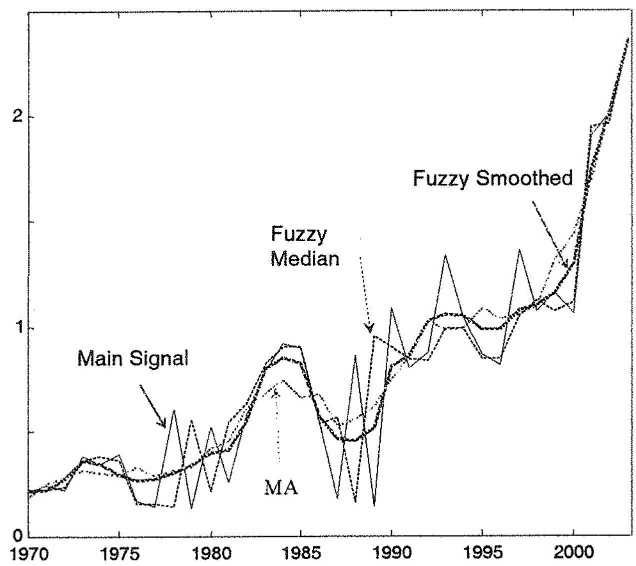

Figure 14: Growth of the maximum simultaneous electric power demand in Iran (×10GW), and its smoothed versions, obtained by FSF, MA, FMF.

8. Conclusion

In this paper, a novel method is introduced based on fuzzy logic to smooth out signals. The proposed method utilizes a single fuzzy rule to share the values of neighboring sharp points. Both time domain and the frequency domain properties of the signal approve superiority of the method in comparison with some conventional and the fuzzy median filters. Application of the Fuzzy Smoothing Filter (FSF) has led to much better results when applying the smoothed signals into modeling processes, two of which are presented in this paper. This method can be generalized into two-dimensional domain, more suitable for image processing applications; this is under investigation and will be reported in near future.

9. Acknowledgement

The authors gratefully acknowledge contributions of the staff of the Shahed University, who have supported financially this research.

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increases, although it improves noise annihilation at high frequencies, the pass-band is decreased.

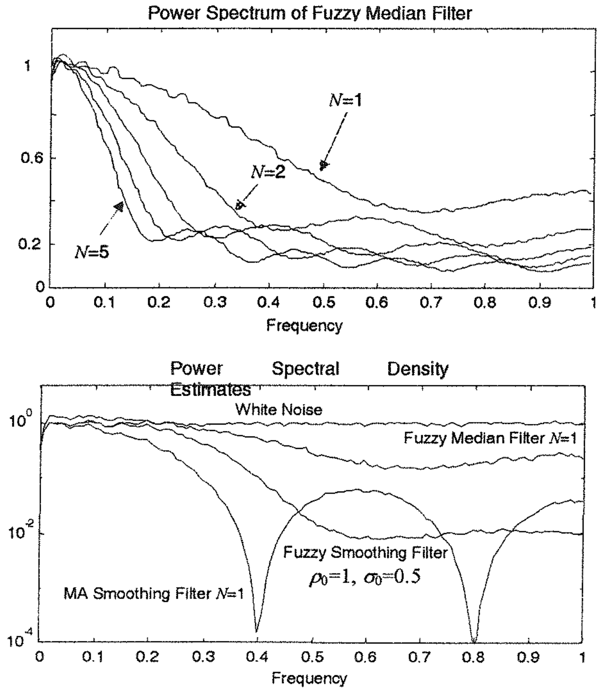


Figure 12: Power Spectrum Estimates of Fuzzy Smoothing filter (FSF) compared to the Fuzzy Median Filter (FMF).

7. APPLICATIONS

The proposed method is applied to smooth out economic signals in modeling economic dynamic systems. Generally, economic data contain uncertainty for both ambiguity in definitions of the economic concepts and measurement inaccuracy. Such uncertainties often cause rapid changes in the signal while a smooth one is expected. Particularly, when derivatives of data signals are used to model socio-economic systems, fluctuating signals are often illogical and undesired, and therefore presence of such signals impedes proper model making. As a fact, smoothing filters, if they have good performance in both frequency and time domains, can help in such problems.

For example, the wealth is usually estimated by the ratio of the whole liquidity to the general prices index: Wealth = Total Liquidity / General Prices. Obviously, variations of a national wealth might not be fast, unless a very destructive event, such as a war, exterminates a considerable portion of a country's economy. The wealth estimate in Iran is depicted in Figure 13, along with smoothed signals obtained through the MA, FSF and FMF filters. The fuzzy smoothed signal by FSF is used to construct macroeconomic model of consumption in Iran [11]. It is not so difficult to observe from the figure that the FSF has outperformed the others by preserving much more information of the original signal. The Euclidean norms of the deviations of the original signal from the smoothed signals are given in Table 1. Although sharpness of the MA filter output is the least, it has lost information at most of the sample points.

Table 1: Comparison of the Wealth signal with its smoothed versions, Sharpness and the Euclidean Difference with the original signal.

	Original	MA	FSF	FMF
Sharpness	0.301	0.059	0.102	0.196
Difference	0	0.585	0.299	0.406

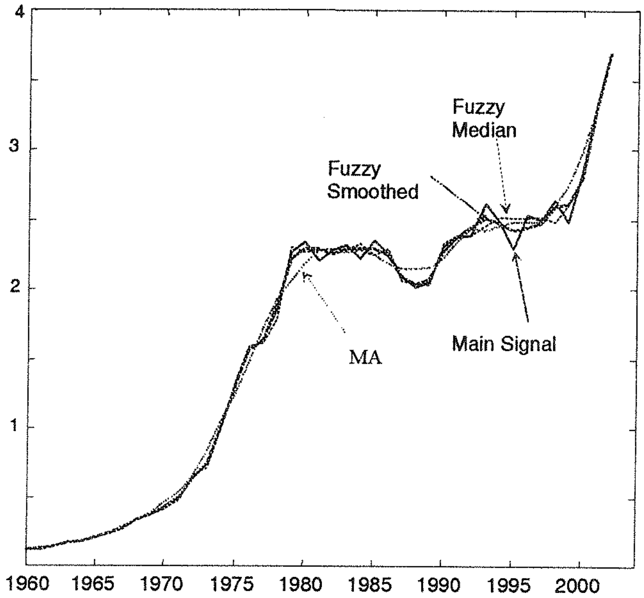


Figure 13: Wealth estimate [Billion Rials/Fixed Prices of 1990] in Iran and its smoothed versions, obtained by FSF, MA, FMF.

As another application, we take an electric power demand signal in Iran. It represents the maximum simultaneous annual demands including blackouts and frequency drops. Since the whole demand for goods in a country is a subjective concept depending on the culture and many other rigid nation lifestyles, it cannot have abrupt changes from one year to the next year. Uncertainties, especially raised by measurement errors, have caused this signal to change rapidly as depicted in Figure 14. By applying the FSF to this signal, we obtained a reasonably varying signal used to model electricity demand in Iran [12]. Table 2 confirms that the proposed filter, FSF, leads to less deviation from the original data in the sense of rms, while it offers lower sharpness index.

Table 2: Comparison of the Wealth signal with its smoothed versions, Sharpness index and the Deviation from the original signal.

	Original	MA	FSF	FMF
Sharpness	0.586	0.247	0.157	0.437
Difference	0	1.211	1.002	1.477

unchanged, with the same kind of oscillations. See, for example, sample points of 1-3, 5-7 and 9-12, which show a descending trend, and sample points 3-5 and 7-9 are ascending, where the MA filter resulted in variations mostly in diverse direction. Monte-Carlo simulations can show that due to the inherent properties of the proposed method, its capability to diminish average sharpness of typical signals is much higher than the other smoothing filters. This will be illustrated more in next sections.

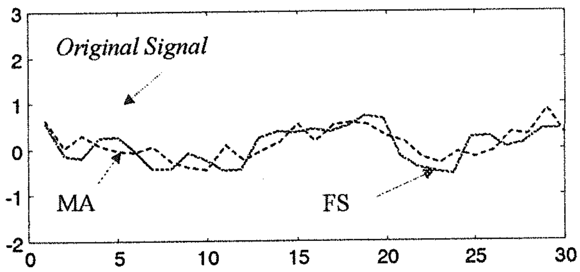


Figure 9: Fuzzy Smoothing filter (FSF) with σ_0=1 and ρ_0=2, in comparison to the Moving Average with M=N=1.

These advantages of the proposed fuzzy filter, FSF, in the time domain are briefly listed bellow:

1) Data processing is performed for only a subset of the original data,
2) Information stored in the original data are mostly preserved and less difference is obtained,
3) Smoothness index of the output signal is higher.

6. Spectral Analysis

The spectral properties of the fuzzy filtered signal are much better in comparison with the MA smoothing filter in (22). In this section, the output power spectrums of both filters are estimated by Fourier transformation. The frequency response of MA filter is given by [10]:

$$P_{MA}(f)=\mathcal{F}\{S_M\{y(t)\}\}=\frac{\sin^2(\beta\pi f)}{(\beta\sin\pi f)^2} \tag{23}$$

where β=M+N+1 denotes the window width and f is the normalized discrete frequency .

Usually, calculation of the power spectrum (frequency response) of a nonlinear filter is not easy, or even may be impossible. Knowing the fact that it is impossible to calculate the frequency response of the fuzzy filter, a Monte-Carlo simulation with thousands of bandwidth white noise signals is designed in order to estimate its power spectrum. Applying the smoothing method on these signals, we obtained a sigmoid type function for the estimated power spectrum as:

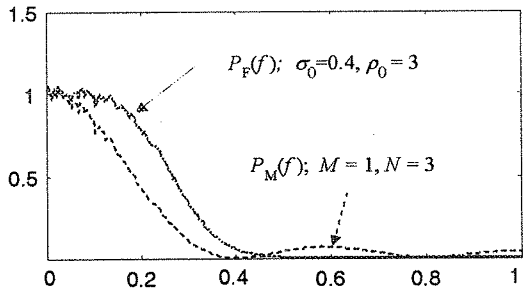
$$P_{FS}(f)=\mathcal{F}\{S_F\{y(t)\}\}=\frac{1+e^{-\mu\alpha}}{1+e^{\alpha(f-\mu)}} \tag{24}$$

for which the parameters α and μ depend on the parameters σ_0 and ρ_0. To find the exact relationship among these parameters, more elaborations are needed. Figure 10 shows the estimated power spectrums with sigmoid shape, and Figure 11 verifies the above claim by exhibiting the reasonable dependence between the estimated bandwidth and σ_0.

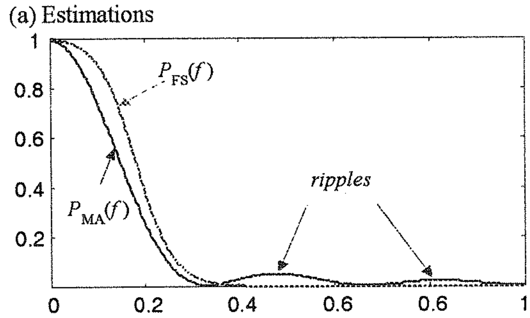
A rough linear relationship (25) can be estimated through the regression analysis though we do need much more accurate spectrum estimates to find a better relationship between parameters; this is currently under investigation and will be reported later.

$$f_b=0.185+0.189\,\sigma_0;\quad 0.1<\sigma_0<1 \tag{25}$$

where f_b is the discrete pass-band frequency.



(a) Estimations



(b) Ideal Shapes

Figure 10: Power Spectrum (Magnitude) of the Fuzzy Smoothing Filter (FSF) compared to the Moving Average (MA) filter.

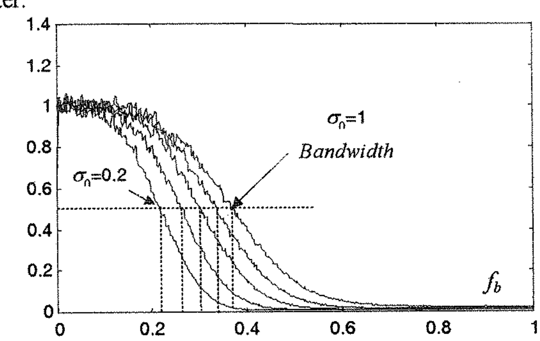


Figure 11: Changes in the Bandwidth of the FSF with σ_0

Below, we summarize our findings related to the power spectrum of the proposed fuzzy filter (FSF):

1) it is much wider at low frequencies,
2) it rapidly falls with a steeper slope,
3) it has a quite flat zero response at high frequencies, where the MA filter exhibits some ripples.

It is evident from (23) that increasing the size of window do not help to resolve the ripples problem of the MA filter. Similar disadvantages even worse are obtained for the Fuzzy Median Filter when compared to the FSF. For the FMF, Figure 12 reveals that as the window width

$$y(t) = S_N\{x(t)\} = \frac{\sum_{k=-M}^{N} F[x(t+k)]x(t+k)}{\sum_{k=-M}^{N} F[x(t+k)]} \tag{13}$$

where $F[.]$ is a nonlinear weighting function computed within a given window of the original signal, from $t\text{-}M$ to $t+N$. For a Moving Average (MA) filter, $F[.]$ is defined as:

$$F_{\mathrm{MA}}[x(t+k)] = \begin{cases} 1; & -M < t < N \\ 0; & otherwise \end{cases} \tag{14}$$

For a Median Filter it becomes:

$$F_{\mathrm{MD}}[x(t+k)] = \begin{cases} 1; & x(t+k) = x_m \\ 0; & otherwise \end{cases} \tag{15}$$

where x_m is the median of $x(t)$ within the window. In a Fuzzy Median Filter (FMF) this function may be modified to [9][13]:

$$F_{\mathrm{FMD}}[x(t+k)] = \begin{cases} 1 - \dfrac{|x(t+k) - x_m|}{x_{mm}}; & |x(t+k) - x_m| < x_{mm} \\ 0; & otherwise \end{cases} \tag{16}$$

where x_{mm} is defined by:

$$x_{mm} = max(|max(x) - x_m|, |min(x) - x_m|) \tag{17}$$

In this paper, for the sake of brevity and similarity of the results, other types of the Fuzzy Median Filter and Fuzzy Mean Filters are skipped.

Furthermore, by comparing (7) with (13), one can conclude that $F[.]$ in the proposed FSF method, is in fact:

$$F_{\mathrm{FS}}[x(t+k)] = \begin{cases} 1; & k \in \{-1,1\} \\ \rho_t; & k = 0 \end{cases} \tag{18}$$

As mentioned earlier, in this formulation both of ρ_{t-1} and ρ_{t+1} are assumed to be one. However, in general we have to calculate the weightings for each point individually as:

$$F_{\mathrm{FS}}[x(t+k)] = \rho_{t+k}\,; \; k = -M, \ldots, N \tag{19}$$

where ρ_{t+k} should be determined by some properly developed fuzzy rule. In this paper, we only study the simple case of $N=M=1$. Next section will compare performance of this simplified version with other types of both conventional and fuzzy filters.

It is evident that for a generalized case, (8) and (9) should be reformulated as:

$$y(t+k) = x(t+k) + [x(t) - y(t)]/(M+N) \tag{20}$$

for $k = -M, \ldots, N$; $k \neq 0$.

5. Simplifications and Comparisons

After a vast number of simulation tests, it has been observed that a constant exclusion index is good enough to escape the calculations of the "then" part of (10). Therefore, in the simplified method, ρ_k is obtained by:

$$\rho_k = \begin{cases} \rho_0\,; & if \;\; s_k > \sigma_0 \\ \infty\,; & otherwise \end{cases} \tag{21}$$

By setting the constant ρ_0 to 2 or 3, a minor improvement in the smoothness index has been gained,(see Figure 8) The trajectory of ρ_t, which varies for each sample point at all iterations, is also depicted in this figure for the first iteration of a case in which the term sets have the membership functions given in Figure 4 and Figure 6 (dashed one). It is clear from Figure 8 (b) that even maximum values of ρ_t do not exceed 2.25, and therefore fixing its value on 2 will not affect results of smoothing so much. Comparison of the two solid and dashed lines in Figure 8 (a) verifies this claim. Samples for which $\rho_t = 0$ are not processed for their satisfying smoothness.

By such simplifications, we have obtained a simple filter without loosing any valuable advantage. In fact, although the smoothness index of a few points is degraded, the average sharpness is reduced 2 percents, though it is not assured for all cases. Clearly, such a simple filter may be compared to the well-known Moving Average (MA) smoothing filter defined by (14); this may be rewritten as:

$$y(t) = S_{\mathrm{M}}\{x(t)\} = \frac{1}{M+N+1}\sum_{k=-M}^{N} x(t+k) \tag{22}$$

which is an especial case of a general nonlinear filter.

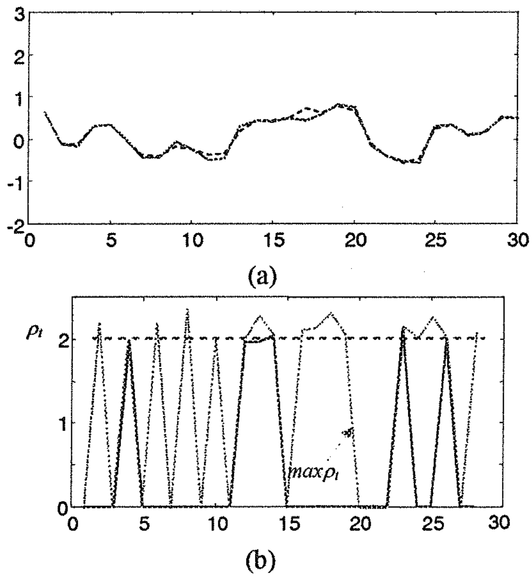


Figure 8: (a) Result of smoothing the random signal in Figure 1, applying (21) into (7)-(9) (simplified method) with σ_0=1 and ρ_0=2 (solid line), ρ_0 =3 (dashed line); (b) ρ_t obtained by the main algorithm (first iteration: solid line, *max* ρ_t: dashed line).

In the sequel of this section, we present smoothness property of the filters in the time domain. Figure 9 easily shows superiority of the fuzzy filter. By generating some unwanted unsmooth points, that the conventional MA filter has caused, it has deteriorated some information of the original signal. While the fuzzy filter, which processes merely the sharp points, has kept the original shape of data the smoothed point is equal to the original data. Actually, the exclusion coefficients for the samples k-1 and k+1 are always assumed to be one. However, to keep overall integral of the curve constant, data values at t_{k-1} and t_{k+1} should be modified as:

$$y(t_{k-1}) = x(t_{k-1}) + [x(t_k) - y(t_k)]/2 \quad (8)$$

$$y(t_{k+1}) = x(t_{k+1}) + [x(t_k) - y(t_k)]/2 \quad (9)$$

This correction is based on the presumption that the additional noise has a mean of zero.

Simply, one may fuzzify the exclusion concept by a membership function as given in Figure 4 for the linguistic variable *not exclusive*.

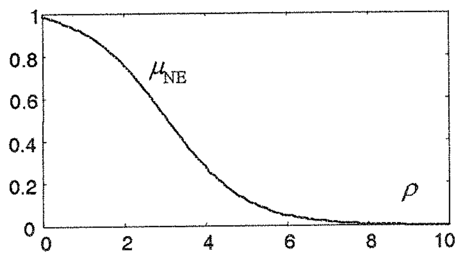


Figure 4: Membership functions defined for the linguistic variable of *not exclusive*.

Now, we can apply the single fuzzy rule given in (2); This is rewritten as the following "if"-"then" statement:

If the k^{th} point of $x(t)$ is in a *very sharp* point,
Then its value in $y(t)$ will be *not exclusive*. (10)

Equally, this rule is implied by a fuzzy relation as:

$$R_1 : s_k \tilde{\in} \mathrm{VS} \to \rho_k \tilde{\in} \mathrm{NE}\ ;\ k = 2, \ldots, n\text{-}1. \quad (11)$$

where $\tilde{\in}$ means inclusion in a fuzzy set. Figure 5 shows how the method removes sharp points of the random data given in Figure 1. Although the average sharpness, which is calculated by averaging (3) over all samples, is reduced to 0.51, some other sharp points are still generated because of (8) and (9). To verify this point bette , look at the sample points 3, 7, 11 and 14 up to 16. To resolve this, the proposed method is modified bellow applying two techniques.

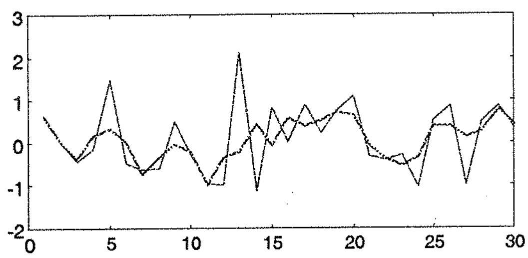


Figure 5: Result of applying (11) to smooth the random signal in Figure 1.

3. The Modified FSF

Two modifications are proposed and tested in this section. First, we apply (11) to the points which have a sharpness of more than an acceptable value, say σ_0=1. This means that $\mu_{VS}(.)$ in Figure 3 should be modified to the one plotted in Figure 6, as an example. In this way, many non-sharp points will be isolated, without being processed. Thus, (11) is replaced by:

$$R_1 : s_k \tilde{\in} \mathrm{VS} \to \rho_k \tilde{\in} \mathrm{NE}\ ;\ \{k \mid s_k > \sigma_0\} \quad (12)$$

This modification discards a considerable number of data points from the process. Nevertheless, many of those points are affected by their neighboring points through (8) and (9). Such a modification may be used to the edge detection problem too, which will not be discussed in this paper.

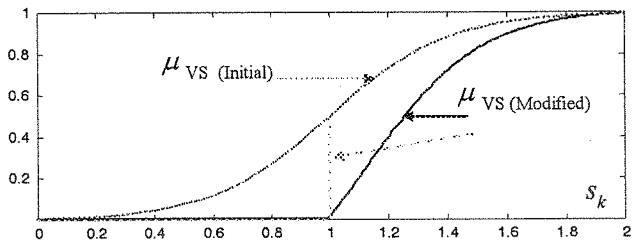


Figure 6: Modified memberships for *very sharp* (compared to Figure 3).

The second modification is to iterate the process until there is no points with sharpness index more than σ_0. The two modifications are applied to the same random signal, and the resulting smoothed signal is depicted in Figure 7. The average sharpness is reduced to 0.418.

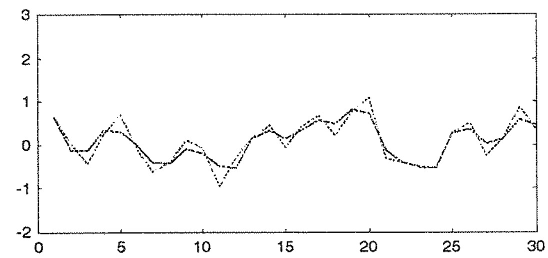


Figure 7: Result of applying (12) to smooth the random signal in Figure 1, assuming σ_0=1; without iteration (dashed line) compared to 5 iterations (solid line).

It should be emphasized that convergence of the iterated process can be approved experimentally, even for long random signals of more than 1000 samples within less than 10 iterations; this intuitively is perceptible too. To prove this claim in a mathematical sense, more elaborations are it required and is may be considered as an open problem.

4. Generalization

Both linear and nonlinear (including fuzzy based) smoothing digital filters may be in general formulated by [10]:

expressed by:

$$y(t) = S_F\{x(t)\} \tag{1}$$

while, the process of $S_F\{.\}$ simply represents the following single rule:

If sharpness of $x(t)$ at point t is high,
then share its value with its neighboring points at t-1 and t+1. (2)

In the "if" part of the rule a distinction between sharp and non-sharp points is recognized. The "then" part should determine how to associate the value of the current point with its neighbors.

The first step to obtain sharpness of each point is to calculate the angle of the discrete signal at that point. Although it is easy to determine the angle, there is a matter that the angle is dependent on the time axis scale. It can be seen for a sample signal in Figure 1 how a change in scale will change the angle at a certain point of the curve.

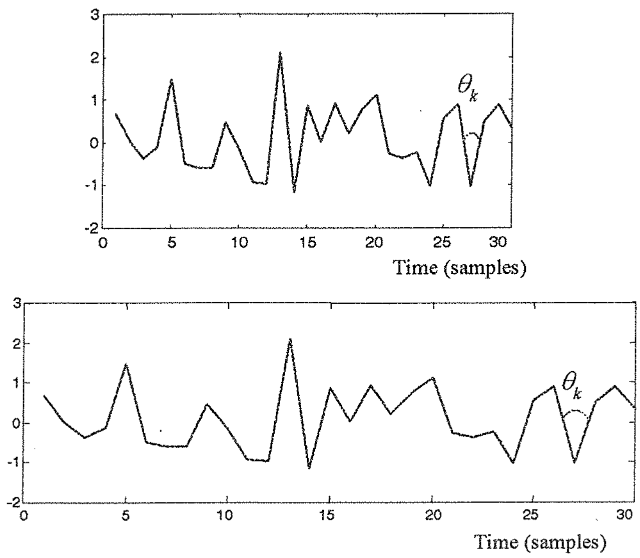


Figure 1: A random discrete signal sketched in two different time scales.

The same problem arises when a change in the vertical axis is considered. To resolve this problem, it is necessary to divide increments of $x(t)$ by an appropriate coefficient which should be proportional to the mean of absolute changes in $x(t)$, if the time variable t is distributed uniformly. This is in fact a normalization pre-processing step, which makes the angles independent of the axes scales. Another point is that the absolute angle value should be obtained and the sharpness must be determined regardless of its sign. Summarizing these concerns leads to the following formulation:

$$s_k = \cos(\theta_k) + 1 \tag{3}$$

$$\theta_k = \tan^{-1}\left|\frac{\gamma(t_k - t_{k-1})}{x_k - x_{k-1}}\right| + \tan^{-1}\left|\frac{\gamma(t_{k+1} - t_k)}{x_{k+1} - x_k}\right| \tag{4}$$

$$\gamma = (\text{constant})\ E\{|\Delta x|\} \tag{5}$$

where γ is the normalizing coefficient and sk represents the sharpness index which varies in the interval [0, 2]. The constant coefficient in (5) is an adjustment parameter, which will be replaced by one hereafter. Figure 2 shows the sharpness indexes calculated for each points of the random signal as plotted in Figure 1. The signal has an average sharpness of 0.622.

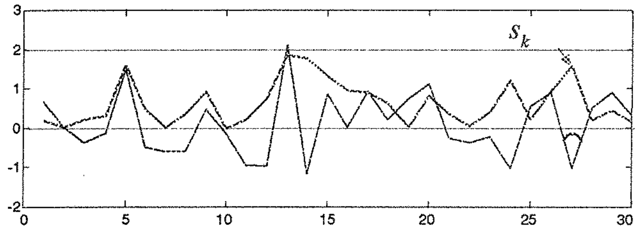


Figure 2: The sharpness index of the random discrete curve of Figure 1 computed by (3) to (5).

At the next step, the points should be categorized according to their sharpness. A simple set of membership functions like the exponential functions in Figure 3 can be defined for three categories of *non-sharp*, *medium* and *very sharp*. Therefore, points having an angle about 0° to 45° are considered as very sharp with a membership degree of more than 0.9. In addition, points with associated angles about 180° are smooth (*non-sharp*) points. Finally, with angles equal to 90° the points are categorized as *medium sharp* point. Anyway, as far as the single fuzzy rule of (2) is concerned, we will merely use $\mu_{VS}(.)$ and for more simplicity, the two other functions have no use in this method.

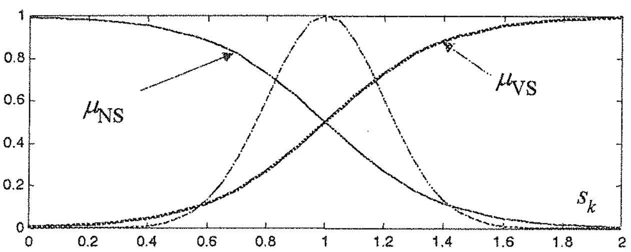


Figure 3: Membership functions defined for the linguistic variables of *not*, *medium* and *very sharp*.

Continuing with the "then" part, the *association* concept, which determines the share of each point at each step, is realized by:

$$y(t_k) = \eta\, x(t_{k-1}) + (1 - 2\eta)x(t_k) + \eta\, x(t_{k+1}) \tag{6}$$

where the coefficient η measures the amount of neighboring points share. It is evident that (6) actually is a weighted mean. To shorten calculations, the above equation is replaced by:

$$y(t_k) = [x(t_{k-1}) + \rho_k\, x(t_k) + x(t_{k+1})]/(2 + \rho_k) \tag{7}$$

whereas ρ_k has a reverse meaning of *monopoly*, which may be named *exclusion*. This parameter should be obtained for each sample point t_k in the interval $[0, \infty)$. As ρ_k tends to zero we have full association, that is, the value of $x(t_k)$ is totally shared with both $x(t_{k-1})$ and $x(t_{k+1})$. On the other hand, $\rho_k \to \infty$ means complete exclusion, for which

Introduction to an Iterative Weighted Mean Smoothing Filter Based on a Simple Fuzzy Rule

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ABSTRACT

Availability of reliable data is an essential part of any scientific investigation. Many of data records contain inaccurate information, not only due to measurement errors but also for ambiguity of the measuring concept. This occurs especially in human science fields, where instead of softly varying signals sharp signals are observed. Since the conventional smoothing methods do not make sense when applying to such fields, fuzzy smoothing methods are preferred. This paper proposes a novel smoothing procedure based on a single fuzzy rule for smoothing out sharpness of data curves, and analyzes the method focusing on power spectrums. The proposed method smoothes out sharp points found in the signal by sharing their value with the neighboring points. A survey of both time domain and frequency domain performances shows superiority of the proposed method compared to the other classical smoothing methods cited in the literature. Some applications are also introduced to highlight better the merit of the proposed method.

KEYWORDS

Signal Processing, Pre-filtering, Smoothing Filter, Nonlinear Digital Filter, Fuzzy Logic.

1. INTRODUCTION

Fuzzy filters are in fact a subset of nonlinear filters widely used during the last two decades in various fields of science and technology [1]-[7]. There are some methods have been developed for smoothing problems. Image processing is on the top of the list of smoothing methods, mostly affected by fuzzy logic [4]-[7]. Fuzzy image enhancement, fuzzy edge detection and impulse noise annihilation using fuzzy filtering have attracted many research as well [6]-[8]. In addition, in speech analysis fuzzy filters played crucial role [1]-[3].

Some other various applications of fuzzy smoothing can be found in other fields of science and technology [11]-[14]. One may refer to fuzzy traffic smoothing in network communications [2], language processing [15] and even bioinformatics [16]. However, the present approach in nature considers a different aspect of the general goal of smoothing. For example, edge detection here is not as important as it is in the field of image processing methods. Nevertheless, such goals may be achieved as well.

This paper proposes a novel direct simple method of signal smoothing. In this method, sharpness of the signal at each point is measured and compared to the average desired sharpness by a linguistic variable. At the next step, very sharp points are distinguished and their values are shared with the two (or more) points in their neighborhood. In the last step, we modify the values of the neighboring points in order to keep the whole integral of the curve equal to its initial value. The above brief implication will be explained in details through next sections. First, we introduce our fuzzy filter. After some modifications and generalizations in the third and forth sections, sections 5 and 6 compares the method with the other known conventional and fuzzy filters, in both time and frequency domains. In section 7, two applications of the proposed method in the field of human science (economics and energy economics) are presented. Finally, section 8 concludes the paper.

2. THE FUZZY SMOOTHING FILTER (FSF)

Denoting the original signal as $x(t)$ and its smoothed version as $y(t)$, the fuzzy smoothing process is then

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