilton's equations, where the reference [7] is an example. Many researchers used Hsu's general results [8] for the stability of coupled Hill's