On the Motion of High-Reynolds Particles in a Quiescent Fluid

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ABSTRACT

Predicting the motion of particles in a quiescent liquid is a key problem in fluid mechanics that has a bearing on wide range of applications in multiphase flow modeling. A significant part of the paper is devoted to an evaluation of gravity, drag, added mass and history force on the motion of high-Reynolds number particles. In this study, the motion of metallic and plastic particles of 5 and 10 mm diameters in liquid media during their sedimentation toward a plate was studied experimentally. Variation of the sphere size and density allow measurements at Reynolds numbers, based on terminal velocity, between 1000 and 95000. The comparison is made by solving the equation of particle motion based on Lagrangian approach. The results showed if gravity is sufficient to describe the particle trajectory in a gas, this is not the case in a liquid where drag and added mass force are important but not sufficient: the history force is shown to be non-negligible. For the small particles, the history force has a $(t-\tau)^{-1/2}$ behavior as predicted by Odar and Hamilton and as observed in numerical simulation. Finally, comparison of the dynamics of particles of different diameters for the same density shows that the diameter has a significant role in treatment of particle sedimentation and that large particles show transitory oscillations in velocity profile.

KEYWORDS:

Particle trajectory, Lagrangian approach, History force, High Reynolds numbers

1. Introduction

The accurate evaluation of the hydrodynamic forces acting on a particle moving in a viscous fluid remains a fundamental question in multiphase flow modeling. This problem arises in many engineering applications, e.g., spray combustion, pollution control, boiling and bubble dynamics, sedimentation, and erosion of turbine blades. All these problems are concerned with interaction of particles with fluids, which requires accurate knowledge of all hydrodynamic forces acting on a particle. Another problem is associated with the ability of dispersed solid particles to follow the fluid motion when their density or initial velocity does not match the fluid velocity or its density; that is, the ability of solid particles to behave as Lagrangian tracers of fluid motion. This issue is of importance for the prediction of dispersion of particles in flows, as well as for measurement techniques such as particle image velocimetry (PIV). So, this motivation comes from work aiming at developing a Lagrangian tracking technique for the motion of solid particles during large intervals of times. It raises the question of the response of a particle to rapid changes in the velocity of the fluid, or to a sudden acceleration. Analytical approaches to the time-dependent motion of a solid particle in a given quiescent fluid have been restricted to zero or small particle Reynolds numbers. However, they provide a general frame of description of the forces acting on the particle. The equation of particle motion in its general form is rather cumbersome to deal with. Usually, various simplified versions are used. In other words, among the forces acting on a particle, the gravity or buoyancy force, the quasi-steady drag and the added mass force are currently included and their adequate expressions are now well defined. The history force, taking into account the vorticity diffusion in the surrounding fluid and the disturbance effect caused by the acceleration of the sphere, is often neglected in simulation of particle trajectory. Nevertheless, the applicability of the equation of motion is still to be clarified. If one considers an equation of the particle motion trajectory with

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parameters corresponding to the creeping flow approximation, one finds that the history force generally should exceed inertial forces.

In several papers, advection of particles with inertia in a fluid was investigated numerically under an assumption that the history force can be neglected [1]-[8]. Ounis and Ahmadi [9] studied the motion of small spherical particles in a random flow field analytically. The equation of motion of asmall spherical rigid particle in a turbulent flow field, including stokes drag, virtual mass and the Basset (history) force effect were considered. Results obtained recently for the motion of a particle in a shock wave showed that the Basset force could be even more significant than stokes drag force [10]. Abbad et al. [11],[12] experimentally studied a free-falling rigid sphere in a quiescent incompressible Newtonian fluid, placed in an oscillating frame. They investigated numerically the effect of the history force acting on the sphere at small particle Reynolds numbers ($Re_p \leq 2.5$). The comparison was made by solving the equation of motion of the sphere with and without the history force. They found that the history force plays a significant role in the momentum balance. Harada et al. [13] studied both experimentally and numerically a spherical nylon particle of diameter 12.7 and 25.4 mm approaching a wall in an incompressible fluid under the action of gravity at particle Reynolds number 6.01 and 25.8, respectively. Their results showed that in addition to the gravity, the drag and the added mass force, the Basset history force also had a significant effect on the particle motion through the sedimentation in both cases. Gondret et al.[14] investigated both experimentally and numerically the bouncing motion of solid spheres onto a solid plate in an ambient fluid. They demonstrated that history forces cannot be neglected for the bouncing trajectories after the collisions for particle Reynolds numbers up to about 10^3.

Most of the previous studies have been performed on the motion of particles in a quiescent fluid at low or moderate particle Reynolds number. The objective of the present paper is to examine the effect of each hydrodynamic force on the motion of spherical metallic and plastic particles of different diameters at large particle Reynolds numbers (Re_p>1000). Both experiments and a numerical analysis were conducted to examine the fluid-forces model in a range of particle Reynolds number between 1000 and 95000. In the present paper, we focused on the trajectory of the particles motion from their start from rest to the wall impact by varying the density and diameter of the solid particles. The particle trajectory was calculated with the Lagrangian method. The equation of particle motion was used and the corresponding condition imposed on the fluid has been taken into account. In our numerical model, gravity, drag, added mass and history forces were considered with proper modification. In order to assess model validation, the numerical simulations were compared with the experimental data obtained with other researches.

2. FORMULATION AND NUMERICAL METHOD

The particle trajectory can be determined by solving its equation of motion, which can be deduced from Newton's Second Law. The equation of motion for small particles in a viscous quiescent fluid dates back to the pioneering work of Basset, Boussinesq and Oseen, and is commonly known as the BBO equation. They solved the Navier Stokes equations for a creeping flow by neglecting the advective acceleration terms and derived the following equation for the acceleration of the sphere [15]:

$$m_p \frac{dU}{dt} = -6a\pi \mu_f U\varphi - \frac{1}{2} m_f \frac{dU}{dt} + (m_p - m_f)g$$
$$-6a^2 \sqrt{\pi \mu_f \rho_f} \int_0^t \frac{dU/dt}{\sqrt{t-\tau}} d\tau \qquad (1)$$

where ρ_f is the density of the fluid, μ_f its viscosity, U is the sphere velocity, a its radius, m_p its mass and m_f is the mass of the fluid displaced by the sphere ($m_f = (4/3)\pi a^3 \rho_f$). The right hand side of equation (1) consists of the summation of all forces exerted on the particle along its trajectory in quiescent fluid. The terms on the right-hand side of equation (1) are, in the order of their appearance, steady drag (F_D), virtual mass force (F_A), Basset or history force (F_H) and gravity force (F_G).

The steady drag is responsible for the limit velocity of a sphere falling under gravity. The expression in equation (1) is valid only for $Re_p = 0$ ($Re_p = 2aU\rho_f/\mu_f$). It is well known that for finite particle Reynolds numbers, the convective inertia increases the drag. The analytic expression is not known for all particle Reynolds numbers but the empirical law for the drag coefficient as a function of Re_p is well documented for a noncreeping flow from $Re_p \to 0$ up to values higher than 10^7. One usually writes the steady drag as [16]:

$$F_D = -6a\pi\mu_f U\varphi \qquad (2)$$

where φ is a function of the particle Reynolds number. Various approximations of $\varphi(Re_p)$ for rigid spherical particles can be found in the book of Clift et al. [16]. In the present study, we used the following approximation of the $\varphi(Re_p)$ valid in a wide range of particle Reynolds numbers [17]:

$$\varphi = (1 + 0.15 Re_p^{0.687})$$
$$+ \frac{1.75 \times 10^{-2} Re_p}{1 + 4.25 \times 10^4 Re_p^{-1.16}} \quad \forall \; Re_p < 3 \times 10^5 \qquad (3)$$

Odar & Hamilton [18] and Odar [19] studied experimentally the force on a guided sphere rectilinearly oscillating in an otherwise stagnant fluid. They proposed an equation for the motion of a sphere based on their experimental study as:

$$m_p \frac{dU}{dt} = -6a\pi\mu_f U\varphi - \frac{1}{2} C_a m_f \frac{dU}{dt} + (m_p - m_f)g$$

$$-6a^2 C_h \sqrt{\pi\mu_f\rho_f} \int_0^t \frac{dU/dt}{\sqrt{t-\tau}} d\tau \qquad (4)$$

with C_a and C_h obtained experimentally and given by:

$$C_a = 2.1 - \frac{0.132}{A_c^2 - 0.12} \qquad (5)$$

$$C_h = 0.48 - \frac{0.32}{(A_c + 1)^3} \qquad (6)$$

The parameter A_C is called the acceleration number and is defined by :

$$A_c = \frac{2U^2}{a|dU/dt|} \qquad (7)$$

Note that in the inviscid limit, the added mass force is modified by the presence of a wall by the factor $(1+3a^3/8(a+h)^3)$, where h is the distance of the bottom apex of the particle to the wall [20].

In this paper, the velocity of a particle is obtained by integrating equation (4) using the Runge-Kutta 4th order method and the particle position is determined according to the velocity (U=dx/dt). In the numerical simulation, the main problem of solving the equation (4) is the history force term. To solve this problem, it assumed that the general temporal variation of particle velocity can be broken up into a series of step changes. At time 0 there is a change Δu_0,at time t_1 a change Δu_1 and at time t_2 a change Δu_2 and so on. For instance, to compute the effect of history force at time t_3, the cumulative effect of the history force can be written as follow [17]:

$$F_{H(t_3)} = 6a^2 \sqrt{\pi\mu_f\rho_f} \int_0^{t_3} \frac{dv/dt}{\sqrt{t-\tau}} d\tau$$

$$= 6a^2 \sqrt{\pi\mu_f\rho_f} \left[\frac{\Delta u_0}{\sqrt{t_3}} + \frac{\Delta u_1}{\sqrt{t_3 - t_1}} + \frac{\Delta u_2}{\sqrt{t_3 - t_2}} \right] \qquad (8)$$

3. Experimental set-up

The trajectory of solid particles falling toward a Plexiglas plate is investigated experimentally. We used solid spheres made of different materials and with the same diameter. Table 1 summarizes the relevant properties of the spheres used in the experiments. The experiments were conducted by dropping the particles in water. The mass density of used water is 998.1 kg/m^3, whereas the viscosity is 1 × 10^{-3} Pa.s (at T=20°). The particle trajectory is recorded by a high speed camera (Photron Fastcam PC1 1024) at 1000 frames/second. The recorded sequences of the particle motion are analyzed by using the Photron Fastcam Viewer. The experiments were conducted in a rectangular Plexiglas tank with base dimensions of 275 mm × 275 mm and a depth of 280 mm. To avoid air entertainment, the particles were initially submerged and held in a place a few millimeters under the water surface by means of a suitable support. Figure 1 depicts the sequences of snapshots of the Delrin particle motion toward the Plexiglas wall in water.

TABLE 1: PROPERTIES OF THE PARTICLES USED IN THE EXPERIMETS

Material	Diameter (mm)	Density (kg/m3)
Delrin	5 , 10	1360
Teflon	5 , 10	2300
Steel	5 , 10	7780

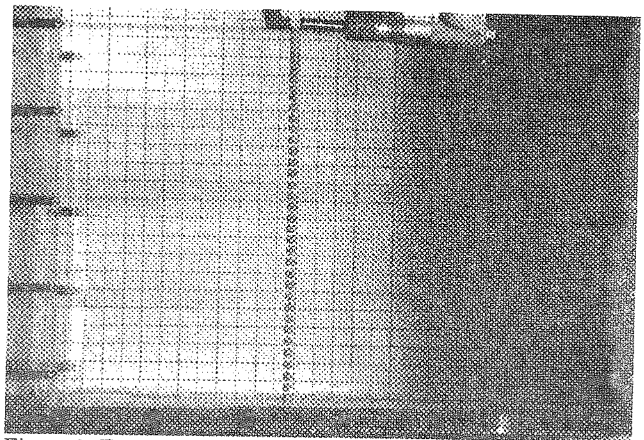

Figure 1. Experimental facility and a sample picture of the 5mm Delrin particle dropped from a height of 0.195 m onto a Plexiglas plate in water

4. Results

In the following section, we compute the particle trajectory numerically and compare the results obtained with the experimental data.

4.1. Trajectory of the small particles

Figure 2 displays the experimental and numerical trajectories of the Delrin sphere of diameter 5 mm falling in water. The experimental result show that due to viscous forces, in addition to gravity, drag and added mass as the hydrodynamic forces must be included. Nevertheless, the predicted trajectories underestimate the experimental ones. As the figure shows, excellent agreement between the measured and predicted particle trajectory was obtained when the history force included in the governing equation. In this case, the particle Reynolds particle number (Re_p) based on terminal velocity is about 1040. The numerical results show that to predict correct trajectory, contribution of the gravity, the drag, the added mass and the history force are 52.9, 34.7, 8.1 and 4.3 percent of hydrodynamic force, respectively. Another interesting attempt to examine the influence of the hydrodynamic forces on the motion of the Delrin particle in water is shown in Figure 3 where the obtained velocity profile for each case from the numerical simulation is compared to the experimental velocity profile. These curves provide clear information about effect of each hydrodynamic force on the motion of the Delrin particle. Here, an excellent agreement is observed when the history force is taken into account.

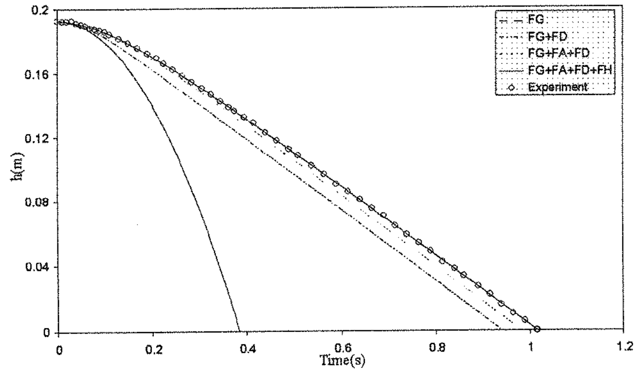

Figure 2. Trajectory of the motion of Delrin particle in water. Experimental measurements and numerical simulations

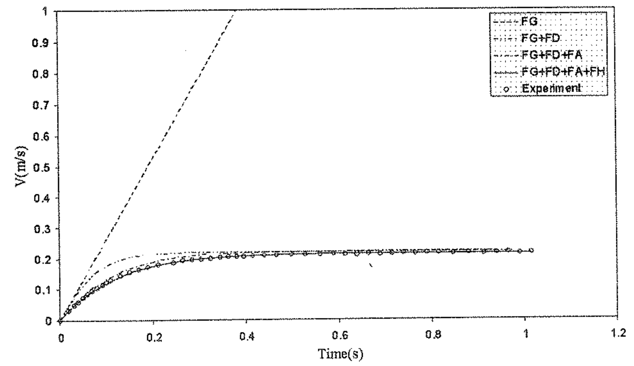

Figure 3. Velocity profile for the motion of Delrin particle in water.

By choosing the Teflon particle of diameter 5mm and changing the value of density from 1360 to 2300 kgm^{-3}, we vary the ratio of inertia to gravitational mass and we expect to observe different dynamical behaviors. In particular, we expect the motion of a lighter bead (Delrin) to be more influenced by the eventual unsteadiness of its wake. Figure 4 shows the trajectory and velocity profile of the Teflon sphere. As it is evident from these figures, taking only the gravity and the dissipating role of the drag force into account explains up to about 89.7 percent of the experimental data. With the addition of the added mass force, the accuracy of the simulation results increases up to about 97.0 percent. But there is still a little discrepancy between the experimental and numerical results. Therefore, in this case the history force appears again necessary to give a good agreement with the experimental data even at $Re_p \approx 1925$. However, in this case, the history force acting on the particle is about 3.0 percent of the hydrodynamic force.

For increasing particle Reynolds number, the steel particle is used. Figure 5 shows the experimental and numerical trajectory and velocity profile of the steel sphere of diameter 5mm at $Re_p \approx 4670$ in the water. The results show that in spite of neglecting the history term in equation (4), the major part of the experimental data can be described with the other terms. However, the figure confirms that to produce curve fit of the experimental trajectory, the history force effect appears again necessary but not considerable and can be ignored. In this case, the results obtained from numerical simulation without the history force explain up to about 99 percent of the experimental data.

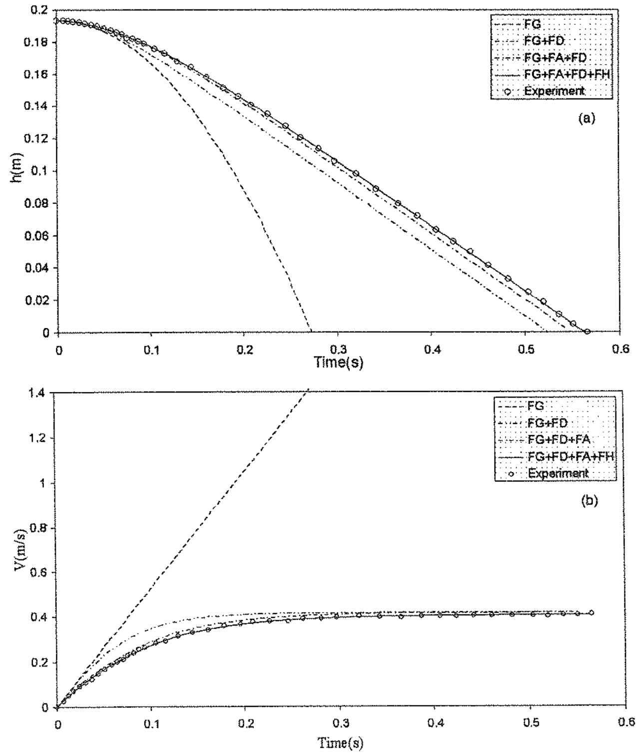

Figure 4. (a) Trajectory and (b) Velocity profile for the motion of Teflon particle in water.

In the following, we have summarized the contribution of each hydrodynamic force which is obtained from numerical simulation on prediction of the correct particle trajectory with respect to the particle Reynolds number (Figure 6). The results of numerical model for the motion of small particles (d=5 mm) with Reynolds number between 1000 to 5000 indicate that in addition to gravity force, the drag force is dominant especially for light particles, then the apparent mass is necessary to increase the accuracy of model with observations, and history force has a small contribution in final results.

Finally, the numerical calculations from our model (including gravity, drag, added mass and history forces) for the motion of steel particles of diameter 3 and 4 mm in the water are compared with the experimental data of Mordant and Pinton [21]. From Figure 7, the good agreement between numerical and experimental results reveals the high accuracy of the present model at high particle Reynolds numbers.

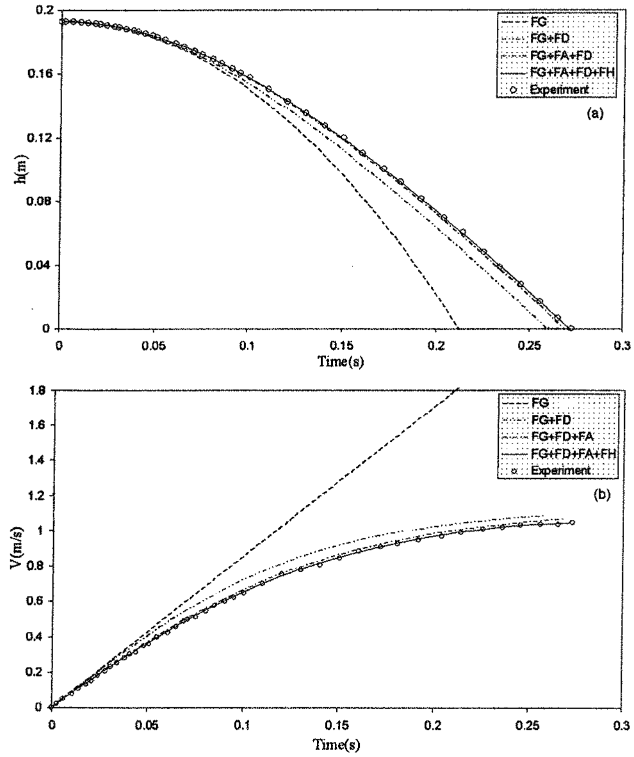

Figure. 5. As Figure 5 but for steel particle of diameter d=5mm.

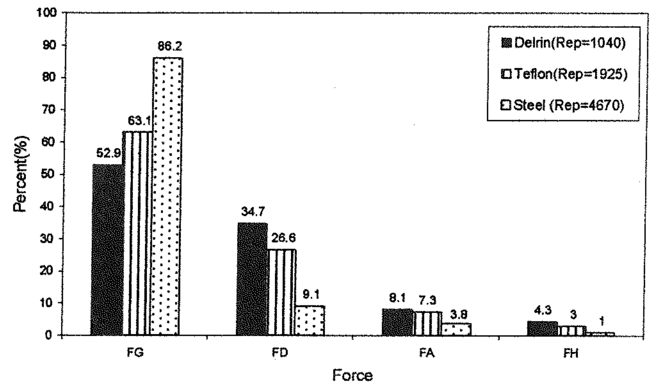

Figure 6. Contribution of each hydrodynamic force on prediction of the correct trajectory of different particles (d=5mm) in water.

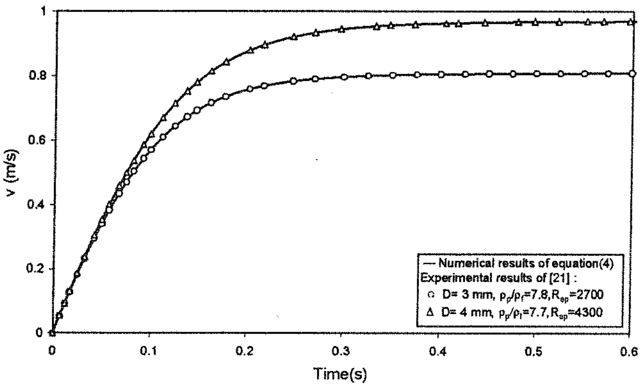

Figure 7. Velocity profiles of steel particles motion in water at rest. The numerical results of the equation (4) are compared with the experimental data of reference [21].

4.2. Trajectory of the large particles

The effect of particle size on the history force appears in equation (4) as the square of radius (a^2). Indeed by changing the size of the particles from 5mm to 10mm for the same density, we expect the motion of the bigger beads to be more influenced by the eventual unsteadiness of their wake. In Figure 8, the velocity of particles of 10 mm in diameter and different materials (Delrin, Teflon and Steel) are shown without any averaging. As the figure displays, in the large cases, the velocity presents oscillations while approaching its terminal value. However, we note that the oscillations for the steel particle are smaller than what is reported for the Delrin and Teflon particles. Such oscillations must be linked to a temporal evolution of the particle wake. One observes that the velocity is no longer a monotonous function of time: it alternates periods of increase and decrease. For this to happen, the acceleration of the particle must change sign; in particular the reaction of the wake on the particle is sufficient to overcome the gravitational force. Note that the particle Reynolds number is close to 0.46×10^4, $10^4$and 9.43×10^4 for the Delrin,Teflon and steel particles, respectively. It is tempting to associate these events with the vortex shedding that may occur at these particle Reynolds numbers for the large particles. This is not observed for the small spheres (d=5 mm).

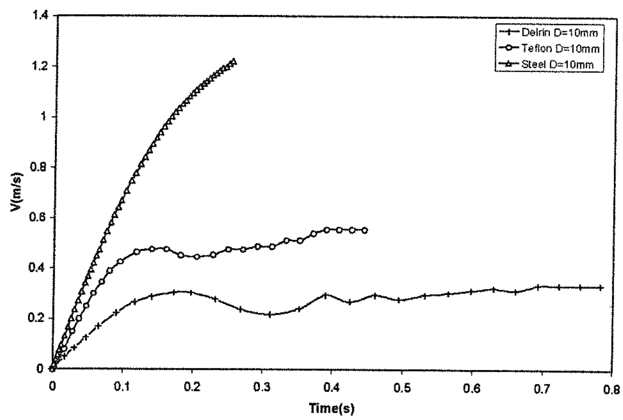

Figure 8. Velocity measurement without averaging for the 10mm spheres.

Figure 9 shows the experimental and numerical trajectories of Delrin particle with 10 mm diameter. As observe, even thought Re_p=4600, the numerical model (includes FG, FD, FA and FH) can predict trajectory of particle coinciding with experimental observation just before the beginning of oscillatory motion and after that, the numerical trajectory deviates from the observation about maximum 9%. With increasing Reynolds number of particle up to 95000 (steel particle), this deviation is reduced to about 0.4% (Figure 10). Therefore, in high Reynolds the numerical model which includes gravity, drag, added mass and history force can predict the experimental observation with very little discrepancy.

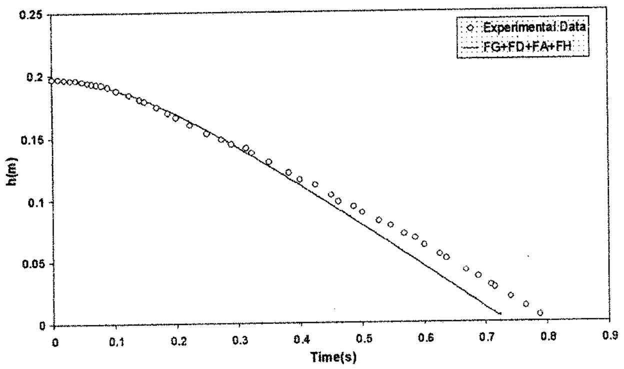

Figure 9. The experimental and numerical trajectories of the 10mm Delrin particle ($Re_p \approx 4600$)

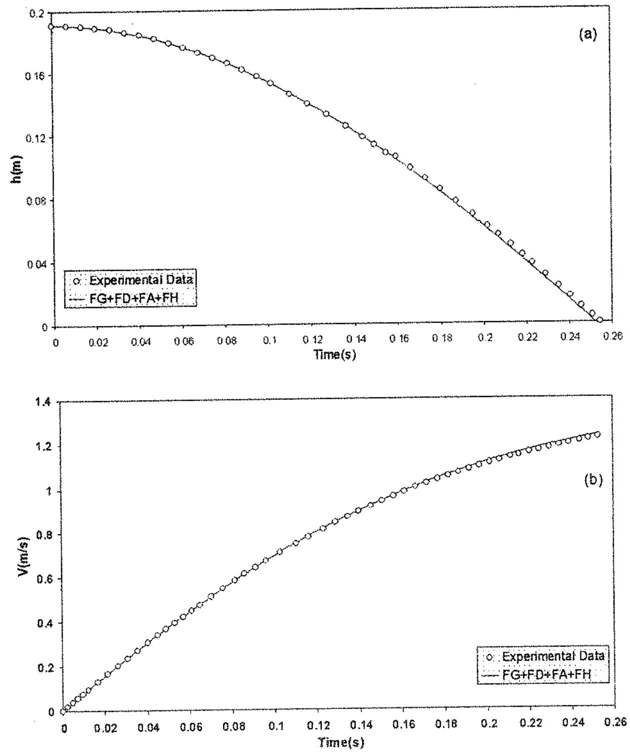

Figure 10. (a) Trajectory and (b) velocity profile for the motion of 10mm Steel particle ($Re_p \approx 94300$)

5. Conclusion

In the literature, there are many experimental studies on the effect of hydrodynamic forces acting on macroscopic particle motion at low and moderate particle Reynolds numbers[10]-[14]. Thus it appeared to us very useful to quantify experimentally and numerically the effect of each hydrodynamic force on the particle motion at high particle Reynolds number. For this, experiments were carried out to study the free motion of spherical particle of 5 and 10 mm diameters and different material (Delrin, Teflon and Steel) in liquid media during their sedimentation toward a plate. Variation of the sphere size and density allowed measurements at particle Reynolds numbers, based on terminal velocity, between 1000 and 95000. By using a video tracking technique and image processing, accurate measurements of the particle position and velocity are carried out. This study showed that in water and with the 5mm particles, taking only the gravity into account leads to an underestimate of the particles trajectory. The dissipating role of the drag force is important to explain the trajectory but not sufficient, and the added mass effect turns out to be non-negligible even for a density ratio of about 7. However, the addition of these terms is not sufficient to reproduce the experimental curve, which clearly showed that the history force is necessary to predict correctly the particle trajectory. In this study, for the smaller particles, the addition of the drag force to the gravity explained up to about 91 percent of the experimental curve. When added mass force was included then the accuracy of the simulation results increased up to about 97 percent. As a result, only about 3 percent of the results was accounted with the history force. It should be note that for the smaller particles, the particle Reynolds number was between 1000 and 5000.

For the particles of 10 mm diameter, we observed a behavior not yet described by the numerical model (includes FG, FD, FA and FH). Even though Re_p varied between 4600 to 95000, the numerical model predicted trajectory of particle coinciding with experimental observation just before the beginning of oscillatory motion and after that the numerical results had very little discrepancy with experimental observation. It is believed that the oscillation may be due to transient vortex shedding in the wake of the particles which reacts on the motion of the particles.

It should be noted that in this study the influence of the particle phase on the fluid considered to be negligible (one-way coupling model). This assumption was true just for the small particles and for the large particles as well, the numerical model needs to be improved by including the effect of particles on the fluid, i.e., developing a two-way coupling model.

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