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The Buckling Analytical of Conical Sandwich Shells with Temperature Dependent and an Improved High-Order Theory

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ABSTRACT: In this research paper, an improved theory is used for buckling analysis of sandwich truncated conical shells with thick core and thin functionally graded material face sheets and homogeny core and with temperature-dependent properties. Section displacements of the conical core are assumed by cubic functions, and displacements of the functionally graded material face sheets are assumed by first-order shear displacements theory. The linear variations of temperature are assumed in the through thick. According to a power-law and exponential distribution the volume fractions of the constituents of the functionally graded material face sheets are assumed to be temp-dependent by a third-order and vary continuously through the thickness. In other words to get the strain components, the nonlinear Von-Karman method and his relation is used. The equilibrium equations are obtained via minimum potential energy method. Analytical solution for simply supported sandwich conical shells under axial compressive loads and thermal conditions is used by Galerkin's solution method. Analysing the results show that the critical dimensionless axial loads are affected by the configurations of the constituent materials, compositional profile variations, thermal condition, semi-vertex angle and the variation of the sandwich geometry. Numerical modeling is made by ABAQUS finite element software. The comparisons show that the present results are in the good and better agreement with the results in the literature and the present finite element modelling.

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1- Introduction

The buckling analysis of cylindrical and conical shells subject to types of loading is of current interest to engineers engaged in engineering practice. It is of great technical importance to clarify the buckling analysis behaviour of conical shells under axial compression [1]. A number of studies have been made of the buckling analyses of composite conical shells under combined external pressure and axial compression loading cases [2]. Sandwich structures find an increasing use in aerospace, transportation, and other industries, which require a lightweight structural component. Several theoretical models have been developed in the recent years to this background. In these theories assume that the height of the core remains unchanged. Classical theories can be found written by Plantemma [3]. Classical theories can often accurately determine the response of the conical sandwich structure, for example, buckling load and fundamental vibration frequencies. Modern conical sandwich structures are usually made of two metallic or composite face sheets and low strength honeycomb or soft core. These properties denoted as localized effects cannot be accurately determined using classical theories. In other words take into account the compressibility of the core, an improved sandwich theory has been developed by Frostige [4]. This theory a beam, plate, or sandwich shell model is used for the face sheets and soft core is considered as elastic medium that has shear and vertical stiffness only, while its longitudinal stress and strain are neglected [5]. Improved theory has successfully been used to analyse various problems of conical sandwich structures that include vibration and static problems well [6]. Nowadays, there are three approaches that are presented to analyse the mechanical behaviour of conical sandwich structures and to predict their static and dynamic responses, That is: three-dimensional elasticity approaches, Equivalent Single Layer (ESL) theories and layer-wise theories [7]. There are a few exact three-dimensional elasticity solutions for static and dynamic analysis of the composite truncated conical sandwich plates. Noor et al. [8] presented the three-dimensional elasticity solution for global buckling of simply supported conical sandwich panels with composite face sheets and soft core. Jie and Wass [9] studied the elastic stability of a conical sandwich panel using two-dimensional classical elasticity theories. They determined buckling and wrinkling load of two-dimensional cylindrical and conical sandwich panels. Some authors analysed the buckling behaviour of cylindrical and conical sandwich plates using ESL theories [10]. Kant et al. [11] presented the analytical solution for buckling of conical sandwich plates using a high-order theory. Nayak et al. [12] studied the global buckling analysis of sandwich shell using improve third-order equivalent single plate theory and Finite Element Model (FEM). Bushnell et al. [13] have carried out experimental and numerical to analyse thermal buckling of conical sandwich shells. As well as, a layer-wise model was presented by Dafedar et al. [14] for global buckling analysis of multi core sandwich cylindrical and conical plates. They assumed the high order polynomial functions for all displacement components in all layers. Also, they proposed a simplified model and calculated critical buckling loads based on the geometric and stiffness matrix concept. The abrupt variation in laminated composites between two neighbouring layers can exit delamination. Functionally Graded Materials (FGMs) composites are a particular kind of layers composite

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materials made up of two or more materials where the mechanical properties as elasticity and thermal factor vary continuously in one or more directions. As a consequence, such detrimental effects as delaminating can be drive out.

Today, functionally graded materials have been used for their excellent mechanical and well thermal properties. FGMs materials are high performing and best thermal resistant, capable of withstanding ultrahigh temperatures and extremely large thermal and cold gradients present in nuclear applications. FGMs are inhomogeneous composite materials in microscopically. FGM was first proposed in 1984 by a group of materials scientists in Sendai Japan, as novel materials with thermal barrier or thermal shielding properties. Research activities have been accelerated within Japan in the recent past [15]. Also, FGMs are usually made from a mixture of metals and ceramics materials through a powder metallurgy process. The advantage of using these materials is that they are able to withstand high temperature gradient environment while maintaining their structural integrity properties. For example, the insulating tile for a re-entry space vehicle can be designed such that the outside is a refractory ceramic material, and the inside a load-carrying structure made of a strong and tough metal [16]. Other advantages of FGMs are reduction of stresses, an improved residual stress, enhanced thermal, higher fracture toughness, and reduced stress intensity factors. The composition is varied from a ceramic surface to a metal surface with a desired variation of the volume fraction of the two materials in between in two layers or surfaces [17]. The first FGMs were designed as thermal materials for aerospace application and fusion nuclear reactors. Afterwards, FGMs were developed for automotive industry, biomedical application, military and general structural element in high thermal environments conditions. Sandwich truncated conical shells are used in various engineering backgrounds such as nuclear reactors, vessel heads, component of missiles, spacecrafts, and other civil, auto-mechanic industry and aerospace engineering applications. A sandwich truncated conical shell is one of the main components of the propulsion system structure in rockets and aeroplanes. More researchers have presented the stability of conical shells under external and internal loads. Singer studied a classical theory for the buckling load of a conical shell under uniform external and internal pressure load [18]. Singer had extended his research to study the effect of axial load constraint on the instability of thin conical sandwich shells under external and internal pressure load [19]. There are even many studies on stability problems of the FGMs conical sandwich shells and the layered, conical sandwich shells containing FGMs layer, There are extremely challenging analytical problems in addition to the other difficulties mentioned above [20]. Instability of truncated conical sandwich shells under thermal and axial loading was investigated by Lu et al. [21,22]. They investigated the parameter research on truncated conical sandwich shells for two cases of thermal distribution, temperature gradient along the generator of the cone thick and temperature changes circular and meridionally. Determine of the critical temperature and its relation to geometric parameters as thickness and reduce was presented. It was show that axial compression load or bending was primary cause for thermal buckling. Dumeir et al. [23] have described truncated conical sandwich shell caps for static and dynamic or free vibration buckling loads using FSDT and classical lamination solution. The problem of buckling of thin conical sandwich shells under uniform pressure has been the subject of considerable number researchers. Chronologically the first among them is a paper by Niordson [24] in which the buckling problem of conical sandwich shells subjected to uniform external and internal lateral pressure is solved by using Rayleigh-Ritz type variational method. Mushtari et al. [25] studied the buckling behaviour of cylindrical and conical sandwich shells under external and internal pressure load. Singer [26] also used the symmetric the buckling mode under axisymmetric external pressure. Singer [27] investigated the effect of axial constraint on the instability of thin conical sandwich shells under external pressure load. The effect of the four boundary condition on the buckling behaviour of a conical sandwich shell under uniform pressure load was studied by Thurston [28] and Baruch et al. [29]. Recently, Kheirikhah and khalili [30] investigated high-order theory for axial buckling analysis of sandwich Functionally Graded (FG) plates with orthotropic soft core. Seidi et al. [31-33] investigated an improver high-order theory for buckling analysis of sandwich truncated conical shell with thin FGMs face sheets and uniform soft core. The variations of temperature are assumed constant in FGMs face sheets and the soft core. The investigations of the buckling response of conical shells under combined loading cases are limited in number. Karpov [34] and A.H. Sofiye [35,36] studied the stability of conical sandwich shell under axial and pressure load. The best of the author's knowledge, there are no research in the open literature on buckling behaviour of truncated conical sandwich shells with FGMs face sheets. By the increased of these materials, it is important to knowledge the buckling process behaviour of FGMs sandwich structures subjected to different mechanical and thermal loads. The conversation of this study is to present the buckling process behaviour of a thin faces sandwich truncated conical panel FGMs shell subjected to uniform thermal and axial load in press. In this research, a high order theory and improve theory is presented for uniform and homogeneous axial and thermal buckling analysis of truncated conical sandwich shell with FGMs material faces sheets and homogenous soft conic core. FSDT is used for the face sheets and cubic function is assumed for the transverse and in-plane displacements of the soft and homogenous core. The linear variations of temperature are assumed in FG face sheets and the soft core. The Von-Karman type formulations are used to determine strains. Continuity transverse and shear stress condition at the interface as well as the condition of zero transverse shear stresses on the up and down surface of the conical shell are satisfied. Also, transverse flexibility and normal strain and stresses on the soft conic core are considered. The equation of boundary conditions are derived via principle of minimum potential energy formula. Analytical solution for static analysis of simply supported truncated conical sandwich shell plates under axial in-plane compressive loads is presented using Galerkin's solution.

2- Material Properties

It is shown in Fig. 1, a sandwich truncated conical shell with thin FGMs face sheets is considered under uniform axial loads. The coordinate system (S, θ, z) structure is referred to a curvilinear, where θ and S the circumferential direction on the reference surface and axes lie along the generator of the cone, respectively. And the z direction, being perpendicular to the plane of the first two axes. In other words in Fig. 1, R_1 and R_2 indicate reduce of the sandwich cone at its small and big ends, respectively, then β denotes the semi-vertex angle of the sandwich cone, and h_0 and h_i are the thicknesses of the outer and inner FGMs face conical sheets, respectively. The face sheets materials are assumed to be FGM and the soft core is assumed as homogeny material with thickness h_c. The variation radius of the cone changes between R_1 and R_2 as:

$$R(S) = R_1 + S.\sin(\beta) \quad (1)$$

It is assumed that the FGMs are made of a mixture of a metal and ceramic phase. Metal phase (denoted by " m") and ceramic phase (denoted by "ce"), with the material composition varying along its thickness direction (i.e. in the z-axis). Hence, the materials properties of FGMs C_{FGM}, same of young's elasticity modulus E_{FGM} , poisons' factor ϑ_{FGM}, density ρ_{FGM} and thermal expansion factor α_{FGM} , can be investigate as [35]:

$$C_{FGM} = C_{ce}V_{ce} + C_m V_m \quad (2)$$

where C_{ce} and C_m are properties of the ceramic and metal, respectively and expressed as a function of high order temperature as:

$$C = C_0(C_{-1}T^{-1} + 1 + C_1T + C_2T^2 + C_3T^3) \quad (3)$$

That T=300K (room temp), C_0, C_{-1}, C_1, C_2, C_3 are the factor of temperature T (K) remark in Kelvin and are constant for any constituent materials [36]. V_{ce} and V_m are the ceramic and metal volume fraction. Also, the properties of the FGMs sandwich face sheets $P(z_j,T)$ like as elasticity modulus, the Poisson's factor, thermal expansion factor and the power-law function density of FGMs sandwich face sheets Eq. (4), are introduced by [35]:

$$P(z_j,T) = g(z_j)P^j_{ce}(T) + [1-g(z_j)]P^j_m(T)\ ,\ j=(o,i)$$
$$g(z_o) = \left(\frac{h_o/2 - z_o}{h_o}\right)^N,\ g(z_i) = \left(\frac{h_i/2 + z_i}{h_i}\right)^N \quad (4)$$

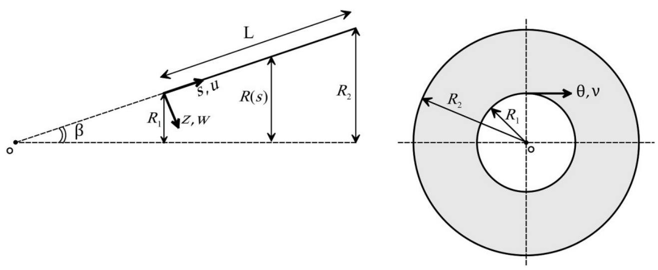

Fig. 1. A typical sandwich truncated conical shell

3- Temperature Distribution

Equations In this paper, for the thermal conditions, it is supposed that one value of the temperature is imposed on the outer surface of the sandwich conical shell and the other value on the inner surface of the sandwich, The temperature variations of the FGMs sandwich face sheets and the core are supposed to vary in the thickness direction only by a linear function as shown in Fig. 2 and Eqs. (5) to (7) [39].

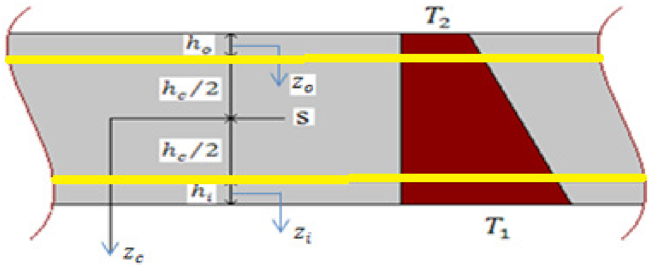

Fig. 2. Linear change of temperature in the thickness of the sandwich panel

$$T_o(z_o) = \frac{1}{h_c + h_o + h_i}\left[(T_1 - T_2)z_o + \frac{h_o}{2}(T_1 + T_2) + T_2(h_i + h_c)\right] \quad , \quad -\frac{h_o}{2} \le z_o \le \frac{h_o}{2} \quad (5)$$

$$T_c(z_c) = \frac{1}{h_c + h_o + h_i}\left[(T_1 - T_2)z_c + \frac{h_c}{2}(T_1 + T_2) + T_2h_i + T_1h_o\right] \quad , \quad -\frac{h_c}{2} \le z_c \le \frac{h_c}{2} \quad (6)$$

$$T_i(z_i) = \frac{1}{h_c + h_o + h_i}\left[(T_1 - T_2)z_i + \frac{h_i}{2}(T_1 + T_2) + T_1(h_o + h_c)\right] \quad , \quad -\frac{h_i}{2} \le z_i \le \frac{h_i}{2} \quad (7)$$

4- Mathematical Equations

4- 1- Kinematic formulations

In the proposed model for sandwich truncated conical shells, FSDT is used for the sandwich face sheets.

$$u_j(s,\theta,z_j) = u_{0j}(s,\theta) + z_j\phi^j_s(s,\theta)$$
$$v_j(s,\theta,z_j) = v_{0j}(s,\theta) + z_j\phi^j_\theta(s,\theta) \quad ,j=(o,i) \quad (8)$$
$$w_j(s,\theta,z_j) = w_{0j}(s,\theta)$$

where u_0, v_0 and w_0 are displacements of mid surface along the S, θ and z direction and φ is rotations of the mid surface in length of S and θ axis. The core of conic is thicker and softer than the sandwich face sheets. Hence the displacement fields for the conic core are supposed as a cubic plan for the in-plane and vertical movement components:

$$u_c(s,\theta,z_c) = u_0(s,\theta) + u_1(s,\theta)z_c + u_2(s,\theta)z_c^2 + u_3(s,\theta)z_c^3$$
$$v_c(s,\theta,z_c) = v_0(s,\theta) + v_1(s,\theta)z_c + v_2(s,\theta)z_c^2 + v_3(s,\theta)z_c^3 \quad (9)$$
$$w_c(s,\theta,z_c) = w_0(s,\theta) + w_1(s,\theta)z_c + w_2(s,\theta)z_c^2 + w_3(s,\theta)z_c^3$$

where u_k and v_k (k=0- 1- 2- 3) are the unclear of the in-plane movement components of the sandwich core and w_k (k=0- 1- 2- 3) are the unclear of its vertical movement, respectively. In this assumption there are twenty eight unclears: ten movements unclears for two face sheets, twelve movement unknowns for the conic core, and six Lagrange coefficients.

4- 2- Compatibility conditions

In the propose model, the core is perfectly limited to the face sheets. Thus, there are three interface movement in both face sheet-core interface which can be get as follows:

$$
\begin{aligned}
&u_o\left(z_o=h_o/2\right)=u_c\left(z_c=-h_c/2\right),\\
&v_o\left(z_o=h_o/2\right)=v_c\left(z_c=-h_c/2\right),\\
&w_o=w_c\left(z_c=-h_c/2\right),\\
&u_c\left(z_c=h_c/2\right)=u_i\left(z_i=-h_i/2\right),\\
&v_c\left(z_c=h_c/2\right)=v_i\left(z_i=-h_i/2\right),\\
&w_c\left(z_c=h_c/2\right)=w_i
\end{aligned}
\tag{10}
$$

4- 3- Strain components

The nonlinear Von-Karman strain-movement formulations for the face sheets (j=o,i) can be determined Eq. (11) as:

$$
\begin{aligned}
&\varepsilon_{ss}^{j}=u_{0j,s}+z_j\varphi_{s,s}^{j}+\frac{1}{2}w_{j,s}{}^{2}-\alpha_j\Delta T_j\\
&\varepsilon_{\theta\theta}^{j}=\frac{1}{r}\left(v_{0j,\theta}+z_j\varphi_{\theta,\theta}^{j}+u_{0j}\sin\beta+z_j\varphi_s^{j}\sin\beta+w_j\cos\beta\right)\\
&+\frac{1}{2r^2}w_{j,\theta}{}^{2}-\alpha_j\Delta T_j\\
&\varepsilon_{zz}^{j}=-\alpha_j\Delta T_j \qquad j=(o,i)\\
&\gamma_{s\theta}^{j}=\frac{1}{r}\left(u_{0j,\theta}+z_j\varphi_{s,\theta}^{j}-v_{0j}\sin\beta-z_j\varphi_\theta^{j}\sin\beta\right)\\
&+v_{0j,s}+z_j\varphi_{\theta,s}^{j}+\frac{1}{r}w_{j,s}w_{j,\theta}\\
&\gamma_{sz}^{j}=\varphi_s^{j}+w_{j,s},\\
&\gamma_{\theta z}^{j}=\frac{1}{r}\left(w_{j,\theta}-v_{0j}\cos\beta-z_j\varphi_\theta^{j}\cos\beta\right)+\varphi_\theta^{j}
\end{aligned}
\tag{11}
$$

The nonlinear Von-Karman strain-movement formulation for the sandwich core can be defined as:

$$
\begin{aligned}
&\varepsilon_{ss}^{c}=u_{0,s}+u_{1,s}z_c+u_{2,s}z_c{}^{2}+u_{3,s}z_c{}^{3}+\frac{1}{2}w_{0,s}{}^{2}-\alpha_c\Delta T_c\\
&\varepsilon_{\theta\theta}^{c}=\frac{1}{r}\Big[v_{0,\theta}+v_{1,\theta}z_c+v_{2,\theta}z_c{}^{2}+v_{3,\theta}z_c{}^{3}+\left(u_0+u_1z_c+u_2z_c{}^{2}+u_3z_c{}^{3}\right)\sin\beta\\
&+\left(w_0+w_1z_c+w_2z_c{}^{2}+w_3z_c{}^{3}\right)\cos\beta\Big]+\frac{1}{2r^2}w_{0,\theta}{}^{2}-\alpha_c\Delta T_c\\
&\varepsilon_{zz}^{c}=w_1+2w_2z_c+3w_3z_c{}^{2}-\alpha_c\Delta T_c,\\
&\varepsilon_{zz}^{c}=w_1+2w_2z_c+3w_3z_c{}^{2}-\alpha_c\Delta T_c,\\
&\gamma_{\theta z}^{c}=\frac{1}{r}\begin{bmatrix}w_{0,\theta}+w_{1,\theta}z_c+w_{2,\theta}z_c{}^{2}+w_{3,\theta}z_c{}^{3}\\-\left(v_0+v_1z_c+v_2z_c{}^{2}+v_3z_c{}^{3}\right)\cos\beta\end{bmatrix}+v_1+2v_2z_c+3v_3z_c{}^{2}\\
&\gamma_{s\theta}^{c}=\frac{1}{r}\begin{bmatrix}u_{0,\theta}+u_{1,\theta}z_c+u_{2,\theta}z_c{}^{2}+u_{3,\theta}z_c{}^{3}\\-\left(v_0+v_1z_c+v_2z_c{}^{2}+v_3z_c{}^{3}\right)\sin\beta+w_{0,s}w_{0,\theta}\end{bmatrix}\\
&+v_{0,s}+v_{1,s}z_c+v_{2,s}z_c{}^{2}+v_{3,s}z_c{}^{3}
\end{aligned}
\tag{12}
$$

4- 4- The stress equations

For definition of stress resultants for each FGMs face sheet (j=o,i), main equations are as follows:

$$
\begin{aligned}
&\begin{Bmatrix}N_{ss}^{j}\\N_{\theta\theta}^{j}\\N_{s\theta}^{j}\end{Bmatrix}=\begin{bmatrix}A_{11}&A_{12}&0\\A_{12}&A_{22}&0\\0&0&A_{66}\end{bmatrix}\begin{Bmatrix}\varepsilon_{ssj}^{(0)}\\\varepsilon_{\theta\theta j}^{(0)}\\\varepsilon_{s\theta j}^{(0)}\end{Bmatrix}\\
&+\begin{bmatrix}B_{11}&B_{12}&0\\B_{12}&B_{22}&0\\0&0&B_{66}\end{bmatrix}\begin{Bmatrix}\varepsilon_{ssj}^{(1)}\\\varepsilon_{\theta\theta j}^{(1)}\\\varepsilon_{s\theta j}^{(1)}\end{Bmatrix}-\begin{Bmatrix}N_{ss}^{Tj}\\N_{\theta\theta}^{Tj}\\0\end{Bmatrix}\\
&\begin{Bmatrix}M_{ss}^{j}\\M_{\theta\theta}^{j}\\M_{s\theta}^{j}\end{Bmatrix}=\begin{bmatrix}B_{11}&B_{12}&0\\B_{12}&B_{22}&0\\0&0&B_{66}\end{bmatrix}\begin{Bmatrix}\varepsilon_{ssj}^{(0)}\\\varepsilon_{\theta\theta j}^{(0)}\\\varepsilon_{s\theta j}^{(0)}\end{Bmatrix}\\
&+\begin{bmatrix}D_{11}&D_{12}&0\\D_{12}&D_{22}&0\\0&0&D_{66}\end{bmatrix}\begin{Bmatrix}\varepsilon_{ssj}^{(1)}\\\varepsilon_{\theta\theta j}^{(1)}\\\varepsilon_{s\theta j}^{(1)}\end{Bmatrix}-\begin{Bmatrix}M_{ss}^{Tj}\\M_{\theta\theta}^{Tj}\\0\end{Bmatrix}\\
&Q_{\theta j}^{(1)}=\frac{\pi^2}{12}B_{55}\gamma_{\theta zj}^{(0)}+\frac{\pi^2}{12}D_{55}\gamma_{\theta zj}^{(1)}\quad,\qquad j=(o,i)
\end{aligned}
\tag{13}
$$

where strain components are defined as:

$$
\begin{aligned}
&\begin{Bmatrix}\varepsilon_{ssj}^{(0)}\\\varepsilon_{\theta\theta j}^{(0)}\\\varepsilon_{s\theta j}^{(0)}\end{Bmatrix}=\begin{Bmatrix}u_{0j,s}+\frac{1}{2}w_{j,s}{}^{2}\\\frac{1}{r}\left(v_{0j,\theta}+u_{0j}\sin\beta+w_j\cos\beta\right)+\frac{1}{2r^2}w_{j,\theta}{}^{2}\\\frac{1}{r}\left(u_{0j,\theta}-v_{0j}\sin\beta\right)+v_{0j,s}+\frac{1}{r}w_{j,s}w_{j,\theta}\end{Bmatrix}\\
&\begin{Bmatrix}\varepsilon_{ssj}^{(1)}\\\varepsilon_{\theta\theta j}^{(1)}\\\varepsilon_{s\theta j}^{(1)}\end{Bmatrix}=\begin{Bmatrix}\phi_{s,s}^{j}\\\frac{1}{r}\left(\phi_{\theta,\theta}^{j}+\phi_s^{j}\sin\beta\right)\\\frac{1}{r}\left(-\phi_\theta^{j}\sin\beta+\phi_{s,\theta}^{j}\right)+\phi_{\theta,s}^{j}\end{Bmatrix}\\
&\begin{Bmatrix}\gamma_{szj}^{(0)}\\\gamma_{\theta zj}^{(0)}\end{Bmatrix}=\begin{Bmatrix}w_{j,s}+\phi_s^{j}\\\frac{1}{r}\left(w_{j,\theta}-v_{0j}\cos\beta\right)+\phi_\theta^{j}\end{Bmatrix},\\
&\begin{Bmatrix}\gamma_{szj}^{(1)}\\\gamma_{\theta zj}^{(1)}\end{Bmatrix}=\begin{Bmatrix}0\\\frac{1}{r}\left(-\phi_\theta^{j}\cos\beta\right)\end{Bmatrix}
\end{aligned}
\tag{14}
$$

Also, N_{ss}^{Tj}, $N_{\theta\theta}^{Tj}$, M_{ss}^{Tj} and $M_{\theta\theta}^{Tj}$ are the thermal resultants and A_{kl}^{j}, B_{kl}^{j} and D_{kl}^{j}, (k,l=1,2,6) are the stiffness matrix components in which for the outer and the inner sandwich face sheets are as follows:

$$
\begin{aligned}
&N_{ss}^{Tj}=N_{\theta\theta}^{Tj}=\int_{-\frac{h_j}{2}}^{\frac{h_j}{2}}\frac{E_j(z_j,T_j)}{1-\upsilon_j(z_j,T_j)}.\alpha_j(z_j,T_j).\Delta T_j\,dz_j\ ,\ j=(o,i)\\
&M_{ss}^{Tj}=M_{\theta\theta}^{Tj}=\int_{-\frac{h_j}{2}}^{\frac{h_j}{2}}\frac{z_j.E_j(z_j,T_j)}{1-\upsilon_j(z_j,T_j)}.\alpha_j(z_j,T_j).\Delta T_j\,dz_j\ ,j=(o,i)\\
&\begin{Bmatrix}A_{11}^{j}\\B_{11}^{j}\\D_{11}^{j}\end{Bmatrix}=\begin{Bmatrix}A_{22}^{j}\\B_{22}^{j}\\D_{22}^{j}\end{Bmatrix}=\int_{-\frac{h_j}{2}}^{\frac{h_j}{2}}\left[\frac{E_j\left(z_j,T_j\right)}{1-\left(\vartheta_j\left(z_j,T_j\right)\right)^2}\right]\begin{Bmatrix}1\\z_j\\z_j{}^{2}\end{Bmatrix}dz_j
\end{aligned}
\tag{15}
$$

$$\begin{Bmatrix} A_{12}^j \\ B_{12}^j \\ D_{12}^j \end{Bmatrix} = \int_{-\frac{h_j}{2}}^{\frac{h_j}{2}} \left[\frac{\vartheta_j(z_j,T_j)\times E_j(z_j,T_j)}{1-(\vartheta_j(z_j,T_j))^2} \right] \begin{Bmatrix} 1 \\ z_j \\ z_j^2 \end{Bmatrix} dz_j$$

where $E_j(z_j,T_j)$, $\vartheta_j(z_j,T_j)$ and $\alpha_j(z_j,T_j)$, $j=(o,i)$ are the young's modulus, the Poisson's factor and the thermal expansion factor of the FGMs sandwich face sheets, and demonstrate by power-law function of FGMs. And, twenty three stress resultants of the core are obtained as:

$$\begin{aligned}
&\left\{Q_{sz}^c, M_{Q1sz}^c, M_{Q2sz}^c, M_{Q3sz}^c\right\} = \int_{-\frac{h_c}{2}}^{\frac{h_c}{2}} \left(1.z_c, z_c^2, z_c^3\right)\tau_{sz}^c dz_c \\
&\left\{Q_{\theta z}^c, M_{Q1\theta z}^c, M_{Q2\theta z}^c, M_{Q3\theta z}^c\right\} = \int_{-\frac{h_c}{2}}^{\frac{h_c}{2}} \left(1.z_c, z_c^2, z_c^3\right)\tau_{\theta z}^c dz_c \\
&\left\{R_z^c, M_{z1}^c, M_{z2}^c\right\} = \int_{-\frac{h_c}{2}}^{\frac{h_c}{2}} \left(1, z_c, z_c^2\right)\sigma_{zz}^c dz_c \\
&\left\{Q_{s\theta}^c, M_{Q1s\theta}^c, M_{Q2s\theta}^c, M_{Q3s\theta}^c\right\} = \int_{-\frac{h_c}{2}}^{\frac{h_c}{2}} \left(1.z_c, z_c^2, z_c^3\right)\tau_{s\theta}^c dz_c \\
&\left\{R_s^c, M_{s1}^c, M_{s2}^c, M_{s3}^c\right\} = \int_{-\frac{h_c}{2}}^{\frac{h_c}{2}} \left(1.z_c, z_c^2, z_c^3\right)\sigma_{ss}^c dz_c \\
&\left\{R_\theta^c, M_{\theta1}^c, M_{\theta2}^c, M_{\theta3}^c\right\} = \int_{-\frac{h_c}{2}}^{\frac{h_c}{2}} \left(1.z_c, z_c^2, z_c^3\right)\sigma_{\theta\theta}^c dz_c
\end{aligned} \tag{16}$$

4- 5- Governing relationship

The governing relation for the sandwich face sheets and the soft core are obtained from the principle of minimum potential energy theory:

$$\delta U + \delta V = 0 \tag{17}$$

That U is the strain energy parameter; V is total potential energy and δ denotes the change operator.
The value of V is equals to:

$$\delta V = -\int_0^L \int_0^{2\pi} (n_s^o \delta u_{0o} + n_s^i \delta u_{0i} + n_s^c \delta u_{0c}) r\, d\theta ds \tag{18}$$

where u_{0o}, u_{0i}, u_{0c} are the movements of middle plane of the outer and inner sandwich face sheets and core in the long direction, respectively. n_s^o and n_s^i are the in-plane external force of the outer and inner sandwich face sheets, respectively. The first changing of the strain energy can be determined in terms of all stresses and strains of the face sheets and the soft core. The adaptability conditions at the face sheet-core interfaces, Eq. (10), are compel through the use of six Lagrange coefficient. Hence, the first change of the internal potential energy is:

$$\begin{aligned}
\delta U =& \int_{V_o} \left(\sigma_{ss}^o \delta\varepsilon_{ss}^o + \sigma_{\theta\theta}^o \delta\varepsilon_{\theta\theta}^o + \tau_{s\theta}^o \delta\gamma_{s\theta}^o + \tau_{sz}^o \delta\gamma_{sz}^o + \tau_{\theta z}^o \delta\gamma_{\theta z}^o\right) dV \\
&+ \int_{V_i} \left(\sigma_{ss}^i \delta\varepsilon_{ss}^i + \sigma_{\theta\theta}^i \delta\varepsilon_{\theta\theta}^i + \tau_{s\theta}^i \delta\gamma_{s\theta}^i + \tau_{sz}^i \delta\gamma_{sz}^i + \tau_{\theta z}^i \delta\gamma_{\theta z}^i\right) dV \\
&+ \int_{V_{core}} \left(\tau_{sz}^c \delta\gamma_{sz}^c + \tau_{\theta z}^c \delta\gamma_{\theta z}^c + \sigma_{zz}^c \delta\varepsilon_{zz}^c + \sigma_{ss}^c \delta\varepsilon_{ss}^c + \sigma_{\theta\theta}^c \delta\varepsilon_{\theta\theta}^c + \tau_{s\theta}^c \delta\gamma_{s\theta}^c\right) dV \\
&+ \delta \int_0^L \int_0^{2\pi} \Big[\lambda_{so}\left(u_o(z_o = h_O/2) - u_c(z_c = -h_c/2)\right) + \lambda_{\theta o}\big(v_o(z_o = h_o/2) \\
&- v_c(z_c = -h_c/2)\big) \\
&+ \lambda_{zo}\left(w_o - w_c(z_c = -h_c/2)\right) + \lambda_{si}\left(u_c(z_c = h_c/2) - u_i(z_i = -h_i/2)\right) \\
&+ \lambda_{\theta i}\left(v_c(z_c = h_c/2) - v_i(z_i = -h_i/2)\right) + \lambda_{zi}\left(w_c(z_c = h_c/2) - w_i\right) \Big] r d\theta ds \\
+& \delta \int_0^L \int_0^{2\pi} \Big[\lambda_{so}\left(u_o(z_o = h_O/2) - u_c(z_c = -h_c/2)\right) + \lambda_{\theta o}\big(v_o(z_o = h_o/2) \\
&- v_c(z_c = -h_c/2)\big) \\
&+ \lambda_{zo}\left(w_o - w_c(z_c = -h_c/2)\right) + \lambda_{si}\left(u_c(z_c = h_c/2) - u_i(z_i = -h_i/2)\right) \\
&+ \lambda_{\theta i}\left(v_c(z_c = h_c/2) - v_i(z_i = -h_i/2)\right) + \lambda_{zi}\left(w_c(z_c = h_c/2) - w_i\right) \Big] r d\theta ds
\end{aligned} \tag{19}$$

where σ_{ss}^j and $\sigma_{\theta\theta}^j$ ($j=o,i$) are the in-plane stresses $\varepsilon_{\theta\theta}^j$ and ε_{ss}^j and ($j=o,i$) are the linear in-plane normal strains of the outer and inner sandwich face sheets; σ_{ss}^c, $\sigma_{\theta\theta}^c$, $\varepsilon_{\theta\theta}^c$ and ε_{ss}^c are the in-plane normal stresses and strains of the core, respectively; $\tau_{s\theta}^j$ and $\gamma_{s\theta}^j$ ($j=o,i,c$) are the in-plane shear stresses and strains in the sandwich face sheets and soft core; σ_{zz}^c and ε_{zz}^c are the normal stress and normal strain in the perpendicular direction of the core; τ_{sz}^c and γ_{sz}^c are the perpendicular shear stress and perpendicular shear strain in the core; V_o, V_i and V_{core} are the volume of outer and inner face sheets and the core, respectively; λ_{sj}, $\lambda_{\theta j}$ and λ_{zj} ($j=o,i$) are six Lagrange factors for stability conditions at the outer and inner sandwich face sheet-core interfaces. By using the Eqs. (17) to (19) and after some algebraic operate, the twenty eight equations of equilibrium are obtained as follows:
For the outer FG face sheet:

$$\begin{aligned}
&rN_{ss,s}^o + \left(N_{ss}^o - N_{\theta\theta}^o\right)\sin\beta + N_{s\theta,\theta}^o - r\lambda_{so} + rn_s^o = 0 \\
&N_{\theta\theta,\theta}^o + rN_{s\theta,s}^o + 2N_{s\theta}^o \sin\beta + Q_{\theta o}^{(0)}\cos\beta - r\lambda_{\theta o} = 0 \\
&rM_{ss,s}^o + \left(M_{ss}^o - M_{\theta\theta}^o\right)\sin\beta + M_{s\theta,\theta}^o - rQ_{so}^{(0)} - r\lambda_{so}\frac{h_o}{2} = 0 \\
&M_{\theta\theta,\theta}^o + rM_{s\theta,s}^o + 2M_{s\theta}^o \sin\beta - rQ_{\theta o}^{(0)} + Q_{\theta o}^{(1)}\cos\beta - r\lambda_{\theta o}\frac{h_o}{2} = 0
\end{aligned} \tag{20}$$

Five equations for the inner FG face sheet:

$$\begin{aligned}
&rN_{ss,s}^i + \left(N_{ss}^i - N_{\theta\theta}^i\right)\sin\beta + N_{s\theta,\theta}^i + r\lambda_{si} + rn_s^i = 0 \\
&N_{\theta\theta,\theta}^i + rN_{s\theta,s}^i + 2N_{s\theta}^i \sin\beta + Q_{\theta i}^{(0)}\cos\beta + r\lambda_{\theta i} = 0 \\
&rM_{ss,s}^i + \left(M_{ss}^i - M_{\theta\theta}^i\right)\sin\beta + M_{s\theta,\theta}^i - rQ_{si}^{(0)} - r\lambda_{si}\frac{h_i}{2} = 0 \\
&M_{\theta\theta,\theta}^i + rM_{s\theta,s}^i + 2M_{s\theta}^i \sin\beta - rQ_{\theta i}^{(0)} + Q_{\theta i}^{(1)}\cos\beta - r\lambda_{\theta i}\frac{h_i}{2} = 0
\end{aligned} \tag{21}$$

Twelve equations for the core:

$$\begin{aligned}
&rR_{s,s}^c + \left(R_s^c - R_\theta^c\right)\sin\beta + Q_{s\theta,\theta}^c + r\lambda_{so} - r\lambda_{si} = 0 \\
&-rQ_{sz}^c + rM_{s1,s}^c + \left(M_{s1}^c - M_{\theta1}^c\right)\sin\beta + M_{Q1s\theta,\theta}^c - r\lambda_{so}\frac{h_c}{2} - r\lambda_{si}\frac{h_c}{2} = 0 \\
&-2rM_{Q1sz}^c + rM_{s2,s}^c + \left(M_{s2}^c - M_{\theta2}^c\right)\sin\beta + M_{Q2s\theta,\theta}^c + r\lambda_{so}\frac{h_c^2}{4} - r\lambda_{si}\frac{h_c^2}{4} = 0 \\
&-3rM_{Q2sz}^c + rM_{s3,s}^c + \left(M_{s3}^c - M_{\theta3}^c\right)\sin\beta + M_{Q3s\theta,\theta}^c - r\lambda_{so}\frac{h_c^3}{8} - r\lambda_{si}\frac{h_c^3}{8} = 0 \\
&Q_{\theta z}^c \cos\beta + R_{\theta,\theta}^c + rQ_{s\theta,s}^c + 2Q_{s\theta}^c \sin\beta + r\lambda_{\theta o} - r\lambda_{\theta i} = 0
\end{aligned} \tag{22}$$

$$
\begin{aligned}
&M^{c}_{Q1\theta z}\cos\beta - rQ^{c}_{\theta z} + M^{c}_{\theta 1,\theta} + rM^{c}_{Q1s\theta,s}\\
&+2M^{c}_{Q1s\theta}\sin\beta - r\lambda_{\theta o}\frac{h_c}{2} - r\lambda_{\theta i}\frac{h_c}{2} = 0\\
&M^{c}_{Q2\theta z}\cos\beta - 2rM^{c}_{Q1\theta z} + M^{c}_{\theta 2,\theta} + rM^{c}_{Q2s\theta,s}\\
&+2M^{c}_{Q2s\theta}\sin\beta + r\lambda_{\theta o}\frac{h_c^{\,2}}{4} - r\lambda_{\theta i}\frac{h_c^{\,2}}{4} = 0\\
&M^{c}_{Q3\theta z}\cos\beta - 3rM^{c}_{Q2\theta z} + M^{c}_{\theta 3,\theta} + rM^{c}_{Q3s\theta,s}\\
&+2M^{c}_{Q3s\theta}\sin\beta - r\lambda_{\theta o}\frac{h_c^{\,3}}{8} - r\lambda_{\theta i}\frac{h_c^{\,3}}{8} = 0\\
&rM^{c}_{Q1sz,s} + M^{c}_{Q1sz}\sin\beta + M^{c}_{Q1\theta z,\theta} -\\
&rR^{c}_{z} - M^{c}_{\theta 1}\cos\beta - r\lambda_{zo}\frac{h_c}{2} - r\lambda_{zi}\frac{h_c}{2} = 0\\
&rM^{c}_{Q2sz,s} + M^{c}_{Q2sz}\sin\beta + M^{c}_{Q2\theta z,\theta} - 2rM^{c}_{z1} -\\
&M^{c}_{\theta 2}\cos\beta + r\lambda_{zo}\frac{h_c^{\,2}}{4} - r\lambda_{zi}\frac{h_c^{\,2}}{4} = 0\\
&rM^{c}_{Q3sz,s} + M^{c}_{Q3sz}\sin\beta + M^{c}_{Q3\theta z,\theta} - 3rM^{c}_{z2} -\\
&M^{c}_{\theta 3}\cos\beta - r\lambda_{zo}\frac{h_c^{\,3}}{8} - r\lambda_{zi}\frac{h_c^{\,3}}{8} = 0
\end{aligned}
$$

Compatibility conditions corresponding to perfect bonding:

$$
\begin{aligned}
&u_{0o} + \frac{h_o}{2}\phi^{o}_{s} - u_0 + u_1\frac{h_c}{2} - u_2\frac{h_c^{\,2}}{4} + u_3\frac{h_c^{\,3}}{8} = 0\\
&v_{0o} + \frac{h_o}{2}\phi^{o}_{\theta} - v_0 + v_1\frac{h_c}{2} - v_2\frac{h_c^{\,2}}{4} + v_3\frac{h_c^{\,3}}{8} = 0\\
&w_o - w_0 + w_1\frac{h_c}{2} - w_2\frac{h_c^{\,2}}{4} + w_3\frac{h_c^{\,3}}{8} = 0\\
&u_0 + u_1\frac{h_c}{2} + u_2\frac{h_c^{\,2}}{4} + u_3\frac{h_c^{\,3}}{8} - u_{0i} + \frac{h_i}{2}\phi^{i}_{s} = 0\\
&v_0 + v_1\frac{h_c}{2} + v_2\frac{h_c^{\,2}}{4} + v_3\frac{h_c^{\,3}}{8} - v_{0i} + \frac{h_i}{2}\phi^{i}_{\theta} = 0\\
&w_0 + w_1\frac{h_c}{2} + w_2\frac{h_c^{\,2}}{4} + w_3\frac{h_c^{\,3}}{8} - w_i = 0
\end{aligned}
\tag{23}
$$

where $N(w_i)$, $N(w_o)$ and $N(w_c)$ are defined as:

$$
\begin{aligned}
N(w_o) &= \hat{N}^{o}_{ss}\left(rw_{o,ss} + w_{o,s}\sin\beta\right)\\
N(w_i) &= \hat{N}^{i}_{ss}\left(rw_{i,ss} + w_{i,s}\sin\beta\right)\\
N(w_c) &= \hat{R}^{c}_{s}\left(rw_{0,ss} + w_{0,s}\sin\beta\right)
\end{aligned}
\tag{24}
$$

and $\hat{N}^{o}_{ss}$, $\hat{N}^{i}_{ss}$ and $\hat{R}_s^{\,c}$ are the total external load, $\hat{N}_0$, that are exerted to the outer face sheet, inner face sheet and the soft core along S direction, respectively.

$$
\hat{N}^{o}_{ss} + \hat{N}^{i}_{ss} + \hat{R}^{c}_{s} = -\hat{N}_0 \tag{25}
$$

It assumed homogeny state of strain for the face sheets and the core. So that by a little simplification we can write:

$$
\frac{\hat{N}^{o}_{ss}}{h_o\bar{E}_o} = \frac{\hat{N}^{i}_{ss}}{h_i\bar{E}_i} = \frac{\hat{R}^{c}_{s}}{h_c E_c} \tag{26}
$$

where E_c is the young modulus of the core; and $\bar{E}_o$ and $\bar{E}_i$ are the equilibrium young moduli in the outer and inner sandwich face sheet, respectively, that are obtained as:

$$
\bar{E}_o = \frac{\int_{-h_o/2}^{h_o/2} E_o(z_o)\,dz_o}{h_o} \quad , \quad \bar{E}_i = \frac{\int_{-h_i/2}^{h_i/2} E_i(z_i)\,dz_i}{h_i} \tag{27}
$$

Also, by used of Eqs. (25) and (26), the external in-plane bar exerted to the face sheets and the core along S direction can be determined as:

$$
\begin{aligned}
\hat{N}^{o}_{ss} &= \frac{-h_o\bar{E}_o\hat{N}_0}{h_o\bar{E}_o + h_i\bar{E}_i + h_c E_c}\\
\hat{N}^{i}_{ss} &= \frac{-h_i\bar{E}_i\hat{N}_0}{h_o\bar{E}_o + h_i\bar{E}_i + h_c E_c}\\
\hat{R}^{c}_{s} &= \frac{-h_c E_c\hat{N}_0}{h_o\bar{E}_o + h_i\bar{E}_i + h_c E_c}
\end{aligned}
\tag{28}
$$

4- 6- Analytical solution and discussion

It assumed the outer and the inner face sheets are simply supported and the perpendicular movements through the depth of the core at the edges of the sandwich truncated conical shell are prevented. Thus, the face sheets are supposed to be FG and the core is supposed to be isotropic. Then, a Galerkin solution theory with twenty eight trigonometric shape functions, which satisfies the boundary conditions, is obtained. The shape functions can be calculated as:

$$
\begin{aligned}
u_{0j} &= C_{uj}\cos\left(\frac{m\pi}{L}s\right)\cos(n\theta) &&, \ j = o,i\\
v_{0j} &= C_{vj}\sin\left(\frac{m\pi}{L}s\right)\sin(n\theta) &&, \ j = o,i\\
w_{j} &= C_{wj}\sin\left(\frac{m\pi}{L}s\right)\cos(n\theta) &&, \ j = o,i\\
\phi^{j}_{s} &= C_{\phi sj}\cos\left(\frac{m\pi}{L}s\right)\cos(n\theta) &&, \ j = o,i\\
v_{K} &= C_{vK}\sin\left(\frac{m\pi}{L}s\right)\sin(n\theta) &&, \ K = 0,1,2,3\\
w_{K} &= C_{wK}\sin\left(\frac{m\pi}{L}s\right)\cos(n\theta) &&, \ K = 0,1,2,3\\
\lambda_{sj} &= C_{\lambda sj}\cos\left(\frac{m\pi}{L}s\right)\cos(n\theta) &&, \ j = o,i\\
\lambda_{\theta j} &= C_{\lambda\theta j}\sin\left(\frac{m\pi}{L}s\right)\sin(n\theta) &&, \ j = o,i
\end{aligned}
\tag{29}
$$

where C_{uj} , C_{vj}, C_{wj}, $C_{\varphi sj}$, $C_{\varphi\theta j}$, C_{uK}, C_{vK}, C_{wK}, $C_{\lambda sj}$, $C_{\lambda\theta j}$ and $C_{\lambda zj}$ are the unclear constants factor of the shape functions. Hence, (m) and (n) are the wave numbers. After substitution of Eq. (29) into Eqs. (20) to (23), due with the stress resultants of the FG face sheets, and the high-order stress resultants of the core, the problem can be investigate in matrix form as:

$$
[L]\{C\} = \{F\} \tag{30}
$$

That vector $\{C\}$ is consist of twenty eight unclear constants and components of matrix $[L]$ and vector $\{F\}$ are not presented here for the sake of brevity. Eq. (30) is calculated for $\{C\}$ by using MATLAB programming software and the twenty eight unclear constants are determined.

5- Mathematical Formulation

5- 1- Numerical solution and discussion

In this section, to verify the presented results for buckling of sandwich conical shell with FGMs face sheets, in matching software outcomes with theoretical results obtained by ABAQUS 6.9, see Fig. 3. It is used C3D8T elements. The reliability and converge of FEM is shown in Fig. 4. The buckling results of the sandwich truncated conical shells with FGMs material face sheets are obtained for a sandwich conical model shell with geometrical shown in Table 1, but, for ceramic and metal structure. Type1: the soft core is complete stainless steel with (ϑ=0.3 and E=210 (GPa), to top and the bottom face sheets are silicon/ nitride/Nickel. Type 2: the soft core is complete titanium and the FGMs face sheets are Zirconia/Titanium material. In the two above cases, the outer surfaces of the FGMs face sheets are supposed to be ceramic rich (Si3N4 or ZrO2) and the FGMs face sheets in the face sheet-core interfaces are supposed to be complete metal (Ni or Ti–6Al–4V). The material satisfied C in the FGMs face sheets can be investigated as a function of thermal as Eq. (3). Temperature dependent silicified of materials of Si3N4, Ni, ZrO2 and Ti–6Al–4V described by five parameters for third-order function of temperature that is shown by Eq. (3), [38].

Table 1. Dimensions of the specimens [37]

h_c	h_o & h_i	R_2/H	$L/R_2 \cos\beta$	β
15mm	1.5mm	300	2	15

H: Total thickness of the sandwich

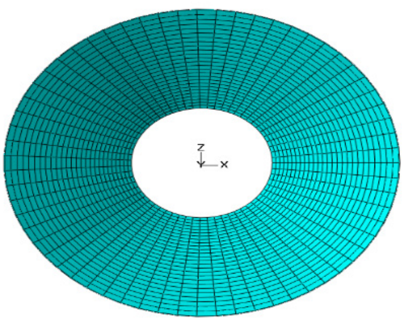

Fig. 3. FE meshing for the proposed research used for verification

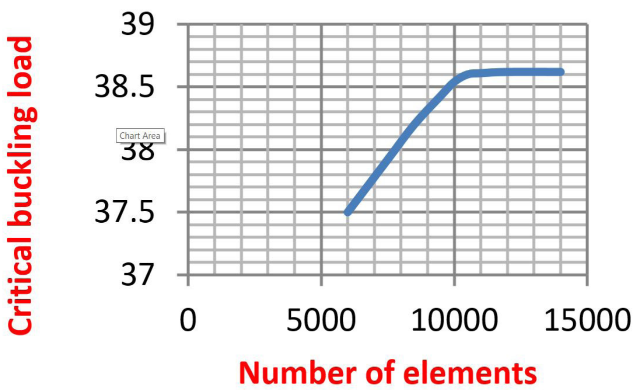

Fig. 4. Converge of FE method result

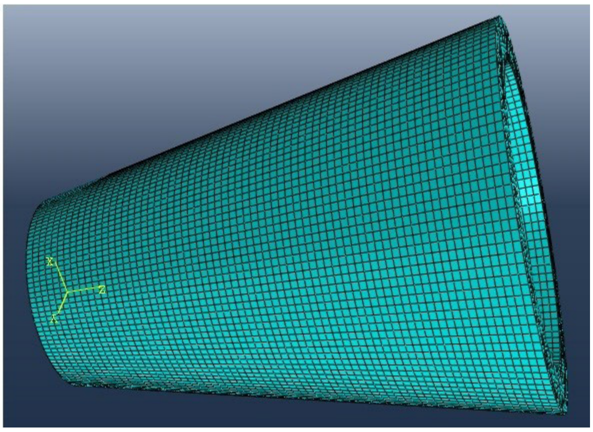

Fig. 5. FEM model of meshing

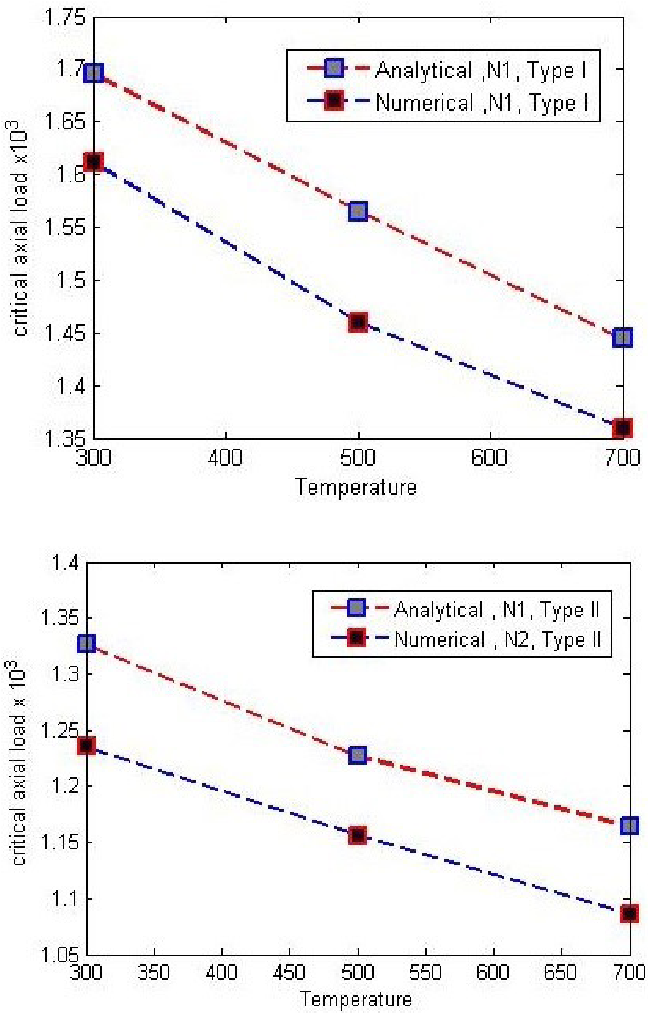

Fig. 6. The amount of valence static axial load with other thermal terms for type 1 and 2.

Tables 2 and 3 show the critical dimensionless uniform axial loads for Types 1 and 2 symmetric sandwich truncate conical shells. A comparison is carried out in Tables 2 and 3 among the results of the proposed research and FEM of ABAQUS software for various thermal conditions and also for three amount of power-law theory index, N. The critical dimensionless uniform axial load is obtained as:

Table 2. The amount of critical static axial force dimensionless for type 1 sandwich conical shells with distinct thermal terms and: *N*.

		$\bar{N}_{cr}\times10^3$		
		T_1=300K T_2=300K	T_1=500K T_2=300K	T_1=700K T_2=300K
	Present	1.757	1.633	1.537
N=1	ABAQUS	1.696	1.565	1.444
	Discrepancy	3.6%	4.16%	6.4%
	Present	1.695	1.562	1.469
N=2	ABAQUS	1.611	1.460	1.359
	Discrepancy	5.2%	7.0%	8.1%
	Present	1.754	1.629	1.525
N=3	ABAQUS	1.793	1.556	1.441
	Discrepancy	2.2%	4.7%	5.8%

Table 3. The amount of valence dimensionless uniform static axial force for second type sandwich conical shells with other thermal term and *N*.

		$\bar{N}_{cr}\times10^3$		
		T_1=300K T_2=300K	T_1=500K T_2=300K	T_1=700K T_2=300K
	Present	1.326	1.227	1.164
N=1	ABAQUS	1.235	1.156	1.086
	Discrepancy	6.8%	6.1%	7.2%
	Present	1.278	1.180	1.117
N=2	ABAQUS	1.230	1.119	1.041
	Discrepancy	3.9%	5.4%	7.3%
	Present	1.323	1.221	1.157
N=3	ABAQUS	1.279	1.159	1.088
	Discrepancy	3.4%	5.3%	6.3%

$$\bar{N}_{cr} = \frac{N_0}{E_{ce}.H} \tag{31}$$

where E_{ce} is the elasticity modulus of the homogeneous ceramic; and (H) is the total width of the sandwich. In Table 3 shown that the minimum and max discrepancies in percent among the propose theory and FEM method results for type 1 sandwich conical shells are 1.9 % and 8.1%, respectively. Also, the minimum and maximum discrepancies in percent among this theory and FEM software results for type 2 sandwich conical shells, shown in Table 4, are 0.7% and 7.3%, respectively. It is shown that illustrated that the FEM results of ABAQUS software and the results of this study are in well matching and agreement with each other.

5- 2- Analytical resultant

In this paragraph, the numerical solution results are calculated for type 1and 2 sandwich truncated conical shells with FG face sheets introduced in previous section. Critical dimensionless uniform static axial bar is calculated in Tables 5 to 9 for different thermal conditions, different core thickness-to-face sheet thickness ratio h_c/h_o , different bigger radius-to-sandwich thickness ratio R_2/H, different semi-vertex angle β, and other word for three virtues repartition of the FGMs face sheets and for two kind of sandwich conic shells, type 1 and 2. In general, Tables 4 to 8 indicate that with change the virtues distribution of the FGMs face sheets from linear state to quadratic, and then from quadratic to cubic repartition, the critical dimensionless axial bar $\bar{N}_{cr}$ is reduction and increased, respectively. Mutation of the critical dimensionless uniform static axial load $\bar{N}_{cr}$ with three compositional profiles, *N*=1,2,3, three semi-vertex angles, β=30,45,60, and three different thermal conditions are presented in Tables 5 and 6, for type 1 and 2 sandwich conical shells, respectively. Tables 4 and 5 indicate with increasing the semi-vertex β angle, the critical dimensionless homogeny static axial bar is decreased. For example, by increasing the semi-vertex angle from 30 to 60, the critical dimensionless static axial loads in Tables 5 and 6 are decreased between 34.3% and 42.1%.

In Tables 6 and 7, the valence of the acute dimensionless uniform static axial loads are shown for type 1 and 2 sandwich truncated conical shells, respectively, with three core thickness-to-face sheet thickness ratio, h_c / h_o=5,10,15, three compositional profiles, *N*=1,2,3, and three different thermal conditions. Tables 7 and 8 show, by grow thing the core thickness-to-face sheet width ratio from 5 to 10, the valence dimensionless static axial bars for type 1 sandwich shells are reduce between 2.6% and 10.2%, and for type 2 sandwich conical shells are decreased between 1.2% and 4.8%. Also, Tables 7 and 8 indicate that by adding the core thickness-to-face sheet width ratio from 10 to 15, the

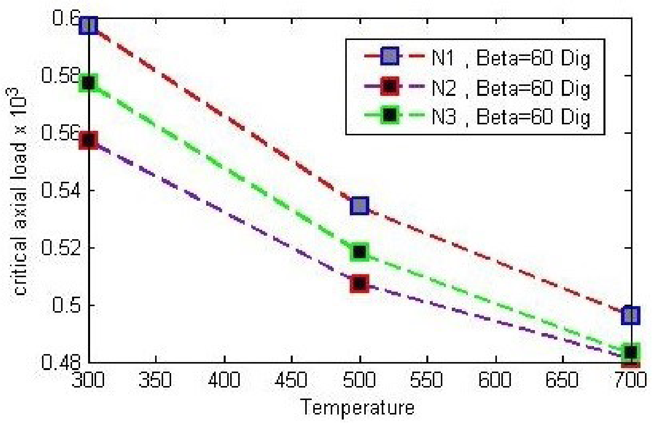

Fig. 7. The valence load for type 1 sandwich shells with other β(Beta), N and different thermal term

critical dimensionless axial loads for type 1 sandwich shells are decreased between 2.5% and 6.7%, and for second type sandwich shells are decreased between 0.7% and 3.4%.

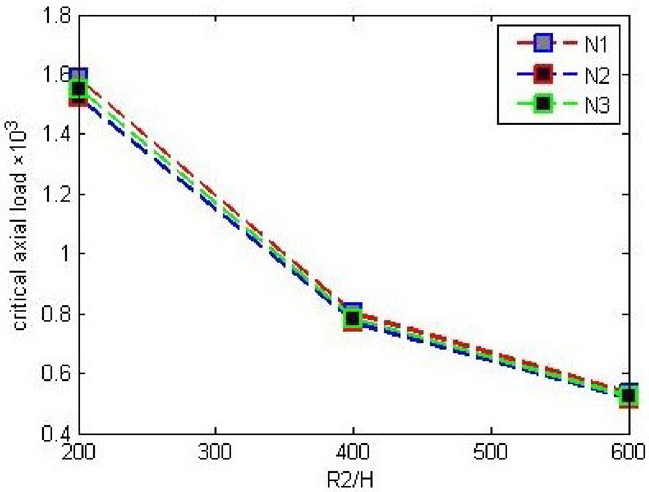

Fig. 8. The valence for type 1 sandwich conical with different R_2/H, N and different thermal term

Table 4. The critical uniform static axial load dimensionless for type 1 sandwich conical shells with other β, N and different thermal term. (R_2/H=400 , h_c/h_o=10 , $(R_2/L)\sin\beta$=4 , T_2=300K)

T_1(K)	$\bar{N}_{cr}\times10^3$								
	N=1			N=2			N=3		
	β=30	β=45	β=60	β=30	β=45	β=60	β=30	β=45	β=60
300	0.909	0.802	0.597	0.916	0.770	0.557	0.946	0.795	0.577
500	0.845	0.723	0.534	0.844	0.711	0.507	0.878	0.735	0.535
700	0.795	0.678	0.496	0.792	0.661	0.481	0.822	0.685	0.483

Table 5. The valence dimensionless axial load for second type sandwich conical shells with other β, N and different thermal conditions. (R_2/H=400 , h_c/h_o=10 , $(R_2/L)\sin\beta$=4 , T_2=300K)

T_1(K)	$\bar{N}_{cr}\times10^3$								
	N=1			N=2			N=3		
	β=30	β=45	β=60	β=30	β=45	β=60	β=30	β=45	β=60
300	0.735	0.622	0.438	0.690	0.581	0.434	0.714	0.599	0.434
500	0.680	0.569	0.402	0.637	0.558	0.395	0.659	0.551	0.399
700	0.646	0.541	0.374	0.602	0.507	0.363	0.624	0.523	0.367

Table 6. The critical dimensionless uniform static load for type 1 sandwich shells with other h_c/h_o , N and other thermal term. (R_2/H=300 , β=15°, $(L/R_2)\cos\beta$=2 , T_2=300K)

T_1(K)	$\bar{N}_{cr}\times10^3$								
	$h_c/h_o=5$			$h_c/h_o=10$			$h_c/h_o=15$		
	N=1	N=2	N=3	N=1	N=2	N=3	N=1	N=2	N=3
300	0.735	0.622	0.438	0.690	0.581	0.434	0.714	0.599	0.434
500	0.680	0.569	0.402	0.637	0.558	0.395	0.659	0.551	0.399
700	0.646	0.541	0.374	0.602	0.507	0.363	0.624	0.523	0.367

Table 7. The valence dimensionless uniform axial load for second type sandwich shells with other h_c/h_o , N and other thermal conditions. (R_2/H=300 , β=15°, $(L/R_2)\cos\beta$=2 , T_2=300K)

T_1(K)	$\bar{N}_{cr}\times10^3$								
	$h_c/h_o=5$			$h_c/h_o=10$			$h_c/h_o=15$		
	N=1	N=2	N=3	N=1	N=2	N=3	N=1	N=2	N=3
300	1.365	1.293	1.362	1.326	1.278	1.323	1.287	1.263	1.286
500	1.289	1.218	1.283	1.239	1.194	1.223	1.217	1.174	1.214
700	1.223	1.158	1.227	1.164	1.129	1.171	1.133	1.091	1.133

Table 8. The valence dimensionless static load ($\bar{N}_{cr}\times10^3$) for type 1 and 2 sandwich shells with distinct R_2/H, N and different thermal conditions. (β:45°, h_c/h_o =10 , $(R_2/L)\sin\beta$=4 , T_2=300K)

N	R_2/H	Type 1			Type 2		
		T_1=300K	T_1=500K	T_1=700K	T_1=300K	T_1=500K	T_1=700K
1	200	1.586	1.495	1.421	1.197	1.113	1.079
	400	0.802	0.738	0.686	0.622	0.572	0.551
	600	0.532	0.492	0.464	0.401	0.373	0.352
2	200	1.523	1.444	1.342	1.154	1.056	0.971
	400	0.770	0.724	0.679	0.581	0.536	0.498
	600	0.513	0.478	0.449	0.386	0.356	0.326
3	200	1.583	1.494	1.344	1.194	1.099	1.054
	400	0.795	0.755	0.695	0.599	0.546	0.516
	600	0.529	0.493	0.468	0.400	0.372	0.358

In Table 8, changes of values of valence dimensionless static load for type 1 and 2 sandwich conical with three bigger radius-to-sandwich thickness ratio, R_2/H=200,400,600, three compositional profiles, N=1,2,3, and three different thermal conditions are presented. In type 1 and 2 sandwich shells, when R_2/H is increased from 200 to 400, the amount of the critical dimensionless uniform axial loads are reduce about 50%, and by increasing R_2/H from 400 to 600, the amount of the critical dimensionless static uniform loads are reduce about 33.5% .

Tables 4 to 8 indicate with increasing the temperature of the internal surface of the sandwich shell, T_1, from 300K to 700K , the critical dimensionless axial loads are decreased between 10.4% and 19.8% for type 1 sandwich conical shells, and the critical dimensionless axial loads are decreased between 10% and 16.3% for type 2 sandwich conical shells. Also, Tables 4 to 8 show the valence dimensionless uniform axial loads calculated for type 1 sandwich conical shells are generally bigger than the critical dimensionless axial loads calculated for second type sandwich conical shells.

6- Conclusions

The buckling of sandwich conical shells with FG face sheets under uniform axial pressure load is studied. The material virtues of FGMs face sheets are varied by power-law repartition along the width. It shown that the values of the critical dimensionless uniform axial load are saddened by the formation of the constituent materials, hybrid profile variations, semi-vertex beta angle, thermal condition and the change of the sandwich shape. Collation the results of this research with FEM results validates the propose analysis. Based on the results obtained, the following outcomes can be drawn:

- By changing the distribution property of FG face sheets from linear to quadratic, and then from quadratic to cubic distribution, the critical dimensionless axial is decreased and increased, respectively.
- By increasing the semi-vertex angle β, the valence dimensionless static uniform axial load is decreased.
- With growing the core thickness-to-face sheet width ratio, the valence dimensionless static axial bars for type 1 and 2 sandwich truncated conical shells are reduced.
- When bigger radius-to-sandwich thickness ratio, R_2/H, is grouted, the values of the valence dimensionless static uniform axial forces are reduced.
- The valence dimensionless forces calculated for type 1 sandwich conical shells are bigger than the critical dimensionless uniform axial bars calculated for second type sandwich conical shells.
- with additional the temperature of the internal surface of the sandwich shell, T_1, from 300K to 700K , the critical dimensionless axial bars are decreased between 10.4% and 19.8% for type 1 sandwich conical shells, and are decreased between 10% and 16.3% for type 2 sandwich conical shells.
- For future work we suggest do this work in experimental and mention the effect of nonlinear thermal.

References

[1] R. Mohammadzadeh, M. M. Najafizadeh, and M. Nejati, “Buckling of 2D-FG Cylindrical Shells under Combined External Pressure and Axial Compression,” *Advances in Applied Mathematics and Mechanics*, vol. 5, pp. 391-406, 2015.

[2] C.-P. Wu, Y.-C. Chen, and S.-T. Peng, “Buckling analysis of functionally graded material circular hollow cylinders under combined axial compression and external pressure,” *Thin-Walled Structures*, vol. 69, pp. 54-66, 2013/08/01/ 2013.

[3] F. Plantema, “*Sandwich Construction*,” New York: Johen Wiley & Inc, 1966.

[4] Y. Frostig, M. Baruch, O. Vilnay, and I. Sheinman, High-Order Theory for Sandwich-Beam Behavior with Transversely Flexible Core vol. 118, 1992.

[5] E. Bozhevolnya and J. Sun, “Free Vibration analysis of Sandwich Beams,” *Sandwich struct Matter*, vol. 6, pp. 47-76, 2004.

[6] Y. Frostig and O. T. Thomsen, “High-order free vibration of sandwich panels with a flexible core,” *International Journal of Solids and Structures*, vol. 41, pp. 1697-1724, 2004/03/01/ 2004.

[7] G. Schwarts and Y. Frostig, “Free Vibrationof Delaminated Unidirectional Panels with a Transversely Flexible Core and General Boundary Conditions-a High-order Approach - Google Search,” *J Sandwich Struct Mater*, vol. 10, pp. 99-131, 2008.

[8] A. Noor, J. M. Peters, and W. Scott Burton, Three-Dimensional Solutions for Initially Stressed Structural Sandwiches vol. 120, 1994.

[9] W. Ji and A. Waas, Global and Local Buckling of a Sandwich Beam vol. 133, 2007.

[10] W. Ji and A. M. Wass, "Elastic analysis of Sandwich Panel Buckl Problem: Benchomark Solutions and Finite Element Formulations," *ZAMP*, vol. 61, pp. 897-917, 2010.

[11] T. Kant and K. Swaminathan, "Analytical solutions for the static analysis of laminated composite and sandwich plates based on a higher order refined theory," *Composite Structures*, vol. 56, pp. 329-344, 2002/06/01/ 2002.

[12] A. K. Nayak, R. A. Shenoi, and S. S. J. Moy, "Dynamic response of composite sandwich plates subjected to initial stresses," *Composites Part A: Applied Science and Manufacturing*, vol. 37, pp. 1189-1205, 2006/08/01/ 2006.

[13] D. Bushnell and S. Smith, "Stress and buckling of nonuniformly heated cylindrical and conical shells," *AIAA Journal*, vol. 9, pp. 2314-2321, 1971/12/01 1971.

[14] J. B. Dafedar, Y. M. Desai, and A. A. Mufti, "Stability of sandwich plates by mixed, higher-order analytical formulation," *International Journal of Solids and Structures*, vol. 40, pp. 4501-4517, 2003/08/01/ 2003.

[15] M. Koizumi, "FGM activities in Japan," *Composites Part B: Engineering*, vol. 28, pp. 1-4, 1997/01/01/ 1997.

[16] S. Suresh and A. Mortnsen, "*Fundementals of Functionally Materials*," IOM Communications, London, 1998.

[17] B. Sankar and J. T. Tzeng, Thermal Stresses in Functionally Graded Beams vol. 40, 2002.

[18] N. FIN, "Buckling of Conical Shells Subjected to External Lateral Pressure," *Trans Roy Inst Technol*, Stockholm, vol. 10, pp. 1-21, 1947.

[19] K. M. Mushtari and A. V. Sachenkov, Stability of Cylindrical and Conical Shells of Circular Cross Section, with Simultaneous Action of Axial Compression and External Normal Pressure vol. 18, 1958.

[20] A. H. Sofiyev, "The stability of functionally graded truncated conical shells subjected to aperiodic impulsive loading," *International Journal of Solids and Structures*, vol. 41, pp. 3411-3424, 2004/06/01/ 2004.

[21] L. K. Chang and S. Y. Lu, "Thermal buckling of conical shells," *AIAA Journal*, vol. 5, pp. 1877-1882, 1967/10/01 1967.

[22] L. K. Chang and S. Y. Lu, "Nonlinear thermal elastic buckling of conical shells," *Nuclear Engineering and Design*, vol. 7, pp. 159-169, 1968/02/01/ 1968.

[23] P. C. Dumir, G. P. Dube, and A. Mallick, Axisymmetric static and dynamic buckling of laminated thick truncated conical cap vol. 38, 2003.

[24] J. Blachut, O. Ifayefunmi, and M. Corfa, *Collapse and Buckling of Conical Shells*, 2011.

[25] J. Kim, S.-R. Lee, and S.-S. Park, "Analyses of thrust bearings in scroll compressors considering keyways," *Tribology International*, vol. 43, pp. 728-736, 2010/04/01/ 2010.

[26] J. Singer, A. Eckstein, and M. Baruch, *BUCKLING OF CONICAL SHELLS UNDER EXTERNAL PRESSURE, TORSION AND AXIAL COMPRESSION*, 1962.

[27] J. Singer, "The Effect of Axial Constraint on the Instability of Thin Conical Shells Under External Pressure," *Journal of Applied Mechanics*, vol. 29, pp. 212-214, 1962.

[28] G. A. Thurston, "Effect of Boundary Conditions on the Buckling of Conical Shells Under Hydrostatic Pressure," *Journal of Applied Mechanics*, vol. 32, pp. 208-209, 1965.

[29] M. Baroch, Harar, and J. Singr, "Infuence of in-plane Boundary Condition on the Stability of Conical Shells Under Pressure,".*Israel J Technol*, vol. 5, pp. 12-24, 1967.

[30] M. M. Kheirikhah, S. M. R. Khalili, and K. Malekzadeh Fard, "Biaxial buckling analysis of soft-core composite sandwich plates using improved high-order theory," *European Journal of Mechanics - A/Solids*, vol. 31, pp. 54-66, 2012/01/01/ 2012.

[31] J. seidi and S. M. R. Khalili, "Analytical of sandwich truncated conical shell with nano FGM layers in high order theory," *Mechanical and Mechatoronic conferene*, Tehran. Iran, 2016.(In Persian)

[32] J. seidi and S. M. R. Khalili, "Buckling Analytcal of sandwich truncated conical shell with three order theory," Civil, *Electric and Mechanical conferene*, Gorgan. Iran, 2014. (In Persian)

[33] J. seidi and S. M. R. Khalili, "Analytcal of sandwich truncated conical shell with soft core and FGM layers in high order theory," *Mechanical engineering and Electric conferene*, Tehran. Iran, 2015(In Persian).

[34] N. I. Karpov and O. A. Karpova, Stability of conical shell under combined load vol. 13, 1981.

[35] A. H. Sofiyev, "The buckling of FGM truncated conical shells subjected to combined axial tension and hydrostatic pressure," *Composite Structures*, vol. 92, pp. 488-498, 2010/01/01/ 2010.

[36] A. H. Sofiyev, "On the buckling of composite conical shells resting on the Winkler–Pasternak elastic foundations under combined axial compression and external pressure," *Composite Structures*, vol. 113, pp. 208-215, 2014/07/01/ 2014.

[37] J. N. Reddy, "*Thermal Mechanical Behavior of Functionally Graded Materials*," Texas, vol. 2, 1998.

[38] F. Javad and M. H. Hajmohammadi, "Study Effect of Geometrical Parameters on the Buckling of Cylindrical Shells under Hydrostatic Pressure " *Indian Journal of Science Technology*, vol. 6(11), pp. 5527-5532, 2013.

[39] B. S. Golzan and H. Showkati, "Buckling of thin-walled conical shells under uniform external pressure," *Thin-Walled Structures*, vol. 46, pp. 516-529, 2008/05/01/ 2008.

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