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In the above two case studies, the same trajectory for the load is considered. However, in each case different additional kinematical constraint is considered for redundancy resolution. It is seen that, the maximum allowable load has a different value in each case. Therefore, the value of maximum allowable load for a given trajectory depends on the additional constraint functions that we apply to redundancy resolution.

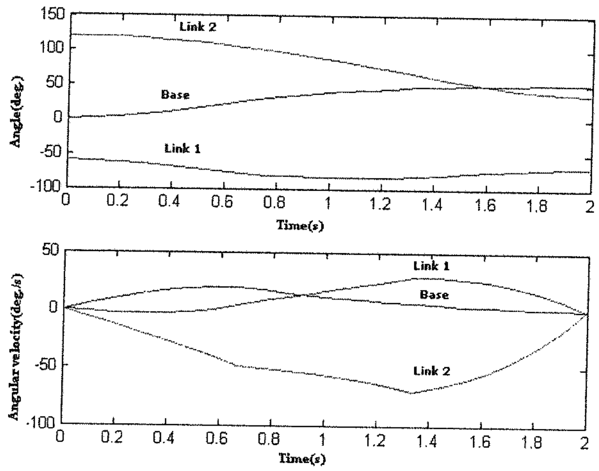

Figure (8) Variations of the base and links angles and angular velocities along the trajectory.

Matlab Ver 6.1

In this paper, the program MATLAB Release 12 Ver. 6.1 is used for dynamic modeling and simulation studies. This program provides many features that are useful in kinematic, dynamic and trajectory planning in robotics as well as useful capabilities for simulation analysis and results from experiments with real robots. Also there are some toolboxes and publications written by the MATLAB that provides many functions and libraries for the kinematic and dynamic analysis of robotic manipulators [12-13].

Conclusion

In this study, by combining null-space dynamics, nonholonomic kinematic constraints and operational space dynamics with together the augmented operational space is proposed for redundant mobile manipulators. Then by combining the motion of a mobile base by a planar two link arm, kinematic model of the overall system is derived. Using the proposed algorithm the maximum load carrying capacity of a mobile manipulator on a desired load trajectory is found. Two case studies are presented within which a two-link planar differentially driven mobile manipulator with a similar trajectory for the load, and different additional constraint functions for redundancy resolution is considered. At the first case the angle between the two links of manipulator is considered as additional constraint and corresponding maximum allowable load is computed as $m_{load} = 20.188$ kg. At the second case, the base coordinates are applied as additional constraints and corresponding maximum allowable load is computed $m_{load} = 37.147$ kg.

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α_e is rejected as a control variable and the base position coordinates (x_f, y_f) are considered as the user specified additional constraints $X_z = g(q)$.

The corresponding differential kinematic equation and augmented Jacobian matrix is derived as

$$\begin{pmatrix} Sin(\theta_0) & -Cos(\theta_0) & l_0 & 0 & 0 \\ 1 & 0 & J_{23} & J_{24} & J_{25} \\ 0 & 1 & J_{33} & J_{34} & J_{35} \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \dot{x}_f \\ \dot{y}_f \\ \dot{\theta}_0 \\ \dot{\theta}_1 \\ \dot{\theta}_2 \end{pmatrix} = \begin{pmatrix} 0 \\ \dot{x}_e \\ \dot{y}_e \\ \dot{x}_f \\ \dot{y}_f \end{pmatrix} \tag{37}$$

By direct calculation, the determinant of the augmented Jacobian matrix on the left hand side of the equation (26) is found $Det(J_a) = l_0 \times l_1 \times l_2 \sin(\theta_2)$.

Hence J_a is non-singular provided that $\theta_2 \neq 0^\circ$ or 180°. That is the two arms are not along the same axis. Suppose that the base length is $l_0 = 40\ cm$, the links length is $l_1 = l_2 = 50\ cm$. Let the initial configuration of the mobile base is $q_i = \{x_f,\ y_f,\ \theta_0\} = \{0cm,\ 0cm,\ 0rad\}$

The initial task vector is considered as $X_i = \{x_t, y_t, x_f, y_f\}_i = \{50,0,0,0\}\ cm$ and desired final task vector at time $t = 2\sec.$ is specified as $X_f = \{x_t, y_t, x_f, y_f\}_f = \{150,75,50,75\}\ \ cm$. Notice that final tool tip position is not feasible without the base motion. The desired task space path is specified as straight lines from initial to final configuration. By simulation study the overall movement of the mobile manipulator is found and shown in Fig. 7. The allowable load carrying capacity for the mobile manipulator at each point of trajectory is determined and maximum allowable load was found $m_{load} = 37.147kg$. The corresponding base and links angles and angular velocity variations along the trajectory are illustrated in Fig. 8.

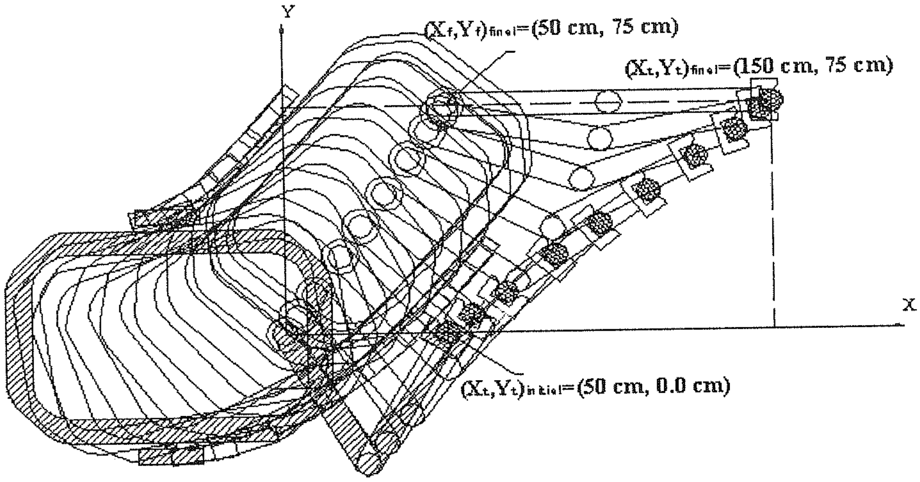
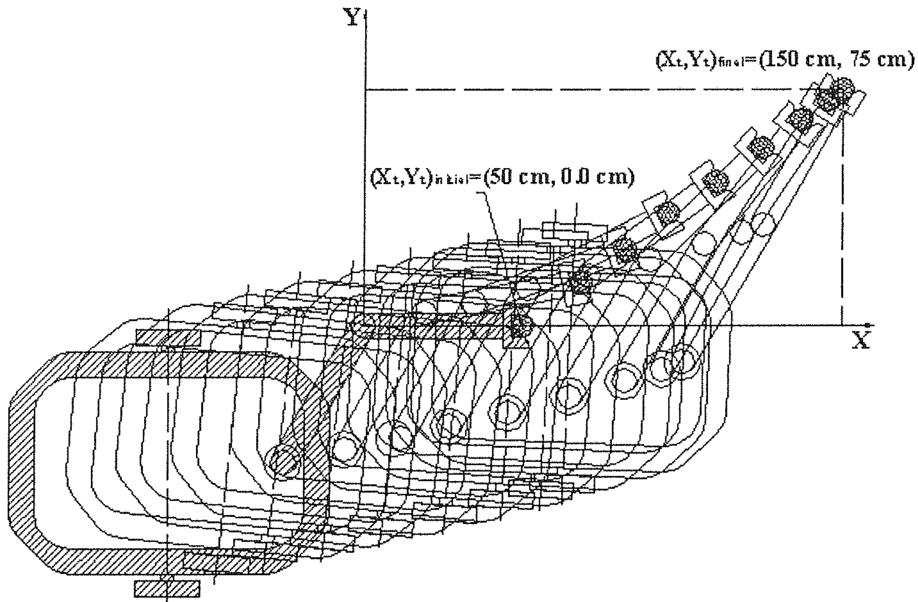


Figure (7) The movement of the mobile manipulator from initial to final configuration alongthe trajectory.

The initial task vector is considered as $X_i = \{x_t, y_t, \alpha, \beta\}_i = \{50cm, 0cm, 0^\circ, 120^\circ\}$ and desired final task vector at time $t = 2Sec$ is specified as $X_f = \{x_t, y_t, \alpha, \beta\}_f = \{150cm, 75cm, 60^\circ, 180^\circ\}$. The final tool tip position is not attainable without the base motion. By considering straight lines from initial to final configuration for the task space variables, overall movement of the mobile manipulator is determined and illustrated by simulation study (Fig. 5).

The allowable load carrying capacity for the mobile manipulator at every point of the trajectory is determined and maximum allowable load is found $m_{load} = 20.188kg$ [11]. The corresponding base and links angle and angular velocity variations along the trajectory are illustrated in the Fig. 6.



Hatched figure-Initial configuration

Figure (5) The movement of the mobile manipulator from initial to final configuration.

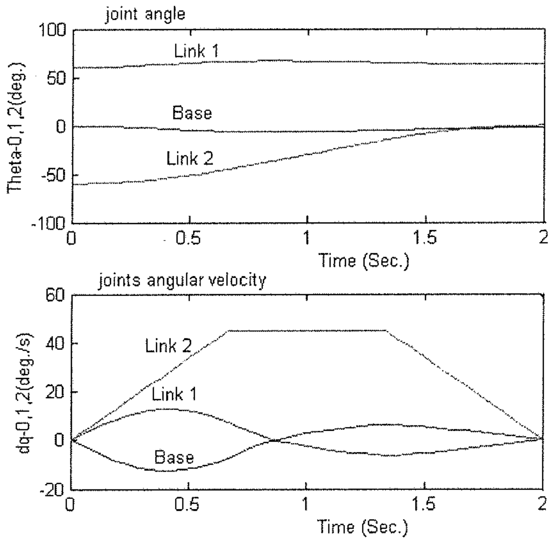


Figure (6) Variations of the base and links angles and angular velocities along the trajectory.

Simulation Model-2

The planar mobile manipulator similar to simulation model-1 is considered. In this case

$$X_z = g(q) \tag{33}$$

Note that the augmented Jacobian equation enables to treat the differential kinematics of redundant mobile manipulators as if they were nonredundant manipulators [9-10]. In the following case study, two special types of kinematic functions are applied to the mobile manipulator and its result is applied for determining the maximum allowable load of mobile manipulators.

Application and Simulation Studies

Full load motion of mobile manipulators while carrying a load is an important consideration in their application and motion planning. The maximum allowable loads that can be achieved by a mobile manipulator during a given trajectory are limited by a number of factors. Dynamic properties of mobile base and mounted manipulator, their actuator limitations and additional constraints applied to resolving the redundancy are probably the most important factors [11]. The above-mentioned two link planar manipulator is considered for simulation study (Fig. 4).

The joints actuator torque constraints are formulated based on the typical joint-speed characteristics of DC motors as follow

$$\begin{aligned} T^{(+)} &= k_1 - k_2\dot{q} \\ T^{(-)} &= -k_1 - k_2\dot{q} \end{aligned} \tag{34}$$

where $k_1 = T_s$ and $k_2 = T_s / \omega_{nl}$, T_s is the stall torque and ω_{nl} is the maximum no-load speed of the motor. Here, the joint actuators are similar and their constants are $\omega_{nl} = 3.5 rad/s$ and $k_1 = 63.22 N.m$.

Simulation Model-1

The planar two-link arm is mounted on mobile base at point F on the main axis of the base (Fig. 4). The position of point F relative to world coordinate frame is denoted by x_f, y_f. The manipulator elbow angle β between two arms is used as additional constraint equation. Thus

$$X_z = \beta = \pi - \theta_2 \tag{35}$$

Using the equation (30) the corresponding differential kinematic equation and augmented Jacobian matrix is derived as

$$\begin{pmatrix} Sin(\theta_0) & -Cos(\theta_0) & l_0 & 0 & 0 \\ 1 & 0 & J_{23} & J_{24} & J_{25} \\ 0 & 1 & J_{33} & J_{34} & J_{35} \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} \dot{x}_f \\ \dot{y}_f \\ \dot{\theta}_0 \\ \dot{\theta}_1 \\ \dot{\theta}_2 \end{pmatrix} = \begin{pmatrix} 0 \\ \dot{x}_e \\ \dot{y}_e \\ \dot{\alpha} \\ \dot{\beta} \end{pmatrix} \tag{36}$$

The determinant of the augmented Jacobian matrix on the left hand side of the equation (30) is found to be $Det(J) = l_0 \neq 0$. Therefore, the matrix J is non-singular regardless of the configuration of the mobile manipulator.

$$\begin{pmatrix} \dot{x}_e \\ \dot{y}_e \\ \dot{\alpha}_e \end{pmatrix} = \begin{pmatrix} \underline{1} & 0 & J_{13} & J_{14} & J_{15} \\ 0 & \underline{1} & \underline{0} & \underline{0} & J^*{}_{25} \\ 0 & 0 & 0 & 0 & \underline{J^*_{35}} \end{pmatrix} \begin{pmatrix} \dot{x}_f \\ \dot{y}_f \\ \dot{\theta}_0 \\ \dot{\theta}_1 \\ \dot{\theta}_2 \end{pmatrix} \tag{29}$$

The values of $J^*{}_{35} = 1 - J_{15} / J_{13}$ and $J^*_{25} = J_{25}(1 + J_{23} / J_{13})$ generally aren't zero. The column space is produced by the first, second and the fifth independent columns of J_e matrix spans which is a three dimensional space. Thus, the velocity task space of the end effector $\dot{X}_e = (\dot{x}_e, \dot{y}_e, \dot{\alpha}_e)^T$ at every instant of time can be selected freely. In special case, when J^*_{35} is equal to zero, then $\dot{\alpha}_e = 0$ resulted and a constraint on velocity of end effector is generated.

C: In the system of equation $J_{m\times n}\dot{q}_{n\times 1} = \dot{x}_{m\times 1}$, the dimension of the column space of $J_{m\times n}$ matrix $R(J)$ is equal with the rank of the matrix r and generally is a subspace of R^m. When $r \prec m$, there are $m - r$ constraint equations. Also when $m \prec n$, the null-space dimension of J matrix is equal to $n - r$.

Therefore in the analysis of combination of base and manipulator, by considering system of combination of base and manipulator, and system of equations (29), the dimension of column space $r = 3$, number of rows $m = 3$ and columns $n = 5$ results $m - r = 0$, thus the number of constraint equations are zero. Also the dimension of null-space that satisfies $J\dot{q} = 0$ equals to $n - r = 5 - 3 = 2$.

The mobile manipulator differential kinematics in augmented form can be defined as

$$\begin{pmatrix} J_c \\ J_e \\ J_z \end{pmatrix} \dot{q} = \begin{pmatrix} 0 \\ \dot{x}_e \\ \dot{x}_n \end{pmatrix} \tag{30}$$

The system of equation (30) is the combination of nonholonomic constraint equation (23), end effector velocity task space Equation (27), and null-space self motion equation in the form of equation (3) described as bellow

$$\dot{x}_n = J_z \dot{q} \tag{31}$$

vector space of J_z must be orthogonal to vector space of J_e such that satisfies the following relation

$$J_e J_z^T = 0 \tag{32}$$

and also are not parallel with vector space J_c. Hence, $\dot{x}_n$ is the projection of $\dot{q}$ in the direction spanned by vector space of J_z. Note that vector space of J_z can be obtained by using singular value decomposition and other methods. But, the simple method is to choose user specified constraint equation and check its validity by the Equations (30). The general form of these equations is

Substituting equation (20) into equation (21), we have

$$\dot{x}_f Sin(\theta_0) - \dot{y}_f Cos(\theta_0) + l_0\dot{\theta}_0 = 0 \qquad (22)$$

or in the matrix form

$$\left(Sin(\theta_0) \quad -Cos(\theta_0) \quad l_0\right)^T \dot{q}_b = 0 \qquad (23)$$

The general compact form of constraints can be written as

$$J_c(q)\dot{q}_b = 0 \qquad (24)$$

Lets θ_1, θ_2 denote angles and l_1, l_2 denote lengths of arm links. By assuming the coordinates of end effector $X_e = (x_e, y_e, \alpha_e)^T$, where x_e, y_e are position of the end-effector and α_e is its orientation relative to X axis of world coordinate frame. Therefore we have:

$$\begin{aligned} x_e &= x_f + l_1 Cos(\theta_0 + \theta_1) + l_2 Cos(\theta_0 + \theta_1 + \theta_2) \\ y_e &= y_f + l_1 Sin(\theta_0 + \theta_1) + l_2 Sin(\theta_0 + \theta + \theta_2) \\ \alpha_e &= \theta_0 + \theta_1 + \theta_2 \end{aligned} \qquad (25)$$

By differentiating equation (25) relative to time, the end effector velocity components are as below:

$$\begin{aligned} \dot{x}_e &= \dot{x}_f - l_1(\dot{\theta}_0 + \dot{\theta}_1)Sin(\theta_0 + \theta_1) - l_2(\dot{\theta}_0 + \dot{\theta}_1 + \dot{\theta}_2)Sin(\theta_0 + \theta_1 + \theta_2) \\ \dot{y}_e &= \dot{y}_f + l_1(\dot{\theta}_0 + \dot{\theta}_1)Cos(\theta_0 + \theta_1) + l_2(\dot{\theta}_0 + \dot{\theta}_1 + \dot{\theta}_2)Cos(\theta_0 + \theta_1 + \theta_2) \\ \dot{\alpha}_e &= \dot{\theta}_0 + \dot{\theta}_1 + \dot{\theta}_2 \end{aligned} \qquad (26)$$

Equation (26) in matrix from is:

$$\begin{pmatrix} \dot{x}_e \\ \dot{y}_e \\ \dot{\alpha}_e^* \end{pmatrix} = \begin{pmatrix} 1 & 0 & J_{13} & J_{14} & J_{15} \\ 0 & 1 & J_{23} & J_{24} & J_{25} \\ 0 & 0 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} \dot{x}_f \\ \dot{y}_f \\ \dot{\theta}_0 \\ \dot{\theta}_1 \\ \dot{\theta}_2 \end{pmatrix} \qquad (27)$$

where $J_{13} = J_{14} = -l_1 Sin(\theta_0 + \theta_1) - l_2 Sin(\theta_0 + \theta_1 + \theta_2)$, $J_{15} = -J_{25} = -l_2 Sin(\theta_0 + \theta_1 + \theta_2)$ and $J_{23} = J_{24} = l_1 Cos(\theta_0 + \theta_1) + l_2 Cos(\theta_0 + \theta_1 + \theta_2)$.

The compact form of the system of equation (27) can be written as

$$\dot{X}_e = J_e(q)\dot{q} \qquad (28)$$

By row operations on matrix J_e, system of equations (27) is changed to constraint equations that manipulator dynamic must satisfied them.

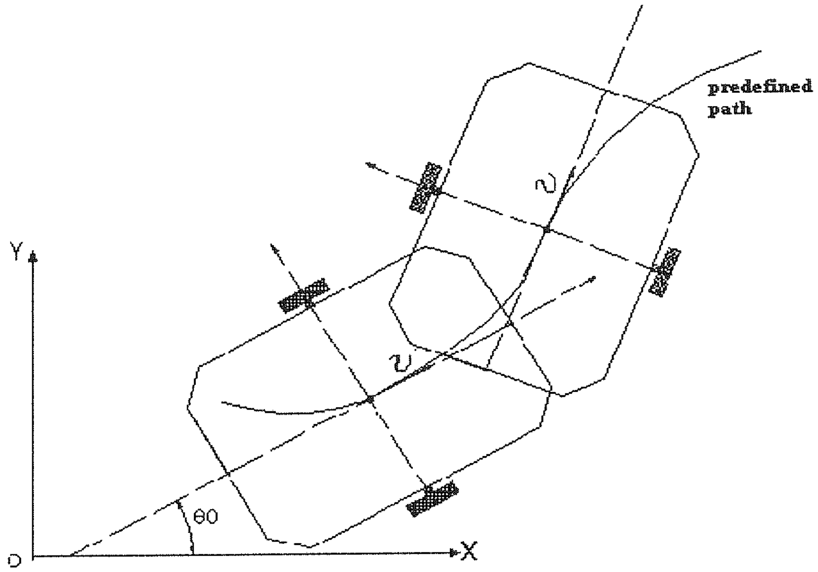


Figure (3) The rotation of mobile base is dependent to its path.

Kinematic Equation of Mobile Manipulator

In this section we consider a planar two-link arm mounted on mobile base at point F on the main axis of the base. The position of point F relative to world coordinate frame is denoted by x_f, y_f. Position of the arm mounted on mobile base is as bellow

$$x_f = x_b + l_0 Cos(\theta_0)$$
$$y_f = y_b + l_0 Sin(\theta_0) \tag{20}$$

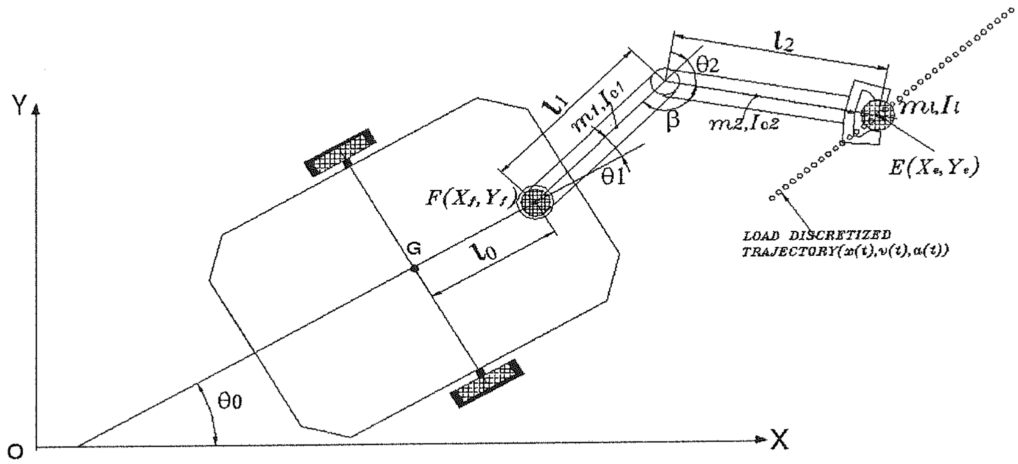


Figure (4) The combination of base and manipulator.

where l_0 is distance between the arm and middle of driving wheels common axis. Therefore velocity of base at point F is as follows.

$$\dot{x}_f = \dot{x}_b - l_0\, \dot{\theta}_0\, Sin(\theta_0)$$
$$\dot{y}_f = \dot{y}_b + l_0\, \dot{\theta}_0\, Cos(\theta_0) \tag{21}$$

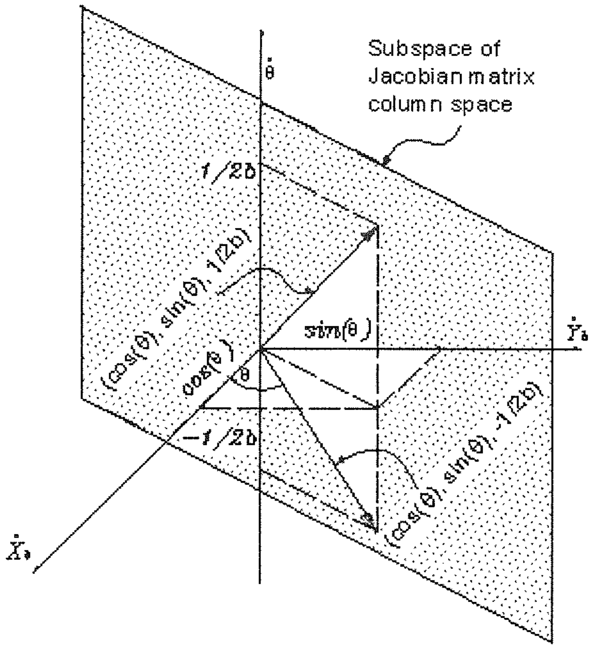


Figure (2) The column space of Jacobian matrix is a plane in the three dimensional space $\dot{X}_b, \dot{Y}_b, \dot{\theta}_0$

$$R\dot{\phi}_r/2\begin{pmatrix} Cos(\theta_0) \\ Sin(\theta_0) \\ 1/2b \end{pmatrix} + R\dot{\phi}_l/2\begin{pmatrix} Cos(\theta_0) \\ Sin(\theta_0) \\ -1/2b \end{pmatrix} = \begin{pmatrix} \dot{x}_b \\ \dot{y}_b \\ \dot{\theta}_0 \end{pmatrix} \tag{17}$$

Equation (17) is determined by changing matrix J to upper triangular form such as bellow:

$$R\begin{pmatrix} Cos(\theta_0) & Cos(\theta_0) \\ 0 & 2/b \\ 0 & 0 \end{pmatrix}\begin{pmatrix} \dot{\phi}_r \\ \dot{\phi}_l \end{pmatrix} = \begin{pmatrix} \dot{x}_b \\ \dot{\theta}_0 - \dot{x}_b/bCos(\theta_0) \\ \dot{y}_b - \dot{x}_b\tan(\theta_0) \end{pmatrix} \tag{18}$$

The third equation of right hand side of equation (18) is compatible only when

$$\dot{y}_b - \dot{x}_b\tan(\theta_0) = 0 \tag{19}$$

This constraint equation defines at any instant the path of the base movement is orthogonal to common axis of driving wheels (Fig. 3). Therefore moving at any instant of time in the direction of wheels axis is not possible and vector at this direction spans the null-space of J matrix. Using the other means, three velocity variables of the configuration space cannot be chosen arbitrarily and that at any time, the constraint equation (19) must be satisfied.

B: In robotic systems by velocity level kinematics equation $J_{m\times n}\dot{q}_{n\times 1} = \dot{x}_{m\times 1}$ when the number of independent actuators are less than D.O.F of manipulator (m>n) then, there are $n-r$

$$\dot{x}_b = vCos(\theta_0)$$
$$\dot{y}_b = vSin(\theta_0) \quad (13)$$
$$\dot{\theta}_0 = \dot{\theta}_0$$

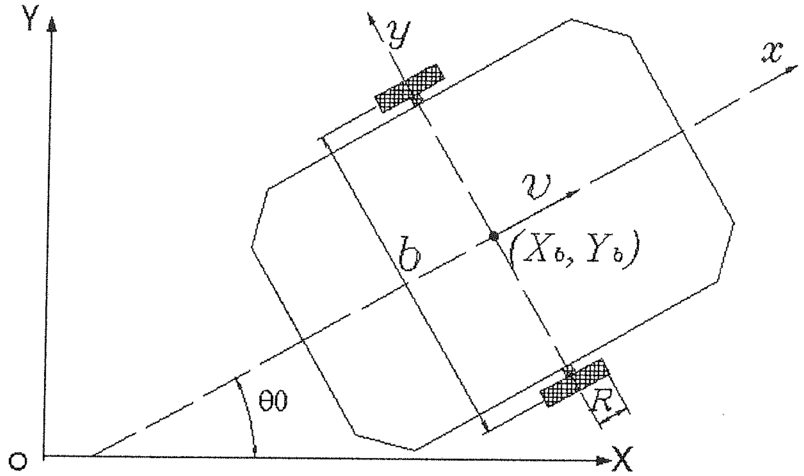


Figure (1) Configuration of wheeled mobile base relative to world coordinate frame OXY.

On the other hand by assuming non slipping (pure rotation) condition for driving wheels relative to ground surface we have

$$v = (R\dot{\phi}_r + R\dot{\phi}_l)/2$$
$$\dot{\theta}_0 = R/b(\dot{\phi}_r - \dot{\phi}_l) \quad (14)$$

where R, b are radius of driving wheels and distance between two driving wheels, respectively. The matrix form of equations is derived by applying equations (14) into (13).

$$\begin{pmatrix} \dot{x}_b \\ \dot{y}_b \\ \dot{\theta}_0 \end{pmatrix} = \begin{pmatrix} R/2Cos(\theta_0) & R/2Cos(\theta_0) \\ R/2Sin(\theta_0) & R/2Sin(\theta_0) \\ R/b & -R/b \end{pmatrix} \begin{pmatrix} \dot{\phi}_r \\ \dot{\phi}_l \end{pmatrix} \quad (15)$$

The compact form of Equation (15) is as bellow

$$\dot{X}_b = J_b(q)\dot{q}_b \quad (16)$$

where $\dot{X}_b \in R^{3\times1}, q_b \in R^{2\times1}$ and $J_b \in R^{3\times2}$.

Using linear algebra and functional analysis, the problem can be extended to general cases. A: System of equations $J_{m\times n}\dot{q}_{n\times1} = \dot{X}_{m\times1}$ is solvable if $\dot{X}$ can be written as a combination of columns of J matrix. The vector space spanning by independent column vectors of J matrix is called column space of J and denoted by $R(J)$ and generally is a subspace of R^m [8]. By other means Equation (13) has a solution, only when $(\dot{x}_b, \dot{y}_b, \dot{\theta}_0)^T$ lies in the plane produced by two column vectors of J. This plane is column space of J matrix (Fig. 2).

$$\dot{X}_r = \begin{pmatrix} \dot{X} \\ \dot{X}_n \end{pmatrix} \tag{7}$$

The kinematic equation of manipulator is determined as

$$\dot{X}_r = J_a \dot{q} \tag{8}$$

when $n \prec m$, there exist constraint equations in the system kinematics belongs to null-space of adjoint of J define as $J_c \in R^{c \times n}$ as a matrix consisting of the orthogonal basis vectors spanning the null-space of adjoint of J, then there is a relationship leads to:

$$J_c \dot{q} = 0 \tag{9}$$

If these equations are nonholonomic, then they cannot be integrated and eliminated from the system dynamics. By using equations (2) and (9) augmented, Jacobian matrix can be defined as:

$$J_a = \begin{pmatrix} J \\ J_c \end{pmatrix} \tag{10}$$

and the kinematic equation of manipulator is determined the same as Equation (8).

In the other case, similar to mobile manipulators, overall system consists of two subsystem, where one of the subsystem (mobile base) has nonholonomic constraints in a situation that $m \succ n$, but overall system has redundancy. In such cases the augmented Jacobian matrix may be defined as:

$$J_a = \begin{pmatrix} J \\ J_z \\ J_c \end{pmatrix} \tag{11}$$

and velocity level kinematic equation by combining equations (2), (8), and (9) can be expressed as:

$$\begin{pmatrix} \dot{X} \\ \dot{X}_n \\ 0 \end{pmatrix} = \begin{pmatrix} J \\ J_z \\ J_c \end{pmatrix} \dot{q} \tag{12}$$

Kinematic Equation of Mobile Base

In this Section, a mobile base is considered by two independent driving wheels at two sides as shown in Fig. 1. In general configuration space of the base is defined by the two positions coordinates (x_b, y_b) at the middle of driving wheels common axis, together with the orientation angle θ_0. Therefore, the 3×1 configuration vector $X_b = (x_b, y_b, \theta_0)^T$ characterizes the position and orientation of the base in the plane of motion relative to fixed world frame OXY. Differential kinematics of the base by relating rotating velocity of the left and right driving wheels $\dot{\phi}_r, \dot{\phi}_l$ to base velocity v can be determined as introduced some optimal criteria for redundancy resolution of mobile manipulators [7].

In this paper, at first, the kinematic equation of nonholonomic mobile base is derived. Then, the kinematic equations of the overall system are derived by considering a two link planar manipulator mounted on mobile base. At each case, the redundancy and constraints on the system dynamics are analyzed. Suitable user specified functions are introduced to overcome the redundancy on the system dynamics. Finally, to evaluate the performance of the proposed method simulation test is carried out to verify and determine the maximum load carrying capacity of a mobile manipulator.

Mathematical Modeling of Mobile Manipulator

The kinematic equation of a manipulator is given by:

$$X = f(q) \tag{1}$$

where $X \in R^m$ denotes the task space of manipulator with respect to the base frame, $q \in R^n$ is joint vector and f a vector consisting of m scalar functions.

If $n \succ m$, then the degree of redundancy $n - m$ is denoted by r. By the other means for solvability generally there must exists r additional function to relate joint vectors to each other. If $n \prec m$, then there exist $m - n$ constraint or dependent relations between joint vectors denoted by c. From the Equation (1), the Jacobian based equation of the manipulator is determined as:

$$\dot{X} = J\dot{q} \tag{2}$$

where $J = \partial f / \partial \theta \in R^{m\times n}$ denotes the Jacobian matrix and maps the velocity vector of the task space to joints velocity space such that for every $x \in X$ and $\theta \in \theta$, $\dot{X}$ is image of $\dot{\theta}$ under J.

In the case of $n \succ m$, $J_z \in R^{r\times n}$ defined as a matrix consisting of the orthogonal basis vectors spanning the null-space of J. Then J_z is orthogonal complement of J and satisfies:

$$J_z J^T = 0 \tag{3}$$

$$J_z J_z^T = I \tag{4}$$

If $\dot{X}_n$ is defined as the null-space velocity, then we have a complementary mapping relationship at velocity level between the joint space and the null-space which is:

$$\dot{X}_n = J_z \dot{q} \tag{5}$$

By using equations (2) and (5) and by augmenting the Jacobian:

$$J_a = \begin{pmatrix} J \\ J_z \end{pmatrix} \tag{6}$$

and augmented velocity space as:

Analysis of Kinematic Redundancy on Mobilen Manipulators

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Abstract

In this paper kinematic model of a nonholonomic mobile manipulator is formulated. The kinematic equations for a mechanical mobile manipulator are developed in such a way that the kinematic interaction between the mobile platform and mechanical manipulator explicitly appears in the equations. In both conditions the motion constraints and kinematic redundancy are considered and analyzed. These concepts are expanded in general cases by using special subjects on linear algebra and functional analysis. A useful and unique kinematic model for redundancy resolution of mobile manipulators is introduced by defining specified variables. Finally, the application of the model is verified by simulation results in order to find the maximum allowable load of a mobile manipulator.

Keywords

Mobile manipulator, redundancy, nonholonomic

Introduction

In recent years variety of applications is introduced for wheeled mobile manipulators. Some of these manipulators are mobile bases with several recognition systems and environmental sensors mounted on them. Typical applications of these type of manipulators are to explore and recognize the unknown environments, such as moving on narrow channels and exploring on other planets, load transfer at dangerous or hazardous places such as nuclear power plants and chemical product companies.

In the mathematical model of wheeled mobile manipulators usually there are kinematic constraints that arise from special moving mechanism at manipulator base. These constraints are not integrable and cannot be eliminated from system's model. On the other hand by combining motion of base and manipulator there are redundancy in the kinematic model of the system. There are many researches and literatures in mathematical modeling of wheeled mobile robots. Campion and et al. [1] presented the kinematic and dynamic models of common types of wheeled mobile robots and classified them into general cases. Yamamoto and Yun analyzed the dynamic interaction of base and manipulator motions at the same time [2]. Chung and Velinsky by combining the Newton-Euler and Lagrange formulation presented a control algorithm for a mobile base by one and two degrees of freedom manipulators [3, 4]. Seraji presented an on-line control algorithm for some types of mobile manipulators by using the concept of augmented Jacobian matrix [5]. Chung and et al. [6] presented the interaction of null-space dynamics and operational space dynamics of redundant manipulators. Honzik