

**Table (2) Comparison of some arbitrary stackings with the optimal.**

Stacking sequence	$F_1$	improvement in optimal %
[K90 <sub>12</sub> ] <sub>s</sub>	846.39	91.90
[Al <sub>12</sub> ] <sub>s</sub>	1099.49	47.73
[K0 <sub>12</sub> ] <sub>s</sub>	1142.60	42.15
[(K0/Al/K90) <sub>4</sub> ] <sub>s</sub>	1534.81	5.83
[(K90/Al/K0) <sub>4</sub> ] <sub>s</sub>	1535.02	5.81
optimal	1624.25	

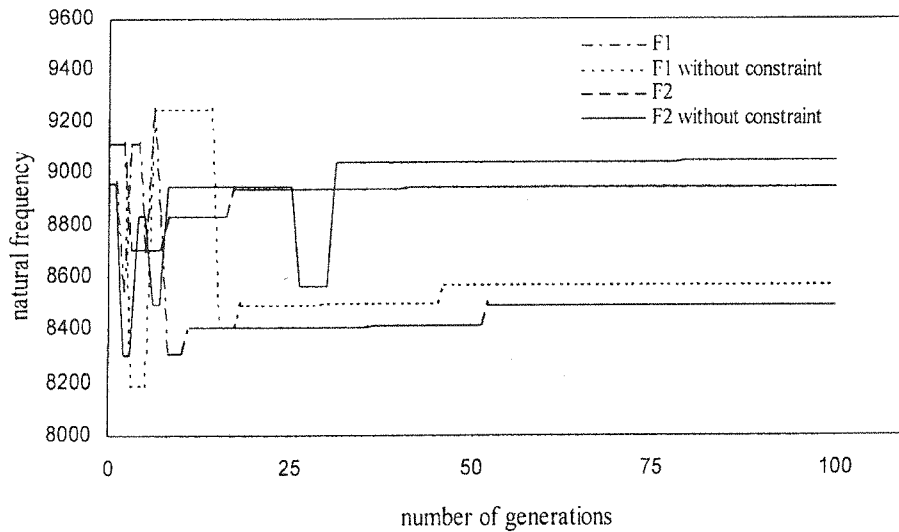
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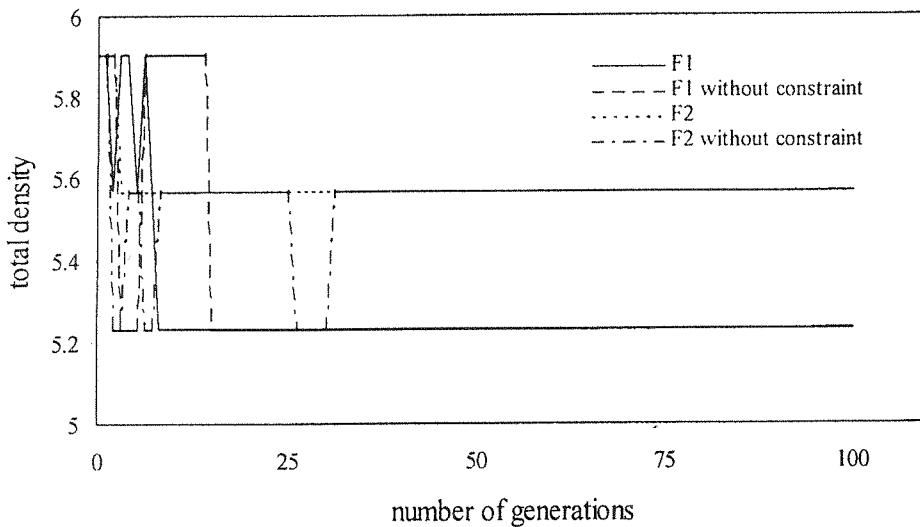
**Table (3) Optimal stacking sequences at different rotational speeds.**

$\Omega$ (rad/s)	criteria	F	$\omega_n$ (rad/s)	density	stacking sequence
10	F1 <sub>C</sub>	1616.99	8458.7	5.23	[Al / K0 / Al / K0 <sub>3</sub> / Al / K0 <sub>3</sub> / K90 / K0] <sub>s</sub>
	F1 <sub>U</sub>	1630.10	8527.3	5.23	[Al / K0 <sub>3</sub> / Al / K0 <sub>2</sub> / Al / K0 <sub>4</sub> ] <sub>s</sub>
	F2 <sub>C</sub>	7420.66	8904.8	5.56	[K0 / Al <sub>3</sub> / K0 <sub>2</sub> / Al / K0 <sub>3</sub> / K90 / K0] <sub>s</sub>
	F2 <sub>U</sub>	7499.83	8999.8	5.56	[Al <sub>3</sub> / K0 <sub>7</sub> / Al / K0] <sub>s</sub>
20	F1 <sub>C</sub>	1602.67	8383.8	5.23	[Al <sub>2</sub> / K0 <sub>2</sub> / K90 / K0 <sub>3</sub> / K90 / K0 <sub>2</sub> / Al] <sub>s</sub>
	F1 <sub>U</sub>	1631.99	8537.2	5.23	[K0 / Al / K0 <sub>2</sub> / Al <sub>2</sub> / K0 <sub>6</sub> ] <sub>s</sub>
	F2 <sub>C</sub>	7429.00	8914.8	5.56	[Al <sub>2</sub> / K0 <sub>2</sub> / Al / K0 <sub>3</sub> / Al / K0 <sub>2</sub> / K90] <sub>s</sub>
	F2 <sub>U</sub>	7506.83	9008.2	5.56	[Al / K0 / Al / K0 <sub>2</sub> / Al / K0 <sub>2</sub> / Al / K0 <sub>3</sub> ] <sub>s</sub>
30	F1 <sub>C</sub>	1620.12	8475.1	5.23	[K0 <sub>2</sub> / Al / K0 <sub>2</sub> / Al / K0 <sub>3</sub> / Al / K0 / K90] <sub>s</sub>
	F1 <sub>U</sub>	1633.64	8545.8	5.23	[Al / K0 <sub>6</sub> / Al <sub>2</sub> / K0 <sub>3</sub> ] <sub>s</sub>
	F2 <sub>C</sub>	7437.25	8924.7	5.56	[Al <sub>2</sub> / K0 <sub>2</sub> / Al / K0 <sub>3</sub> / Al / K0 / K90 / K0] <sub>s</sub>
	F2 <sub>U</sub>	7515.75	9018.9	5.56	[Al / K0 / Al / K0 / Al / K0 / Al / K0 <sub>5</sub> ] <sub>s</sub>
40	F1 <sub>C</sub>	1622.34	8486.7	5.23	[Al / K0 <sub>3</sub> / Al / K0 <sub>3</sub> / K90 / K0 <sub>2</sub> / Al] <sub>s</sub>
	F1 <sub>U</sub>	1635.53	8555.7	5.23	[Al / K0 <sub>5</sub> / Al / K0 <sub>4</sub> / Al] <sub>s</sub>
	F2 <sub>C</sub>	7444.08	8932.9	5.56	[K0 / K90 / Al <sub>2</sub> / K0 <sub>2</sub> / Al / K0 <sub>3</sub> / Al / K0] <sub>s</sub>
	F2 <sub>U</sub>	7523.66	9028.4	5.56	[Al / K0 / Al <sub>2</sub> / K0 <sub>7</sub> / Al] <sub>s</sub>
50	F1 <sub>C</sub>	1624.25	8496.7	5.23	[K0 / Al / K0 <sub>2</sub> / Al / K0 / Al / K0 <sub>3</sub> / K90 / K0] <sub>s</sub>
	F1 <sub>U</sub>	1638.05	8568.9	5.23	[K0 / Al <sub>3</sub> / K0 <sub>8</sub> ] <sub>s</sub>
	F2 <sub>C</sub>	7453.50	8944.2	5.56	[Al <sub>2</sub> / K0 <sub>2</sub> / K90 / K0 / Al / K0 <sub>3</sub> / Al / K0] <sub>s</sub>
	F2 <sub>U</sub>	7533.25	9039.9	5.56	[Al <sub>2</sub> / K0 / Al / K0 / Al / K0 <sub>6</sub> ] <sub>s</sub>
60	F1 <sub>C</sub>	1626.01	8505.9	5.23	[K0 <sub>2</sub> / Al <sub>2</sub> / K0 <sub>3</sub> / K90 / K0 <sub>2</sub> / Al / K0] <sub>s</sub>
	F1 <sub>U</sub>	1639.64	8577.2	5.23	[K0 <sub>2</sub> / Al <sub>3</sub> / K0 <sub>7</sub> ] <sub>s</sub>
	F2 <sub>C</sub>	7538.00	9045.6	5.56	[K0 <sub>2</sub> / Al <sub>2</sub> / K0 <sub>2</sub> / Al / K0 <sub>3</sub> / Al / K0] <sub>s</sub>
	F2 <sub>U</sub>	7540.41	9048.5	5.56	[Al <sub>2</sub> / K0 <sub>3</sub> / Al / K0 / Al / K0 <sub>4</sub> ] <sub>s</sub>
70	F1 <sub>C</sub>	1628.17	8517.2	5.23	[Al / K0 <sub>3</sub> / Al / K0 / Al / K0 <sub>3</sub> / K90 / K0] <sub>s</sub>
	F1 <sub>U</sub>	1641.71	8588.1	5.23	[Al <sub>2</sub> / K0 <sub>7</sub> / Al / K0 <sub>2</sub> ] <sub>s</sub>
	F2 <sub>C</sub>	7469.16	8963.4	5.56	[Al / K0 / Al / K0 <sub>3</sub> / Al / K0 / Al / K0 <sub>2</sub> / K90] <sub>s</sub>
	F2 <sub>U</sub>	7546.83	9056.2	5.56	[K0 <sub>2</sub> / Al <sub>3</sub> / K0 <sub>5</sub> / Al / K0] <sub>s</sub>
80	F1 <sub>C</sub>	1629.64	8524.9	5.23	[K0 / K90 / K0 / Al / K0 <sub>3</sub> / Al / K0 <sub>2</sub> / Al / K0] <sub>s</sub>
	F1 <sub>U</sub>	1643.46	8597.2	5.23	[Al / K0 / Al / K0 <sub>8</sub> / Al] <sub>s</sub>
	F2 <sub>C</sub>	7477.00	8972.4	5.56	[Al / K0 <sub>2</sub> / Al / K0 <sub>2</sub> / Al / K0 / Al / K0 / K90 / K0] <sub>s</sub>
	F2 <sub>U</sub>	7555.25	9066.3	5.56	[Al / K0 <sub>3</sub> / Al / K0 / Al / K0 / Al / K0 <sub>3</sub> ] <sub>s</sub>
90	F1 <sub>C</sub>	1631.44	8534.3	5.23	[K0 <sub>2</sub> / Al / K0 <sub>3</sub> / Al / K0 <sub>2</sub> / Al / K0 / K90] <sub>s</sub>
	F1 <sub>U</sub>	1645.43	8607.5	5.23	[Al / K0 / Al / K0 <sub>5</sub> / Al / K0 <sub>3</sub> ] <sub>s</sub>
	F2 <sub>C</sub>	7562.91	9075.5	5.56	[K0 / Al / K0 / Al / K0 <sub>3</sub> / Al / K0 <sub>3</sub> / Al] <sub>s</sub>
	F2 <sub>U</sub>	7561.33	9073.6	5.56	[K0 / Al / K0 <sub>6</sub> / Al <sub>3</sub> / K0] <sub>s</sub>
100	F1 <sub>C</sub>	1646.60	8613.6	5.23	[K0 <sub>3</sub> / Al / K0 <sub>2</sub> / Al / K0 <sub>3</sub> / Al / K0] <sub>s</sub>
	F1 <sub>U</sub>	1647.55	8618.6	5.23	[K0 / Al <sub>3</sub> / K0 <sub>8</sub> ] <sub>s</sub>
	F2 <sub>C</sub>	7571.66	9086.0	5.56	[Al / K0 <sub>2</sub> / Al / K0 <sub>3</sub> / Al / K0 <sub>3</sub> / Al] <sub>s</sub>
	F2 <sub>U</sub>	7572.83	9087.4	5.56	[Al / K0 <sub>2</sub> / Al / K0 / Al <sub>2</sub> / K0 <sub>5</sub> ] <sub>s</sub>

subscripts <sub>c</sub> and <sub>u</sub>, refer to contiguity constrained and unconstrained objective functions



**Figure (4) Natural frequency of laminated cylindrical shell rotating at 50 rad/sec**

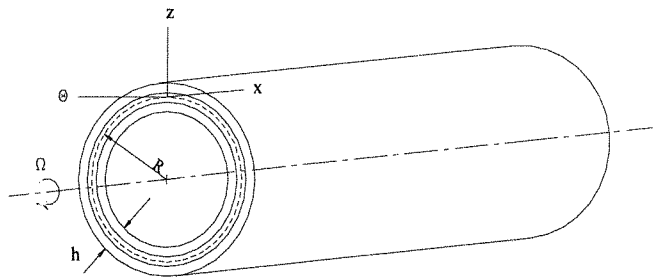


**Figure (5) Total density of laminated cylindrical shell rotating at 50 rad / sec.**

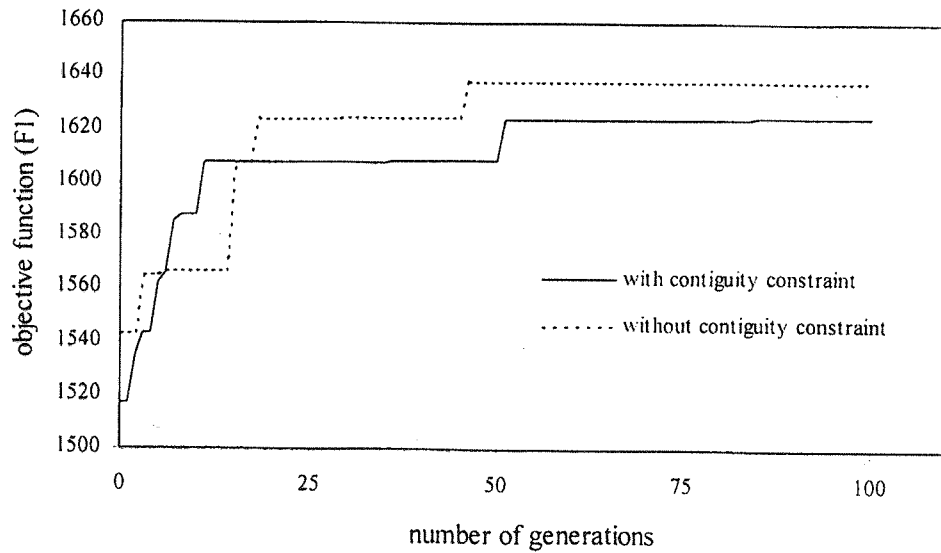
**Tabel (1) Non-dimensional frequency parameter,  $\omega R(\rho/E_2)^{1/2}$  for a [0,90,0] simply supported-simply supported rotating laminated cylindrical shell.**

$\Omega$ (rev/sec)	Love <sup>1</sup>	Sanders <sup>1</sup>	Flugge <sup>1</sup>	Donnell <sup>1</sup>	Present
0.1	0.248917	0.248917	0.248917	0.248917	0.248917
0.4	0.249765	0.249764	0.249765	0.249765	0.249765
1.0	0.251458	0.251458	0.251458	0.251458	0.251461
10	0.276620 <sup>2</sup>				0.276886
25	0.317525 <sup>2</sup>				0.319236
50	0.382460 <sup>2</sup>				0.389740

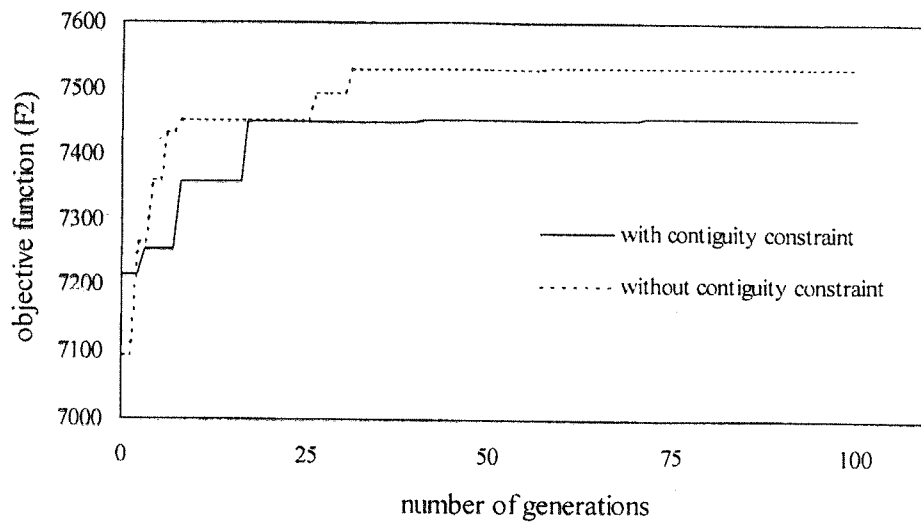
$E_{22}=7.6$  MPa,  $E_{11}/E_{22}=2.5$ ,  $G_{12}=4.1$  GPa,  $\nu_{12}=0.26$ ,  $\rho=1643$  kg/m<sup>3</sup>,  $m=1$ ,  $n=1$ ,  $h/R=0.002$ ,  $L/R=5$ , (1) from Ref. [3], (2) from Ref. [4].



**Figure 1. Geometry of the laminated cylindrical shell**



**Figure (2) Constrained and unconstrained objective functions (F1), 24-ply symmetric laminate with  $L/R = 5$  and rotating at 50 rad/sec**



**Figure (3) Constrained and unconstrained objective function (F2), 24-ply symmetric laminate with  $L/R = 5$  and rotating at 50 rad / sec**

#### 4-2-Optimal stacking sequence of rotating laminated cylinder

To start a genetic algorithm optimization, the operators should be specified first. LeRiche and Haftka [12] have used Goldberg's [21] derivation of the optimum population size for binary coded genetic algorithm and have adapted it for 1, 2 3 coding, where the optimum population size for a string length of 12, is shown to be 22. In this paper, probabilities of crossover and mutation are taken to be 0.9 and 0.01 respectively and the algorithm is stopped after processing 100 generations. The shell is 24-ply symmetric laminate with  $L/R=5$  and rotating at 50 rad/sec.

Optimization is performed on functions  $F_1$  and  $F_2$ , mentioned in previous sections, with contiguity constraint and without it. The objective functions at different generations are shown in Fig. 2 and Fig. 3 and the optimal designs are presented in Table 3, at the rotational speed of 50 rad/sec. In Table 2, constrained objective functions of some arbitrary stacking sequences are compared with the best result obtained using  $F_1$  criteria. The improvement in the objective function of the optimal design is between 5.81% (respect to  $[K90/A1/K0]_{4k}$ ) and 91.9% (respect to  $[K90]_{12}$ ).

In Fig. 4, maximum natural frequencies at different generations for the four possible objective functions are presented. Considering  $F_1$ , the natural frequencies of the best design are 8496.7 rad/sec and 8568.9 rad/sec for the constrained and unconstrained cases, respectively, while for  $F_2$  these values are 8944.2 rad/sec and 9039.9 rad/sec respectively. So the results obtained using  $F_2$  have higher natural frequencies.

The same comparison is made for mass of the cylinder in Fig. 5. Both the constrained

and unconstrained optimizations have reached the same values for  $F_1$  and  $F_2$ . These values are 5.231 kg/m<sup>2</sup> for  $F_1$  and 5.568 kg/m<sup>2</sup> for  $F_2$ . So the designer is able to choose between different stacking sequences, depending on whether the natural frequency is of greater importance or the weight of the structure. Optimal design obtained using  $F_2$  has higher natural frequency, but it is heavier too.

The optimal stacking sequences for rotational speeds from 10 to 100 rad/sec are presented in Table 3.

At all rotational speeds, optimal designs obtained using  $F_2$  have greater natural frequency and are heavier. Also, all of the laminates that are optimized using  $F_1$  have the same total mass, and it is the same for the laminates that have used  $F_2$  as objective function.

#### 5-Conclusions

The use of genetic algorithm in optimization of the stacking sequence of a hybrid rotating composite cylinder was presented. For the analysis of the free vibration of rotating laminated cylindrical shell, all of the nonlinear terms of circumferential strain were included. The effect of these terms are more obvious at higher rotational speeds. The stacking sequence of the laminated shell is optimized for the objective functions including the effects of natural frequency and weight of the structure. Depending on the function used, more improvements in each of these terms could be obtained.

6061-T6 A1 (A1) and Kevlar 49/Epoxy at 90° (K90). The GA begins with an initial population of designs randomly generated and fitness function value for each individual design in the population is evaluated to determine its performance. The individuals are then ranked according to their objective function values. Using a roulette wheel strategy [19], parents are selected as to give the most fit designs better chance of reproduction. This allows the most desirable characteristics of the designs to be copied into the next generation of strings by the crossover process.

Crossover allows selected individuals to trade characteristics of their designs by exchanging parts of strings. We use a 2-point crossover where two break points are chosen randomly in the string. Two offsprings are created by swapping the parents substrings. The crossover is applied with a probability usually between 0.7 and 1. If crossover is not applied, the parents are copied into the next generation. For example when  $[A1_3/K90/K0/A1/K0/K90_3]_s$  laminate is mated with  $[K90_3/A1/K0_2/A1/A1_4]_s$  with the crossover points after the fourth and the eighth positions, selected randomly, the resulting children are  $[A1_3/K90/K0_2/A1_2/K90_2]_s$  and  $[K90_3/A1/K0/A1/K0/K90/A1_2]_s$ :

parents selected based on fitness

2 2 2 3 | 1 2 1 3 | 3 3

3 3 3 2 | 1 1 2 2 | 2 2

children after crossover

2223 | 1 1 2 2 | 33

3332 | 1 2 1 3 | 2 2

The mutation operation is then performed with a low probability on the newly created children. This is done by first deciding randomly whether to perform mutation to a string or not. Then a bit in the string is randomly switched to one of the other three

choices available,

before mutation

1 2 3 1 | 2 2 1 3 | 3 3

after mutation

1 2 2 1 | 2 2 1 3 | 3 3

Enough children are created in each generation, to replace all, but the best member of the old population. This approach to the GA is called an elitist plan since the best design is always carried into the next generation. This procedure is repeated until a specified number of generations has passed or after a specified number of generations without any improvement in the fitness of the best design is reached.

## 4-Numeical Results and Discussions

### 4-1-Free vibration of laminated cylindrical shell

To validate the formulations and the computer program developed, the natural frequency of the cylindrical shell is compared with some available solutions. Table 1 shows the results from the present study and Ref. [3], that Donnell' s, Flugge' s, Love's and Sanders' theories are used to determine the natural frequency of the rotating multilayered shell. The shell considered is very thin ( $h/R = 0.002$ ) and the results from all theories are almost the same. Ref. [3] has considered only low rotational speeds, up to 1 rev/sec. For higher speeds, Love's formulation mentioned in Ref. [4] is used for comparison. It is seen that even for very thin shells, difference between natural frequencies obtained from these theories, increases at higher rotational speeds. This is because of the full nonlinear terms of  $\epsilon_{\theta\theta}$  that are considered in the present study. In the governing equations, the effect of these nonlinear terms are associated with the initial circumferential stress,  $N_{\theta}^i$  and so the rotational speed of the shell.

$$\begin{bmatrix} L_{11} & L_{12} & L_{13} \\ L_{21} & L_{22} & L_{23} \\ L_{31} & L_{23} & L_{33} \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad (11)$$

where  $L_{ij}$  are differential operators on  $u, v$  and  $w$ .

considering the simply supported-simply supported boundary conditions, the displacement field which satisfies the boundary conditions can be written as

$$\begin{aligned} u &= A \cos\left(\frac{m\pi x}{L}\right) \cos(n\theta + \omega t) \\ v &= B \sin\left(\frac{m\pi x}{L}\right) \sin(n\theta + \omega t) \\ w &= C \sin\left(\frac{m\pi x}{L}\right) \cos(n\theta + \omega t) \end{aligned} \quad (12)$$

where  $A, B$  and  $C$  are the displacement amplitudes,  $m$  and  $n$  are the axial and circumferential wavenumbers, respectively and  $\omega$  is the natural frequency. Substituting the displacement field, equation (12), into equation (11) yields

$$\begin{bmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{23} & K_{33} \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad (13)$$

Equation (13) can be solved for the roots  $\omega$ , by imposing the condition of non-trivial solution, i.e., equating the determinant of the characteristic matrix  $[K_{ij}]$  to zero.

## 2-2- Optimization formulation

In the optimization problem, it is desired to have the maximum possible natural frequency while the weight should be kept as low as possible. So, the objective function should increase, as the weight of the laminate decreases or the natural frequency increases. In order to alleviate matrix cracking problems, number of contiguous plies of the

same orientation is limited to four by imposing a penalty parameter  $p_{con}$ . The optimization problem is formulated in standard form using two different objective functions as

$$\text{Maximize } F_1 = (1 - p_{con}) \frac{\omega}{\rho_t} \quad (14)$$

or

$$\text{Maximize } F_2 = (1 - p_{con}) p_w \omega \quad (15)$$

$$p_w = 1 - \left( p_{min} \frac{\rho_t - \rho_{min}}{\rho_{max} - \rho_{min}} \right)$$

where  $\omega, \rho_t, \rho_{min}$  and  $\rho_{max}$  are the natural frequency, total density, minimum and maximum densities of the laminae, for the cylindrical laminated shell, respectively.  $p_w$  is the penalty parameter for the weight of the laminate, a value between  $p_{min}$  and 1. Results presented in this paper are obtained for 6061-T6-A1 [ $E_{11}=68.28$  GPa,  $E_{22}=E_{11}$ ,  $G_{12}=26.21$  GPa,  $\nu_{12}=0.33$ ,  $\rho=2713.0$  kg/m<sup>3</sup>] and Kevlar 49/Epoxy [ $E_{11}=75.86$  GPa,  $E_{22}=5.517$  GPa,  $G_{12}=2.069$  GPa,  $\nu_{12}=0.34$ ,  $\rho=1384.0$  kg/m<sup>3</sup>] where each ply has a thickness of 0.0127 cm. Also a contiguity penalty parameter of 8% is used [12].

## 3-Genetic Algorithm

Genetic algorithm (GA) is a probabilistic optimization method that works on population of designs, first introduced by Holland [18]. In the genetic algorithm approach, a solution (i. e., a point in the search space) is called a chromosome or string. A genetic search requires a population of chromosomes (strings) each representing a combination of features from the set of features. In our case, each string represents an alternative stacking sequence where 1, 2 and 3, stands for the three possible stacks, Kevlar 49 /Epoxy at 0° (K0),

where  $\rho$  is density,  $\forall$  the total volume of the laminate,  $V$  the velocity vector and  $\delta$  the variational symbol. The velocity vector for the rotating cylindrical shell is

$$V = \dot{U} e_x + (\dot{V} - \Omega(R+W)) e_\theta + (\dot{W} + \Omega V) e_z \quad (4)$$

where  $e$  and  $(\cdot)$  represent the unit vector and differentiation with respect to time.

Substituting Eqs. (1), (2) and (4) into Eq. (3) and integrating through the thickness, the governing equations are obtained as

$$\begin{aligned} N_{x,x} + \frac{1}{R} N_{x\theta,\theta} + N_\theta^i \left( \frac{1}{R^2} u_{,\theta\theta} \right) &= \rho_t a_x \\ N_{x\theta,x} + \frac{1}{R} N_{\theta,\theta} + \frac{1}{R^2} M_{\theta,\theta} + \frac{1}{R} M_{x\theta,x} + \\ \frac{1}{R^2} N_\theta^i (v_{,\theta\theta} + 2w_{,\theta} - v) &= \rho_t a_\theta \\ M_{x,xx} + \frac{2}{R} M_{x\theta,x\theta} + \frac{1}{R^2} M_{\theta,\theta\theta} - \frac{1}{R} N_\theta + \frac{1}{R^2} N_\theta^i \\ (w_{,\theta\theta} - 2v_{,\theta} - Ru_{,x} - w) &= \rho_t a_z \end{aligned} \quad (5)$$

where

$$\begin{aligned} a_x &= \ddot{u} \\ a_\theta &= \ddot{v} + 2\Omega \dot{w} - \Omega^2 v \\ a_z &= \ddot{w} - 2\Omega \dot{v} - \Omega^2 w \end{aligned}$$

$$\rho_t = \int_{-h/2}^{h/2} \rho dz \quad (6)$$

where the terms in paranthesis are due to the nonlinear parts of  $\epsilon_{\theta\theta}$ . In deriving the governing equations, it is assumed that due to the rotation, the circumferential force is large in comparison with other stress resultants and so, its product with the

displacements has been retained. The circumferential force,  $\bar{N}_\theta$  is decomposed to the sum of the initial value,  $N_\theta^i$  and the vibratory force,  $N_\theta$ . Assuming that the boundary conditions are enforced only after spinning has been attained, it can be shown [2] that for a thin shell

$$N_\theta^i = \rho_t R^2 \Omega^2 \quad (7)$$

In addition, to take into account the stretching of the middle surface,  $\bar{N}_\theta$  is replaced by  $\bar{N}_\theta(1+u_{,x})$  in the second and third equations of motion [17].

The stress and moment resultants are expressed in terms of the stress components as

$$\begin{aligned} \{N_x, N_\theta, N_{x\theta}\} &= \int_{-h/2}^{h/2} \{\sigma_x, \sigma_\theta, \sigma_{x\theta}\} dz \\ \{M_x, M_\theta, M_{x\theta}\} &= \int_{-h/2}^{h/2} \{\sigma_x, \sigma_\theta, \sigma_{x\theta}\} z dz \end{aligned} \quad (8)$$

For a thin laminated orthotropic layer, the stress-strain relation is

$$\{\sigma_x, \sigma_\theta, \sigma_{x\theta}\}^T = [Q] \{\epsilon_x, \epsilon_\theta, \epsilon_{x\theta}\}^T \quad (9)$$

where for cross-ply laminates, the stiffness matrix  $[Q]$  is given by

$$[Q] = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \quad (10)$$

Substituting eqs. (8), (9) and (10) into equation (5), yields



problems [10, 11]. In the area of composite structural design, genetic algorithms are used to optimize the stacking sequence of laminated plates for buckling load [12], to design stiffened composite panels against buckling [13] and to solve the optimal material tailoring problem [14]. Also some changes to the basic genetic algorithm are suggested to reduce computational cost [15,16].

In this paper, for the optimal stacking sequence of a hybrid laminated rotating cylinder, first the governing equations are derived where all of the nonlinear terms of circumferential strain  $\epsilon_{\theta\theta}$ , are included in the theory. For the case of simply supported boundary conditions, series solution is used to solve the eigenvalue problem. Next, using genetic algorithm, the optimal stacking sequence of the laminated shell is obtained. Two different objective functions that include the natural frequency and weight of the laminated cylinder are optimized at different rotational speeds. The number of contiguous layers is limited by implementing a penalty parameter.

## 2-Analysis and Problem Formulation

### 2-1 Free vibration of rotating laminated cylindrical shell

Consider a thin laminated cylindrical shell composed of  $N$  plies each of thickness  $t$ , perfectly bonded together, as shown in Figure 1. The cylinder has a length  $L$ , thickness  $h$ , mean radius  $R$  and is rotating about its longitudinal axis at a constant angular speed of  $\Omega$ . The coordinate system  $(x, \theta, z)$  is fixed at the middle surface of the laminate. The displacement components are  $U, V$  and  $W$  in  $x, \theta$  and  $z$  directions respectively.

Concerning the description of the shell

motion, the following assumptions are made in developing the governing equations:

- (i) The layer material is linearly elastic and macroscopically homogeneous.
- (ii) The shell is circular cylindrical with a uniform thickness.
- (iii) The shell is rotating at a constant angular velocity.
- (iv) Dissipative effects such as damping are negligible.

The displacements  $U, V$  and  $W$  in the shell at time  $t$ , are assumed to be of the form

$$\begin{aligned} U(x, \theta, z, t) &= u(x, \theta, t) - z \frac{\partial w}{\partial x}(x, \theta, t) \\ V(x, \theta, z, t) &= \left(1 + \frac{z}{R}\right) v(x, \theta, t) - \frac{z}{R} \frac{\partial w}{\partial \theta}(x, \theta, t) \\ W(x, \theta, z, t) &= w(x, \theta, t) \end{aligned} \quad (1)$$

where  $(u, v, w)$  are the displacements of a point  $(x, \theta, 0)$  on the reference plane of the shell. The strain displacement equations in an orthogonal Cartesian coordinate system are

$$\begin{aligned} \epsilon_{xx} &= \frac{\partial U}{\partial x} \\ \epsilon_{\theta\theta} &= \frac{1}{(R+z)} \left( \frac{\partial V}{\partial \theta} + W \right) + \frac{1}{2(R+z)^2} \\ &\quad \left[ \left( \frac{\partial V}{\partial \theta} + W \right)^2 + \left( \frac{\partial U}{\partial \theta} \right)^2 + \left( \frac{\partial W}{\partial \theta} - V \right)^2 \right] \\ \epsilon_{x\theta} &= \frac{\partial V}{\partial x} + \frac{1}{(R+z)} \frac{\partial U}{\partial \theta} \end{aligned} \quad (2)$$

The Hamilton variational principle is used to derive the equations of motion, that can be stated in the absence of body forces and tractions, as

$$\int_0^T \left[ \int_V (\sigma_x \delta \epsilon_{xx} + \sigma_\theta \delta \epsilon_{\theta\theta} + \sigma_{x\theta} \delta \epsilon_{x\theta}) dV - \int_V \rho V \cdot \delta V dV \right] dt = 0 \quad (3)$$

# Optimal Stacking Sequence of a Hybrid Laminated Rotating Cylindrical Shell using Genetic Algorithm

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## Abstract

Hybrid laminates, materials that consist of alternate aluminium alloy and composite laminae, are functionally graded material systems of particular interest to the aerospace industry. In this study, the use of genetic algorithm in stacking sequence optimization of thin rotating hybrid laminated cylindrical shells is studied. The centrifugal and Coriolis forces are included in the theory and the Navier-type solutions are presented for simply supported boundary conditions. To validate the formulation, results are compared to those from other shell theories. For the optimization problem, two different objective functions that include the weight and natural frequency of the rotating cylinder are considered. The objective functions are constrained for the maximum number of contiguous plies. Optimal stacking sequences at different rotational speeds are presented.

## Key words

optimization, genetic algorithm, hybrid laminates, rotating cylinder, free vibration

## 1-Introduction

The vibration of composite shells is of importance in view of the current interest in designing with composite materials. Thin cylindrical shells are widely used as structural elements. There are various engineering applications of rotating cylindrical shells such as high-speed centrifugal separators, gasturbines for high-power aircraft engines and spinning satellite structures. Until recently, most of the research efforts had been devoted to the dynamic behavior of rotating isotropic shells. Padovan [1] was seemingly the first to consider the effects of material anisotropy. In most studies on the vibration of the spinning laminated shells, linear strain relations are considered [2-8].

Composite constructions offer many

opportunities for engineers and designers to optimize structures for a particular or even multiple tasks. The problem is often formulated as a continuous optimization problem with the thickness and orientation of plies, as design variables [9], but for most practical problems, ply thicknesses are fixed and orientations are limited to a small set of angles, so the design problem becomes a stacking sequence optimization. The design space usually contains many local extrema, even singular ones and also many near optima designs may exist. Thus there is a need for optimization techniques that can identify multiple and singular extrema.

Optimization methods based on genetic algorithms have been applied to structural