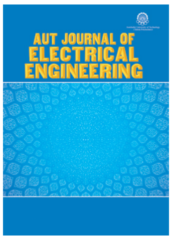

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On Joint Sub-Channel Allocation, Duplexing Mode Selection, and Power Control in Full-Duplex Co-Channel Femtocell Networks

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ABSTRACT: As one of the promising approaches to increase the network capacity, Full-duplex (FD) communications have recently gained a remarkable attention. FD communication enables wireless nodes to simultaneously send and receive data through the same frequency band. Thanks to the recent achievements in the self-interference (SI) cancellation, this type of communication is expected to be potentially utilized in cellular systems, especially in small cell networks. In this paper, we integrate the FD communications into femtocell networks where femtocell users (FUs) share the same spectrum with macro users (MUs). In particular, aiming to maximize the number of admitted FUs in the network and satisfying the target rate for FUs, we jointly study the problem of sub-channel allocation, duplexing-mode selection, and power control for FUs in both uplink (UL) and downlink (DL) transmissions. Moreover, we address the power control problem for macro-tier where the main goal is to minimize the transmission power of prioritized MUs while guaranteeing a target rate for them. To jointly address these problems for both tiers, we propose a distributed algorithm in which FBSs and admitted FUs choose whether to operate in half-duplex (HD) or FD mode so as to meet their target rate. The convergence and performance of the proposed algorithm are evaluated through simulation where it is demonstrated that the average admission ratio of FUs in our proposed distributed scheme surpasses the existing traditional HD approaches.

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1- Introduction

In recent years, there has been an enormous increase in the demand of mobile users of wireless networks which gives a great impetus to the further enhancement of the network capacity. Full-duplex (FD) communication as one of the emerging revolutionary paradigms has recently gained a significant attention owing to its potential of further improving or even doubling the overall spectral efficiency of traditional half-duplex (HD) communications. In FD communication, wireless nodes are able to simultaneously transmit and receive information over the same frequency band [1]. In practice, however, FD-enabled wireless nodes severely suffer from the Self-Interference (SI) in the sense that the presence of concurrent transmission and reception over the same frequency band may negatively affect the performance of communication made by FD nodes. To address such challenges, some SI cancellation approaches such as analog, digital, and antenna SI cancellation approaches have been recently proposed [2], which allow overcoming the limitation of implementing FD communications in wireless networks.

The SI density strongly depends on the transmit power level of the wireless node. That is, the lower the transmit power used at the transmitter, the more manageable SI the receiver is exposed to. In small cell networks such as femtocells, the SI could be handled more efficiently due to the presence of short distance and low power communications between femto base stations (FBSs) and femtocell users (FUs).

In a two-tier OFDMA network consisting of MBSs and FBSs, two interference scenarios may occur (i) the cross-tier interference between the macrocell and femtocells' communications, and (ii) the co-tier interference either within the macrocells or among femtocells. Considering FD communications, in addition to the SI, inter-user interference may also exist. As a result, transmit power and sub-channel allocation along with duplexing mode selection become challenging. Under such scenario, the duplexing mode selection is defined as the procedure in which either a user or a BS can switch between FD and HD modes.

1- 1- Related works

The resource allocation problem in OFDMA-based femtocell networks has been extensively studied in the literature of which the main focus is on improving the throughput of users [4-10], [17-20], [25-26], improving the fairness [11], [16], minimizing the transmission power [12], [24], optimizing outage and energy consumption [13], first satisfying higher priority users and then serving the remaining users [15], and maximizing the number of admitted users [14]. Only a few current works have considered FD communication, namely, [17-20] and [26]. In [17], FD communication is considered in cellular networks where the SI is considered to be constant; however, in practice, the SI changes with varying transmit power. In [18], the transmit power of annoying femtocells is controlled so as to mitigate the cross-tier interference received at either MUs or the MBS. Although, the allocation of resources in [19], [20], and [26] have been investigated in FD-enabled femtocell networks, the authors of [19] and [26] did not allow the switching between duplexing modes, and they considered FD communication mode for FBSs and HD communication mode for FUs. The authors of [20] investigated the joint sub-channel allocation and duplexing mode selection for a femtocell.

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In summary, several research gaps have been left unaddressed in the literature; addressing each one has a prominent role in improving the performance of the network. For example, (i) fixed resources are considered for macro-tier, and resource allocation problem is studied only for femto-tier [5,6,8,13,19,20,24,26], (ii) only cross-tier interference is considered (co-tier interference is ignored) [8,13,14,18,26], or only co-tier interference is considered (cross-tier interference is ignored) [9,10,12,15,16], (iii) a target rate is not guaranteed for FUs [4-6], [10,11,15,18], [25-26], and (iv) joint selection of duplexing modes, allocation of sub-channels and transmit power for users have been studied only in [18]. Specifically, some works have studied the power control problem [4, 6, 13], some have addressed sub-channel allocation problem [7,10,15,16]. These works and the papers [5, 8, 9, 12, 14], [24-26] have not studied the joint duplexing mode selection, sub-channel allocation, and power control, whereas taking both radio resource allocation (sub-channel allocation and power control) and duplexing mode selection into account improves the performance of the network.

1- 2- Contribution

In this paper, despite the most existing works referred to in sub-section 1-1, the sub-channel allocation, duplexing mode selection, and power control problem are jointly addressed for users in FD-enabled femtocells underlying cellular networks. We focus on maximizing the number of admitted FUs in the network, in both uplink (UL) and downlink (DL) transmissions. We consider a network in which both FUs and FBSs can switch between HD and FD modes. In contrast to the existing works, both co-tier and cross-tier interference among users are considered, and a target rate is guaranteed for the users of both tiers.

Our main contributions are summarized as follows. We study the problem of downlink power control for macro-tier while for femto-tier, the joint sub-channel allocation, duplexing mode selection, and power control problem are addressed, aiming to maximize the number of admitted FUs in both UL and DL transmissions subject to target rate for MUs and FUs. To address the optimization problems of both tiers in a practical manner, we propose a sub-optimal distributed algorithm with a low complexity. Our simulation results demonstrate that our proposed scheme outperforms the traditional HD femtocells in terms of femtocell's average admission ratio.

The organization of this paper is as follows. The system model is introduced in section 2. The optimization problem for both macro-tier and femto-tier is formally stated in section 3. The power control algorithm for macro-tier and joint sub-channel allocation, duplexing mode selection, and power control algorithm for femto-tier is presented in section 4. Finally, simulation results and conclusions are presented in section 5 and section 6, respectively.

2- System Model and Equations

As shown in Fig. 1, we consider a two-tier OFDMA network, including one MBS and K FBSs under co-channel deployment where the macrocell and femtocells share the whole spectrum. Femtocells connect to the backbone through the wired backhaul. We assume that the users in this network are already associated with MBS and FBSs by using, for example, Reference Signal Received Power (RSRP) scheme. The network consists of a set of $M = M^m + M^f$ users denoted by $\mathcal{M} = \mathcal{M}^m \cup \mathcal{M}^f$ including a set of M^m MUs denoted by $\mathcal{M}^m = \{1,2,...,M^m\}$ and a set of M^f FUs denoted by $\mathcal{M}^f = \{1,2,...,M^f\}$. The network is provided with a set of N sub-channels denoted by $\mathcal{N} = \{1,2,...,N\}$. Furthermore, the set of BSs is denoted by $\mathcal{B} = \{1,2,...,K+1\}$ wherein the first element corresponds to the MBS and the rest represent FBSs. Let user i be served by the BS b_i and the set of users served by the BS k be denoted by $\mathcal{U}_k$. Let q_i^n be the transmit power of the corresponding BS of the user $i \in \mathcal{M}$, i.e., b_i, on sub-channel n in DL transmission and p_i^n be the transmit power of user $i \in \mathcal{M}^f$ on sub-channel n in UL transmission. We define the matrices $\mathbf{Q} = \left[q_i^n\right]_{M \times N}$ and $\mathbf{P} = \left[p_i^n\right]_{M \times N}$ for the power allocation in DL and UL transmissions, respectively. Additionally, let $\tilde{h}_{i,j}^n$ and $h_{i,k}^{n,\text{dl}}$ be the path gain between user i and user j and downlink path gain between user i and BS k, respectively. Also, let $h_{b_i,j}^{n,\text{ul}}$ and $h_{b_i,k}^n$ be the uplink path gains between the serving BS of user i, i.e. b_i, and user j and between the serving BS of user i and the BS k on sub-channel n, respectively. Also, we denote the noise power by N_0.

We define sub-channel allocation matrices $\mathbf{C} = \left[c_i^n\right]_{M \times N}$ and $\mathbf{A} = \left[a_i^n\right]_{M \times N}$ for DL and UL transmissions, respectively, where the binary variable $c_i^n = 1$, if the sub-channel n is allocated to either MU i or FU i in DL, and $a_i^n = 1$ if the sub-channel n is allocated to FU i in UL.

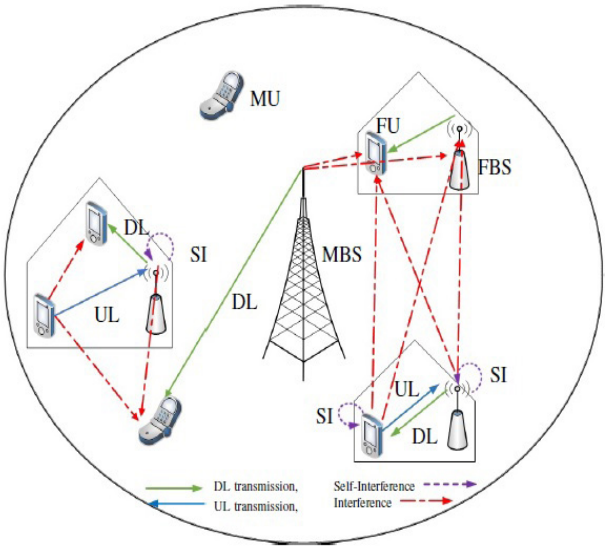

Fig. 1. A system model of Full-duplex femtocells underlying cellular network.

Duplexing mode is defined as the radio signal direction based on which a pair of wireless nodes exchange data between each other. In this paper, it is assumed that the MBS serves MUs in half-duplex communication mode while FBSs and FUs are allowed to switch between HD and FD modes. Thus, according to whether an FU and its associated FBS operate in HD or FD mode on a given sub-channel in UL and/or DL transmissions, four independent duplexing modes can be established as FD-FD, FD-HD, HD-FD, and HD-HD modes. Here, the first and second terms stand for the duplexing modes associated with FBS and FU, respectively. It is worth noting that establishing HD-FD mode needs to consider base station assignment to FUs which is out of the scope of this paper. Thus, in this paper, we define FD-FD, FD-HD, and HD-HD duplexing modes as follows:

- FD-FD: both FU i and its associated FBS b_i operate in FD mode on sub-channel n, i.e. $a_i^n = c_i^n = 1$.
- FD-HD: if a given FU i operates in HD mode on sub-channel n, i.e. $a_i^n + c_i^n = 1$; then there exists only one FU $j \in \mathcal{U}_k$, $j \neq i$, which operates in HD mode at sub-channel n; therefore, $a_j^n + c_j^n = 1, a_i^n \neq a_j^n$.
- HD-HD: as in traditional cellular networks, both FU i and its associated FBS b_i operate in HD mode on sub-channel n, and there exists no FU $j \in \mathcal{U}_k$, $j \neq i$ which operates in HD mode at the same sub-channel n, i.e. $a_i^n + c_i^n = 1$, $\left\{ \nexists j \in \mathcal{U}_k \text{ and } j \neq i \,\middle|\, a_j^n + c_j^n = 1 \right\}$.

In the DL transmission, the SINR received at MBS due to the transmission of MU $i \in \mathcal{M}^m$ on sub-channel $n \in \mathcal{N}$ is given by

$$\gamma_i^{n,\mathrm{dl}}\left(\mathbf{P},\mathbf{Q},\mathbf{A},\mathbf{C}\right) = \frac{c_i^n q_i^n h_{i,1}^{n,\mathrm{dl}}}{I_i^n\left(\mathbf{P},\mathbf{Q},\mathbf{A},\mathbf{C}\right) + N_0} \tag{1}$$

where I_i^n is the interference caused by femtocells (FUs and FBSs) to MU i on sub-channel n, and I_i^n is expressed as

$$I_i^n\left(\mathbf{P},\mathbf{Q},\mathbf{A},\mathbf{C}\right) = \sum_{j \in \mathcal{M}^f} \left(a_j^n p_j^n \tilde{h}_{i,j}^n + c_j^n q_j^n h_{i,b_j}^{n,\mathrm{dl}} \right) \tag{2}$$

Recall that MUs operating in HD mode and the sub-channels allocated to MUs in DL transmission are shared by admitted FUs in UL and DL transmissions.

Considering FBSs and FUs are allowed to switch between FD and HD modes, the SINR for each FU $i \in \mathcal{M}^m$ in UL and DL transmissions are obtained as explained in what follows.

In DL transmission, the SINR received at FU $i \in \mathcal{M}^m$ on sub-channel $n \in \mathcal{N}$ is given by

$$\gamma_i^{n,\mathrm{dl}}\left(\mathbf{P},\mathbf{Q},\mathbf{A},\mathbf{C}\right) = \frac{c_i^n q_i^n h_{i,b_i}^{n,\mathrm{dl}}}{I_i^{n,\mathrm{dl}}\left(\mathbf{P},\mathbf{Q},\mathbf{A},\mathbf{C}\right) + I_i'^{n,\mathrm{dl}}\left(\mathbf{P},\mathbf{A}\right) + N_0} \tag{3}$$

where $I_i^{n,\mathrm{dl}}$ is the interference caused by the MBS and co-channel femtocells on sub-channel n and is expressed as,

$$I_i^{n,\mathrm{dl}}\left(\mathbf{P},\mathbf{Q},\mathbf{A},\mathbf{C}\right) = \sum_{j \in \mathcal{M} \setminus \mathcal{U}_{b_i}} \left(a_j^n p_j^n \tilde{h}_{i,j}^n + c_j^n q_j^n h_{i,b_j}^{n,\mathrm{dl}} \right) \tag{4}$$

and $I_i'^{n,\mathrm{dl}}$ denotes the interference received at FU i on sub-channel n due to full-duplex communication. It can be easily realized that the amount of $I_i'^{n,\mathrm{dl}}$ strongly depends on the duplexing mode in which FU i and its associated FBS operate. In particular, whenever FD-FD mode is exploited, i.e. $a_i^n = c_i^n = 1$, $I_i'^{n,\mathrm{dl}}$ is given by

$$I_i'^{n,\mathrm{dl}}\left(\mathbf{P},\mathbf{A}\right) = p_i^n \Delta, \tag{5}$$

where $\Delta \leq 1$ denotes SI gain. Under FD-HD mode, however, the FBS b_i serves FU i in DL transmission and FU $j \in \mathcal{U}_{b_i}, j \neq i$ in UL transmission, i.e. $a_i^n = 0, c_i^n = 1$ and $a_j^n = 1, c_j^n = 0$. Therefore, $I_i'^{n,\mathrm{dl}}$ is obtained by

$$I_i'^{n,\mathrm{dl}}\left(\mathbf{P},\mathbf{A}\right) = p_j^n \tilde{h}_{i,j}^n \tag{6}$$

Finally, in HD-HD mode, i.e. $a_i^n + c_i^n = 1$ and $a_j^n + c_j^n = 0, \forall j \in \mathcal{U}_k, j \neq i$, $I_i'^{n,\mathrm{dl}}\left(\mathbf{P},\mathbf{A}\right) = 0$.

In UL transmission, the received SINR of FU $i \in \mathcal{M}^m$ on sub-channel $n \in \mathcal{N}$ is expressed as follows

$$\gamma_i^{n,\mathrm{ul}}\left(\mathbf{P},\mathbf{Q},\mathbf{A},\mathbf{C}\right) = \frac{a_i^n p_i^n h_{b_i,i}^{n,\mathrm{ul}}}{I_i^{n,\mathrm{ul}}\left(\mathbf{P},\mathbf{Q},\mathbf{A},\mathbf{C}\right) + I_i'^{n,\mathrm{ul}}\left(\mathbf{Q},\mathbf{C}\right) + N_0}, \tag{7}$$

where $I_i^{n,\mathrm{ul}}$ is the interference caused by the MBS and co-channel femtocells on sub-channel n, i.e

$$I_i^{n,\mathrm{ul}}\left(\mathbf{P},\mathbf{Q},\mathbf{A},\mathbf{C}\right) = \sum_{j \in \mathcal{M} \setminus \mathcal{U}_{b_i}} \left(a_j^n p_j^n h_{b_i,j}^{n,\mathrm{ul}} + c_j^n q_j^n h_{b_i,b_j}^n \right) \tag{8}$$

and $I_i'^{n,\mathrm{ul}}$ represents the interference due to FD communication in UL transmission. Similar to the DL transmission, $I_i'^{n,\mathrm{ul}}$ in FD-FD mode, where $a_i^n = c_i^n = 1$, is obtained as

$$I_i'^{n,\mathrm{ul}}\left(\mathbf{Q},\mathbf{C}\right) = q_i^n \Delta \ . \tag{9}$$

Under FD-HD mode, where $a_i^n = 1, c_i^n = 0$ and $a_j^n = 0, c_j^n = 1$, $I_i'^{n,\mathrm{ul}}$ is given by

$$I_i'^{n,\mathrm{ul}}\left(\mathbf{Q},\mathbf{C}\right) = q_j^n \Delta \ . \tag{10}$$

Eventually, in HD-HD mode, i.e. $a_i^n + c_i^n = 1$ and $a_j^n + c_j^n = 0, \forall j \in \mathcal{U}_k, j \neq i$, $I_i'^{n,\mathrm{ul}} = 0$.

Based on (1), (3), and (7), the achievable rate of user i on sub-channel n in DL and UL transmissions, denoted by $R_i^{n,\mathrm{dl}}$ and $R_i^{n,\mathrm{ul}}$, respectively, are obtained as follows,

$$R_i^{n,\mathrm{dl}} = \log_2\left(1 + \gamma_i^{n,\mathrm{dl}}\right) \tag{11}$$

$$R_i^{n,\mathrm{ul}} = \log_2\left(1 + \gamma_i^{n,\mathrm{ul}}\right) \tag{12}$$

In the next section, we formulate the power control problem for the macrocell, and joint sub-channel allocation, duplexing mode selection, and power control for femtocells. In the former, the objective is to minimize the aggregate transmit power of MUs subject to the constraint that the target rate of MUs is met. In the latter, we aim to maximize the number of admitted FUs to FBSs by guaranteeing a target rate for every admitted FU.

3- Problem Formulation

Generally, in two-tier networks in which the femtocells are overlaid on a macrocell, one of the most well-known approaches for utilizing the spectrum space more efficiently is to maximize the number of admitted FUs by FBSs. An FU is admitted to communicate with an associated FBS whenever its target rate is satisfied during the UL/DL transmissions. Clearly, the number of admitted FUs may severely affect the performance of the network. Specifically, by increasing the number of the admitted FUs on a sub-channel, the total interference exposed to MUs by FBSs and FUs would also increase. As a result, a subset of admitted FUs should be removed so as to protect the target rate of MUs. According to the above discussion, in this section, assuming that the available sub-channels are already allocated to MUs and MBS, we first formulate the power control problem for macro-tier under HD communication mode. Then, we formally state the joint sub-channel allocation, duplexing mode selection, and power control for femto-tier.

3- 1- Downlink Power Control Problem for Macro-tier

Assume that available sub-channels are pre-allocated to MUs by round-robin scheduling and are fixed during the power control phase. The transmit power levels for MUs over the allocated sub-channels are dynamically adjusted to tackle the cross-tier interference due to concurrent transmission of FUs and FBSs and, also, to decrease the interference caused by MBS at FUs and FBSs. Therefore, aiming to minimize the aggregate transmit power of the MBS corresponding to MUs, the power control optimization problem for macro-tier can be formulated as follows,

$$\begin{aligned} &\min_{q_i^n} \sum_{n\in\mathcal{N}} \sum_{i\in\mathcal{M}^m} q_i^n \\ &\text{subject to.} \quad \text{C1:} \ \sum_{n\in\mathcal{N}} c_i^n R_i^{n,\mathrm{dl}} \geq R_{i,\mathrm{MU}}^{\min,\mathrm{dl}}, \ \forall i\in\mathcal{M}^m, \\ &\qquad\qquad\quad \text{C2:} \ \sum_{n\in\mathcal{N}} \sum_{i\in\mathcal{U}_1} c_i^n q_i^n \leq q_1^{\max}, \\ &\qquad\qquad\quad \text{C3:} \ q_i^n \geq 0, \ \forall i \in \mathcal{M}^m, \ \forall n \in \mathcal{N}, \end{aligned} \tag{13}$$

where C1 corresponds to the protection of each MU i that is protected if $\sum_{n\in\mathcal{N}} c_i^n R_i^{n,\mathrm{dl}} \geq R_{i,\mathrm{MU}}^{\min,\mathrm{dl}}$ where $R_{i,\mathrm{MU}}^{\min,\mathrm{dl}}$ is the acceptable target rate of MU i in DL transmission. The constraint C2 indicates that the aggregate transmit power of MBS for all MUs on all sub-channels should not exceed the threshold $q_1^{\max}$. Finally, C3 represents that the allocated transmit power on each sub-channel n should be non-negative.

3- 2- Joint Sub-channel Allocation, Duplexing Mode Selection, and Power Control Problem for Femto-tier

To improve the spectral efficiency in femtocells, the FBSs aim to admit as more FUs as possible. Let $R_{i,\mathrm{FU}}^{\min,\mathrm{dl}}$ and $R_{i,\mathrm{FU}}^{\min,\mathrm{ul}}$ denote the target rate of FU i in DL and UL transmissions, respectively. Let $\mathcal{S}^{\mathrm{dl}} \subseteq \mathcal{M}^f$ be the FUs that meet their target rate in DL transmission, i.e.

$$\mathcal{S}^{\mathrm{dl}} = \left\{ i\in\mathcal{M}^f \,\middle|\, \sum_{n\in\mathcal{N}} c_i^n R_i^{n,\mathrm{dl}} \geq R_{i,\mathrm{FU}}^{\min,\mathrm{dl}} \right\},$$

and $\mathcal{S}^{\mathrm{ul}} \subseteq \mathcal{M}^f$ be the FUs that meet their target rate in UL transmission, i.e.

$$\mathcal{S}^{\mathrm{ul}} = \left\{ i\in\mathcal{M}^f \,\middle|\, \sum_{n\in\mathcal{N}} a_i^n R_i^{n,\mathrm{ul}} \geq R_{i,\mathrm{FU}}^{\min,\mathrm{ul}} \right\}.$$

Accordingly, we formulate the joint sub-channel allocation, duplexing mode selection, and power control problem in femtocells as follows,

$$\begin{aligned} &\max_{p_i^n, q_i^n, a_i^n, c_i^n} \left|\mathcal{S}^{\mathrm{dl}}\right| + \left|\mathcal{S}^{\mathrm{ul}}\right| \\ &\text{subject to.} \quad \text{C1:} \ \sum_{i\in\mathcal{M}^f} a_i^n p_i^n \tilde{h}_{j,i}^n + c_i^n q_i^n h_{j,b_i}^{n,\mathrm{dl}} \leq I_{\mathrm{th}}^n, \ \forall n\in\mathcal{N}, \\ &\qquad\qquad\quad \text{C2:} \ \sum_{n\in\mathcal{N}} \sum_{i\in\mathcal{U}_k} c_i^n q_i^n \leq q_{\mathrm{k}}^{\max}, \ \forall k\in\mathcal{B}\backslash\{1\}, \\ &\qquad\qquad\quad \text{C3:} \ \sum_{n\in\mathcal{N}} a_i^n p_i^n \leq p_i^{\max}, \ \forall i\in\mathcal{M}^f, \\ &\qquad\qquad\quad \text{C4:} \ \sum_{i\in\mathcal{U}_k} c_i^n \leq 1, \ \forall n\in\mathcal{N}, \ \forall k\in\mathcal{B}\backslash\{1\}, \\ &\qquad\qquad\quad \text{C5:} \ \sum_{i\in\mathcal{U}_k} a_i^n \leq 1, \ \forall n\in\mathcal{N}, \ \forall k\in\mathcal{B}\backslash\{1\}, \\ &\qquad\qquad\quad \text{C6:} \ c_i^n, a_i^n \in \{0,1\}, \ \forall n\in\mathcal{N}, \ \forall i\in\mathcal{M}^f, \\ &\qquad\qquad\quad \text{C7:} \ q_i^n, \ p_i^n \geq 0, \ \forall i \in \mathcal{M}^f, \ \forall n \in \mathcal{N}, \end{aligned} \tag{14}$$

where C1 corresponds to the MUs' protection on each sub-channel n. MU i is protected on the sub-channel n if the total interference received from co-channel FUs and FBSs is lower than a given interference temperature limit that is denoted by I_{th}^n. The constraints C2 and C3 refer to the maximum transmit power of FBS k and FU i in DL and UL transmissions, respectively. Constraints C4 and C5 indicate that each sub-channel n is allowed to be allocated to at most one FU in DL and UL transmissions, respectively. Constraint C6 represents the state space of sub-channel allocation variables, and, finally, C7 indicates that the transmit power on each sub-channel n in DL and UL should be non-negative.

4- Our Proposed Distributed Resource Allocation Algorithm

Hitherto, we have stated the resource allocation problems for macro-tier and femtocells in (13) and (14), respectively. Since the constraint C6 in problem (14) is with integer variables, this optimization problem is a Mix-Integer Non-Linear Programming problem (MINLP) and, thus, it is mathematically intractable to solve it. In this section, we propose sub-optimal algorithms to address both the power control problem for macrocell and the joint sub-channel allocation, duplexing mode selection, and power control problem for femtocells, in a distributed manner. For simplicity, we make the following assumption.

Assumption 1: We assume that the target rate of user i is equally divided across all of its allocated sub-channels. Hence, the minimum rate of MU i on each allocated sub-channel n is given by

$$\hat{R}_{i,\mathrm{MU}}^{n,\mathrm{dl}} = \frac{R_{i,\mathrm{MU}}^{\min,\mathrm{dl}}}{\tau_{i,1}^{\mathrm{dl}}},$$

where $\hat{R}_{i,\mathrm{MU}}^{n,\mathrm{dl}}$ is the minimum rate for MU i on sub-channel n and $\tau_{i,1}^{\mathrm{dl}}$ is the number of allocated sub-channels to MUi. Also, the minimum rate of FU i on each allocated sub-channel n in UL transmission is given by

$$\hat{R}_{i,\mathrm{FU}}^{n,\mathrm{ul}} = \frac{R_{i,\mathrm{FU}}^{\min,\mathrm{ul}}}{\sum_{n\in\mathcal{N}} a_i^n}$$

and in DL transmission is given by

$$\hat{R}_{i,\mathrm{FU}}^{n,\mathrm{dl}} = \frac{R_{i,\mathrm{FU}}^{\min,\mathrm{dl}}}{\sum_{n\in\mathcal{N}} c_i^n}$$

where $\hat{R}_{i,\mathrm{FU}}^{n,\mathrm{ul}}$ and $\hat{R}_{i,\mathrm{FU}}^{n,\mathrm{dl}}$ are the minimum rates for FU i on sub-channel n in UL and DL transmissions, respectively.

4- 1- The Proposed power control algorithm for Macro-tier

In (13), the optimization problem has to minimize the aggregate transmit power of MUs by satisfying the MUs' protection and maximum transmit power constraints. According to Assumption 1, the target rate of MU i is mapped on the minimum rate of each allocated sub-channel n corresponding to the SINR received on sub-channel n, namely, target-SINR. We denote the target-SINR of MU i on sub-channel n by $\hat{\gamma}_{i,\mathrm{MU}}^{n,\mathrm{dl}}$ as

$$\hat{\gamma}_{i,\mathrm{MU}}^{n,\mathrm{dl}} = 2^{\hat{R}_{i,\mathrm{MU}}^{n,\mathrm{dl}}} - 1, \tag{15}$$

where

$$\hat{R}_{i,\mathrm{MU}}^{n,\mathrm{dl}} = \frac{R_{i,\mathrm{MU}}^{\min,\mathrm{dl}}}{\tau_{i,1}^{\mathrm{dl}}},$$

and $\tau_{i,1}^{\mathrm{dl}}$ is the number of allocated sub-channels to MU i. With using round-robin scheduling for MUs, the number of allocated sub-channels to each MU i is the same and given by $\tau_{i,1}^{\mathrm{dl}}=\left\lfloor \frac{N}{M^m} \right\rfloor$ [21].

Hence, we can assume that the MU i is protected if $\gamma_i^{n,\mathrm{dl}} \geq c_i^n \hat{\gamma}_{i,\mathrm{MU}}^{n,\mathrm{dl}}$. Additionally, the MBS's maximum transmit power is assumed to be equally divided among all sub-channels allocated to its users, that is

$$\bar{q}_{i,\mathrm{MU}}^{n}=\frac{q_1^{\max}}{\sum\limits_{i\in\mathcal{M}^m} \tau_{i,1}^{\mathrm{dl}}}$$

where $\bar{q}_{i,\mathrm{MU}}^{n}$ is the maximum transmit power of MBS on each sub-channel n. Therefore, problem (13) can be divided by N sub-problems as follows,

$$\begin{aligned} &\min_{q_i^n} \sum_{n\in\mathcal{N}} \sum_{i\in\mathcal{M}^m} q_i^n \\ &\text{subject to.} \quad \mathrm{C1:}\ \gamma_i^{n,\mathrm{dl}} \geq c_i^n \hat{\gamma}_{i,\mathrm{MU}}^{n,\mathrm{dl}},\ \forall i\in\mathcal{M}^m,\ \forall n\in\mathcal{N}, \\ &\qquad\qquad\quad\ \mathrm{C2:}\ 0 \leq q_i^n \leq \bar{q}_{i,\mathrm{MU}}^{n},\ \forall i \in \mathcal{M}^m,\ \ \forall n \in \mathcal{N}. \end{aligned} \tag{16}$$

In problem (16), it is desirable to minimize the aggregate transmit power of MUs subject to the MUs' target-SINR on each allocated sub-channel n and, at the same time, the maximum transmit power of MBS to its associated MUs on each allocated sub-channel n is limit. To address this problem, we adopt the well-known distributed Foschini-Miljanic power control algorithm (TPC) proposed in [22]. TPC is a distributed power updating algorithm proposed for co-channel cellular networks to minimize the aggregate transmit power of users so that their target-SINR are met. The transmit power update function for MU $i\in\mathcal{M}^m$ on sub-channel $n\in\mathcal{N}$ is given by

$$q_i^n(t+1) = \min\left\{ \bar{q}_{i,\mathrm{MU}}^{n},\ \hat{\gamma}_{i,\mathrm{MU}}^{n,\mathrm{dl}} \frac{I_i^n(t)+N_0}{h_{i,1}^{n,\mathrm{dl}}} \right\} \tag{17}$$

where

$$I_i^n(t)=\sum_{j\in\mathcal{M}^f}\left(a_j^n p_j^n(t)\tilde{h}_{i,j}^n + c_j^n q_j^n(t) h_{i,b_j}^{n,\mathrm{dl}}\right)$$

is the instantaneous interference caused by co-channel femtocells (FBSs and FUs) to MU i.

4- 2- The Proposed Joint Sub-channel Allocation, Duplexing Mode Selection, and Power Control Algorithm for Femto-tier

When the sub-channels allocated to MUs in DL transmission are shared by admitted FUs in UL and DL transmissions, the amount of interference originating from both FBSs and FUs at the MUs may cause performance degradation for macro-tier in the sense that some MUs may not meet their target rate. For this issue, the resource allocation has to protect every MU on its allocated sub-channel n, i.e. C1 in (14), by maintaining the total interference received at MU i bellow the interference temperature limit, I_{th}^n. As noted before, MU i is protected on sub-channel n if $\gamma_i^{n,\mathrm{dl}} \geq c_i^n \hat{\gamma}_{i,\mathrm{MU}}^{n,\mathrm{dl}}$, additionally the target-SINR of MU i on sub-channel n is attainable if

$$0 \leq \hat{\gamma}_{i,\mathrm{MU}}^{n,\mathrm{dl}} \frac{I_i^n(\mathbf{P},\mathbf{Q},\mathbf{A},\mathbf{C})+N_0}{h_{i,1}^{n,\mathrm{dl}}} \leq \bar{q}_{i,\mathrm{MU}}^{n}.$$

The interference temperature limit, i.e. I_{th}^n, can be obtained by

$$I_{\mathrm{th}}^n=\frac{\bar{q}_{i,\mathrm{MU}}^{n} h_{i,1}^{n,\mathrm{dl}}}{\hat{\gamma}_{i,\mathrm{MU}}^{n,\mathrm{dl}}}-N_0. \tag{18}$$

Based on (18), I_{th}^n is a function of noise power, target-SINRs, path gains, and maximum transmit power of MBS to MU i on sub-channel n. To protect MUs, the interference temperature limit is divided by the number of FUs co-existing on sub-channel n. Hence, the maximum cross-tier interference caused by each femtocell k at MU j on each sub-channel n can be expressed as

$$\bar{I}^n=\frac{I_{\mathrm{th}}^n}{\sum\limits_{i\in\mathcal{M}^f}\left(a_i^n+c_i^n\right)}.$$

For simplicity, the maximum power of FBS k is divided among all of the sub-channels. Accordingly, the maximum transmit power of FBS b_i on each sub-channel n is given as follows

$$\bar{q}_{i,\mathrm{FU}}^{n}=\min\left\{ \frac{q_k^{\max}}{\sum\limits_{i\in\mathcal{M}^m}\sum\limits_{n\in\mathcal{N}} c_i^n}, \frac{\bar{I}^n}{h_{j,b_i}^{n,\mathrm{dl}}} \right\} \tag{19}$$

where $\bar{q}_{i,\mathrm{FU}}^{n}$ is the maximum transmit power of FBS b_i on sub-channel n in DL transmission, and $h_{j,b_i}^{n,\mathrm{dl}}$ is the path gain from FU i serving FBS, i.e. b_i to MU j. Likewise, for UL transmission assuming equal division of FU i's maximum power among its allocated sub-channels, the maximum transmit power of FU i on each sub-channel n is given by

$$\bar{p}_{i,\mathrm{FU}}^{n}=\min\left\{ \frac{p_i^{\max}}{\sum\limits_{n\in\mathcal{N}} a_i^n}, \frac{\bar{I}^n}{\tilde{h}_{j,i}^n} \right\} \tag{20}$$

where $\bar{p}_{i,\mathrm{FU}}^{n}$ is the maximum transmit power of FU i on sub-channel n in UL transmission and $\tilde{h}_{j,i}^n$ is the path gain from FU i to MU j.

Similar to (15) and according to Assumption 1, the target-SINR of FU i on sub-channel n at DL and UL transmissions are expressed by

$$\hat{\gamma}_{i,\mathrm{FU}}^{n,\mathrm{dl}}=2^{\hat{R}_{i,\mathrm{FU}}^{n,\mathrm{dl}}}-1, \tag{21}$$

where $\hat{R}_{i,\mathrm{FU}}^{n,\mathrm{dl}}=\frac{R_{i,\mathrm{FU}}^{\min,\mathrm{dl}}}{\sum\limits_{n\in\mathcal{N}} c_i^n}$, and

$$\hat{\gamma}_{i,\mathrm{FU}}^{n,\mathrm{ul}}=2^{\hat{R}_{i,\mathrm{FU}}^{n,\mathrm{ul}}}-1, \tag{22}$$

where $\hat{R}_{i,\mathrm{FU}}^{n,\mathrm{ul}}=\frac{R_{i,\mathrm{FU}}^{\min,\mathrm{ul}}}{\sum\limits_{n\in\mathcal{N}} a_i^n}$, respectively.

According to the above discussions, the problem in (14) can be rewritten as follows,

$$\begin{aligned} &\max_{p_i^n,q_i^n,a_i^n,c_i^n} \left|\mathcal{S}^{\mathrm{dl}}\right| + \left|\mathcal{S}^{\mathrm{ul}}\right| \\ &\text{subject to.} \quad \mathrm{C1:}\ 0\leq q_i^n \leq \bar{q}_{i,\mathrm{FU}}^{n}\ \ \forall n\in\mathcal{N},\ \forall i\in\mathcal{M}^f, \\ &\qquad\qquad\quad\ \mathrm{C2:}\ 0\leq p_i^n \leq \bar{p}_{i,\mathrm{FU}}^{n}\ \ \forall n\in\mathcal{N},\ \forall i\in\mathcal{M}^f, \\ &\qquad\qquad\quad\ \mathrm{C3:}\ \sum_{i\in\mathcal{U}_k} c_i^n \leq 1,\ \forall n\in\mathcal{N},\ \forall k\in\mathcal{B}\backslash\{1\}, \\ &\qquad\qquad\quad\ \mathrm{C4:}\ \sum_{i\in\mathcal{U}_k} a_i^n \leq 1,\ \forall n\in\mathcal{N},\ \forall k\in\mathcal{B}\backslash\{1\}, \\ &\qquad\qquad\quad\ \mathrm{C5:}\ c_i^n, a_i^n \in \{0,1\},\ \forall n \in \mathcal{N},\ \forall i \in \mathcal{M}^f, \end{aligned} \tag{23}$$

where

$\mathcal{S}^{\text{dl}}=\left\{i\in\mathcal{M}^f \middle| \gamma_i^{n,\text{dl}}\geq c_i^n\hat{\gamma}_{i,\text{FU}}^{\min,\text{dl}}\right\}$ and $\mathcal{S}^{\text{ul}}=\left\{i\in\mathcal{M}^f \middle| \gamma_i^{n,\text{ul}}\geq a_i^n\hat{\gamma}_{i,\text{FU}}^{\min,\text{ul}}\right\}$.

Note that according to FBSs' and FUs' maximum transmit power on each sub-channel n are given by (19) and (20), respectively; if the constraints C1 and C2 in (19) are satisfied, the constraint C1 in (14) would be also satisfied.

In order to address the MINLP problem in (23), we break it into two sub-problems, one for sub-channel allocation and duplexing mode selection and the other for power control which will be addressed one-by-one in the following.

4- 2- 1- The Proposed Sub-channel Allocation and Duplexing Mode Selection Algorithm for Femto-tier

Considering fixed pre-allocated transmit power, the sub-channel allocation and duplexing mode selection sub-problem is formulated as follows

$$
\begin{aligned}
&\max_{a_i^n,c_i^n} \left|\mathcal{S}^{\text{dl}}\right|+\left|\mathcal{S}^{\text{ul}}\right| \\
&\text{subject to.}\quad \text{C1: } \sum_{n\in\mathcal{N}} c_i^n=\left\lfloor\frac{N}{|\mathcal{U}_k|}\right\rfloor, \forall k\in\mathcal{B}\backslash\{1\},\ \forall i\in\mathcal{U}_k, \\
&\qquad\qquad\quad \text{C2: } \sum_{n\in\mathcal{N}} a_i^n=\left\lfloor\frac{N}{|\mathcal{U}_k|}\right\rfloor, \forall k\in\mathcal{B}\backslash\{1\},\ \forall i\in\mathcal{U}_k, \\
&\qquad\qquad\quad \text{C3: } \sum_{i\in\mathcal{U}_k} c_i^n\leq 1,\ \forall n\in\mathcal{N},\ \forall k\in\mathcal{B}\backslash\{1\}, \\
&\qquad\qquad\quad \text{C4: } \sum_{i\in\mathcal{U}_k} a_i^n\leq 1,\ \forall n\in\mathcal{N},\ \forall k\in\mathcal{B}\backslash\{1\}, \\
&\qquad\qquad\quad \text{C5: } c_i^n, a_i^n\in\{0,1\},\ \forall n\in\mathcal{N},\ \forall i\in\mathcal{M}^f,
\end{aligned}
\tag{24}
$$

where C1 and C2 imply that the number of allocated sub-channels to every FU i in each femtocell k in DL and UL transmissions are the same. By doing so, the problem in (24) becomes a well-known assignment problem and can be solved in polynomial time by employing *Hungarian Algorithm* [23]. The combinatorial Hungarian algorithm takes a $N\times N$ matrix as an input, where its element in i-th row and j-th column stores the payoff value achieved by assigning the j-th job to i-th employee. These N jobs are assigned to N employees provided that the total payoff is maximized. To adopt the Hungarian algorithm, we define the matrices $\mathbf{D}_k=\left[d_i^n\right]_{N\times N}$ and $\mathbf{U}_k=\left[u_i^n\right]_{N\times N}$ as the sub-channel allocation matrices for the FUs in femtocell k in DL and UL transmissions, respectively, where d_i^n and u_i^n are defined as follows,

$$
d_i^n=\begin{cases}1, & \text{if } \gamma_i^{n,\text{dl}}\geq\hat{\gamma}_{i,\text{FU}}^{n,\text{dl}} \\ 0, & \text{otherwise}\end{cases}, \tag{25}
$$

$$
u_i^n=\begin{cases}1, & \text{if } \gamma_i^{n,\text{ul}}\geq\hat{\gamma}_{i,\text{FU}}^{n,\text{ul}} \\ 0, & \text{otherwise}\end{cases} \tag{26}
$$

In our proposed sub-channel allocation and duplexing mode selection algorithm for femto-tier, similar to [11], we assume that the number of allocated sub-channels to each FUs in each femtocell is equal to $\left\lfloor N/|\mathcal{U}_k|\right\rfloor$. Since by employing Hungarian algorithm, each job is allocated to only one employee and each employee can do only one job, we define $\left\lfloor N/|\mathcal{U}_k|\right\rfloor$ virtual "employees" for each FU i in femtocell k on DL and UL transmissions. Therefore, each FU i is equivalent with $\left\lfloor N/|\mathcal{U}_k|\right\rfloor$ virtual FU $i_1,i_2,\ldots,i_{\lfloor N/|\mathcal{U}_k|\rfloor}$ on DL and UL transmissions. By adopting the Hungarian algorithm that the matrices $\mathbf{D}_k$ and $\mathbf{U}_k$ are given as inputs into it and allocating the same number of sub-channels to each FU, the number of admitted FUs is maximized. If the virtual FU i_f is matched by sub-channel n, $c_i^n=1$ and $a_i^n=1$ in DL and UL transmissions, respectively. Otherwise, $c_i^n=0$ and $a_i^n=0$. It is worth mentioning that the sub-channel allocation on DL and UL transmissions are obtained separately by giving $\mathbf{D}_k$ and $\mathbf{U}_k$ as inputs into the Hungarian algorithm, respectively. In what follows, an example is presented to illustrate the steps of the proposed sub-channel allocation by employing Hungarian algorithm.

- *Example 1:*

Assume that there exists one femtocell with two FUs and four sub-channels. We define for each FU, two virtual employees. Suppose that based on (25) and (26), the sub-channel allocation matrices for the FUs in DL and UL transmissions are equal to

$\mathbf{D}_k=\begin{bmatrix}1,1,1,1\\1,1,1,0\\0,1,1,0\\0,1,1,0\end{bmatrix}$ and $\mathbf{U}_k=\begin{bmatrix}0,1,1,1\\0,1,0,1\\1,0,1,0\\0,1,1,0\end{bmatrix}$, respectively.

We employ Hungarian algorithm on matrix $\mathbf{D}_k$ step-by-step as follows.

Step 1- By multiplying each element by -1, we have

$$\mathbf{D}_k=\begin{bmatrix}-1,-1,-1,-1\\-1,-1,-1,\ 0\\0\ ,-1,-1,\ 0\\0\ ,-1,-1,\ 0\end{bmatrix}.$$

Step 2- By reducing the rows by subtracting the minimum value of each row from that row, we have

$$\mathbf{D}_k=\begin{bmatrix}0,0,0,0\\0,0,0,1\\1,0,0,1\\1,0,0,1\end{bmatrix}.$$

Step 3- We cover the zero elements with the possible minimum number of lines.

Step 4- A matching is made by choosing a set of zeros such that each row or column has only one selected item. According to this matching, the sub-channel allocation matrix on DL transmission is obtained as

$$\mathbf{D}_k=\begin{bmatrix}0,0,0,1\\1,0,0,0\\0,0,1,0\\0,1,0,0\end{bmatrix}.$$

In a similar way, the sub-channel allocation on UL transmission is obtained as

$$\mathbf{U}_k=\begin{bmatrix}0,1,0,0\\0,0,0,1\\1,0,0,0\\0,0,1,0\end{bmatrix}.$$

Once sub-channels are allocated to FUs, the duplexing mode for each FU on each allocated sub-channel n requires being selected. For this purpose, we calculate the received SINR of each FU i on its allocated sub-channel n in both DL and UL transmissions so as to ensure that each FU i meets its target-SINR in DL and UL transmissions. That is,

- if sub-channel n is allocated to FU i in both DL and UL transmissions, i.e., $a_i^n=c_i^n=1$, SINR for FU i on sub-channel n at DL is obtained by (3) in which $I_i'^{n,\text{dl}}(\mathbf{P},\mathbf{A})$ is given by (5). Also, SINR of FU i on sub-channel n at UL is given by (7) in which $I'^{n,\mathrm{ul}}_i(\mathbf{Q},\mathbf{C})$ is obtained by (9). In this case, one of the following conditions may occur,

  - if SINR of FU i on sub-channel n is more than the target-SINR in both UL and DL transmissions, i.e. $\gamma_i^{n,\mathrm{ul}} \geq \hat{\gamma}_{i,\mathrm{FU}}^{n,\mathrm{ul}}$ and $\gamma_i^{n,\mathrm{dl}} \geq \hat{\gamma}_{i,\mathrm{FU}}^{n,\mathrm{dl}}$, respectively, the FD-FD mode is selected.
  - Otherwise, if the SINR of FU i on sub-channel n is less than the target-SINR in UL or DL or both of them, the HD-HD mode is selected as follows:
    - If $\gamma_i^{n,\mathrm{dl}} \geq \gamma_i^{n,\mathrm{ul}}$, FBS b_i transmits to FU i on sub-channel n and FU i transmits to FBS b_i on another sub-channel, i.e. $c_i^n = 1$ and $a_i^n = 0$.
    - If $\gamma_i^{n,\mathrm{ul}} > \gamma_i^{n,\mathrm{dl}}$, FU i transmits to FBS b_i on sub-channel n and FBS b_i transmits to FU i on another sub-channel, i.e. $a_i^n = 1$ and $c_i^n = 0$.
- If sub-channel n is allocated to FU i in DL transmission and FU j in UL transmission, i.e. $c_i^n = 1, a_j^n = 1$, where $i,j \in \mathcal{U}_k$ and $i \neq j$ [1], SINR for FU i on sub-channel n at DL is obtained by (3) in which $I'^{n,\mathrm{dl}}_i(\mathbf{P},\mathbf{A})$ is given by (6). Also, SINR of FU j on sub-channel n at UL is given by (7), in which $I'^{n,\mathrm{ul}}_i(\mathbf{Q},\mathbf{C})$ is obtained by (10). In this case, one of the following conditions may occur,
  - If the SINR of FU i on sub-channel n is more than the target-SINR, $\gamma_i^{n,\mathrm{dl}} \geq \hat{\gamma}_{i,\mathrm{FU}}^{n,\mathrm{dl}}$ and the SINR of FU j on sub-channel n is more than its target-SINR, i.e. if $\gamma_j^{n,\mathrm{ul}} \geq \hat{\gamma}_{j,\mathrm{FU}}^{n,\mathrm{ul}}$, the FD-HD mode is selected.
  - Otherwise, if the SINR of FU i on sub-channel n or the SINR of FU j on sub-channel n is less than their target-SINR, HD-HD mode is selected as follows
    - .If $\gamma_i^{n,\mathrm{dl}} \geq \gamma_j^{n,\mathrm{ul}}$, FBS b_i transmits to FU i on sub-channel n, i.e. $c_i^n = 1$ and $a_j^n = 0$.
    - If $\gamma_j^{n,\mathrm{ul}} > \gamma_i^{n,\mathrm{dl}}$, FU j transmits to FBS b_i on sub-channel n, i.e. $c_i^n = 0$ and $a_j^n = 1$.

The proposed sub-channel allocation and duplexing mode selection scheme are described in detail in Algorithm 1. As a result of duplexing mode selection, the number of allocated sub-channels for each FU i in DL and UL transmissions is determined. According to the number of allocated sub-channels for FU i, its target-SINR on sub-channel n in DL and UL is updated using (21) and (22), respectively. Also, the maximum transmit power for FBSs and FUs at DL and UL are obtained by (19) and (20), respectively.

4- 2- 2- The Proposed Power Control Algorithm for Femto-tier

So far, sub-channels have been allocated to each FU and duplexing modes have been selected. Given the allocated sub-channels and selected duplexing modes, now, the power control sub-problem for FUs can be formulated as follows,

$$\max_{p_i^n, q_i^n} \left|\mathcal{S}^{\mathrm{dl}}\right| + \left|\mathcal{S}^{\mathrm{ul}}\right|$$
$$\text{subject to.} \quad \begin{aligned} &\text{C1: } 0 \leq \mathrm{q}_i^n \leq \overline{\mathrm{q}}_{i,\mathrm{FU}}^n \ \ \forall n \in \mathcal{N},\ \forall i \in \mathcal{M}^f, \\ &\text{C2: } 0 \leq \mathrm{p}_i^n \leq \overline{\mathrm{p}}_{i,\mathrm{FU}}^n \ \ \forall n \in \mathcal{N},\ \forall i \in \mathcal{M}^f. \end{aligned} \tag{27}$$

Based on our analysis described in the previous sub-section 4-1, the transmission power of FBS $b_i \in \mathcal{B}\backslash\{1\}$ to FU i on sub-channel n can be obtained according to TPC algorithm by

$$q_i^n(t+1) = \min\left\{ \overline{q}_{i,\mathrm{FU}}^n, \hat{\gamma}_{i,\mathrm{FU}}^{n,\mathrm{dl}} \frac{I_i^{n,\mathrm{dl}}(t) + I'^{n,\mathrm{dl}}_i(t) + N_0}{h_{i,\mathrm{b}_i}^{n,\mathrm{dl}}} \right\}, \tag{28}$$

where $I_i^{n,\mathrm{dl}}(t)$ is the instantaneous interference caused by MBS, co-channel FBSs, and FUs to FU i. $I'^{n,\mathrm{dl}}_i(t)$ is the instantaneous interference received at FU i due to full-duplex communication. Likewise, the transmit power of each FU $i \in \mathcal{M}^f$ on each sub-channel $n \in \mathcal{N}$ is obtained as,

$$p_i^n(t+1) = \min\left\{ \overline{p}_{i,\mathrm{FU}}^n, \hat{\gamma}_{i,\mathrm{FU}}^{n,\mathrm{ul}} \frac{I_i^{n,\mathrm{ul}}(t) + I'^{n,\mathrm{ul}}_i(t) + N_0}{h_{\mathrm{b}_i,i}^{n,\mathrm{ul}}} \right\} \tag{29}$$

where $I_i^{n,\mathrm{ul}}(t)$ is the instantaneous interference caused by MBS, co-channel FBSs and FUs to FBS b_i. $I'^{n,\mathrm{ul}}_i(t)$ is the instantaneous interference received at FBS b_i due to full-duplex communication.

Algorithm 1. Proposed sub-channel allocation and duplexing mode selection algorithm for femto-tier

```
Initialization:
Set $q_i^n = \overline{q}_{i,\mathrm{MU}}^n$, for all $i \in \mathcal{M}^m$; allocate sub-channels to MUs by round-robin scheduling. Set $q_i^n = \frac{q_k^{\max}}{N}$ and $p_i^n = \frac{p_i^{\max}}{N}$ for all $i \in \mathcal{M}^f$. Set $\hat{\gamma}_{i,\mathrm{FU}}^{n,\mathrm{dl}} = 2^{\hat{R}_{i,\mathrm{FU}}^{n,\mathrm{dl}}} - 1$ and $\hat{\gamma}_{i,\mathrm{FU}}^{n,\mathrm{ul}} = 2^{\hat{R}_{i,\mathrm{FU}}^{n,\mathrm{ul}}} - 1$, for all $i \in \mathcal{M}^f$ on each sub-channel $n$.
For each femtocell $k$:
Sub-channel allocation in femtocell $k$
  Obtain the matrices $\mathbf{D}_k$ and $\mathbf{U}_k$ by (25) and (26), respectively and give them to Hungarian algorithm as inputs to obtain sub-channel allocation for femtocell $k \in \mathcal{B}\backslash\{1\}$ at DL and UL transmission, respectively.
  Duplexing-mode selection in femtocell $k$
      for each $i \in \mathcal{U}_k$ on each sub-channel $n$
    if $a_i^n = c_j^n = 1$ and $i = j$ then
        $\gamma_i^{n,\mathrm{dl}}$ and $\gamma_i^{n,\mathrm{ul}}$ are obtained by (3) and (7) in which $I'^{n,\mathrm{dl}}_i$ and $I'^{n,\mathrm{ul}}_i$ are given by (5) and (9), respectively.
        if $\gamma_i^{n,\mathrm{dl}} \geq \hat{\gamma}_{i,\mathrm{FU}}^{n,\mathrm{dl}}$ and $\gamma_i^{n,\mathrm{ul}} \geq \hat{\gamma}_{i,\mathrm{FU}}^{n,\mathrm{ul}}$ then
            Select FD-FD mode
        else if $\gamma_i^{n,\mathrm{dl}} \geq \gamma_i^{n,\mathrm{ul}}$ then
            Select HD-HD mode and set $a_i^n = 0$.
        else if $\gamma_i^{n,\mathrm{ul}} > \gamma_i^{n,\mathrm{dl}}$ then
            Select HD-HD mode and set $c_i^n = 0$.
        end if.
    else if $a_i^n = c_j^n = 1$ and $i \neq j$ then
        $\gamma_i^{n,\mathrm{dl}}$ and $\gamma_i^{n,\mathrm{ul}}$ are obtained by (3) and (7) in which $I'^{n,\mathrm{dl}}_i$ and $I'^{n,\mathrm{ul}}_i$ are given by (6) and (10), respectively.
        if $\gamma_i^{n,\mathrm{dl}} \geq \hat{\gamma}_{i,\mathrm{FU}}^{n,\mathrm{dl}}$ and $\gamma_j^{n,\mathrm{ul}} \geq \hat{\gamma}_{j,\mathrm{FU}}^{n,\mathrm{ul}}$ then
            Select FD-HD mode.
        else if $\gamma_i^{n,\mathrm{dl}} \geq \gamma_j^{n,\mathrm{ul}}$ then
            Select HD-HD mode and set $a_j^n = 0$.
        else if $\gamma_j^{n,\mathrm{ul}} > \gamma_i^{n,\mathrm{dl}}$ then
            Select HD-HD mode and set $c_i^n = 0$.
        end if.
    end if.
Update $\hat{\gamma}_{i,\mathrm{FU}}^{n,\mathrm{dl}}$ and $\hat{\gamma}_{i,\mathrm{FU}}^{n,\mathrm{ul}}$, for all FU $i \in \mathcal{U}_k$ on each allocated sub-channel $n$ according to (21) and (22), respectively.
Update $\overline{q}_i^n$ and $\overline{p}_i^n$, for all FU $i \in \mathcal{U}_k$ on each allocated sub-channel $n$ according to (19) and (20), respectively.
```

---

[1] If $c_i^n = 1, a_j^n = 1$, then $a_i^n = 0, c_j^n = 0$, because each sub-channel n is allocated at most one user in DL and one user in UL at each femtocell k.

Applying the TPC algorithm, an FU meets its target-SINR on its allocated sub-channel with consuming the minimum transmit power. As a result, the interference received at co-channel FUs and FBSs either in DL or UL transmissions is mitigated to a remarkable extent for which femtocells admit the largest number of FUs. The proposed power control scheme for macro-tier and femtocells is described in detail in Algorithm 2.

Algorithm 2. Proposed power control algorithm for macro-tier and femto-tier

```
1 Initialization:
Set $q_i^n(1) = \bar{q}_{i,\mathrm{MU}}^n$, for all $i \in \mathcal{M}^m$; allocate sub-channels to MUs by round-robin scheduling. Set $q_i^n(1) = \frac{q_k^{\max}}{N}$ and $p_i^n(1) = \frac{p_i^{\max}}{N}$ for all $i \in \mathcal{M}^f$. Do sub-channel allocation and duplexing mode selection for FUs in DL and UL by Algorithm 1. Obtain $\hat{\gamma}_{i,\mathrm{FU}}^{n,\mathrm{dl}}$ and $\hat{\gamma}_{i,\mathrm{FU}}^{n,\mathrm{ul}}$, $\bar{q}_i^n$ and $\bar{p}_i^n$ for all $i \in \mathcal{M}^f$ on each sub-channel $n$ by Algorithm 1.
2  $t \leftarrow 2$
3  Power control for macro-tier at MBS :
4  Calculate $I_i^n(t)$ and $q_i^n(t)$ using (2) and (17) for each $i \in \mathcal{M}^m$,
   where $c_i^n = 1$.
5  Power control for femto-tier at femtocell $k$ :
6  Estimate $I_i^{n,\mathrm{dl}}(t)$ and $I_i^{n,\mathrm{ul}}(t)$ for each FU $i \in \mathcal{U}_k$ on its
   allocated sub-channel $n$ using (4) and (8).
7  Compute $I'^{n,\mathrm{dl}}_i(t)$ and $I'^{n,\mathrm{dl}}_i(t)$ for each FU $i \in \mathcal{U}_k$ on its
   allocated sub-channel $n$ based on the selected duplexing
   mode by Algorithm 1
8  Obtain $q_i^n(t)$ and $p_i^n(t)$ according to (28) and (29),
   respectively.
9  Set $t = t + 1$, return to step 2 until convergence. The
convergence condition is                and              .
```

4- 3- Convergence and Complexity Analysis of the Proposed Algorithm

The convergence of the proposed algorithm is investigated in the following theorem.

Theorem 1. The proposed algorithm converges to a feasible solution $(\mathbf{P},\mathbf{Q},\mathbf{A},\mathbf{C})$.

Proof. The allocated sub-channels to MUs are considered to be fixed and only transmit power for MUs on allocated sub-channels is updated using TPC algorithm. The proof for the convergence of TPC algorithm is studied in [22]. Additionally, the sub-channel allocation and duplexing mode selection for FUs are obtained at the first iteration. In the rest of iterations, while the allocated sub-channel and selected duplexing mode remain constant for FUs, the transmit power of FUs on allocated sub-channels are updated according to TPC algorithm. Based on the above discussion, it can be realized that the proposed algorithm converges to a feasible solution $(\mathbf{P},\mathbf{Q},\mathbf{A},\mathbf{C})$.

Allocation of sub-channels to FUs using Hungarian algorithm contributes into the major portion of computational overhead of our proposed algorithm, i.e. $O(N^3)$ [23]. The total complexity of our proposed scheme is $O(KN^3)$, where K is the number of femtocells.

5- Simulation Results

To evaluate the performance of our proposed scheme, we consider a macrocell with $1000\mathrm{m} \times 1000\mathrm{m}$ coverage area and a central BS. The co-channel femtocells with $10\mathrm{m} \times 10\mathrm{m}$ coverage area are uniformly deployed in the macrocell coverage area. The MUs and FUs are randomly scattered in every cell. The path gains from users to MBS and FBSs are modeled by $h_{i,k}^{n,\mathrm{dl}} = h_{k,i}^{n,\mathrm{ul}} = x_n d_{i,k}^{-\alpha}$ where $d_{i,k}$ is the distance between user i and BS k. The path gain among users is obtained by $\tilde{h}_{i,j}^n = x_n d_{i,j}^{-\alpha}$ where $d_{i,j}$ is the distance between users i and j. The path gain between BSs is given by $h_{b_i,k}^n = x_n d_{b_i,k}^{-\alpha}$ where $d_{b_i,k}$ is the distance between BSs b_i and k. x_n is a random variable with Rayleigh distribution and $\alpha = 3$ is the path loss exponent. Simulation parameters are listed in Table 1.

Table 1. Simulation Parameters

Parameter	value
Number of MUs (M^m)	4
Number of sub-channels (N)	8
Number of femtocells (K)	4
Number of FUs in femtocell k ($\lvert\mathcal{U}_k\rvert$)	2
MU's target rate ($R_{i,\mathrm{MU}}^{\min,\mathrm{dl}}$)	5 bps\Hz
FU's target rate in UL ($R_{i,\mathrm{FU}}^{\min,\mathrm{ul}}$)	5 bps\Hz
FU's target rate in DL ($R_{i,\mathrm{FU}}^{\min,\mathrm{dl}}$)	5 bps\Hz
SI gain (Δ)	-30 dB
maximum transmit power of MBS ($q_1^{\max}$)	1 w
maximum transmit power of FBSs ($q_k^{\max}$)	1 w
maximum transmit power of FUs ($p_i^{\max}$)	1 w
noise power (N_0)	10^{-13} w

Throughout the simulation, the uplink and downlink admission ratios refer to the number of admitted FUs in UL and DL transmissions to the total number of FUs, respectively. Likewise, the average admission ratio stands for the average of uplink admission ratio and downlink admission ratio. Our experiments are averaged from 500 snapshots where in every snapshot we randomly relocated users. We first evaluate the performance of our proposed algorithm in HD-HD mode and duplexing mode selection in sub-section A. Then, in sub-section B, we compare the performance of the proposed algorithm with the tier-aware resource allocation scheme in [14].

5- 1- Evaluation of Proposed Algorithm in HD-HD Mode and Duplexing Mode Selection

The convergence of the proposed algorithm is shown in Fig. 2. In this experiment, holding the allocated sub-channels and selected duplexing modes for FUs in UL and DL transmissions, the transmit power is updated on each sub-channel, i.e. based on TPC algorithm. As seen, the average transmit power for femtocells at DL and UL in our algorithm converges in a few iterations.

In Fig. 3, we illustrate the average admission ratio of femtocells of the proposed algorithm under HD-HD mode and duplexing mode selection versus MU's different target rates, i.e. $R_{i,\mathrm{MU}}^{\min,\mathrm{dl}}$. Note that when $R_{i,\mathrm{MU}}^{\min,\mathrm{dl}}$ increases, the interference temperature limit for MU i, i.e., I_{th}^{n}, decreases. As a result, the average admission ratio decreases with I_{th}^{n}.

Fig. 4 depicts the average admission ratio of the proposed algorithm with and without enabling the duplexing mode selection with respect to the target rate for FUs in UL and DL transmissions, i.e. $R_{i,\mathrm{FU}}^{\min,\mathrm{ul}}$ and $R_{i,\mathrm{FU}}^{\min,\mathrm{dl}}$, respectively. In this experiment, we consider the same target rate for FUs in UL and DL transmissions. Note that as $R_{i,\mathrm{FU}}^{\min,\mathrm{ul}}$ and $R_{i,\mathrm{FU}}^{\min,\mathrm{dl}}$ increase, FUs may need a transmit power higher than $\bar{p}_i^n$ and $\bar{q}_i^n$ and therefore they may not attain their target rates. Moreover, the mutual interference among femtocells increases with the target rate of FUs. Thus, the average admission ratio in both cases of the proposed algorithm decreases.

Fig. 5 compares the femtocells' average admission ratio of the present algorithm with and without enabling the duplexing mode selection versus a different number of sub-channels. When the number of sub-channels increases, the average admission ratio in both cases increases as well. It is seen that the average admission ratio in HD-HD mode is less than that of the proposed algorithm with duplexing mode selection. This is because when duplexing mode selection is enabled in our proposed algorithm, each sub-channel may be allocated to two users, one in UL and the other in DL transmission and therefore two FUs may reach at their target rates; however, with enabling only HD-HD mode, each sub-channel is allowed to be allocated to at most one FU in DL transmission.

In Fig. 6, we compare the average admission ratio in the proposed scheme under HD-HD mode and duplexing mode selection versus the self-interference gain (Δ). The performance of the algorithm under HD-HD mode is independent of SI gain and every FU meets its target rate only on DL transmission. However, in the proposed algorithm with duplexing mode selection, each FU may be satisfied with its target rate in both UL and DL transmissions. Hence, the average admission ratio in the proposed algorithm with duplexing mode selection is more than the HD-HD mode. It is worth noting that when SI gain increases, the behavior of our proposed algorithm with duplexing mode selection is close to HD-HD mode. The reason is that by increasing the SI gain, HD-HD mode achieves a better performance in terms of average admission ratio when compared to FD-FD and FD-HD modes.

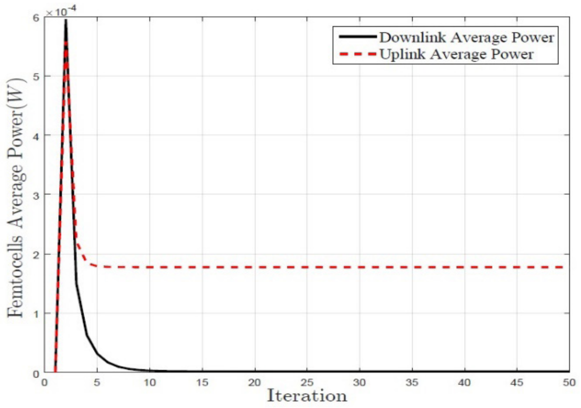

Fig. 2. Convergence of proposed algorithm.

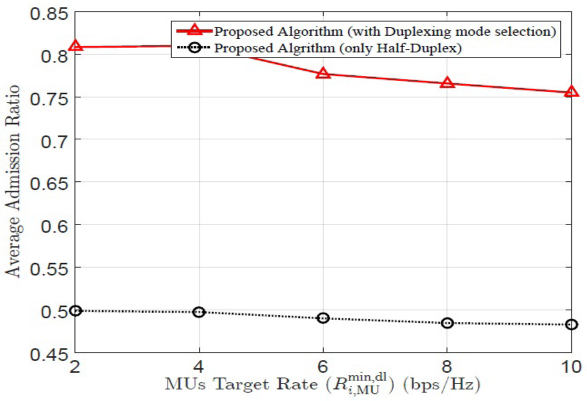

Fig. 3. Average admission ratio versus different MUs' target rates.

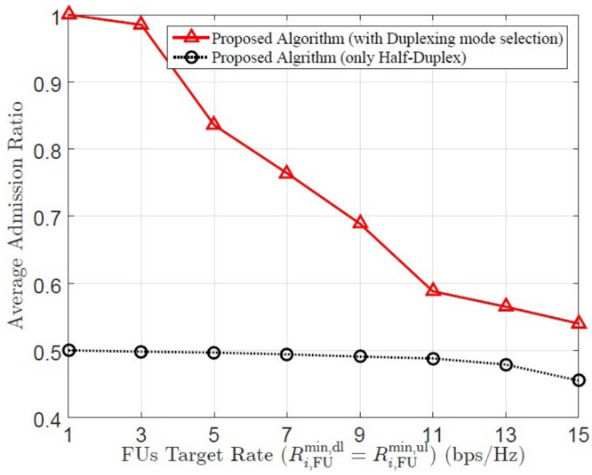

Fig. 4. Average admission ratio versus different FUs' target rate.

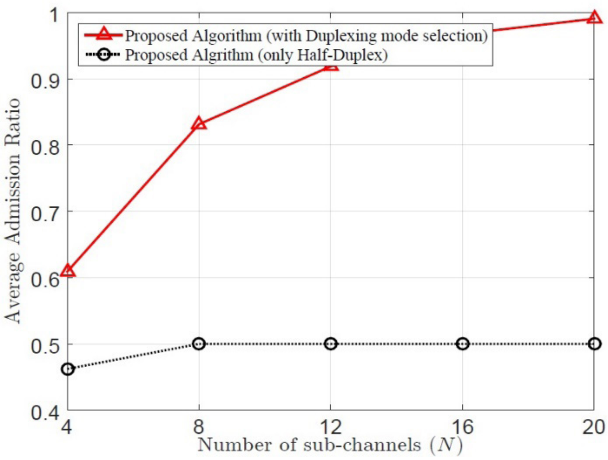

Fig. 5. Average admission ratio versus number of sub-channels.

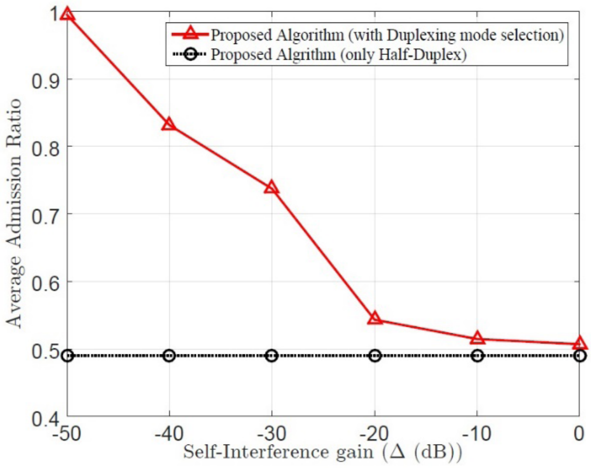

Fig. 6. Performance of proposed algorithm versus different Self-Interference gains.

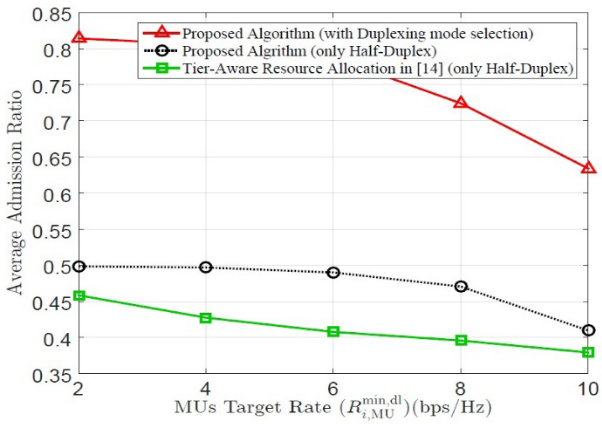

Fig. 7. Femtocells' average admission ratio versus different MUs' target rates for proposed algorithm and algorithm given in [14].

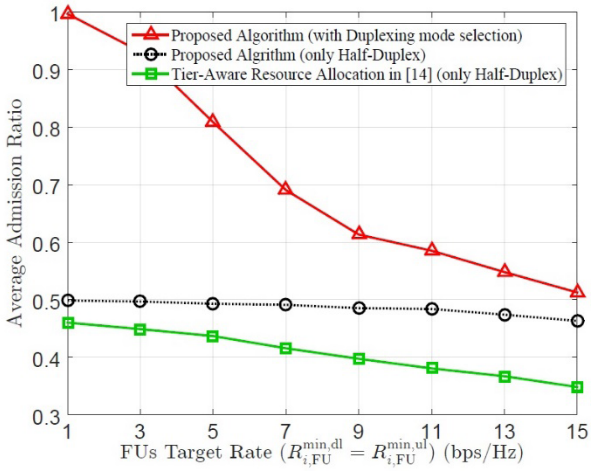

Fig. 8. Femtocells' average admission ratio versus different target rates of FUs on UL and DL for proposed algorithm and algorithm given in [14].

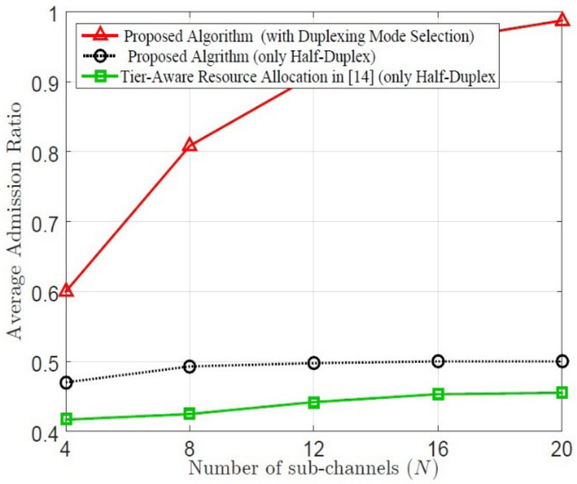

Fig. 9. Femtocells' average admission ratio versus different numbers of sub-channels for proposed algorithm and algorithm given in [14].

5- 2- Comparison of Proposed Algorithm with Resource Allocation Algorithm given in [14]

In this section, we compare the performance of our algorithm with tier-aware resource allocation scheme introduced in [14]. To carry out a fair comparison, we simulate the sub-channel allocation for macro-tier users proposed in [14]. Moreover, since the co-tier interference is ignored in the tier-aware resource allocation given in [14], for having a fair comparison with our proposed algorithm, we consider the case in which the co-tier interference is ignored. However, ignoring the co-tier interference is not practically correct.

Fig. 7 shows the performance of the proposed algorithm and the tier-aware resource allocation scheme in [14] when the target rate of MUs varies from 2 to 10. As observed, the proposed algorithm outperforms the resource allocation algorithm in terms of average admission ratio.

In Fig 8, we compare the average admission ratio achieved by our algorithm with the algorithm proposed in [14] with respect to the target rate of FUs. It is seen that our algorithm outperforms the tier-aware resource allocation scheme.

Fig. 9 illustrates the average admission ratio achieved by our algorithm and tier-aware resource allocation algorithm proposed in [14] when the number of sub-channels varies. It can be observed that our algorithm outperforms the algorithm of [14]. Note that in [14] the transmit power of MUs on theirs allocated sub-channels is a constant value while in the present power control algorithm for macro-tier, the transmit power of each MU on its allocated sub-channel is obtained by (17). Thanks to the power control for MUs, the proposed algorithm outperforms the resource allocation algorithm in [14] in terms of average admission ratio.

6- Conclusion

In this paper, we studied the resource allocation problem for full-duplex co-channel femtocells. Aiming to maximize the number of admitted FUs, we proposed a joint sub-channel allocation, duplexing mode selection, and power control for femto-tier in UL and DL transmissions. Moreover, to minimize the aggregate transmit power of MUs, we proposed a power control algorithm. While the maximum transmit power of FBSs and FUs are adaptively calculated according to MUs' interference temperature limit, in our proposed distributed scheme, a target rate is guaranteed for every user. Simulation results demonstrate that the proposed algorithm outperforms the traditional half-duplex communication mode in terms of femtocells' average admission ratio. It is worth noting that applying the proposed joint sub-channel allocation, duplexing mode selection, and power control in multi MBSs scenario is of interest, which remains as a future work of this paper.

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