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Thermal Analysis Circular Couette Flow of Non-Newtonian Fluid with Viscous Dissipation

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ABSTRACT: The forced convection heat transfer in the circular couette flow of Non-Newtonian fluid is investigated when the inner cylinder is rotated at angular speed and the outer cylinder is fixed. The fluid viscosity is considered concurrently to be dependent on the temperature and shear rate. The temperature dependency of viscosity is modeled exponentially according to the Nahme law and dependence of viscosity on shear is modeled with the Carreau equation. The Viscous dissipation term is adding intricacy to the already highly interdependent set of governing motion and energy equations. The highly nonlinear governing equations are derived for the steady state base flow in the narrow gap limit. The perturbation method has been applied to obtain an approximate solution for these equations. The effect of governing parameter such as Brinkman numbers and Deborah number on the thermal stability is examined. In addition, the analysis illustrated that the Nusselt number of the outer cylinder increases as the Deborah number increases. It, although, decreases by increasing Brinkman number. The pseudoplastic fluid between concentric cylinders is heated as Brinkman number and increases due to frictional loss and it is cooled as Deborah number increases due to the fluid elasticity behavior.

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1- Introduction

The Circular Couette flow consists of a viscous fluid bounded in the gap between two rotating cylinders. Circular Couette flow has wide applications ranging from filtration devices to magneto-hydrodynamics and also in the viscosimetric analysis. Different flow regimes have been classified over the years, including twisted Taylor vortices, wavy outflow boundaries, etc. It has been a well-researched and documented flow in fluid dynamics [1]. The detailed investigation and analysis of Circular Couette flow have also been a charming topic in the field of non-linear fluid researches. For instance, the effect of an axial flow on the stability of the Taylor-Couette flow is investigated for pseudoplastic fluids mentioned in the literature [2].

Typically, industrial liquid substances such as polymeric fluids and melts have zero-shear-rate viscosities that are several orders of magnitude greater than the viscosity of water. Heat conduction in polymeric solutions is poor; as a matter of fact, their thermal conductivity is in order of 0.1W/m.K. The frictional heat transfer can cause a major temperature rise in a flowing polymeric solution [3]. In most cases, these temperature rises exponentially diminish the local viscosity. Commonly, viscous dissipation is often ignored when studying the mechanics of nonpolymeric solutions, although thermally-induced gradients in viscosity can significantly change the isothermal flow, which in turn can lead to a new type of instability. The corresponding flows for complex materials modeled by viscoelastic constitutive equations have been investigated only over the past 40 years, except for some simple cases that have been known for a long time. There are several news studies in the literature for the shear flow with the Phan-Thien-Tanner model [4, 5] under fully developed conditions in channels, pipes, and annulus [6–10]. Oliveira [12] examined an exact solution for tube and slit flow of a FENE-P fluid [11] and Cruz [13] extended some of these solutions for Phan-Thien–Tanner and FENE-P fluids. Tasnim [14] investigated the entropy generation in a porous channel with hydromagnetic effects. Governing equations in Cartesian coordinate are simplified and solved to develop analytical expressions for velocity and temperature, entropy generation number and irreversibility distribution ratio. Mahmud and Fraser [15] derived an analytical solution to study the second law of thermodynamics inside a channel made of two parallel plates and channel with a circular cross-section. Carrington and Sun [16] presented an analytical expression for the entropy generation and Bejan number to analysis second law in internal and external flows. The entropy generation effects in boundary layer flows were investigated by Arpaci and Selamet [17]. They illustrated that both temperature gradient and fluid friction contribute to the entropy generation. Abu-Hijleh [18] presented a numerical work to evaluate the entropy generation due to heat transfer from a cylinder in an air crosses flow for different values of the buoyancy parameter, Reynolds number and cylinder diameter. Khalkhali et al. [19] examined the entropy generation in a heat pipe system. Ashrafi [20] examined the rotational flow of viscoplastic fluids between concentric annulus while friction loss due to viscous effects through the energy equation. Hazbavi [21] presented a numerical work to study the effect of an applied magnetic field for a nonlinear viscoelastic fluid obeying the isothermal model.

These studies have been mainly restricted, to the entropy generation characteristics and second law analysis. Yet, the heat transfer of non-linear fluids inside the annulus is thought-

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out in several studies. For instance, Khellaf and Lauriat [22] studied convective heat transfer characteristics for the Carreau fluid flow between vertical concentric annulus. The laminar forced convection heat transfer of purely viscous, non-Newtonian fluid flow in both eccentric and concentric annuli was analyzed by Manglik and Fang [23]. The viscous dissipation term is always positive for Newtonian liquids. It adopts positive or negative values with non-Newtonian liquids (viscoelastic material). On the other hand, for Newtonian fluids, the quantity of viscous dissipation is always positive and therefore represents an irreversible depreciation of mechanical into internal energy. For viscoelastic fluids, the quantity of viscous dissipation does not have to be positive, since some energy may be stockpiled as elastic energy [24].

However, the research is limited for the non-linear fluid flow due to the complexities originating from both the governing equations and geometry. As for Circular Couette flow of non-isothermal viscoelastic fluids with friction loss, no forced convection heat transfer research was found. This has been the main motivation for the present research to emerge in which the effect of friction loss on the flow is investigated. The objective of the current study is to specify heat transfer characteristics while considering friction loss term in energy equation and the resulting Nusselt number in Circular Couette flow of pseudoplastic fluid obeying the non-isothermal Carreau equation. The viscosity of non-isothermal Carreau fluid is considered simultaneously dependent on temperature and shear rate. The governing equations are simplified in the narrow-gap limit and analytical expressions are presented for dimensionless temperature and Nusselt number in both isothermal and isoflux cases.

2- Problem Formulation

In this section, the constitutive relations are derived and solution procedure is explained. The viscous fluid flow is considered between two infinite, coaxial cylinders of inner and outer radii R_1 and R_2 respectively (Fig. 1). The outer cylinder is stationary and inner cylinder is rotated at a constant angular velocity, Ω, where D is the gap width $D=R_2-R_1$. The incompressible pseudoplastic fluid flow is governed by the following mass conservation, momentum conservation and, energy conservation equations [24]:

$$\nabla \boldsymbol{U} = 0 \tag{1}$$

$$\rho\left(\boldsymbol{U}_{,t} + \boldsymbol{U}.\nabla \boldsymbol{U}\right) = -\nabla \boldsymbol{P} + \nabla.\left(\mu \dot{\Gamma}\right) \tag{2}$$

$$\rho C_p\left(T_{,t} + U.\nabla T\right) = k_c \nabla^2 T + \Phi \tag{3}$$

where a comma denotes partial differentiation. Here $U=(U_R, U_\theta, U_Z)^T$ is the velocity vector, t is the time, P is the pressure, $\tau=\mu\dot{\Gamma}$ is the shear stress, μ is the thermal-shear-rate dependent viscosity, ρ is the density, $\dot{\Gamma}=\nabla U+(\nabla U)^T$ is the shear rate tensor, $\Phi=\tau:\dot{\Gamma}$ is friction loss term, and ∇ is the divergent operators. The fluid is assumed to have a zero-shear-rate viscosity, μ_0, and $\nu_0=\mu_0/\rho$ is the zero-shear-rate kinematic viscosity. The governing equations are simplified in the narrow-gap limit by introducing dimensionless parameters as follows:

$$r = \frac{R-R_1}{D},\ z = \frac{Z}{D},\ \eta = \frac{\mu}{\mu_0},\ \bar{t} = \frac{\nu_0}{D^2}t,\ p = \frac{D^2}{\rho \nu_0^2}P,$$
$$u_r = \frac{D}{\nu_0}U_R,\ u_\theta = \frac{1}{R_1\Omega}U_\theta,\ u_z = \frac{D}{\nu_0}U_Z,\ \Theta = \frac{T-T_0}{\Delta T_0} \tag{4}$$
$$\dot{\gamma} = \frac{R_1\Omega}{D}\dot{\Gamma},$$

where ΔT_0 is a characteristic temperature difference defined as $\Delta T_0=1/\beta$, and β is called the viscosity temperature sensitivity [24, pp. 207]. The fluid viscosity is assumed concurrently dependent on temperature and shear rate. The fluid elasticity behavior is modeled by the Carreau equation [24] and the temperature dependency of viscosity is modeled exponentially according to the Nahme law [24] as:

$$\eta\left(\dot{\gamma}, \Theta\right) = e^{-\beta\Theta}\left[1 + De^2\dot{\gamma}^2\right]^{(n-1)/2} \tag{5}$$

where n is the "power-law exponent" which is less than one for pseudoplastic fluids, $\dot{\gamma}$ is the dimensionless rate-of-strain tensor, De is the Deborah number that indicates the ratio of elasticity force to viscous force specified as $De=\lambda R_1\Omega/D$, λ is the material relaxation time. The Carreau model is adopted, especially for poorly pseudoplastic fluids (with small relaxation time).

In dimensionless equations, the Taylor number, Ta, is defined as $Ta=Re^2\varepsilon$ in terms of main Reynolds number and the gap-to-radius ratio, $\varepsilon=D/R_1$, the Peclet number, Pe, is defined as $Pe=\rho C_p\Omega R_1 D/k_c$, the Nahme number, Na, is defined as $Na=\beta Br$, the Brinkman number, Br, is defined as $Br=\mu_0(\Omega R_1)^2/k_c\Delta T_0$ and k_c is the thermal conductivity. The narrow-gap limit is suppose simplification which is defined as the gap-to-radius ratio very small so that the terms of $O(\varepsilon/Ta)$ can be neglected. In the current study, it is assumed that $Ta=O(1)$, so that the governing equations are simplified

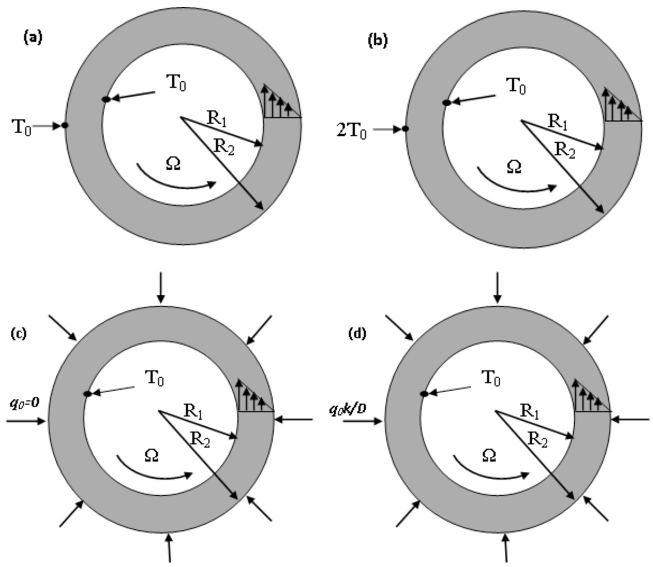

Fig. 1. Schematic of circular couette flow. (a) The Isothermal case with the same wall temperature, (b) The Isothermal case with the different wall temperature, (c) The Isoflux case with an isolated outer cylinder and (d) The Isoflux case with applied constant heat flux to the outer cylinder.

in the narrow-gap limit by ignoring the terms of $O(\varepsilon/Ta)$. Therefore dimensionless governing equations are reduced as follows:

$$u_{r,r}+u_{z,z}=0 \tag{6-a}$$

$$\begin{aligned}&u_{r,t}+u_r u_{r,r}+u_z u_{r,z}-Ta\left(u_\theta\right)^2\\&=-p_{,r}+\eta\left(u_{r,rr}+u_{r,zz}\right)+2\eta_{,r}u_{r,r}+\eta_{,z}\left(u_{r,z}+u_{z,r}\right)\end{aligned} \tag{6-b}$$

$$\begin{aligned}&u_{\theta,t}+u_r u_{\theta,r}+u_z u_{\theta,z}\\&=u_r+\eta\left(u_{\theta,rr}+u_{\theta,zz}\right)+\eta_{,r}u_{\theta,r}+\eta_{,z}u_{\theta,z}\end{aligned} \tag{6-c}$$

$$\begin{aligned}&u_{z,t}+u_r u_{z,r}+u_z u_{z,z}\\&=-p_{,z}+\eta\left(u_{z,rr}+u_{z,zz}\right)+\eta_{,r}\left(u_{r,z}+u_{z,r}\right)+2\eta_{,z}u_{z,z}\end{aligned} \tag{6-d}$$

$$\begin{aligned}&Pe\left(\Theta_{,t}+u_r\Theta_{,r}+u_z\Theta_{,z}\right)\\&=\left(\Theta_{,rr}+\Theta_{,zz}\right)+Na\left(u_{\theta,r}+u_{\theta,z}\right)\eta\end{aligned} \tag{6-d}$$

For the steady-state base flow, only axisymmetric flow is considered, so that the dependence on θ is ignored. The dimensionless velocity components $(u_r,\ u_\theta,\ u_z)^T$ and temperature are given explicitly as:

$$u_r=0,\ u_\theta=f\left(r\right),\ u_z=0\ ,\ \Theta=g\left(r\right) \tag{7}$$

For steady state base flow in the narrow-gap limit, the dimensionless governing equations are reduced as follows:

$$\frac{dp}{dr}=Ta\left(u_\theta\right)^2 \tag{8-a}$$

$$\frac{d}{dr}\left(e^{-\beta\Theta}\left[1+De^2\left(\frac{du_\theta}{dr}\right)^2\right]^{(n-1)/2}\frac{du_\theta}{dr}\right)=0 \tag{8-b}$$

$$\frac{d^2\Theta}{dr^2}=-Na\,e^{-\beta\Theta}\left[1+De^2\left(\frac{du_\theta}{dr}\right)^2\right]^{(n-1)/2}\left(\frac{du_\theta}{dr}\right)^2 \tag{8-c}$$

In the current work, thermal boundary conditions with constant temperature and constant heat flux are applied to the outer cylinder which is considered together with hydrodynamic boundary conditions, ($u_r=0$, $u_\theta=R_1\Omega$, $u_z=0$) at $r=0$ (inner cylinder) and ($u_r=0$, $u_\theta=0$, $u_z=0$) at $r=1$ (outer cylinder).

3- Solution Procedure

We first work through the simplified problem in which viscosity, as well as the thermal conductivity, is not varying with the temperature. The problem can be written in dimensionless form as:

$$\frac{d}{dr}\left(\left[1+De^2\left(\frac{du_\theta}{dr}\right)^2\right]^{(n-1)/2}\frac{du_\theta}{dr}\right)=0 \tag{9-a}$$

$$\frac{d^2\Theta}{dr^2}=-Br\left[1+De^2\left(\frac{du_\theta}{dr}\right)^2\right]^{(n-1)/2}\left(\frac{du_\theta}{dr}\right)^2 \tag{9-b}$$

subject to the boundary conditions:

$$at\qquad r=0\quad u_\theta=1,\qquad \Theta=0 \tag{10-a}$$

$$at\qquad r=1\quad u_\theta=0,\qquad \Theta=0 \tag{10-b}$$

Equations (9) through (10) are easily solved to give:

$$u_\theta=1-r \tag{11-a}$$

$$\Theta=\frac{Br}{2}\left[1+De^2\right]^{(n-1)/2}\left(r-r^2\right) \tag{11-b}$$

The maximum temperature occurs at r=0.5 and is given by:

$$\Theta_{max}=\frac{Br}{8}\left[1+De^2\right]^{(n-1)/2} \tag{12}$$

The simple result can be used for making rough estimates of the temperature rise that can be expected in the gap between the two moving surface in the absence of an axial pressure gradient.

Equations (8) are perturbed form of equations (9) which are presented as:

$$\begin{aligned}&\left[1+De^2\left(\frac{du_\theta}{dr}\right)^2\right]^{(n-1)/2}\frac{du_\theta}{dr}=ce^{\beta\Theta}\\&=c\left(1+\beta\Theta+\frac{1}{2}\beta^2\Theta^2+\frac{1}{6}\beta^3\Theta^3+\ldots\right)=C\end{aligned} \tag{13-a}$$

$$\frac{d^2\Theta}{dr^2}=-Na\,e^{-\beta\Theta}\left[1+De^2\left(\frac{du_\theta}{dr}\right)^2\right]^{(n-1)/2}\left(\frac{du_\theta}{dr}\right)^2 \tag{13-b}$$

in which c is an integration constant. Thus, for small values of β the aforementioned solution of velocity is expected to be accurate. This can be inserted into the energy equation (13b) and it is also necessary to expand the integration constant in a similar series to give:

$$\begin{aligned}&\frac{d^2\Theta}{dr^2}=-\beta\,Br\,C=-Br\left(\beta C_0+\beta^2C_1+\ldots\right)\\&=-Na\left(C_0+\beta C_1+\beta^2C_2+\ldots\right)\end{aligned} \tag{14}$$

Equations (13) are solved by a perturbation procedure, using the temperature sensitivity of the viscosity, β, as the perturbation parameter. This considered the form of the expansion solution to the following:

$$\Theta = \Theta_0(r) + \beta\Theta_1(r) + \beta^2\Theta_2(r) + \ldots \qquad (15)$$

When this expansion is inserted into equation (14), sets of differential equations are obtained by equating coefficients of equal powers of perturbation parameter. The resulting differential equations are solved with the related boundary conditions. The handy third order approximation solution for non-isothermal pseudoplastic fluid flow governed by equations (8) is obtained, resulted in dimensionless velocity and temperature expressions as in the following form:

$$u_\theta = 1 - r \qquad (16)$$

$$\Theta_{T1} = \frac{NaC_0}{2}\left(r - r^2\right) - \frac{Na^2C_0{}^2\beta}{24}\left(r - 2r^3 + r^4\right) \qquad (17\text{-a})$$

$$\Theta_{T2} = \beta r - \frac{NaC_0}{2}\left(r - r^2\right) + \frac{NaC_0\beta^2}{6}\left(r^3 - r\right) - \frac{NaC_0\beta^4}{24}\left(r^4 - r\right) - \frac{Na^2C_0{}^2\beta}{24}\left(r - 2r^3 + r^4\right) \qquad (17\text{-b})$$

$$\Theta_{q1} = \frac{NaC_0}{2}\left(2r - r^2\right) - \frac{Na^2C_0{}^2\beta}{24}\left(8r - 4r^3 + r^4\right) \qquad (17\text{-c})$$

$$\Theta_{q2} = r + \frac{NaC_0}{2}\left(2r - r^2\right) + \frac{NaC_0\beta}{6}\left(r^3 - 3r\right) - \frac{NaC_0\beta^2}{24}\left(r^4 - 4r\right) - \frac{Na^2C_0{}^2\beta}{24}\left(8r - 4r^3 + r^4\right) \qquad (17\text{-d})$$

Here C_0 is equal to $[1+De^2]^{((n-1)/2)}$. The equation (17.a), assigned as Θ_{T1} which is the dimensionless temperature for the isothermal case with the same boundary conditions, Θ=0 at r=0 and Θ=0 at r=1. Equation (17b), assigned as Θ_{T2} which is the dimensionless temperature for the isothermal case with different boundary conditions, Θ=0 at r=0 and $\Theta=\beta$ at r=1. The equation (17.c), assigned as Θ_{q1} which is the dimensionless temperature for the isoflux case with boundary conditions, Θ=0 at r=0 and Θ'=0 at r=1 (the uniform constant temperature is applied over the inner cylinder and the outer cylinder is maintained at a thermal insulation boundary condition). Equation (17.d), assigned as Θ_{q2} which is the dimensionless temperature for the isoflux case with boundary conditions, Θ=0 at r=0 and $\Theta'=\beta$ at r=1 (the uniform constant temperature is applied over the inner cylinder and the outer cylinder is maintained at a uniform constant heat flux).

The practical interest parameter is the heat transfer rate across the cylinders. The global Nusselt number expresses the overall heat transfer rate across the cylinders and is defined as follows:

$$Nu_1 = \frac{h_1 D}{k_c} \quad , \quad Nu_2 = \frac{h_2 D}{k_c} \qquad (18)$$

where h_1 and h_2 are average convection heat transfer coefficients considered at the inner and outer cylinder, respectively. The difference between the two values is the expression of the error resulted in satisfying global energy conservation. Therefore, assessment and analysis are carried out for the value of:

$$Nu = \frac{1}{2}\left(Nu_1 + Nu_2\right) = \frac{hD}{k_c} \qquad (19)$$

In equation (19), convection heat transfer coefficient is determined as:

$$h = \frac{-k_c}{T_b - T_w}\left(\frac{dT}{dR}\right)_{wall} \qquad (20)$$

where T_b is the mean fluid temperature within the domain. After dimensionless, the right-hand side of Equation (20), the heat transfer coefficient h, is reduced to the following form:

$$h = -\frac{k_c}{D}\left(\frac{d\Theta}{dr}\right)_{wall} \qquad (21)$$

Inserting euation (21) into equation (19), one finally obtains:

$$Nu = -\left(\frac{d\Theta}{dr}\right)_{wall} \qquad (22)$$

Inserting Eq. (17) into Eq. (22), the Nusselt number is given by:

$$Nu_{T1} = \frac{NaC_0}{2}\left(2r - 1\right) + \frac{Na^2C_0{}^2\beta}{24}\left(1 - 6r^2 + 4r^3\right) \qquad (23\text{-a})$$

$$Nu_{T2} = -b\,r + \frac{NaC_0}{2}\left(1 - 2r\right) - \frac{NaC_0\beta^2}{6}\left(3r^2 - 1\right) + \frac{NaC_0\beta^4}{24}\left(4r^3 - 1\right) + \frac{Na^2C_0{}^2\beta}{24}\left(1 - 6r^2 + 4r^3\right) \qquad (23\text{-b})$$

$$Nu_{q1} = \frac{NaC_0}{2}\left(2r - 2\right) + \frac{Na^2C_0{}^2\beta}{24}\left(8 - 12r^2 + 4r^3\right) \qquad (23\text{-c})$$

$$Nu_{q2} = -1 + \frac{NaC_0}{2}\left(2r - 2\right) - \frac{NaC_0\beta}{6}\left(3r^2 - 3\right) + \frac{NaC_0\beta^2}{24}\left(4r^3 - 4\right) + \frac{Na^2C_0{}^2\beta}{24}\left(8 - 12r^2 + 4r^3\right) \qquad (23\text{-d})$$

Equation (23.a), assigned as Nu_{T1} which is the Nusselt number for the isothermal case with the same thermal conditions, Θ=0 at r=0 and Θ=0 at r=1. Equation (23.b), assigned as Nu_{T2} which is the Nusselt number for the isothermal case with different thermal conditions, Θ=0 at r=0 and $\Theta=\beta$ at r=1. Equation (23.c), assigned as Nu_{q1} which is the Nusselt number for the isoflux case with thermal conditions, Θ=0 at r=0 and Θ'=0 at r=1 (the inner cylinder is maintained at a uniform constant temperature and a thermal insulation boundary condition is applied on the outer cylinder). The equation (23.d), assigned as Nu_{q2} which is the Nusselt number for the isoflux case with boundary conditions, Θ=0 at r=0 and Θ'=1 at r=1 (the inner cylinder is maintained at a uniform constant temperature and a uniform constant heat flux is applied on the outer cylinder).

Using equations elaborated so far, to evaluate the flow parameters. Here, the parameters for a given practical situation are selected. Therefore, assessment and analysis are carried out for polystyrene solution with n=0.538 and β=0.1 [24].

4- Results and Discussion

In this work, the perturbation method has been applied to highly nonlinear governing equations for finding an approximate solution. The comparisons between approximations (17) and the four order Runge Kutta (RK4) numerical results are shown in Figs. 2 and 3. In fact, Fig. 2 indicates that the maximum absolute error is about *6E-6* at r=0.2 and r=0.8. Fig. 3 indicates that the maximum absolute error is about *1.4E-7* at r=0.5. These comparisons prove the proficiency of perturbation method, particularly in the view of the fact that it only applied the third-order approximation, by considering four terms in the series expansion of a nonlinear part.

The stress-shear rate curve is depicted for different Deborah numbers in Fig. 4. It shows the stress gradually declines by enhancing the shear rate and at low Deborah number; the

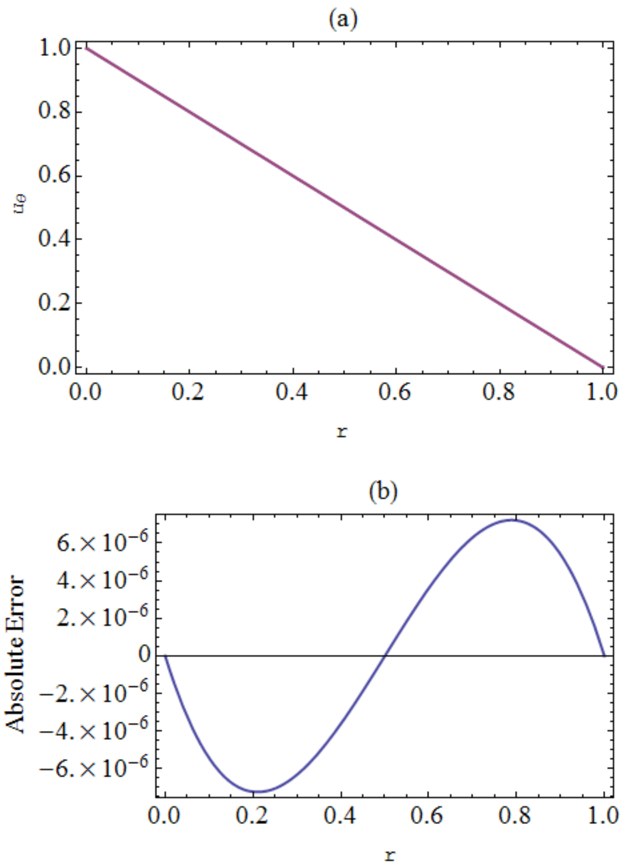

Fig. 2. Approximate solution and four order Runge Kutta numerical solution (a) and Absolute Error (b) of a velocity profile for polystyrene solution with *n*=0.538, *β*=0.1, *Br*=1, and *De*=0.1.

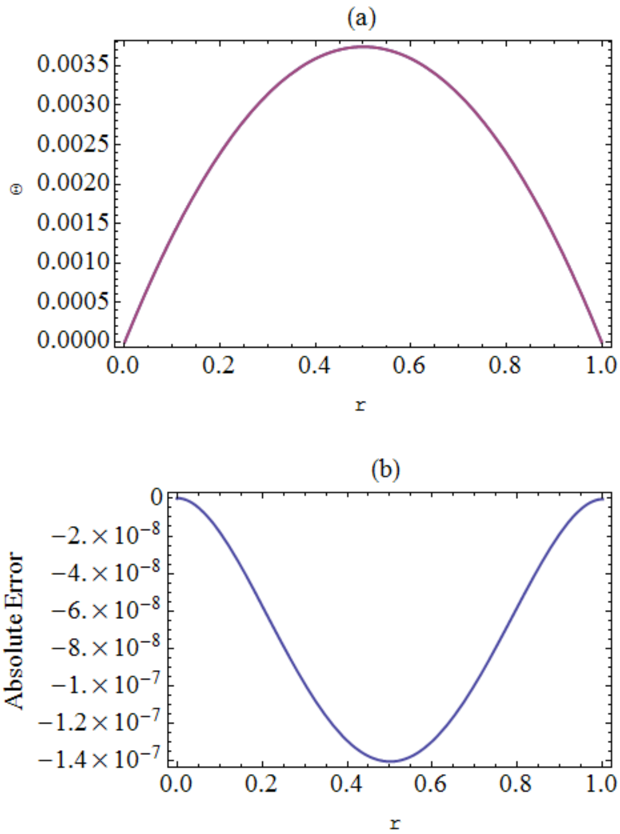

Fig. 3. Approximate solution and four order Runge Kutta numerical solution (a) and Absolute Error (b) of temperature distribution for polystyrene solution with *n*=0.538, *β*=0.1, *Br*=1, and *De*=0.1.

stress approaches a Newtonian plateau where the variation of shear rate does not affect the viscosity, also it exhibits a similar plateau at very high shear rates. This effect is much more pronounced for various Deborah numbers than that for various Brinkman numbers, for this reason, the stress curve is depicted as radial distance functions for different Brinkman numbers and is presented in Fig. 5.

The temperature distributions are depicted as radial distance functions for different Brinkman numbers and are presented in Figs. 6 and 7, respectively. In the isothermal case when the uniform different temperature is applied over the inner and

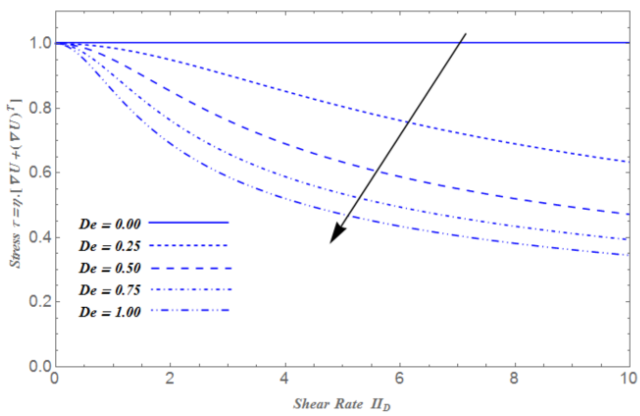

Fig. 4. Deborah number effect on stress-shear rate curve with *n*=0.538, *β*=0.1 and *De*=0.1

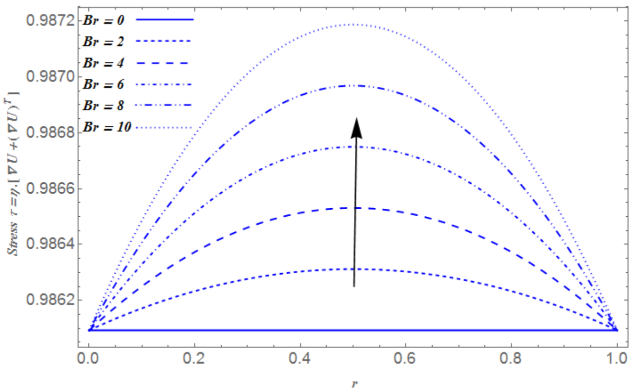

Fig. 5. Brinkman number effect on stress curve with n=0.538, β=0.1 and De=0.1

the outer cylinder (T_0 and $2T_0$ for the inner cylinder and the outer cylinder, respectively), the temperature profile (Fig. 6) shows a maximum value within the gap width between inner and outer cylinders. As the shear rate enhances by enhancing Brinkman number, the base flow temperature increases monotonically due to the frictional loss.

This is attributed to the fact that the value of the friction loss term in equation (8.c) enhances by enhancing the Brinkman number. The position of maximum temperature is also momentous since at this location the heat transfer is zero. It is seen from equation (17.b) that the temperature expression contains a linear term which is the same as the instance of a fluid at rest without any heat generation source. Superimposed on it there is a hyperbolic expression which is owing to the heat produced through friction. It is worthy of note that for a given magnitude of temperature difference of the two walls, the heat is flowed from the outer cylinder wall to the fluid only as long as the speed of inner cylinder does not exceed a specified magnitude ($Br=Br_{cri}$). A reversal of the direction of the flow of heat at outer cylinder wall occurs when the gradient of its temperature alters the sign. Hence, the mentioned principle is applied to determine the direction of the heat flow at the outer cylinder wall:

1. The heat flows from outer cylinder wall to fluid (the outer cylinder wall cooled): $Br<Br_{cri}$
2. The heat flows from fluid to outer cylinder wall (the outer cylinder wall heated): $Br>Br_{cri}$

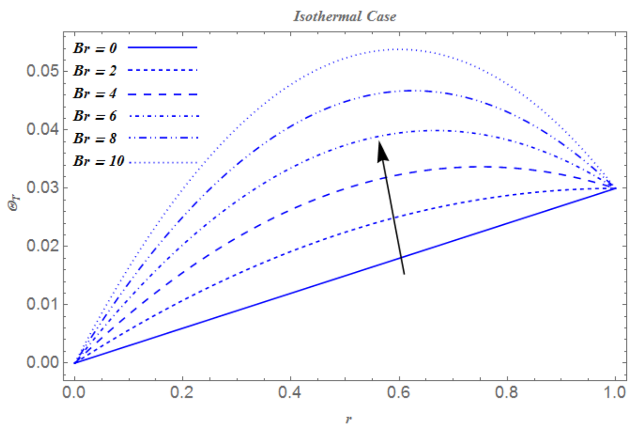

Fig. 6. Brinkman number effect on temperature distribution for isothermal case with n=0.538, β=0.1 and De=0.1

This illustrates that the heat generation due to the friction loss exerts a large effect on the cooling process and that at high velocities the warmer wall may become heated instead of being cooled. This effect is of fundamental importance for the consideration of cooling at high velocities. The same expression can be presented for the isoflux case (as is illustrated in Fig. 7). However, the distributions of temperature for two cases are differentiated. This trend is much more obvious for the Isothermal case than that for the isoflux case.

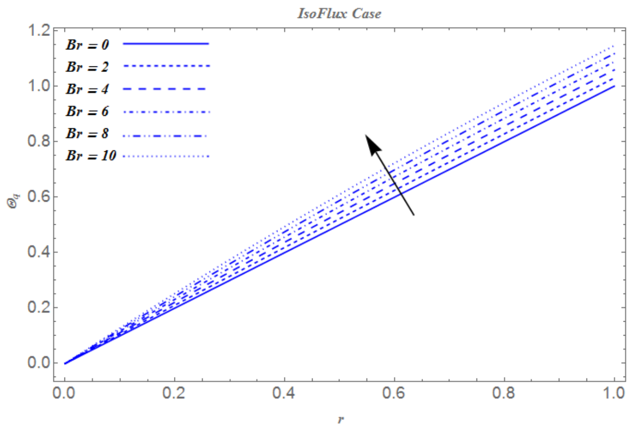

Fig. 7. Brinkman number effect on temperature distribution for Isoflux case with n=0.538, β=0.1 and De=0.1

In the isothermal case when the same uniform temperature is applied over the inner and the outer cylinder (T_0), the fluid elasticity effect on temperature distribution is depicted as radial distance functions for different Deborah numbers and is presented in Fig. 8. This trend is much more obvious for the same uniform temperature case than the different uniform temperature case. As the fluid elasticity increases by enhancing Deborah number, the value of maximum temperature decreases inside the annular gap (as illustrated in Fig. 8). This fluid behavior resulted from the elasticity effect of Carreau fluid, where the fluid viscosity reduces by enhancing Deborah number. The viscosity decreases correspondingly so that more and more fluid above the annular space center (less below) is pulled along with the inner cylinder. Keunings [25] examined the elasticity effect of a Non-Newtonian fluid which was also investigated by and Pinho and Oliveira [7] a greater detail. The same expression

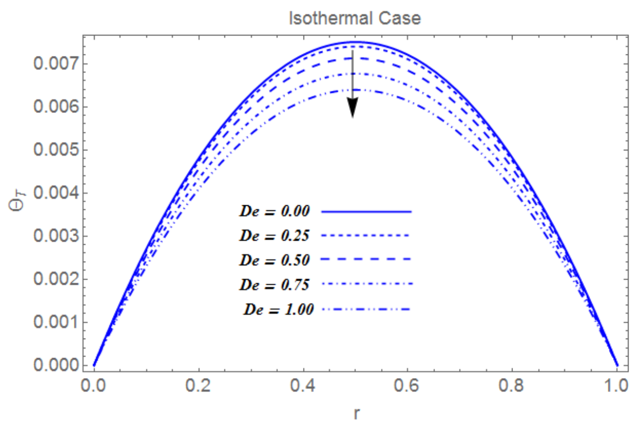

Fig. 8. Fluid elasticity effect on temperature distribution for an isothermal case with n=0.538, β=0.1 and Brinkman number Br=1.

can be presented for the Isoflux case (as illustrated in Fig. 9), however, the configuration of the temperature distributions are not like the isothermal case.

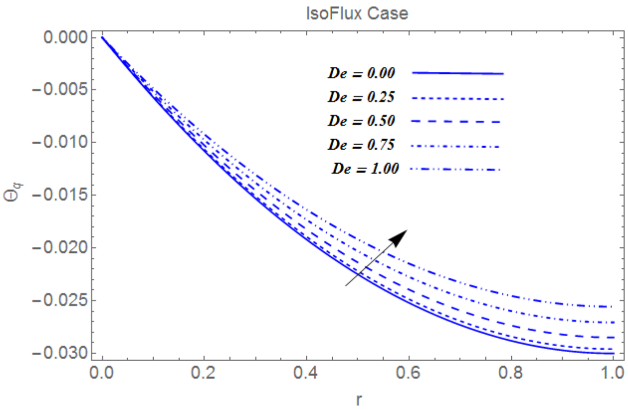

Fig. 9. Fluid elasticity effect on the temperature distribution for Isoflux case with *n*=0.538, *β*=0.1 and Brinkman number *Br*=1.

Next, the fluid elasticity effect in Nusselt number curves of the outer cylinder (Nu_T and Nu_q) are depicted as Brinkman number for different Deborah numbers and are presented in Figs. 10 and 11 respectively. As shown in Fig. 10, the Nusselt number (Nu_T) increases monotonically by enhancing the Brinkman number due to the frictional loss. This is attributed to the fact that the value of the friction loss term in equation (8.c) increases by enhancing the Brinkman number. The heat flows from the outer cylinder wall to the fluid only as long as the speed of inner cylinder does not exceed a specified magnitude ($Br=Br_{cri}$). A reversal of the heat flow direction at outer cylinder wall occurs when Brinkman number exceeds a certain value ($Br=Br_{cri}$) and gradient of its temperature alters the sign.

As shown in Fig. 11, the configuration of the Nusselt number curves of outer cylinder versus the Brinkman number are not similar in shape to the isothermal case. In Isoflux case, the Nusselt number of outer cylinder for all Brinkman number and Deborah number values is constant and equals minus of $\Theta'(0)=1$.

The fluid elasticity effect in Nusselt number curves of the inner cylinder (Nu_T and Nu_q) are depicted as Brinkman number for different Deborah numbers and are presented in Figs. 12 and 13 respectively. As shown in Fig. 12, by enhancing the Brinkman number, the magnitude of Nusselt number (Nu_T) enhances monotonically in all dominant of the Brinkman number due to frictional dissipation. The heat transfer rate (Nu_T) has increased due to the dependence of the fluid elasticity temperature. The same expression can be presented for the Isoflux case (as shown in Fig. 13). It is worthy of note that the rotating cylinder is cooled but in the stationary cylinder is heated.

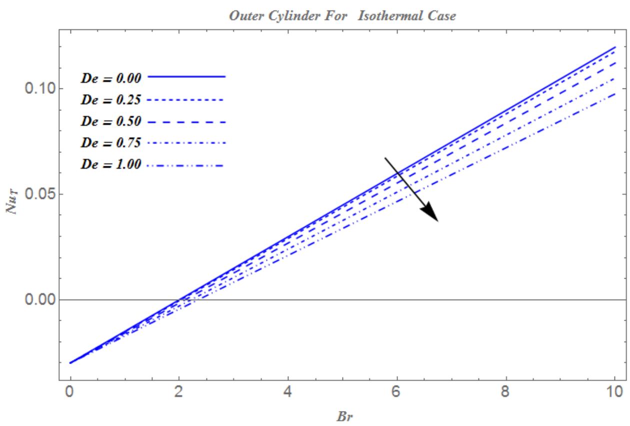

Fig. 10. Nusselt number of outer cylinder versus Brinkman number in the Isothermal case for different *De* number.

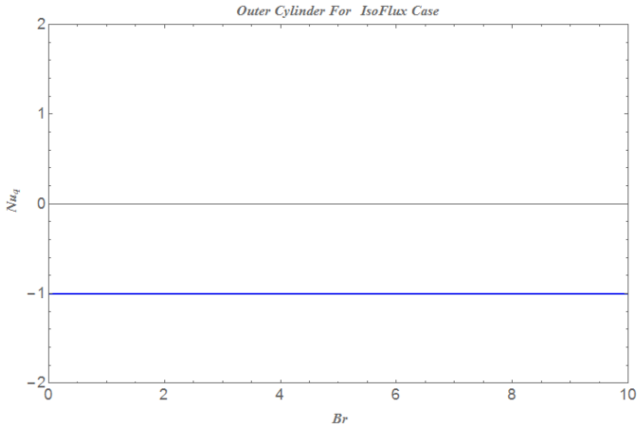

Fig. 11. Nusselt number of outer cylinder versus Brinkman number in the isoflux case for different *De* number.

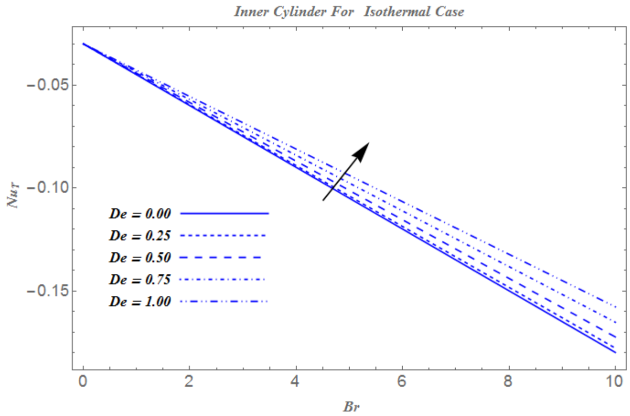

Fig. 12. Nusselt number of inner cylinder versus Brinkman number in the Isothermal case for different *De* number.

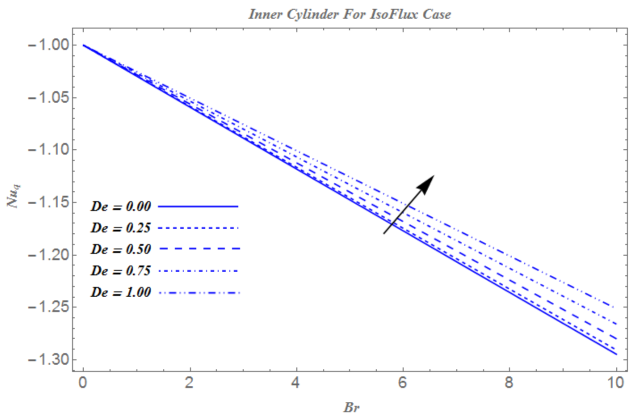

Fig. 13. Nusselt number of inner cylinder versus Brinkman number in isoflux case for different *De* number.

5- Conclusion

In this work, the forced convection heat transfer in circular couette flow of Non-Newtonian fluid is investigated when the inner cylinder is revolved at angular speed and the outer cylinder is fixed. The fluid viscosity is considered concurrently dependent on the temperature and shear rate. The temperature dependency of viscosity is modeled exponentially according to the Nahme law and dependence of viscosity on shear is modeled with the Carreau equation. Four different types of thermal conditions are carried out in two cases (Isothermal case and Isoflux case). In Isothermal case, the same and different uniform temperature is applied over the inner and the outer cylinders. In Isoflux cases, the inner cylinder is maintained at a uniform constant temperature and a uniform constant and zero heat flux is applied to the outer cylinder. The perturbation method is presented to construct analytical approximation expressions for temperature and Nusselt number in the exponential series form. Instead of employing other difficult and sophisticated methods, the achievement of the method for this case, must be anticipated as a possibility for its application to other nonlinear problems. It is concluded that the drafted method is highly accurate.

The effect of viscous dissipation and the fluid elasticity are examined on the temperature and Nusselt number curves as Brinkman number and Deborah number, respectively. In the isothermal case, as the shear rate increases by enhancing Brinkman number, the base flow temperature increases monotonically due to frictional loss and is maximized within the annular gap. The value of maximum temperature reduces inside the annular gap, as the fluid elasticity increases by enhancing Deborah number which is resulted from the elasticity effect of Carreau fluid, where the fluid viscosity reduces by enhancing Deborah number. The same expression can be presented for the isoflux case; however, these trends are much more obvious for the Isothermal case than that for the isoflux case. In the isothermal and Isoflux cases, the heat transfer rate (Nu_T and Nu_q) in inner cylinder is enhanced due to the temperature dependence of the fluid elasticity while it is reduced in the outer cylinder.

Nomenclature

Br	Brinkman number
C_n	Integration constants, n=0, 1, 2
C_p	Specific heat at constant pressure
D	Annular space
De	Deborah number ($De=\lambda Ri\Omega/D$)
h	Convection Heat Transfer Coefficient
k_c	Conductivity Coefficient
n	power-law exponent
Na	Nahme number
Nu	Nusselt Number
P	pressure
Pe	Peclet number
r	Radial distance
R	Cylinder radius
Re	Reynolds number ($Re=R_i\Omega D/v_0$)
t	Time
T	Temperature
Ta	Taylor number
u	Dimensionless velocity vector
U	velocity vector
z	Axial distance

Greek symbols

Γ	Rate-of-strain tensor ($\dot{\Gamma}=\nabla U+(\nabla U)^T$)
Θ	Dimensionless temperature
Φ	Thermal dissipation ($\Phi=\tau:\nabla U$)
Ω	Angular velocity of inner cylinder
β	Viscosity temperature sensitivity
ε	Gap-to-radius ratio ($\varepsilon=D/R_1$)
η	Dimensionless viscosity
θ	Tangential coordinate
λ	Fluid Elasticity (Relaxation time)
μ	Viscosity
μ_0	Zero-shear-rate viscosity
μ_∞	Infinite-shear-rate viscosity
ρ	Density
τ	Shear stress ($\tau=\mu\dot{\Gamma}$)

Subscripts

$,t$	Refers to partial differentiation respect to time
i	Refers to the inner cylinder
o	Refers to the outer cylinder
r	Refers to the radial direction
q	Refers the isoflux boundary condition
T	Refers to the isothermal boundary condition
z	Refers to the axial direction
θ	Refers to the tangential direction

References

[1] C.D. Andereck, S.S. Liu, Swinney, H.L. Flow regimes in a circular Couette system within dependently rotating cylinders. *Journal of Fluid Mechanics*, 164 (1986) 155-183.

[2] N. Ashrafi, A. Hazbavi, Flow pattern and stability of pseudoplastic axial Taylor–Couette flow, *International Journal of Non-Linear Mechanics*, 47 (2012), 905-917.

[3] J. R. A. Pearson, Mechanics of Polymer Processing, *Elsevier*, London, (1985).

[4] N. Phan-Thien, R.I. Tanner, New constitutive equation derived from network theory, *Journal Non-Newtonian Fluid Mech*, 2 (1977) 353–365.

[5] N. Phan-Thien, A non-linear network viscoelastic model, *Journal Rheol*, 22 (1978) 259–283.

[6] P.J. Oliveira, F.T. Pinho, Analytical solution for fully developed channel and pipe flow of Phan-Thien–Tanner fluids, *Journal Non-Newtonian Fluid Mech*, 387 (1999)

271–280.

[7] F. T. Pinho, P. J. Oliveira, Axial annular flow of a nonlinear viscoelastic fluid an analytical solution, *Journal Non-Newtonian Fluid Mech*, 93 (2000) 325–337.

[8] M. A. Alves, P.J. Pinho, F.T. Oliveira, Study of steady pipe and channel flows of a single-mode Phan Thien–Tanner fluid, *Journal Non-Newtonian Fluid Mech*, 101 (2001) 55–76.

[9] D.O.A. Cruz, F.T. Pinho, Skewed Poiseuille-Couette flows of PTT fluids in concentric annuli and channels, *Journal Non-Newtonian Fluid Mech*, 121 (2004) 1–14.

[10] M. Mirzazadeh, M.P. Escudier, F. Rashidi, S.H. Hashemabadi, Analytical solution of purely tangential flow for PTT viscoelastic fluid through concentric annulus, *Journal Non-Newtonian Fluid Mech*, 129 (2005) 88–97.

[11] R.B. Bird, P.J. Dotson, N.L. Johnson, Polymer solution rheology based on a finitely extensible bead-spring chain model, *Journal Non-Newtonian Fluid Mech*, 7 (1980) 213–235.

[12] .J. Oliveira, An exact solution for tube and slit flow of a FENE-P fluid, *Act a Mech*, 158 (2002) 157–167.

[13] D.O.A. Cruz, F.T. Pinho, P.J. Oliveira, Analytical solutions for fully developed laminar flow of some viscoelastic liquids with a Newtonian solvent contribution, *Journal Non-Newtonian Fluid Mech*, 132 (2005) 28–35.

[14] Tasnim, S. H., Mahmud S. and Mamun, M. A. H., Entropy generation in a porous channel with hydromagnetic effect, Exergy, *an Int. Journal*, 2 (2002) 300-308.

[15] Mahmud, S. and Fraser R. A., The second law analysis in fundamental convective heat transfer problems, *Int. J. of Therm. Sci.*, 42 (2003) 177–186.

[16] Carrington, C. G. and Sun, Z. F., Second law analysis of combined heat and mass transfer in internal flow and external flows. *Int. J. Heat and Fluid Flow*, 132 (1992) 65–70.

[17] Arpaci, V.S. and Selamet A., Entropy production in boundary layers, *J. Thermo phys. Heat Transfer*, 4 (1990) 404–407.

[18] Abu-Hijleh, B. A. K., entropy generation in laminar convection from an isothermal cylinder in cross flow, *energy*, 23 (1998) 851-857.

[19] Khalkhali, H. Faghri, A. and Zuo, Z. J., Entropy generation in a heat pipe system, *Applied Thermal Eng.*, 19 (1999) 1027-1043.

[20] N. Ashrafi, A. Hazbavi, Heat transfer in flow of nonlinear fluids with viscous dissipation, *Archive of Applied Mechanics*, 83 (2013) 1739-1754.

[21] A. Hazbavi, Second Law Analysis of Magnetorheological Rotational Flow with Viscous Dissipation, *Journal of Thermal Science and Engineering Applications*, 8 (2016) 021020.

[22] K. Khellaf, G. Lauriat, Numerical study of heat transfer in a non-Newtonian Carreau-fluid between rotating concentric vertical cylinders, *Journal Non-Newtonian Fluid Mech*, 89 (2000) 45–61

[23] R.M. Manglik, P. Fang, Thermal processing of viscous non-Newtonian fluids in annular ducts: effects of power-law rheology, duct eccentricity, and thermal boundary conditions, *Int. J. Heat Mass Transfer*, 45 (2002) 803–814.

[24] R.B. Bird, R.C. Armstrong, *Dynamics of Polymeric Liquids*, Wiley, New York, (1987).

[25] R. Keunings, M. J. Crochet, Numerical simulation of the flow of a viscoelastic fluid through an abrupt contraction, *J. Non-Newtonian Fluid Mech*, 14 (1984) 279–299.

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