Reference

[1] Bažant ZP, Jirasek M. Softening-induced dynamic localization instability: seismic damage in frames. Journal of Engineering Mechanic; ASCE 1996; **122:** 1149-1158.

[2] Zeris CA, Mahin SA. Analysis of reinforced concrete beam-columns under uni-axial excitation. Journal of Structural Engineering; ASCE 1988; **114:** 804-820.

[3] Neuenhofer A, Filippou FC. Evaluation of nonlinear frame finite-element models, Journal of Structural Engineering; ASCE 1997; **123:**
models for EL Centro 1940 and Mexico 1985 earthquakes. Probabilistic Engineering Mechanics 1990; **5:** 171-181.

[7] Bažant ZP, Planas J. Fracture and size effect in concrete and other quasibrittle materials. 1th edn, CRCPress, 1998.

[8] Bažant ZP. Instability, ductility, and size effect in strain softening concrete. Journal of Engineering Mechanic; ASCE 1976; **102:** 331-334.

[9] Bažant ZP, Pan J, Pijaudier-Cabot G. Softening in reinforced concrete beams and frames. Journal of Structural Engineering; ASCE 1987; **113:** 2333-2347.

[10] Foliente GC. Hysteresis modeling of wood joints and structural systems. Journal of Structural Engineering; ASCE 1995; **121:** 1013-1022.

[11] Tariverdilo S. An assessment of the response of softening RC frame structures. PhD dissertation; Sharif University of Technology; Tehran; Iran; 2002.

[12] Kulkarni M, Belytschko T, Bayliss A. Stability and error analysis for time integrators applied to strain softening materials. Computational
958-966.

[4] Spacone E, Filippou FC, Taucer FF. Fiber beam-column model for non-linear analysis of R/C frames: Part I. Formulation. Earthquake Engineering Structural Dynamics 1996. **25:** 711-725.

[5] Bažant ZP, Xiang Y, Prat PC. Microplane model for concrete I: Stress-strain boundaries and finite strain. Journal of Engineering Mechanic; ASCE 1996; **122:** 245-254.

[6] Fan FG, Ahmadi G. Nonstationary Kanai-Tajimi methods in applied Mechanic and Engineering 1995; **124:** 335-363.

[13] Sheikh SA, Yeh CC. Tied concrete columns under axial load and flexure. Journal of Structural Engineering; ASCE 1990; **116:** 2780-2800.

[14] Khaloo AR, Tariverdilo S. Evaluation of nonlinear dynamic response of structures based on stochastic ground motion process. 11th European Conference on Earthquake Engineering; Balkema: Rotterdam; 1998.

[15] Jirasek M. Analytical and numerical solutions for frames with softening hinges. Journal of Engineering Mechanic; ASCE 1997; **123:** 8-14.

[16] Teh LH, Clarke MJ. Tracing secondary equilibrium path of elastic framed structures, Journal of Engineering Mechanic; ASCE 1999; **125:**1358-1364.

[17] Fujii F, Okazawa S. Pinpointing bifurcation points and branch-switching. Journal of Engineering Mechanic; ASCE 1997; **123:**179-189.

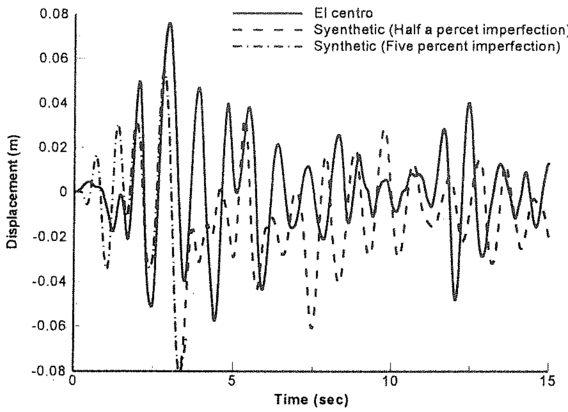
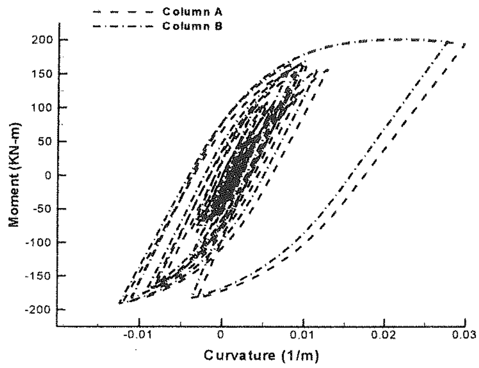
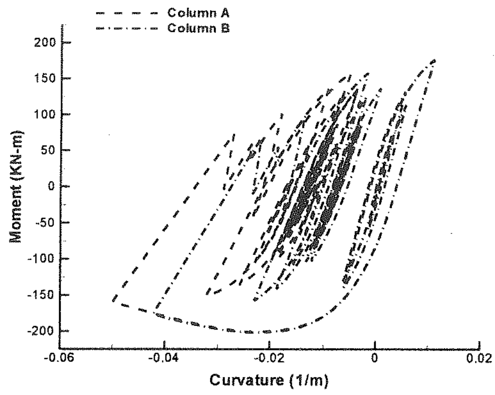


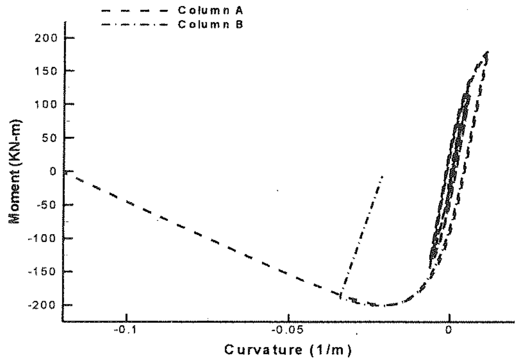
Figure (3) Displacement time history for El centro and synthetic ground motions in structure of first example.



(a)



(b)



(c)

Figure (4) Moment-curvature of columns obtained in dynamic analysis for, a) El centro excitation, b) synthetic ground motion with half a percent imperfection in columns strength, c) synthetic ground motion with five percent imperfection.

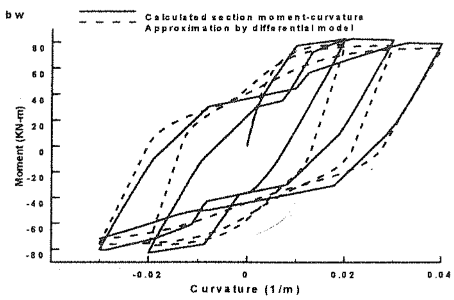
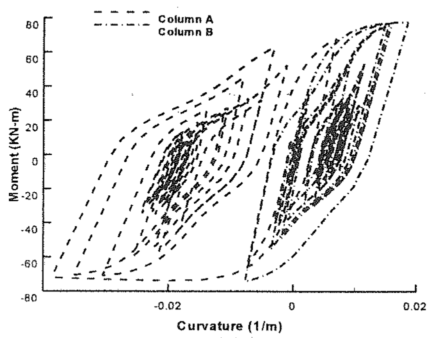
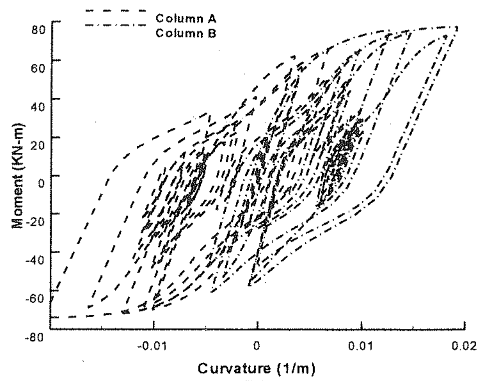


Figure (5) The calculated moment-curvature of column section versus its approximation by differential model.



(a)



(b)

Figure (6) The moment-curvature of critical column sections for, a) Elcentro, b) synthetic ground motions.

$h(z)$: pinching function
k: section stiffness (EI)
M: dimensionless section resisting moment
M_d: section resisting moment with dimension of KN-m
n: parameters of differential hysteretic model
$n(t)$: Gaussian white noise process
X_f: filter response
X_g : ground displacement
z : section hysteretic curvature
$\mathbf{a}(x)$: displacement interpolation function vector
$\mathbf{b}(x)$: force interpolation function vector
$\mathbf{d}(x)$: section displacement vector
$\mathbf{D}(x)$: section force vector
$\mathbf{f}(x)$: section flexibility matrix
$\mathbf{F}$: element flexibility matrix
$\mathbf{q}$: element displacement vector
$\mathbf{Q}$: element force vector
α : ratio of post-yield to pre-yield stiffness
β, γ: parameters of differential hysteretic model
δ_1 , δ_2 , δ_{10} : constants of pinching function
ε_{cr} : tensile cracking strain
η: stiffness degradation parameter of differential hysteretic model
λ : constant
ρ: degree of efficiency of passive confinement
σ_{st}^{i} : stress in the transverse reinforcement of layer I
ω_g : ground frequency
ξ_g : ground damping
φ : dimensionless section curvature
φ_p, φ_n : section yield curvature with dimension 1/m

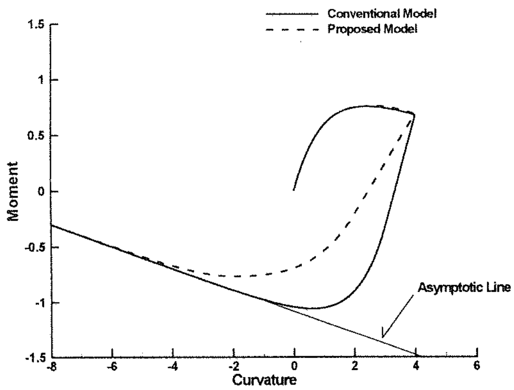


Figure (1) Behavior of the conventional and proposed differential models in cyclic loading.

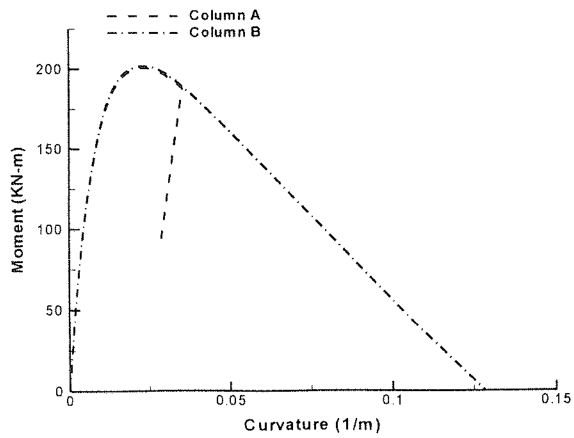


Figure (2) Moment-Curvature of columns of first example obtained by static pushover analysis.

Table (I) Section Properties of the structure of first example.

	Beam	Column
Section yield moment (KN-m)	205	202
Section stiffness (EI, KN-m^2)	41250	30000
Post yield stiffness ratio	0.001	-0.07

Table (II) Material and Section Properties of the structure of Second example.

Reinforcements yield strength (KN/m^2)	300
Column reinforcements hardening ratio	.001
Beam reinforcements hardening ratio	.05
Concrete strength (MPa)	21
Beam dimensions (m)	0.40*0.30
Column dimension (m)	0.30*0.30

To obtain an assessment of the possibility of localized behavior for code-designed structures due to seismic excitation, a second example is considered. The structure has the same dimensions of example one, except that here columns are fixed base. The tributary width of the frame is 4 m and distributed dead and live loads have the values of 7.5 and 2.5 KN/m^2, respectively. This frame structure is designed according to Iranian seismic code and ACI-318 regulations. It is assumed that the structure is located in seismic zone with very high seismic activity and designed as ordinary moment resisting frame (OMRF). Note that design of structure as OMRF in regions with high seismic activity is prohibited in the UBC, however in Iranian seismic code and some other codes as well, it is allowed. The parameters of the hysteretic model used in dynamic analysis are obtained from the material and section properties given in Table II.

The moment-curvature of the column critical sections evaluated using the microplane material model for concrete and bilinear elasto-plastic model for reinforcements is shown in Figure 5. In evaluating section moment-curvature fluctuation of axial force during seismic excitation is not considered. Also shown in this figure is the differential hysteretic model's approximation of the calculated moment-curvature. The observed softening behavior is mainly due to large spacing of transverse reinforcement in critical section and subsequent lack of confinement. In fact due to this large spacing of ties allowed by code, in column sections midway of the ties, there is no confined concrete core (as evaluated using Sheikh and Uzumeri method). Note that the hardening ratio for column reinforcements is taken approximately equal to zero, however for beam reinforcements, to account for contribution of slab, a five percent hardening ratio is assumed.

To initiate localization, five percent imperfection is introduced. The same considerations similar to those used in first example are taken into account to avoid fictitious localization. Figure 6 shows the moment-curvature of the two columns under EL Centro and synthetic ground motion excitations. In this case localization occurs for EL Centro ground motion.

Note that considering the common range of possible changes in the strength of materials, imperfections even greater than that assumed here may happen in practice. Also considering the sensitivity of the structure to the change in characteristics of excitation, there is high probability of deformation localization. Two reasons for this observed behavior might be mentioned here. The first is that the code regulations, which allows the use of OMRF system in high seismic activity regions and the second the large spacing of transverse reinforcements in critical sections.

Conclusions

The possibility of localization in frame structures in static and dynamic analysis investigated. A finite element model with differential constitutive equations for each section is employed. The problem with conventional differential model in the case of softening behavior is addressed and a new differential model introduced. The parameters of the differential model are obtained by evaluating the section moment-curvature using microplane and bilinear elasto-plastic material models for concrete and reinforcements, respectively. A nonstationary model is used to generate artificial ground motions in order to study the sensitivity to possible minor changes in characteristics of input ground motion. Also the effect of imperfection on the response of the structure is investigated. It is shown that for code-designed structures with some extreme assumptions, early collapse of structure due to localization in dynamic analysis may occur.

Acknowledgments

The writers gratefully acknowledge the support provided by research committee of Sharif University of Technology.

NOTATION

A_{st}^{i}: area of transverse steel of the concrete layer i

$e(t)$: amplitude modulating function

f_{cr}: tensile cracking stress

$(F_{ct}^{i})_{avg.}$: average of lateral forces on concrete layer i

to ground motions generated by Fan-Ahmadi model are inconsistent with that of EL Centro ground motion. For example, although energy absorption increases for high periods due to EL Centro ground motion, ensemble average of energy absorption of the synthetic ground motions generated by Fan-Ahmadi model remains nearly constant or may even decrease. To overcome this problem the ground frequency in equations (18) is taken equal to a time function that approximates dominant frequency of the base ground motion instead of frequency of zero crossings. Also the modulating function is changed such that the ensemble average of energy of the generated ground motions becomes equal to that of the EL Centro 1940 ground motion. In doing so the modulating function takes the following form

$$e(t)=46t^{1.85}\exp(-1.3t^{0.75}) \quad \text{for} \quad t \leq 8.6 \text{ sec} \qquad (19a)$$

$$e(t) = 3.6 \quad \text{for} \quad t \geq 8.6 \quad \text{sec} \qquad (19b)$$

Numerical Examples

The first example is one story, one bay frame structure with columns pinned at base. The possibility of bifurcation in static analysis for this structure is discussed in Bažant and Jirasek[1] and Jirasek[15]. The story height and bay length are 3.5 and 4 m respectively. The material and section properties of the structural elements used in study are presented in Table I. Softening behavior for columns and hardening behavior for beam are assumed. The mass of the structure lumped at story level is taken equal to 50000 kg.

Entering the post peak region, negative eigenvalues appear in the stiffness matrix of the structure, which shows the possibility of the bifurcation. The primary equilibrium path corresponds to the unlocalized response, in which both columns undergo softening, while in the secondary path, localization occurs. The energy required for secondary path is smaller than that for the primary one, so the secondary path is the path that structure actually follows up. To trigger secondary equilibrium path in static loading a branch switching technique is required. There are various branch switching methods (e.g., Teh and Clarke [16] and Fujii and Okazawa [17]). Most of branch switching techniques employ displacement perturbation to trigger the secondary path. However the force perturbation method is simpler to employ than the displacement perturbation methods. In this case a force perturbation, proportional to the eigenvector of the lowest (negative) eigenvalue of the stiffness matrix is used for branch switching purpose.

To prevent from localization into unrealistically small regions, in softening elements a minimum element length equal to element depth is used. Assuming an element depth of 0.3 m, in each column, twelve elements with two Gauss points per element are used. For beam, in which hardening behavior is assumed, one element with three Gauss points is employed.

Figure 2 shows the moment-curvature of the critical sections of the two columns obtained by displacement control static pushover analysis. As can be seen localization occurs and while one of the critical sections undergoes unloading the other one goes into softening.

In dynamic analysis to initiate the localized behavior it is necessary to introduce some imperfections into model. In doing so a half a percent decrease in the strength of one of the columns, hereafter called column A, is employed. In this example pinching in hysteretic loops is not considered. Figure 3 shows the time history of the displacement of structure at story level due to EL Centro and a synthetic ground motions. Only the first fifteen seconds of the response, which includes the strong ground motion portion, is shown. Figure 4 shows the moment-curvature of the critical sections of the two columns. As can be seen appreciable localization does not occur. To trigger more localization the imperfection is increased to five percent and the structure analyzed under synthetic ground motion excitation. In this case full localization occurs, i.e., column A goes into softening while unloading in column B occurs and the result of this localized response is early collapse of column and consegently of structur.

Consequently

behavior. Each section is discretized into a number of layers representing concrete and steel rebars. Concrete at each layer of the section is modeled as a three-dimensional Gauss point. At this stage only the behavior of one section is considered, so the Gauss points, representing concrete layers, are in parallel coupling. So there is no need for any localization limiter at this stage. Displacement control on curvature and load control on axial force is used to develop the section load-deformation behavior and for iterative solution of the equilibrium equations the initial stiffness method is employed.

In Gauss points, representing concrete layers, where lateral equilibrium at each layer is imposed separately. By this assumption it possible to consider the different confining conditions due to different shape and spacing of transverse reinforcements by changing the amount of effective transverse reinforcement. Also it is assumed that the strains in the two transverse directions are equal. Therefore lateral equilibrium equation for each concrete layer (Gauss point) will be

$$(F_{ct}{}^{i})_{avg.} + \rho\sigma_{st}{}^{i}A_{st}{}^{i} = 0 \qquad (17)$$

in which $(F_{ct}{}^{i})_{avg.}$ is the average of the two lateral forces in concrete layer i, and ρ denotes the degree of efficiency of passive confinement, $\sigma_{st}{}^{i}$ is the stress in the transverse reinforcements of layer i and $A_{st}{}^{i}$ is the area of the transverse reinforcement per unit length of the flexural member. The degree of efficiency of the passive confinement is defined as the ratio of the confined concrete area to the gross area of each layer. The method of Sheikh and Uzumeri[13] that takes into account the shape and spacing of the transverse reinforcements, is used for calculating the confined concrete area in each layer. For transverse reinforcements also a bi-linear elasto-plastic model is employed.

Stochastic Ground Motion Generation Process

Development of Fan-Ahmadi[6] model is based on some modifications on Kanai-Tajimi stationary filter. They modified conventional Kanai-Tajimi filter, by introducing variable angular frequency $\omega_g(t)$ and modulating function $e(t)$ as follows:

$$\ddot{X}_f + 2\xi_g\varpi_g(t)\dot{X}_f + \varpi_g^2(t)X_f = n(t) \qquad (18a)$$

$$\ddot{X}_g = -(2\xi_g\varpi_g(t)\dot{X}_f + \varpi_g^2(t)X_f)e(t) \qquad (18b)$$

where X_g , X_f are ground displacement and filter response, $\omega_g(t)$, ξ_g and $e(t)$ are time dependent ground frequency, ground damping and amplitude modulating function, respectively, and, $n(t)$ is stationary Gaussian white noise process with zero mean and constant power spectrum intensity of one. For modulating function equal to one and constant ground frequency and damping, the model reduces to the original Kanai-Tajimi filter. In the Fan-Ahmadi model the modulating function is taken equal to evolutionary mean square of amplitude based on two seconds window of the base earthquake scaled by a constant. In this model ground motion frequency is determined by the number of zero crossings of base ground motion evaluated by two seconds time averaging. The value of the ground damping is taken constant and equal to 0.42.

As pointed out by Khaloo and Tariverdilo[14] this model suffers from the following two deficiencies:

I. Ground motions generated by the Fan-Ahmadi model have finite amplitude in near zero frequencies. This holds in spite of nearly zero frequency content of the EL Centro 1940 Ground motion in these frequencies. This frequency contents introduce a nearly static force, which causes high response for structures in the aforementioned range and moreover the displacement at the end of earthquake does not reach zero. To get rid of this frequency contents, a seventh order highpass butterworth filter with cutoff frequency of 0.1 Hertz is applied to the generated ground motion.

II. The second deficiency of the Fan-Ahmadi model is that the dominant angular frequency of EL Centro ground motion (also based on two seconds window) does not coincide with angular frequency of zero crossings. The result of this mismatch is that the nonlinear response of structure due stiffness degradation introduced in the model. For sections with softening behavior φ_m should also satisfy the following equations

$$\min(\varphi_m) = -\ln(-\alpha/(1-\alpha)) \text{ for } \dot{\varphi} \geq 0 \qquad (13a)$$

$$\max(\varphi_m) = \ln(-\alpha/(1-\alpha)) \text{ for } \dot{\varphi} < 0 \qquad (13b)$$

and for sections with hardening behavior

$$\min(\varphi_m) = -\ln(.05) \text{ for } \dot{\varphi} \geq 0 \qquad (14a)$$

$$\max(\varphi_m) = \ln(.05) \quad \text{for } \quad \dot{\varphi} < 0 \qquad (14b)$$

The result of application of the proposed evolution equation in differential model is also presented in Figure 1.

To model different strength for positive and negative moments which is common in RC beams, it is sufficient to evaluate the section moment with dimension of force-length, M_d, by

$$M_d = [(\varphi_p - \varphi_n)z + (\varphi_p + \varphi_n)]kM/2 \qquad (15)$$

in which φ_p and φ_n are section positive and negative yield curvatures, and k is section flexural stiffness (EI). In this case better results can be obtained if we use different values for κ when positive or negative increment of curvature occurs.

Spurious unloading occurs in numerical integration, if displacement increment in iterations is calculated with respect to the condition at the end of the previous iteration. This is due to the fact that the unloading stiffness is always equal to or larger than the loading stiffness. To avoid this problem, in iterations the condition at the end of previous converged *loadstep* is taken as initial condition and displacement increment is calculated with respect to that.

Rung-Kutta integration schemes of different orders are usually used for numerical integration of the rate equations. However for softening there is the problem of error propagation associated with conventional numerical integration schemes (e.g., Kulkarni *et al.*[12]). It should be noted that for n equal to one and two, assuming constant $h(z)$ and η during each loadstep, there is the possibility of direct integration of the rate equations. Use of direct integration scheme (assuming constant $h(z)$ and η) provides exact solution of the rate equations and prevents the error propagation. In this study for numerical integration of the rate equations the direct integration scheme is used.

Derivation of Section load-Displacement Behavior

To determine the parameters of the differential model, the section load-deformation (moment-curvature) behavior is required. The microplane model of Bažant *et al.*[5] is used as constitutive law for concrete and for reinforcements a bi-linear elasto-plastic material model is employed. In the following some considerations associated with the use of microplane model and derivation of section behavior are discussed (Tariverdilo[11]).

In compression reloading, after tension unloading the microplane model shows incorrect behavior. In the microplane model concrete enters compression incorrectly before the tensile cracks are closed, even without any stiffness degradation. The following steps are taken to avoid this problem: (1) Value of the all microplanes variables in the Gauss points, in the last converged compression status are stored, (2) A macroscopic constitutive law is used for concrete in tension, and (3) In compression reloading, after closing of the tensile cracks, the last converged compression state is used as initial condition. The stress-strain relationship in tension is taken linear up to the cracking and after that the following exponential function is employed

$$f = f_{cr} \exp(-\lambda(\varepsilon - \varepsilon_{cr})) \qquad (16)$$

in which f and ε are stress and strain of concrete layer, f_{cr} and ε_{cr} are the stress and strain corresponding to the crack initiation and λ is a positive constant. Also it is assumed that tensile unloading and reloading follow secant slope.

Euler hypothesis of plane sections remain plane after deformation is assumed to obtain the section load-deformation method of Newmark. Viscous damping is modeled using the Rayleigh proportional damping.

Differential Hysteretic Model

A differential hysteretic model is used to model the section load-deformation behavior. This eliminates the need for discretization of the section into layers and tracing the behavior of the each layer during analysis, which results in substantial computational efficiency of the model in comparison with layered finite element models. Based on the differential hysteretic model of Bouc-Wen, dimensionless rate equation for section resisting moment will be

$$\dot{M} = \alpha\dot{\varphi} + (1-\alpha)\dot{z} \qquad (6)$$

in which α is the ratio of post-yield to pre-yield stiffness, φ is dimensionless curvature, z is the hysteretic curvature and dot denotes derivative with respect to time. Using the Foliente[10] differential model the rate equation for z is

$$\dot{z} = h(z)[1-(\beta\,\mathrm{sgn}(\dot{\varphi})\,\mathrm{sgn}(z)+\gamma)|z|^{n}]\dot{\varphi}/\eta \qquad (7)$$

where β, γ and n are the parameters of the hysteretic model, η is stiffness degradation parameter, $h(z)$ is the pinching function and sgn is the signum function. Assuming that sum of β and γ is equal to one, the pinching function $h(z)$ in unloading is equal to one (i.e., there is no pinching) and in loading and reloading is given by

$$h(z) = 1-\delta_1\exp(-(z\,\mathrm{sgn}(\dot{\varphi})-q_1)^2/\delta_2^2) \qquad (8)$$

in which δ_1 and δ_2 evolve as functions of dissipating energy and control the spread and severity of pinching. The rate equation for dissipating energy is

$$\dot{u}_e = (1-\alpha)z\dot{\varphi} \qquad (9)$$

Now consider a section that goes into relatively large positive curvatures. In the differential model the moment-curvature curve tends asymptotically to two inclined lines with slope of α (Figure 1). When load reversal occurs, for negative curvatures there is a strength gain of approximately equal to the strength loss due to softening in positive direction. In addition to this fictitious increase in the strength, the section reaches its peak resistance even before entering the negative curvatures. This is not a realistic description of the response for RC elements. The same misbehavior also occurs for hardening behavior. Also due to poor simulation of stiffness degradation for hardening and softening cases, there will be fictitious drift in the moment-curvature diagram, and consequent asymmetric (shifted) moment-curvature.

To overcome this deficiency, a new evolution equation for η is introduced (Tariverdilo[11]). From hereon it is assumed that n is equal to one. This restriction does not influence the model's capability. We introduce the following evolutionary equation for stiffness degradation parameter η when unloading occurs

$$\eta = (\kappa\varphi_m-\varphi_0)/(-\mathrm{sgn}(\dot{\varphi})\ln(\theta_u)-z_0) \qquad (10)$$

and the following equation for loading

$$\eta = (\kappa\varphi_m-\varphi_0)/(-\mathrm{sgn}(\dot{\varphi})\ln(\theta_l)) \qquad (11)$$

where θ_u and θ_l are defined as

$$\theta_u = 1+[(1-\alpha)(-1+\exp(-\mathrm{sgn}(\dot{\varphi})\kappa\varphi_m))+\mathrm{sgn}(\dot{\varphi})(M_0-\alpha\varphi_0-(1-\alpha)z_0)]/(1-\alpha) \qquad (12a)$$

$$\theta_l = 1+[\mathrm{sgn}(\dot{\varphi})(1-\alpha)(1-\exp(-\mathrm{sgn}(\dot{\varphi})\kappa\varphi_m))-M_0+\alpha\varphi]/[(1-\alpha)(z_0-\mathrm{sgn}(\dot{\varphi}))] \qquad (12b)$$

in which M_0, φ_0 and z_0 are the values of moment, curvature and hysteretic curvature in the beginning of loadstep, respectively, and φ_m is the maximum or minimum curvature that section experienced and κ is a constant that controls the severity of the deformation behavior are described. The synthetic ground motion generation process is then addressed. Finally the results of the application of the described procedure to two frame structures are presented.

Fiber Element Model

Interpolation of deformation by Hermitain interpolation functions in stiffness-based finite element models results in linear curvature distribution, where the actual curvature distribution in large localizations is nonlinear. The flexibility-based models, can approximate this nonlinear curvature distribution using variable flexibility interpolation functions

$$\Delta\mathbf{d}(x)=\mathbf{f}(x)\Delta\mathbf{q}\ ;\ \Delta\mathbf{D}(x)=\mathbf{b}(x)\Delta\mathbf{Q}\ ;\ \Delta\mathbf{Q}=\mathbf{F}^{-1}\Delta\mathbf{q} \qquad (1a,b,c)$$

Now using equations (1) the section deformation increment vector $\mathbf{d}(x)$ will be

$$\Delta\mathbf{d}(x)=\mathbf{f}(x)\mathbf{b}(x)\mathbf{F}^{-1}\Delta\mathbf{q}=\mathbf{a}(x)\Delta\mathbf{q} \qquad (2)$$

where $\mathbf{d}(x)$ denotes section deformations vector, $\mathbf{f}(x)$ is section flexibility matrix, $\mathbf{b}(x)$ is the force interpolation vector, $\mathbf{F}$ is the element flexibility matrix, $\mathbf{q}$ is the element nodal displacement vector, $\mathbf{a}(x)$ is equivalent variable deformation interpolation vector, $\mathbf{D}$ is the section resisting force vector and $\mathbf{Q}$ is the element nodal resisting force vector.

The flexibility-based finite element model of Neuenhofer and Filippou[3] is used for nonlinear analysis of frame structures. The model is able to satisfy equilibrium along member length even when softening occurs. By some changes in the representation, this model is described in the following. After evaluation of the element nodal displacements, following conventional procedure in the stiffness based finite element, the member nodal forces in iteration i is obtained as

$$\Delta\mathbf{Q}^{i}=(\mathbf{F}^{i-1})^{-1}\Delta\mathbf{q}^{i} \qquad (3)$$

Now the section deformations, considering the section unbalance forces at the end of iteration i-1, is calculated as

$$\Delta\mathbf{D}^{i}(x)=\mathbf{b}(x)\Delta\mathbf{q}^{i}+\mathbf{b}(x)\mathbf{Q}^{i-1}-\mathbf{D}^{i-1}(x) \qquad (4a)$$

$$\boxed{\begin{aligned}&\Delta\mathbf{d}^{i}(x)=\mathbf{f}^{i-1}(x)\Delta\mathbf{D}^{i}(x)\,;\\&\mathbf{d}^{i}(x)=\mathbf{d}^{i-1}(x)+\Delta\mathbf{d}^{i}(x)\end{aligned}} \qquad (4b,c)$$

where the member nodal resisting force is determined using the variable deformation interpolation function $\mathbf{a}$(x) as follows

$$\mathbf{Q}^{i-1}=\int_0^l \mathbf{a}^{T}(x)\mathbf{D}^{i-1}(x)dx \qquad (5)$$

The Gauss-Lobatto scheme is used for numerical integration. In this scheme two integration points coincide with the member end sections, where significant inelastic deformations usually take place.

If the described finite element model is applied for softening problems, the slope of the descending branch of the load-deformation curve will be strongly dependent on the finite element discretization. Increase in number of the Gauss points or the number of the elements, results in steeper descending branch. This shows the need for a kind of localization limiters to prevent localization of the deformations into regions with zero volume (Bažant and Planas[7]). There are various localization limiters models, such as the nonlocal and gradient models. However, there is also a simpler way based on the concept of crack band model (Bažant[8]). This concept defines a minimum localization length that the finite element discretization should not be finer than that. For frame elements, Bažant *et al.*[9] suggested that the discretization length should not be less than the member depth.

Initial stiffness method is used for the iterative solution of the nonlinear systems of equations. Energy increment in iterations is used as convergence criteria. For axial force elastic constitutive equation is employed. For time integration of equilibrium equations is done using trapezoidal numerical integration also occur at some other sections along member length. This makes it possible to capture the gradual increase of nonlinear behavior. (3) Microscopic models, in this case the structural elements discretized into two or three dimensional finite elements and reinforcements and concrete (usually) modeled separately. The computational demand of this models, makes them inappropriate for analyze of frame structures with relatively large number of members. The finite element models are a good compromise between lumped and microscopic models. This paper uses a finite element model in which a differential hysteretic constitutive equation is used for each section. This results in substantial computational efficiency with respect to layered finite element models where the behavior of each layer in section should be traced. The traditional stiffness-based finite element models have difficulty with softening, where the element is unable to recognize unloading in interior sections due to softening at end sections. This is mainly due to use of deformation interpolation functions. Here to avoid this problem the flexibility-based finite element model of Neuenhofer and filippou[3], which is a modification of Spacone *et al.*[4] finite element model, is used.

Conventional differential models, based on extensions of Bouc-Wen differential model, are not able to consider properly the softening behavior RC elements in the cyclic loading. The reason for this misbehavior is addressed and as a remedy to this and also to obtain a better estimation of stiffness degradation in cyclic loading in element with softening or hardening behavior a differential model specially suitable for RC beam-column elements introduced. RC beam elements usually have different flexural strength for positive and negative curvatures. A simple way to use differential model for these types of elements is also developed.

To obtain an estimation of the behavior of the real structures and to study the effect of softening on their response, it is necessary to obtain the parameters of hysteretic model from available section properties. In doing so there is a need to have good material model for concrete and steel rebars. The hysteretic modeling of the steel rebars is relatively simple and usually a bilinear elasto-plastic constitutive law is use for this purpose. The material model for concrete should be able to provide good simulation of the concrete volume dilation, path dependency and strength gain due to confining pressure. In this work the microplane model of Bažant *et al.*[5] with slight modification is used.

Temporal variations of amplitude and frequency in earthquake due to zero initial start and dispersion of the propagating seismic waves result in nonstationarity of amplitude and frequency. Numerous synthetic ground motion generation models have been proposed with different assumptions on stationarity and nonstationarity of the amplitude and frequency. In this study a modified version of Fan and Ahmadi[6] generalized nonstationary Kanai-Tajimi filter is used. This model results in a process nonstationary in amplitude and frequency. One of the interesting features of this model is the generation of synthetic ground motions with relatively same amplitude and frequency content evolution as those of base ground motion. So it is appropriate for analysis of the sensitivity of the structure to slight change in the characteristics of a base ground motion. In this study, the N00W component of the EL Centro 1940 ground motion is taken as base ground motion.

An assessment of the possibility and importance of softening in the behavior of RC frame structure is obtained by considering two case studies. The first is investigated.
In the following sections, the finite element, the differential hysteretic model and the procedure used to obtain the section load-deformation behavior are described. The synthetic ground motion generation process is then addressed. Finally the results of the application of the described procedure to two structure is also studied by Bažant and Jirasek[7] and Jirasek[8]. The second case is a code-designed structure, which the possibility of localization in its dynamic response is investigated.

In the following sections, the finite element, the differential hysteretic model and the procedure used to obtain the section load-

Static and Dynamic Localization in Softening RC Frame Structures

A. R. Khaloo
Professor

S. Tariverdilo
Research Assistant

Department of Civil Engineering,
Sharif University of Technology

Abstract

This paper studies the possibility of localizations for frame structures in static and dynamic analysis. A finite element model is used in which the sections resisting force is calculated using an introduced differential hysteretic model. A microplane constitutive law for concrete and bi-linear elasto-plastic material model for reinforcements evaluates parameters of the differential hysteretic model in sections. The sensitivity to probable changes in ground motion characteristics is assessed by a nonstationary ground motion generation process to obtain excitations with approximately the same amplitude and frequency content evolution as those of a base ground motion. The procedure is applied in two case studies and possibility of localization in response is investigated. A measure for the probability of occurrence of this behavior in code-designed structures is obtained. This study indicates that localization and resulting early collapse of structure is possible. It is concluded that some modifications in design code provisions are required.

Keyword

Differential hysteretic model; Finite element model; Localization; Microplane model; Reinforced concrete flexural members; Synthetic ground motions

Introduction

In usual design of structures, it is assumed that the structural elements have adequate deformation ductility. It is equivalent to assumption of a hardening behavior at least in early stages of post yielding behavior. However this is not always the case and softening behavior may be observed even in early stage of post yielding behavicr. Softening causes ocalization of deformations and increase in deformation demand, which usually accelerates the collapse of the structure. If the primary path of the structure is symmetric, localization represents symmetry breaking response and possibility of bifurcation and secondary equilibrium path. Using second law of thermodynamic it can be shown that the secondary path is the actual path that followed up by the structure. In dynamic analysis bifurcation does not occur and localization results in temporal dynamic instability in the form of exponential increase of deformations.

To analyze the effect of the localization on the response of frame structures, various models are available in the literature. This models can be categorized with increasing level of refinement as (1) Lumped models, in which nonlinear behavior is assumed to be lumped at the end of the frame elements. (2) Finite element models, where in addition to element end sections, the nonlinearity can