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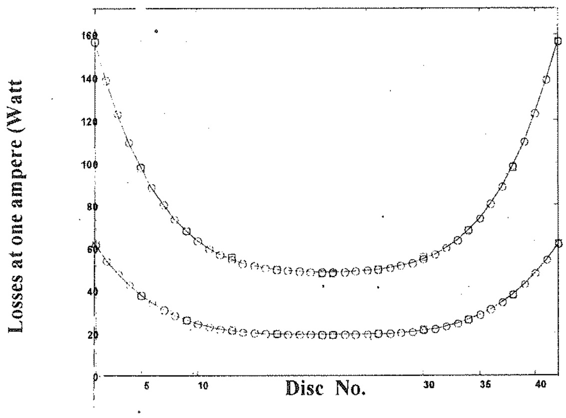

Figure (4) variation of losses for individual discs of actual winding at 1.6 MHz and 10 MHz
□ Calculated by model winding
o Estimated by curve fitting.

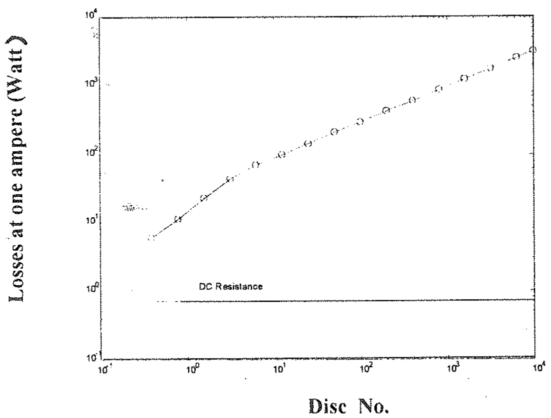

Figure (5) variation of ac resistance versus frequency for 42-disc winding, o Calculated

IV-Conclusion

In this paper a technique is introduced for calculating eddy current losses of transformer windings or reactors by 2-dimensional finite element method, which includes a method for calculating the ac resistance of windings. In this approach a small geometry for calculating the losses of any turn or disc of actual winding is used. The presented method is accurate because it is based on FEM results.

It was shown that the skin effect and proximity effect losses are not orthogonal and in the cases that the currents of all conductors are the same, eddy currents produce more losses in the outer discs of windings and for the case of the middle disc, it is smaller than the other discs.

With this method, finding the hot spots of winding, without using the whole of discs of winding in the finite element model, is possible. This method is applicable at high frequency too.

Appendix

Actual winding data

Core: air
Number of discs: 42
Number of conductors in each disc: 10
Length of winding: 918 mm
Inner radius: 750 mm
Outer radius: 798 mm
Gap between discs: 6 mm
Gap between conductors in discs: 2 mm
Dimensions of cross section of conductors: 3 mm X 16 mm
Conductivity: 5.8E+7

Model winding data

Core: air
Number of discs: 9
Number of conductors in each disc: 10
Length of winding: 192 mm
Inner radius: 750 mm
Outer radius: 798 mm
Gap between discs: 6 mm
Gap between conductors in discs: 2 mm
Dimensions of cross section of conductors: 3 mm X 16 mm
Conductivity: 5.8E+7

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Table (III) Calculated losses of some discs of actual winding by using the model winding for curve fitting

	Losses of actual winding discs at one ampere (Watt)					
Freq. (Hz)	Disc 1	Disc 5	Disc 9	Disc 13	Disc 17	Disc 21
390.625	0.4804	0.2243	0.1146	0.0668	0.0440	0.0362
781.25	0.7811	0.4057	0.2254	0.1473	0.1101	0.0975
1562.5	1.2947	0.7474	0.4785	0.0798	0.3096	0.2912
3125	2.1433	1.3209	0.9404	0.7807	0.7065	0.6816
6250	3.2990	2.0393	1.4821	1.2497	1.1422	1.1061
12500	4.8146	2.9401	2.1054	1.7563	1.5944	1.5401
25K	7.0472	4.2783	3.0464	2.5308	2.2916	2.2115
50K	10.331	6.2573	4.4260	3.6581	3.3019	3.1822
100K	14.740	8.8995	6.3061	5.2199	4.7151	4.5462
200K	21.136	12.758	9.0216	7.4559	6.7289	6.4847
400K	30.231	18.256	12.880	10.630	9.5866	9.2340
800K	43.171	26.165	18.421	15.177	13.669	13.156
1600K	61.656	37.405	26.286	21.619	19.443	18.715
3200K	87.912	53.540	37.540	30.812	27.689	26.637
6400K	125.48	77.353	54.039	44.221	39.704	38.102
10000K	156.94	97.637	68.047	55.583	49.783	47.827

Table (IV) ac resistance of actual 42-disc winding at different frequencies

Frequency (Hz)	Resistance (Ohm)
0	0.708
390.625	5.969
781.25	11.192
1562.5	22.669
3125	43.432
6250	68.290
12500	97.391
25K	141.18
50K	205.34
100K	292.72
200K	418.90
400K	598.23
800K	855.01
1600K	1219.8
3200K	1740.9
6400K	2500.5
10000K	3143.1

Table (V) Estimation of ac resistance of 42-disc winding at 10 MHz

f_2(kHz)	R_2(Ohm)	R_1(Ohm) $f1$=10 (MHz)	Erreor (Ohm)	Error (%)
6400	2500.5	3125.6	17.5	0.55
3200	1740.9	3077.6	65.5	2.08
1600	1219.8	3049.5	93.6	2.97
800	855.01	3022.9	120.2	3.82
400	598.23	2991.2	151.9	4.83
200	418.90	2962.0	181.1	5.76
100	292.72	2927.2	215.9	6.87
50	205.34	2904.0	239.1	7.61
25	141.18	2823.6	319.5	10.16

discs indidualy. One can calculate the losses of a few discs which equqly distributed along the winding. For example in this simulation according to simmetries the calculation of losses of discs 1, 5, 9, 13, 17, and 21 for curve fitting is adequate and consequently we need to solve the model winding five times in different equivalent current cases. To prevent the determining the current distribution in each disc analytically, we have used the equation 2, which gives the equivalent current based on filamentary assumption of discs. According to discussions in previous section the results by this way will not be accurate and they should be corrected by using the equation 3. In this equation we need the losses of actual winding at low frequency. Meshing the whole winding at the frequency of 25 kHz by finite element method because of limittion in the requried nodes in order to calculate the losses of the actual winding is possible. In this frequency the skin depth is seven times smaller than the thickness of conductors and therfore the varition of magnetic field intensity at frequencies higher than it because of slight redistribution of currents is neglegible. Table II shows the currents that should be used in the discs of model winding for calculating the losses of desired discs of actual winding. For example in the forth colmn of this table the losses of disc-5' is coresponds to losses of disc-9 and the equalent currents of disc-1' and disc-9' are 3.5 and 9.02 ampere respectivly that have been calculated by equation 2. The shadow blocks in this table also show the discs that their losses correspond with the discs of actual winding. Tables III and IV show the curve fitting data and total losses, in the case of one ampere in each turn, at different frequencies. In Table IV, it is seen that at frequencies in which the skin depth is much smaller than the thicnesses of the conductors, the ac resistance (total loss at one ampere) can easily be approximated as below;

$$R_1 = R_2 \cdot \sqrt{\frac{f_1}{f_2}} \quad (4)$$

Where R_2 and f_2 are known.

The total eddy current losses of the actual winding at 10 MHz, by using this estimation at different frequencies, has shown in Table V. This Table states that, at high frequencies this approximation is satisfactory. Figures 4 shows the profile of losses distribution at one ampere along with the winding at 1.6 MHz, and 10 MHz. This figure shows that the losses due to the outer discs is more than the others are, and also the middle discs experience less than others do, while for infinite length windings the profile of losses along with the winding is flat. Figure 5 shows the variation of ac resistance versus frequency and as it is seen at frequencies that the skin depth is less than 1/7 of thickness of conductor, the variation of loss in logarithmic scale becomes linear.

Table (II) Equivalent current of discs of model winding for calculating the loss of discs 1,5,9,13,17, and 21 of actual winding (Shadow blocks).

Discs of Model Winding	Desired discs					
	Disc1	Disc5	Disc9	Disc13	Disc17	Disc21
Disc-1'	1A	1A	3.54A	5.08A	6.19A	7.06A
Disc-2'	1A	1A	1A	1A	1A	1A
Disc-3'	1A	1A	1A	1A	1A	1A
Disc-4'	1A	1A	1A	1A	1A	1A
Disc-5'	1A	1A	1A	1A	1A	1A
Disc-6'	1A	1A	1A	1A	1A	1A
Disc-7'	1A	1A	1A	1A	1A	1A
Disc-8'	1A	1A	1A	1A	1A	1A
Disc-9'	13.6A	9.47A	9.02A	8.51A	7.93A	7.25A

meshing a layer with thickness of 5δ for conducting regions will reduce the number of nodes considerably.

In the special case, inwhich the currents of whole turns are equal, the profile of losses along the winding is smooth and simple. The loss calculation of a number of discs results in the estimation of the losses of whole discs, by curve fitting. The required data for curve fitting can be the losses of one disc from every few discs of actual winding including the middle and end discs. To have better results, five order polynomial curve fitting over the data is

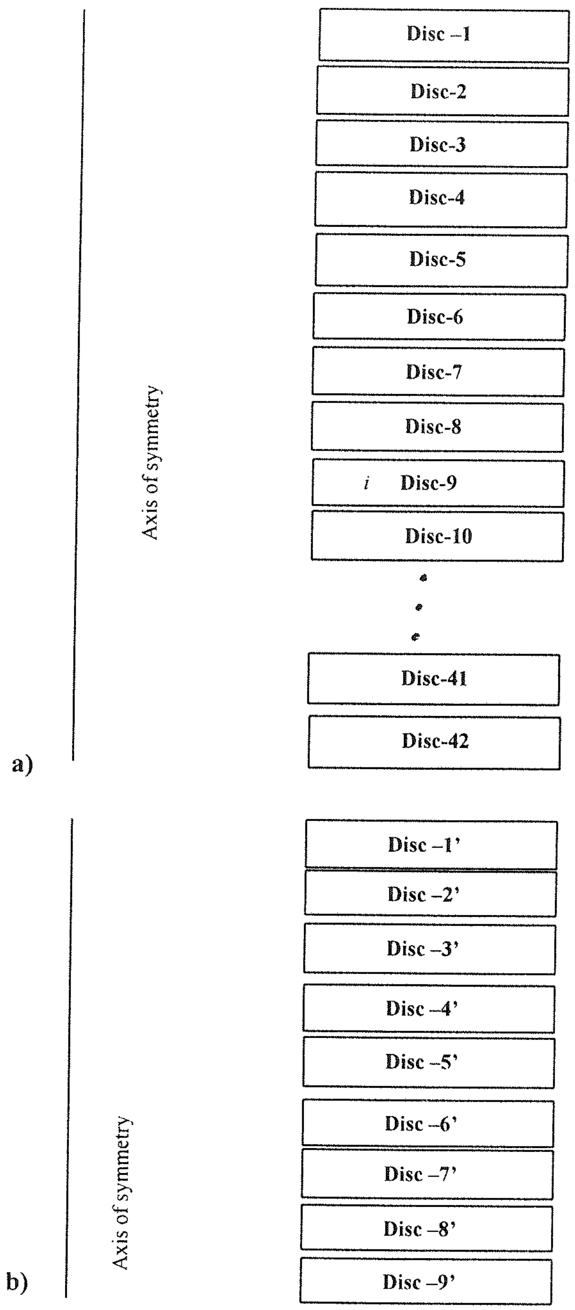


Figure (3) a-Cross-section of actual winding b- Cross-section of model winding.

suggested. As the currents of whole turns are one ampere, the total calculated losses by this manner can be used as the ac resistance of winding.

It is obvious that, this is acceptable only at low frequencies in which the effects of displacement currents are negligible. In the next section, the method will be evaluated by simulation.

V-Simulation Results

As already mentioned the calculation of ac resistance or local losses for finding possible hot spots of winding during transients requires the currents of whole conductors. These currents are computable by using the detailed model of the winding. Since our aim in this paper is evaluating the introduced method, therefore we assume that all conductors are carrying the same current. On the other hand, the capacitive effects between winding turns are ignored.

Figure 3a shows the cross section of a actual winding with 42 discs in which, each disc consists of 10 turns and Figure 3b shows a 9-disc model winding, which is used for calculating the eddy current losses of individual discs of the actual winding. The data for these windings have shown in the appendix.

In this example we assign one ampere for each turn and begin the discussion about the calculation of the eddy current losses for disc 9 of actual winding.

In order to calculate the losses of the disc 9 of actual winding, we will calculate the loss of disc 5' in model winding. For this purpose the currents of discs 1,2,3,4 and 5 are transformed to disc 1' of the model winding and similarly the currents of discs 13, 14, 15... and 42 are transformed to disc 9' of model winding. Currents in discs 2', 3', 4', 5', 6', 7', and 8' are the same as the currents of discs 6, 7, 8, 9, 10, 11, and 12 respectively. In order to find the losses of disc 5' of the model winding that corresponds to the losses in disc 9 of actual winding; the model winding is solved by 2-dimensional finite element. Meanwhile it is assumed that the geometry is symmetrical with respect to axis of winding and the plane which perpendicular to the axis of winding in its middle. Since the currents of turns are equal one ampere then the varition of losses along the winding is smouth and there is no need to calculate the losses of all based on the calculation of losses at individual discs and consequently the loss of each turn of a disc, will be available.

Consider a system of conductors that have arranged in three similar groups: g_0, g_1, and g_2 as shown in Fig. 2 and carry sinusoidal current. Assume that the distance r_1 is at least two times larger than the thickness of groups, i.e.; t. Analytically it is possible to determine the currents of g_1 such that, by removing the g_2 the magnetic field intensity H at position of g_0 remains unchanged [8]. The transformation equation for the filamentary current case, for having the same field at g_0 can be written as below:

$$I_1=I_2 \cdot r_1/r_2 \tag{2}$$

It is obvious that because of non-zero dimensions of conductors, the produced H by equivalent current I_1 at g_0 includes error. The magnitude of magnetic field intensity at position of g_0 is independent of the frequency. By assuming that the magnetic field intensity at the entire surface of g_0 is uniform, the dissipated power at g_0 will be proportional with $|H|^2$ [11]. By knowing the value of losses of region g_0 at low frequency, in which calculation by FEM is easier, one can correct the losses at high frequencies, which calculated by equivalent current method, by introducing constant k.

$$k=\frac{P_{0.Fem}}{P_{0.ecm}} \tag{3}$$

Where $P_{0.Fem}$ is the calculated losses at frequency f_0 by FEM and $P_{0,ecm}$ is the calculated losses at f_0 by equivalent current method.

As a result, at any frequency, by multiplying k at the losses that calculated by equivalent current method, the results can be corrected.

Now, this idea can be used for a winding. In order to calculate the losses of an arbitrary disc i of an actual winding (Fig. 3a), the far discs are removed and a new disc is replaced instead of them. The current in each turn of this new disc should be determined so that its produced magnetic field on disc i, be the same as the currents of removed discs were already producing on the disc i. It is obvious that for calculating the loss of disc i by this simplification, we need two equivalent discs, one for the far discs under it and another for the far discs above it. For two reasons the distance of equivalent discs from disc i should be at least two times of radial thickness of a disc. The first, that is much important, the surrounding medium from the viewpoint of disc i is almost unchanged and the second is because of having a simple current transformation rule from far discs to new disc. On the other hand for transforming the currents from far turns to new disc, the field calculations can be done by assumption of filamentary currents.

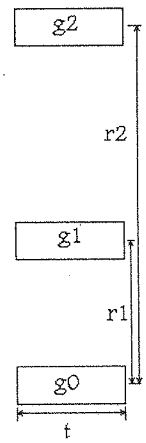


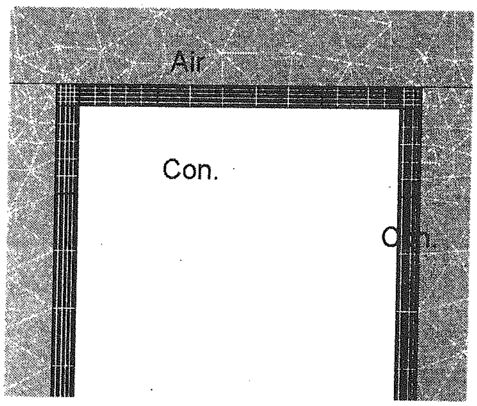
Figure (2) Three similar groups of conductors.

By this method the finite element model of a winding with a lot of discs, in order to calculate the loss of arbitrary disc, reduces to a few discs which is usually less than 10 discs. Our suggestion for the number of discs of model winding is 9. The middle disc of this model winding corresponds to the disc of actual winding that we intend to compute its losses, and currents of the upper and lower discs of the model winding are the transformed current of discs of actual winding which have been removed. In the case of the upper or bottom disc of actual winding, the lower or upper disc of model winding can be used.

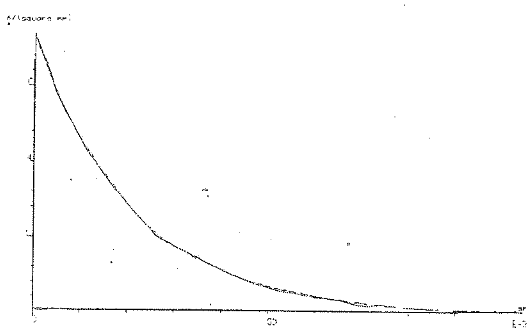
By using this method, the geometry considerably becomes smaller. And also there is no need to fine meshing the discs of model winding except three discs whose middle one is used for calculating the loss of disc i. Also, according to the discussions in section II, current inside the conductors at high frequencies, can be neglected and it in the meshing process has not effect in the results.

There is also another way to reduce the number of elements in the conductive regions. To mesh the fine areas (e.g., skin depths) the quadrilateral elements can be used instead of triangular elements and also the highly stretched rectangular elements can be used in the situations that the variable varies only slightly in the longitudinal direction of a region.

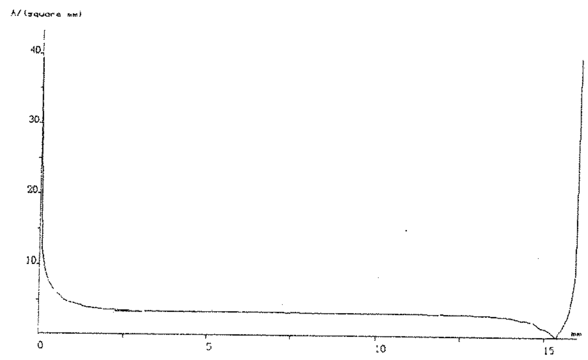
To justify the above discussion, by the program FLUX2D a few turns of a winding, which have rectangular cross section and carry one ampere at frequency 10 MHz have been meshed. Fig. 1a shows two-dimensional mesh elements for the part of a conductor and its surrounding in the winding. As it can be seen in this figure, 5δ second-order quadrilateral elements have been used in the thickness of 5δ. Fig. 1b shows the distribution of current density in the skin depth direction. It is obvious that the variation of current density is exponential and value of it at 5δ vanishes.



(a)



(b)



(c)

Figgure (1) a) Mesh elements at part of conductive and nonconductive area; inside the conductive area has not been meshed, b) Variation of current density in the skin depth direction, c) Variation of current density in the longitudinal direction

And also small quadrilateral elements have been used at the corners of conductors because of rapidly variation of the current density (Fig. 1c). In the longitudinal directions stretched elements have been used because of slightly variation of the current density (Fig. 1c). The interior of conductor has not solved by finite element since the current in that volume is almost zero.

The results of this study show that managing the mesh process by this manner reduces the number of nodes considerably.

III-Calculation of the Eddy Current Losses by FEM

The high voltage winding of power transformers are constituted of many discs and modeling of it by finite element, in order to calculate the variables especially at high frequencies is difficult because of the required large number of nodes and usually impossible. In this section a finite element method is developed for calculating the eddy current losses at any turn or disc of the high voltage winding of power transformers by taking into account the effect of currents in the all turns. In order to calculate the losses, the currents in each turn should be known. As it can be deduced from the detailed model of HV transformer windings [9-10], at high frequencies (because of capacitances) the currents in the turns are not equal and can be calculated by using the detailed model of HV winding for a known excitation. The suggested formulation is this relationship is the finite element method. This method generally requires a great deal of calculation time, much memory and in some configurations, very high mesh refinement for highly accurate results. As a result for big devices such as windings, especially at high frequencies, is not applicable.

In this paper, the finite element method will be used for calculating the eddy current losses at any disc or turn (hot spots) of high voltage winding of power transformers which includes the ac resistance of winding. In the presented method, there is a large amount of reduction in the number of nodes in the finite element model of winding. This is an important point for the computation of eddy current losses at high frequencies by finite element method, which will be discussed in the sections II and III of this paper.

Table (I) Loss in each turn of winding, *f*=10 MHz, I=1 A.

Conductor Number	Loss (watt)	Loss(watt) (Skin + prox.)	Error %
1 (top)	.5512	.1507+.4902	16.27
2	.4198	.1507+.3147	10.86
3	.3296	.1507+.2199	12.44
4	.2704	.1507+.1538	12.61
5	.2136	.1507+.1087	21.44
6	.1876	.1507+.0780	21.91
7	.1646	.1507+.0574	26.43
8	.1487	.1507+.0428	30.13
9	.1393	.1507+.0326	31.57
10	.1334	.1507+.0271	33.28
11(middle)	.1324	.1507+.0255	33.08
12	.1334	.1507+.0271	33.28
13	.1393	.1507+.0326	31.57
14	.1487	.1507+.0428	30.13
15	.1646	.1507+.0574	26.43
16	.1876	.1507+.0780	21.91
17	.2136	.1507+.1087	21.44
18	.2704	.1507+.1538	12.61
19	.3296	.1507+.2199	12.44
20	.4198	.1507+.3147	10.86
21(bottom)	.5512	.1507+.4902	16.27

II-Meshing a Region Involving Skin Effects

For problems involving skin depth, at least two second-order finite elements should make up the thickness of the skin. In fact, the electromagnetic variable in this area varies exponentially. In the interior of second order element, the approximation is parabolic and therefore the size of the elements must be sufficiently small that the arc of the parabola is comparable to the arc of an exponential. In the case of sinusoidal sources and linear materials, the skin depth for the eddy currents can be calculated using:

$$\delta = \sqrt{\frac{2}{\mu\sigma\omega}} \tag{1}$$

Where ω is the angular frequency, σ the conductivity and μ the absolute permeability.

At high frequencies, according to (1), the skin depth is very small and as a general rule, the elements should become small. Therefore for the conductive regions of a device, such as high voltage winding, we need a lot of nodes and this makes the computation of the variable is very difficult or impossible. Since the variable varies exponentially therefore the value of it for depths of more than 5δ are very small and of rectangular cross section, d) the curvature is zero, e) the value of the magnetic field intensity is zero at the outer surface for solenoids.

Even though the end effects are partially taken into account in three-dimensional approximation, the approximation has another restrictions as follows [2]:

-The end effects, as far as the individual coils of winding are concerned, are reduced by assuming constant axial magnetic field intensity at inner and outer constant-radius surfaces of individual layers.

-The end effects for the whole winding are completely ignored. The number of coils within the winding can be taken into account only by repeating the distribution that are valid for the middle coil as many times as there are series coils within the winding.

-No gaps are allowed for between layers, nor between coils.

Different methods have been presented for calculating the ac resistance of round wire windings. In [3-5] the round conductor is replaced with a square conductor of equal cross-sectional area, and then equations related to the rectangular cross-sectional conductor are used. Two areas of erroneous prediction of this method include the potential error due to a geometrical dependency of the skin effect parameter, and a prediction error which becomes prominent at frequencies larger than $f_{norm}=3$ (i.e., the frequency where the skin depth is one third the wire diameter). On the positive side, it appears as if the equivalent rectangular conductor-based solution gives good predictions, provided that the winding is tightly packed and $f_{norm}<3$ [6].

Most often authors categorize the eddy current losses based on two physical phenomena that occur simultaneously in the winding [7]:

-Skin effect, the non-uniform distribution of the current in a conductor (with corresponding increase in losses) due to the magnetic field produced by the current in the conductor itself, and

-Proximity effect, the non-uniform distribution of the current in a conductor (with corresponding increase in losses) due to the magnetic field produced by the current in neighboring conductors.

Based on this definition, the losses due to the skin effect and proximity effect are calculated separately and then simply are added together. This means that the skin effect and proximity effect are orthogonal. This concept is acceptable as if the applied field due to proximity effect is uniform, which never occurs for a finite length winding. Even though by using this idea, handling of the eddy current losses analytically becomes easy, but it is valid only for simple arrangements and causes error for a complicated magnetic deviceas such as HV transformer winding.

In this section it will be shown that the orthogonality concept for calculating the eddy current losses of actual winding is not correct. It will be shown by an example for a single layer air core winding with length of 456 mm whit 21 turns and rectangular cross section of 3 mm$\times$16 mm. Radius of winding, gap between two conductors, frequency and conductivity are respectively 775 mm, 6 mm, 10 MHz and 5.8E07. The finite element software "FLUX2D" has been used to calculate the losses. Table I shows a comparison between the results obtained directly by the finite element calculation and those obtained by using orthogonality assumption for the skin effect and proximity effect. The first, second and third columns show respectively the number of a turn, actual loss of each turn, and loss of each turn that have carried out by orthogonallity assumption for skin effect and proximity effect. In this Table 0.1507 is the value of skin effect loss of individual turn in the absence of other turns. In this table, the calculated total loss at one ampere by means of orthogonallity assumption is %19.23 larger than actual value of loss and the error for middle turns is also larger than %30. Therefore, using the superposition for the skin effect and proximity effect produces error. Furthermore, unlike the expectation, the actual total loss is smaller than the sum of the skin effect loss and proximity effect loss. As a result, designing based on this assumption causes an over design plan.

Another way to tackle the problem is by numerical computation. The accurate calculation of eddy current losses in each region requires precisely determination of magnetic field intensity at that region [8]. The most well known and accurate tool in

Finite Element Technique for Calculating the AC Losses of Transformer Windings at High Frequencies

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Abstract

The known measuring techniques are difficult to apply for determining the power loss of transformer windings at high frequencies. None of them can be used to obtain the distribution of the power loss among and within the layers or discs. Analytical methods use simplification on the geometry of problems, therefore applicable in special cases. In this paper a finite element technique is introduced for calculating the ac losses of disc windings at high frequencies. This includes a method to calculate the ac resistance of windings which not only applicable for any winding with arbitrary shape of conductors, but also gives more accurate answers than analytical methods.

Keywords
Eddy current, Proximity effect, Skin effect, Power transformer, Finite element, Disc winding

I-Introduction

The accurate calculation of eddy current losses is important for predicting the damping during transients. The eddy current losses of actual windings at high frequencies maybe several orders of magnitude larger than its low frequency value.

There are three methods to calculate the eddy current losses of windings of high voltage power transformers. These are measurement, analytical, and numerical methods.

The first one is based on measurements. Determining the loss of individual disc or turn (hot spots) of a winding by this approach is difficult and the accuracy of the results depends on the measuring technique and the frequency of interest. One of the problems at high frequencies is the technical difficulty involved with the measurements such as low power factor. Also experimental techniques require much time and can be expensive. Another problem arises in the design phase.

The second one is the analytical methods. Even though these methods make the computation of the losses easy, in general they use some simplifications in concepts and the shape and the boundary conditions of conductors. Since the eddy current losses depend heavily on the shape of conductors, therefore they only can be applicable for particular problems. Meanwhile by analytical methods, generalization of the formulation to more conductors is difficult.

M.Stafl [1] in his two dimensional approximate solution gives relationships for the power losses, ac-to-dc resistance ratios, and current density distributions in the layers of middle coil of a transformer or of a reactor winding. In this method, these windings are represented by sets of infinitely long straight conductors of rectangular cross section. Some of assumptions of his approach are; a) there is no gap between layers, b) the length of conductors is infinite, c) the conductors are