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The Effect of Road Quality on Integrated Control of Active Suspension and Anti-lock Braking Systems

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ABSTRACT: This paper investigates the effect of road quality on the control strategies of active suspension system integrated with anti-lock braking system in a quarter-car vehicle model. To this aim, two optimal control laws for active suspension system and anti-lock braking system are analytically designed using the responses prediction of a continuous 4 degree of freedom non-linear vehicle model including longitudinal and vertical dynamics. The optimal feature of the suspension controller provides the possibility of adjusting the weighting factors to meet the ride comfort and road holding criteria on roads with various qualities. It is shown that, regulating the tire deflection in a constant value to increase the tire normal load leads to instability of suspension system. Therefore, the active suspension system cannot influence on the anti-lock braking system performance on flat roads in a quarter car model. The same effect is observed for hard braking on irregular roads with good quality. In this condition, the active suspension system should be focused on the ride comfort as its first aim. However, for braking on irregular roads with poor quality, decreasing the variation of tire deflection to avoid the tire from jumping is effective in reducing the stopping distance.

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1- Introduction

The purpose of integrated vehicle control is to combine and supervise two or more control systems affecting vehicle dynamic responses to enhance vehicle safety, ride comfort and handling performances. The Active Suspension System (ASS) and the Anti-lock Braking System (ABS) are two common control systems employed to attain the ride comfort and safety of vehicle during braking, respectively.

The ABS is employed to generate the maximum braking force and improves the vehicle safety by preventing the wheel from being locked. In the past, different control methods have been employed for the design of the conventional ABS controller [1-4]. In these studies, only the longitudinal dynamics is employed for braking control on flat roads. However, on rough roads, the braking can be influenced by tire normal force which is variable [5].

In normal driving conditions on rough roads, the ASS improves the ride performance by isolating the vehicle from road irregularities and reducing the sprung mass acceleration [6, 7]. For the other purpose during hard braking, the ASS controller can keep a good contact between the tire and road by controlling the tire deflection for improved safety. Because of high importance of vehicle safety, it is preferred for ASS to focus on controlling the tire deflection rather than the body acceleration when needed.

It is found that the ASS is a multi-objective system that its control strategy is dependent on driving conditions [7, 8]. For integrated suspension with other control systems, the interaction of tire normal force with tire longitudinal and lateral forces is a key factor. In what follows, some researchers were focused on the interaction between the ASS and ABS. Lin and Ting [9] used two back-stepping controllers for decentralized integration of ASS and ABS in a quarter car model. In this study, the stopping distance was improved by increasing tire normal force, but the performance of suspension system was ignored. Wang et al. [10] investigated a Takagi-Sugeno (T-S) fuzzy neural network for integration of ASS and ABS. In another work, fuzzy and sliding mode control was used for integration of braking and steering control systems [11]. The results indicated that the proposed switching multi-layer control strategies improved the vehicle performance in different situations. In order to control the vertical and yaw motions, Vassal et al. [12] used the gain-scheduled method for both suspension and braking control systems. Kaldas and Soliman [13] investigated the influence of preview controller for both ASS and ABS systems in half car model. In this work, the ride performance of vehicle is considered in terms of ride comfort and road holding and the braking performance is evaluated in terms of braking distance. Riaz and Khan [14] designed a neuro-fuzzy based adaptive control scheme for ASS and a sliding mode control for ABS, for each station of vehicle. Zhang et al. [15] proposed a linear quarter model of active suspension system and used barrier Lyapunov function control method for both ASS and ABS systems. The results of this work showed that, the ASS can assist the ABS by increasing the tire vertical force to reduce the braking distance.

There is no doubt that a good contact between the tire and road leads to a positive interaction between suspension and braking systems [16]. This point has encouraged the researchers to enhance the ABS performance via the ASS control. However, in some cases, the possibility of this interaction has not

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been discussed regarding road qualities and considering all practical aspects of the problem. For example, in [9] the tire normal force is increased by controlling the tire deflection to decrease the stopping distance, is unfeasible for ASS. The feasibility of the mentioned strategy has been ignored yet. On the other hand, the integration policy may be different for braking on different roads including flat roads and irregular roads with good and poor qualities. This important issue has been paid less attention by the researchers in the past.

In this paper, two strategies are proposed for optimal control of ASS and ABS based on a 4 Degree Of Freedom (DOF) non-linear quarter-car model. The minimized performance indexes related to braking and suspension control are defined separately and the control laws are analytically derived using a prediction approach. In the derived optimal control laws, the control policy of suspension system can be determined by adjusting the controller weighting factors to focus on the safety or ride comfort criteria. On the other hand, the ABS controller is designed to follow the reference wheel slip such a way that the maximum braking force can be achieved at each time. Here, the interaction of ASS and ABS during braking on flat roads and irregular roads with different qualities are investigated. Also, the feasibility of existing integrated strategies in a quarter car model is discussed considering all practical aspects of the problem. Finally, the simulation studies are conducted using the 4-DOF non-linear vehicle model excited by the standard good and poor road profiles according to ISO-8608 and the results are compared by the frequency-weighted Root Mean Square (RMS) of the vertical body acceleration according to ISO-2631 standard.

2- Modeling

2- 1- Vehicle dynamics

According to Fig. 1, a 4-DOF non-linear quarter car model including longitudinal and vertical dynamics is used to design the integrated control system.

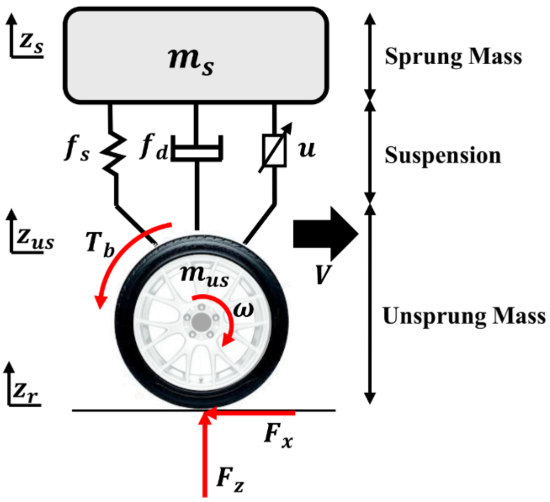

Fig. 1. 4DOF non-linear vehicle model

For the vehicle model, 2-DOF are related to the vertical motion of the sprung mass, m_s and un-sprung mass, m_{us} . The other two degrees of freedom involving brake parts include the wheel angular speed, ω , and the vehicle velocity, V . In this model, f_s is the nonlinear force of suspension spring and f_d is the nonlinear damper force. In addition, z_r is the road profile input, z_s and z_u are the sprung and un-sprung mass displacements from free length of springs.

The equations of motion for sprung and un-sprung masses shown in Fig. 1 can be obtained by using Newton's second law as,

$$m_s \ddot{z}_s = -f_s - f_d - m_s g + u \tag{1}$$

$$m_{us} \ddot{z}_u = f_s + f_d - f_{st} - f_{dt} - m_{us} g - u \tag{2}$$

In real conditions and for hard road irregularities, f_s and f_d are nonlinear functions of suspension deflection, z_s-z_u , and suspension velocity, $\dot{z}_s$-$\dot{z}_u$, respectively, as [6]:

$$f_s = K_{s1}(z_s - z_u) + K_{s2}(z_s - z_u)^2 + K_{s3}(z_s - z_u)^3 \tag{3}$$

$$f_d = C_{s1}(\dot{z}_s - \dot{z}_u) + C_{s2}(\dot{z}_s - \dot{z}_u)^2 \tag{4}$$

The elastic and damping forces of tire can be modeled as,

$$f_{st} = K_t(z_u - z_r) \tag{5}$$

$$f_{dt} = C_t(\dot{z}_u - \dot{z}_r) \tag{6}$$

where K_{s1} , K_{s2} and K_{s3} are the suspension spring coefficients, C_{s1} , C_{s2} and C_{s3} are the suspension damper coefficients, K_t is the tire spring coefficient, C_t is the tire damper coefficient and u is the active suspension force determined by the control law. Note that the summation of f_{st} and f_{dt} cannot be positive and its positive value is saturated to zero when tire jumps.

The governing equations for the motions of wheel are as follow [1]:

$$\dot{V} = -\frac{1}{M_t}(F_x) \tag{7}$$

$$\dot{\lambda} = -\frac{1}{V}\left[\frac{F_x}{M_t}(1-\lambda) + \frac{R^2 F_x}{I_t}\right] + \left(\frac{R}{VI_t}\right)T_b \tag{8}$$

where R is the wheel radius, I_t denote the total moment of inertia of the wheel, V is the longitudinal velocity of the vehicle, λ is the wheel longitudinal slip, T_b is the braking torque, F_x is the longitudinal tire force and M_t is the total mass of the quarter vehicle. Note that, the tire radius variation is assumed to be neglected.

For driving Eq. (8), the wheel longitudinal slip during braking, $V>R\omega$, is calculated as,

$$\lambda = \frac{V - R\omega}{V} \tag{9}$$

in which ω is the angular velocity of the wheel.

In order to describe the saturation property of the tire force in the prescribed vehicle model and because of its simplicity and its good fitness to experimental data [17], the nonlinear Dugoff's tire model is used in this study. In the Dugoff's model, the relation for longitudinal force of tire in terms of longitudinal slip, road friction coefficient and normal force of tire is as follows [1]:

$$F_x = \frac{C_i \lambda}{1-\lambda} f(s) \tag{10}$$

where

$$f(s) = \begin{cases} S(2-S) & \text{if} \quad S < 1 \\ 1 & \text{if} \quad S \geq 1 \end{cases} \tag{11}$$

and

$$S = \frac{\mu F_z \left(1 - \varepsilon_r V \sqrt{\lambda^2 + \tan^2\alpha}\right)(1-\lambda)}{\sqrt{C_\lambda^2 \lambda^2 + C_\alpha^2 \tan^2\alpha}} \tag{12}$$

In the above tire model, μ is the friction coefficient of the road, C_λ and C_α are the tire longitudinal stiffness and cornering stiffness of the tire, respectively. ε_r is the factor of road adhesion reduction. The Dugoff's tire model considers the interaction of longitudinal and lateral forces of tire during combined braking and steering. For the straight-line braking, the slip angle α is applied to be zero.

It should be noted that the tire normal force F_z is the interaction point of the braking dynamics and suspension. The longitudinal braking force is directly dependent on the tire normal force. Also, F_z is the sum of f_{st} and f_{dt} forces in Eq. (2). This shows that the ASS can influence on the tire longitudinal dynamics by controlling the tire normal force.

2- 2- Road spectrum

Following the aim of this paper, the vehicle should be excited by different kinds of road profiles for simulating the performance of integrated braking and suspension control systems. However, in some studies [9], the road irregularity is ignored for integrated control systems and the road profile is assumed to be flat. The feasibility of integrated active suspension and braking systems on flat roads in a quarter-car model needs to be investigated.

According to the standard ISO8608, the road irregularities have been classified in classes A-H based on the Power Spectral Density (PSD) [18]. The relationship between the PSD, $S(\Omega)$, and the spatial frequency, Ω, for different classes of road roughness is shown in Fig. 2 by two straight lines with different slopes on a logarithmic scale.

In this paper, two different profiles with the qualities of poor (D) and good (B) are chosen to evaluate the proposed control strategies. According to the PSDs of these road profiles, two random road inputs are generated and transformed in terms of displacement as shown in Fig. 3a and 3b. These inputs are generated by inverse transform of PSD functions and their heterodyning via random initial phases [18, 19]. Fig. 3c shows the PSD transform of generated random road profiles to verify the PSD inverse and heterodyning functions.

Note that the transformation of the road profile expressed in terms of displacement to that in terms of time is through the vehicle speed.

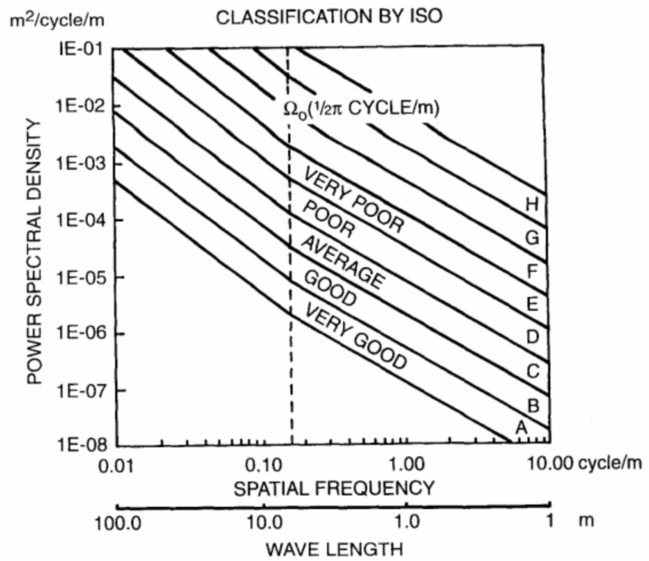

Fig. 2. ISO 8608 road quality clasification [18]

3- Controller Design

For integrated ASS and ABS, two optimal control laws with adjustable weighting factors are analytically designed using the prediction of non-linear responses of a 4-DOF quarter-car model including vertical and longitudinal vehicle dynamics. Firstly, the responses of nonlinear system in the next time interval are predicted based on Taylor series expansion. Then, the control input is calculated using a continuous optimization problem defined in terms of predicted tracking or regulation errors [1, 2, 6, 7, 20]. The proposed method leads to an analytical solution for the control problem that is suitable for online implementation and reduces the requirement of real-time embedded computer hardware. In what follows, the optimal nonlinear control method is employed for the design of ASS and ABS, respectively. After that the possible interaction of these systems on different road profiles including flat road and irregular roads with the qualities described in section 2 will be investigated and discussed.

3- 1- ASS controller design

The purpose of the ASS control system is to keep the sprung mass acceleration, suspension deflection and tire deflection close to their rest situations (static equilibriums) by using a minimum external control force [5-7, 9, 18, 20]. To achieve this aim, a prediction-based method is applied for calculating the external control force.

Considering vertical dynamics, the equations of motions (Eqs. (1) and (2)) in the state space forms are derived as,

$$\dot{z}_1 = z_2 - z_4 \tag{13}$$

$$\dot{z}_2 = f_1 + \frac{u}{m_s} \tag{14}$$

$$\dot{z}_3 = z_4 - \dot{z}_r \tag{15}$$

$$\dot{z}_4 = f_2 - \frac{u}{m_{us}} \tag{16}$$

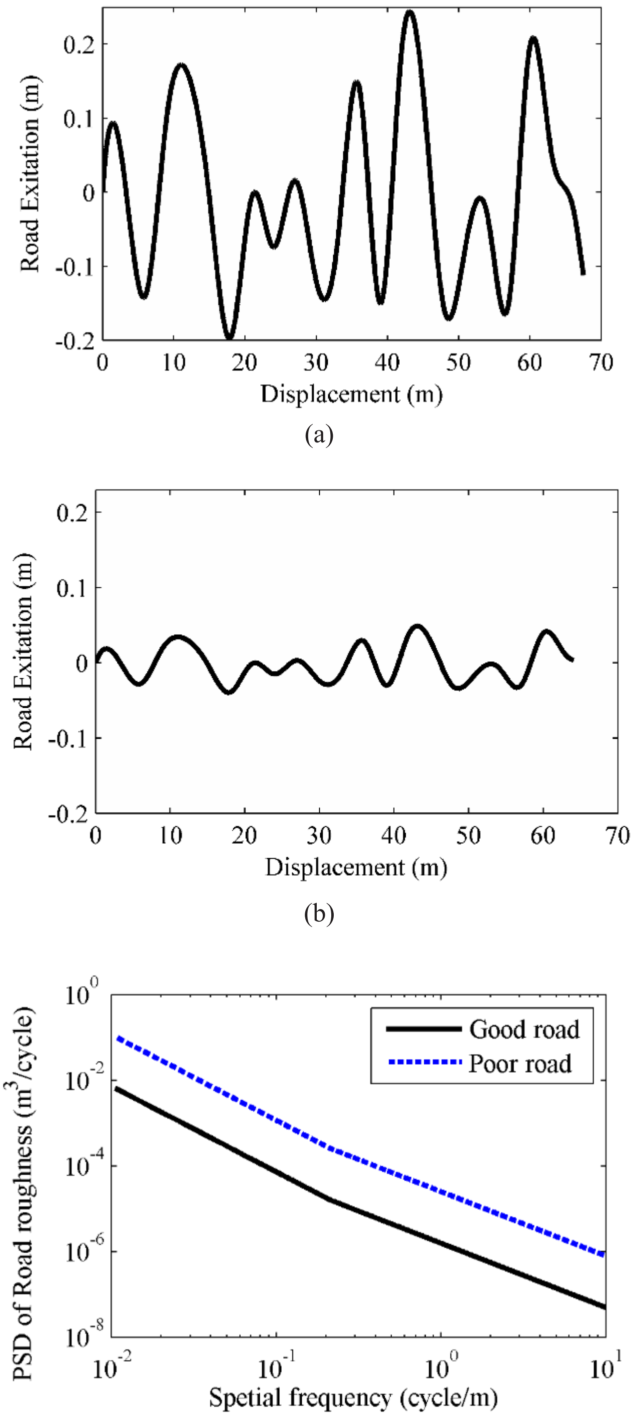

Fig. 3. Road excitation (a) poor road (b) good road and (c) PSD of road roughness

where $z_1=z_s-z_u$, $z_2=\dot{z}_s$, $z_3=z_u-z_r$ is the tire deflection, $z_4=\dot{z}_u$ is the tire vertical velocity. The functions f_1 and f_2 are defined in terms of suspension and tire spring and damper forces.

According to the requirements of suspension system, three control variables, z_1, z_2 and z_3, are taken as the outputs of the system, $y_1=[z_1, z_2, z_3]^T$. Therefore, a point-wise performance index minimizing the next instant output tracking errors is defined as follows [1, 4-7, 20]:

$$J_1 = \frac{1}{2}\sum_{i=1}^{3}\eta_i e_i^2(t+h_1) + \frac{1}{2}\eta_4 u^2(t) \tag{17}$$

where e_i is the defined as the error of each output as,

$$e_i = z_i - z_{i0} \tag{18}$$

and $\eta_i(i=1,2,3,4) \geq 0$ are weighting factors and h_1 is the predictive period. z_{i0} is the rest situations of z_i (static equilibriums) and should be tracked by each output. These reference values have been derived by solving Eqs. (13) to (16) in static situation ($z_2=z_4=u=0$). In the same way, z_{s0} is calculated from Eq. (1).

The predicted response for each output in the next interval, $t+h_1$, is approximated by a kth-order Taylor series expansion at t. The expansion order is taken to be equal to the relative degree of the system [1, 7, 20]. This selection which forces the control input to be constant in the predictive interval leads to small control efforts for nonlinear systems with small relative degrees.

According to Eqs. (13) to (16), the system dynamics has the well-defined relative degree, $r_1=1$ for z_2 and $r_2=2$ for both z_1 and z_3. Therefore, the first-order Taylor series is sufficient for z_2 and the second-order Taylor series is sufficient for z_1 and z_3 as,

$$z_1(t+h_1) = z_1(t) + h_1(z_2 - z_4) + \frac{h_1^2}{2!}\left[f_1 - f_2 + u\left(\frac{1}{m_s} + \frac{1}{m_{us}}\right)\right] \tag{19}$$

$$z_2(t+h_1) = z_2(t) + h_1\left(f_1 + \frac{u}{m_s}\right) \tag{20}$$

$$z_3(t+h_1) = z_3(t) + h_1(z_4 - \dot{z}_r) + \frac{h_1^2}{2!}\left(f_2 - \frac{u}{m_{us}} - \ddot{z}_r\right) \tag{21}$$

By using Eqs. (19) to (21), the performance index Eq. (17) can be obtained as a function of control input u. Now, the ASS control input, u, is derived by applying the optimality condition as follows:

$$\frac{\partial J_1}{\partial u} = 0 \tag{22}$$

which leads to:

$$u(t) = c\left\{\eta_1 d_1\left[e_1(t) + h_1(z_2 - z_4) + \frac{h_1^2}{2!}(f_1 - f_2)\right] + \eta_2 d_2\left[e_2(t) + h_1 f_1\right] + \eta_3 d_3\left[e_3(t) + h_1(z_4 - \dot{z}_r) + \frac{h_1^2}{2!}(f_2 - \ddot{z}_r)\right]\right\} \tag{23}$$

where

$$c = -\frac{1}{\eta_1 d_1^2 + \eta_2 d_2^2 + \eta_3 d_3^2 + \eta_4},$$

$$d_1 = \frac{h_1^2}{2!}\left(\frac{1}{m_s} + \frac{1}{m_{us}}\right), \quad d_2 = \frac{h_1}{m_s}, \quad d_3 = -\frac{h_1^2}{2m_{us}}$$

The control law (Eq. (23)) with adjustable parameters can be used to meet ride comfort and road holding criteria. In integration with the ABS, various strategies for the ASS can be considered regarding different road qualities in the quarter car vehicle model. In the following, the possibility of various strategies on the flat road and irregular roads with good and poor qualities are presented.

3- 1- 1- ASS strategy on flat road

The only way for integration of ASS and ABS on the flat road is to regulate the tire deflection in a constant nonzero value via active suspension system in order to increase the tire normal force. In this strategy presented by Lin and Ting [9], the vehicle stopping distance is decreased on the flat road by increasing the tire normal force in a quarter car model. However, there are some defects in this strategy which will be revealed by some simple analyses. Before that, a definition used in the analysis is given in the following.

Definition 1: Tire deflection variation is defined as the difference between the static loaded radius and the dynamic loaded radius of the tire, which will be denoted by $\hat{z}_u = z_u - z_{u0}$ [9, 21].

In the quarter car model, the extra normal force of tire can be generated by tracking a constant nonzero value of tire deflection variation via the active suspension system. This strategy called tire squeezing [9] may be able to affect the ABS controller performance by remarkably reducing the stopping distance. However, the stability of the suspension system is deteriorated in this strategy and the control force goes to infinity according to the following theorem. Note that when the road irregularity is ignored, the performance of suspension controller has been discussed only by tracking the desired tire deflection which is usually a constant negative value, although the suspension controller performance has been often studied in the presence of road profiles [10, 20, 22].

Theorem 1: In the quarter car model, tracking a constant nonzero value of tire deflection variation yields to instability in the suspension system and needs an infinite suspension actuator force.

Proof: Eqs. (2) and (3) are rewritten as follow:

$$m_s \ddot{z}_s = -F_{Strut} + u - m_s g \tag{24}$$

$$m_{us} \ddot{z}_u = F_{Strut} - u - F_{Tire} - m_{us} g \tag{25}$$

where the forces are defined as,

$$F_{\text{Tire}} = F_z = f_{st} + f_{dt} \tag{26}$$

$$F_{\text{Strut}} = f_s + f_d \tag{27}$$

Due to the lack of road irregularities, $z_r = \dot{z}_r = 0$. When the tire deflection variation tracks a constant value, the steady state form of Eq. (26), by applying $\dot{z}_u = \ddot{z}_u = 0$ and using Eq. (25), yields

$$F_{\text{Tire}} = K_t z_u = K_t \left(\hat{z}_u + z_{u0} \right) = K_t \hat{z}_u + \left(m_{us} + m_s \right) g \tag{28}$$

Substituting Eq. (28) into Eq. (25) in the above situation yields,

$$F_{\text{Strut}} - u = K_t \hat{z}_u + m_s g \tag{29}$$

Eq. (24) can be rewritten according to Eq. (29) as

$$\ddot{z}_s = \frac{-K_t}{m_s} \hat{z}_u \tag{30}$$

When the tire deflection $\hat{z}_u$ takes a negative constant value, the sprung mass acceleration will be a positive constant value. This means that the sprung mass moves up rapidly with increasing speed and its height will be physically infinite during the short time of braking. Therefore, the suspension system is unstable and theorem 1 is proved.

As an example of Theorem 1, if assume that the controller regulates -0.005*m* for the tire deflection, it causes acceleration in the sprung mass about 2.5 m/s^2. Therefore, within about 2.5 seconds which takes to stop the vehicle, the sprung mass height (z_s) is increased more than 7.8 meters and its vertical speed (x_5) reaches up to 6.3 m/s from zero initial conditions. This condition is impossible for vehicle during braking and indicates the impractically of tire squeezing strategy. The above analysis will be simulated in the last section of the paper. From Theorem 1, it is concluded that squeezing the tire must not be used in the integrated ASS and ABS to decrease the stopping distance.

3- 1- 2- ASS strategy on irregular roads

As seen in the previous section, the tire squeezing yields to instability in suspension dynamics. Therefore, the proposed controller for ASS should track the static equilibrium points of suspension states, i.e. zero value for the tire deflection variation and suspension acceleration. According to the control law (Eq. (23)), various criteria can be provided by regulation of the weighting factors η_1 , η_2 and η_3 as the free parameters. By increasing the weighting factor of each output, the better tracking for the corresponding output is provided. The exact tracking for each output can be achieved by setting the other weighting factors to zero.

The primary purpose of controlling the suspension system is to ensure the ride comfort. Limiting the tire deflection to improve the road holding is of secondary importance. The weighting factor of each output considered in the proposed control law determines the importance of each criterion relative to the other. For example, when the ride comfort is considered as the only design criterion, the special case of the control law, which tracks only the sprung mass motion, is derived by applying $\eta_1 = \eta_3 = 0$. This case denotes “first mode” of suspension control in this paper. Also, to provide a good tire-road adhesion, the “second mode” is proposed to only reduce the tire deflection variations by applying $\eta_2 = \eta_3 = 0$. The tire-road adhesion is of great importance in hard braking maneuverer. In all the cases mentioned above, the cheap control strategy is considered in which the weighting factor of control input is equal to zero, $\eta_4 = 0$, and there is no limitation on the control input. In the simulation results, the two modes of ASS are evaluated in integration with ABS considering road irregularity with different qualities.

3- 2- ABS controller design

The purpose of ABS control is to maintain the wheel slip, $\lambda(t)$, close to its desired response, λ_d . This control variable is considered as the output of the system, $y_2=\lambda$. Again, a performance index that penalizes the next instant tracking error and the current control input is considered in the following form:

$$J_2 = \frac{1}{2}\rho_1 e_\lambda^{\,2}\left(t+h_2\right) + \frac{1}{2}\rho_2 T_b^{\,2} \tag{31}$$

where $e_\lambda=\lambda-\lambda_d$ is the tracking error of wheel slip, $\rho_1>0$, $\rho_2\geq 0$ are weighting factors and h_2 is the predictive period.
According to Eq. (8), the system longitudinal dynamics has the well-defined relative degree, $r_3=1$, for λ. Therefore, the first-order Taylor series is sufficient for expansion of λ,

$$\lambda\left(t+h_2\right) = \lambda\left(t\right) + h_2\left[\xi\left(\lambda, V\right) + \frac{R}{VI_t}T_b\right] \tag{32}$$

where

$$\xi\left(\lambda, V\right) = -\frac{1}{V}\left[\frac{F_x}{M_t}\left(1-\lambda\right) + \frac{R^2 F_x}{I_t}\right] \tag{33}$$

The control input, T_b , is derived by applying the optimality condition as follows.

$$\frac{\partial J_2}{\partial T_b} = 0 \tag{34}$$

which leads to,

$$T_b\left(t\right) = a\left[\rho_1 b\left\{e_\lambda + h_2\left(\xi - \dot{\lambda}_d\right)\right\}\right] \tag{35}$$

where

$$a = -\frac{1}{\rho_1 b^2 + \rho_2}, \quad b = \frac{h_2 R}{VI_t} \tag{36}$$

For the case of cheap control with no limitation on the control input, the weighting factor ρ_2 is taken to be zero. Applying the slip control law (Eq. (35)) with $\rho_2=0$, which is based on the nominal model, in the actual model (Eq. (8)) leads

$$\dot{\lambda} = \xi - \frac{1}{h_2}\left[e_\lambda + h_2\left(\hat{\xi} - \dot{\lambda}_d\right)\right] \tag{37}$$

Therefore, the error dynamics is derived as follows:

$$\dot{e}_\lambda + \frac{1}{h_2}e_\lambda = \left(\xi - \hat{\xi}\right) \tag{38}$$

Deviation of ξ from the nominal model $\hat{\xi}$ can be as a result of uncertainties in tire model and road condition. Because of saturation property of tire forces, it can be supposed that there exist a constant $\eta>0$, such that $|\xi-\hat{\xi}|<\eta$. In order to evaluate of the stability of controller in the presence of uncertainty, a candidate for Lyapunov function is considered as $V=(1/2)e_\lambda^{\,2}$.

Using Eq. (38) in the derivative of Lyapunov function gives

$$\dot{V} = -\frac{1}{h_2}e_\lambda^2 + \left(\xi - \hat{\xi}\right)e_\lambda \leq -\frac{1}{h_2}e_\lambda^2 + \eta\left|e_\lambda\right| \tag{39}$$

The second term on the right-hand side of Eq. (39) can be replaced by using the well-known inequality $ab \leq na^2+b^2/4n$ for any real a , b and $n>0$. Considering $n=1/4h_2$ leads,

$$\begin{aligned}\dot{V} &\leq -\frac{1}{h_2}e_\lambda^2 + \frac{1}{4h_2}e_\lambda^2 + h_2\eta^2 \\ &\leq -\frac{3}{4h_2}e_\lambda^2 + h_2\eta^2 \\ &\leq -\frac{3}{2h_2}V + h_2\eta^2\end{aligned} \tag{40}$$

By using the comparison lemma [23] and solving the first order differential equation, we have,

$$V = \frac{1}{2}e_\lambda^2 \leq \left[V\left(0\right) - \frac{2}{3}h_2^2\eta^2\right]e_\lambda^{-\frac{2}{3h_2}t} + \frac{2}{3}h_2^2\eta^2 \tag{41}$$

which implies that $e_\lambda(t)$ is uniformly bounded for all times and converges to the compact set $|e_\lambda| \leq 2h_2\eta/\sqrt{3}$. For any given $\varepsilon>0$, we can choose $0<h_2<\sqrt{3}\varepsilon/2\eta$ in the control law so that e_λ converges to $|e_\lambda| \leq \varepsilon$. Therefore, the stability of the controller in the sense of Lyapunov is proved.

4- Simulation Results

In this section, simulation results of the non-linear 4-DOF vehicle model with parameters of Table 1 are presented to show the integration policy of ABS and ASS on different road qualities. For the simulation study, the dynamic equations and control strategies are implemented and solved in MATLAB Simulink software. In the simulation, the initial velocity of the vehicle is considered 30 m/s at the start of hard braking. Since a flat road is not able to excite the vertical dynamics of quarter car model, two standard roads with the qualities of C and E, as shown in Fig. 3, are generated to study the effect of road profile on the ASS strategies. However, all the presented strategies for integrating braking and suspension systems on the flat and irregular roads are simulated and discussed. For all cases, the vehicle stopping distance together with the ride comfort indexes will be reported and compared.

Table 1. Parameters for the case study vehicle [1, 20]

Parameter	Value	Parameter	Value
M_t	390 kg	K_t	175500 N/m
m_s	350 kg	C_t	1500 N.s/m
m_{us}	40 kg	I_t	1.7 kg.m^2
R	0.3 m	C_i	50000
K_{s1}	19960 N/m	ε_r	0.015
K_{s2}	-73696 N/m^2	μ	0.8
K_{s3}	3170400 N/m^3	g	9.81 m/s^2
C_{s1}	1385 N.s/m	C_{s2}	524 N.s/m^2

4- 1- Simulation results on the flat road

Fig. 4 shows the vehicle stopping distance for hard braking on the flat road. The strategy of tire squeezing discussed in section 3.1.1, is employed in which the tire deflection variation tracks a constant negative value, namely -0.005 m. It is considered that the integrated ABS and ASS with the strategy of tire squeezing remarkably decreases the stopping distance in comparing with the case of stand-alone ABS. However, according to Theorem 1, some of the suspension states show instability in the controlled system according to Fig. 5. For example, the sprung mass displacement or body displacement is calculated about 11 meters and actuator force is $2.5*10^5$ (N) which is impossible in reality. It is concluded that the ASS cannot influence on the ABS performance on flat roads in a quarter car model. The same result can be seen in the paper [9] using the same strategy. But, unfortunately, the performance of suspension system has not been discussed there.

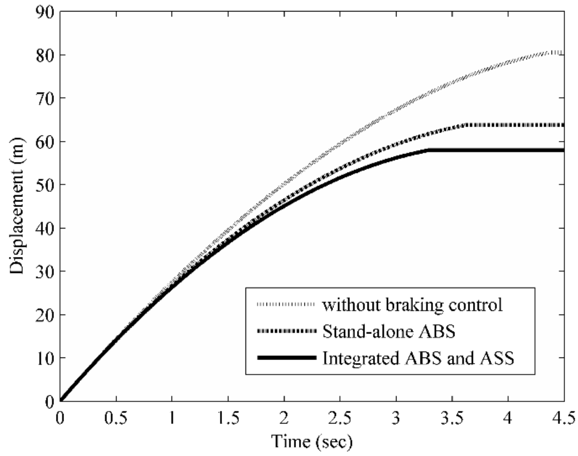

Fig. 4. Required braking distances with the use of different braking schemes

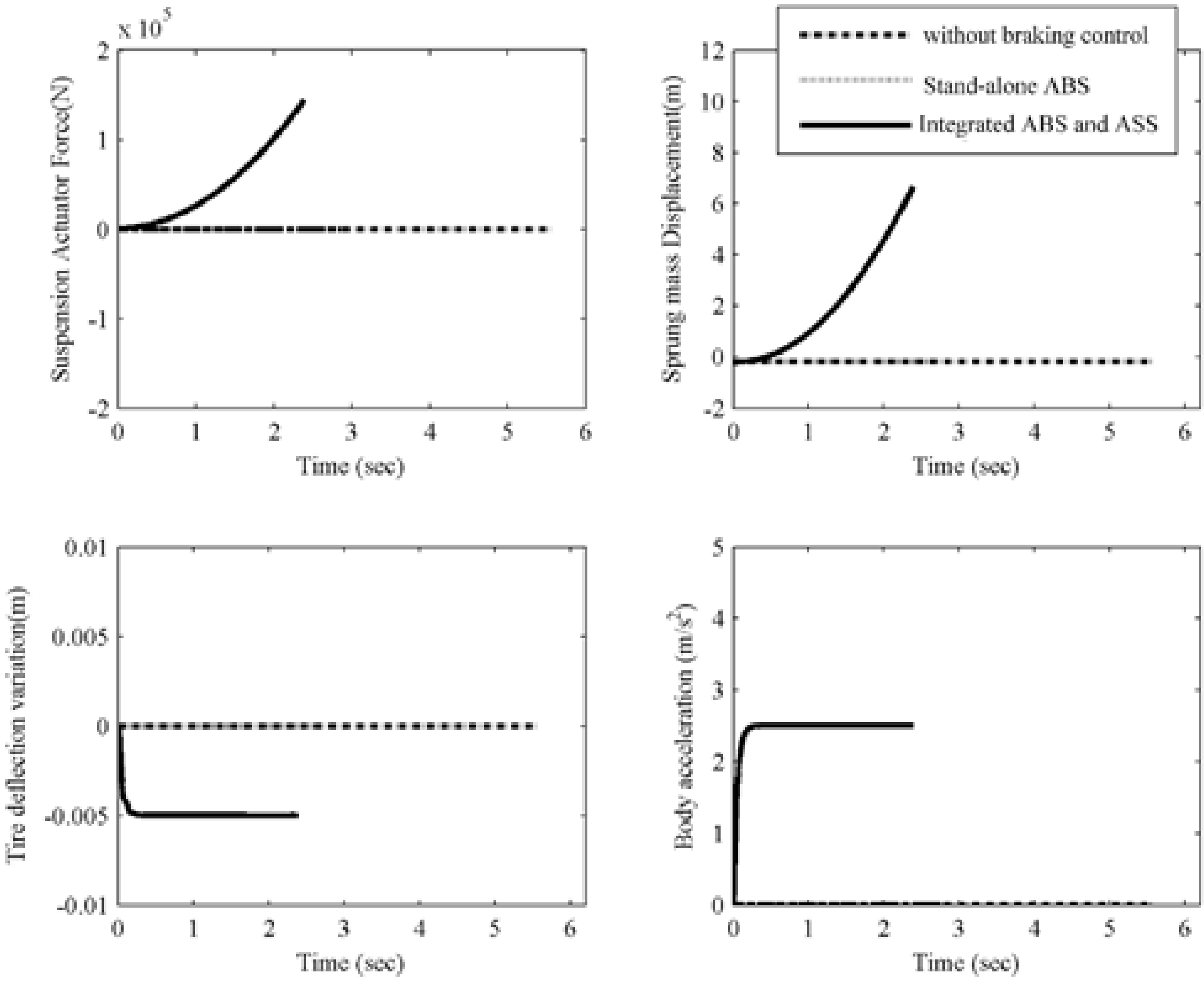

Fig. 5. Suspension system responses on the flat road with different braking schemes

4- 2- Simulation results of irregular roads

For braking on irregular roads, firstly, the simulation results of vehicle with manual braking and passive suspension system on the roads with poor and good qualities are carried out. The obtained responses have shown in Fig. 6 gives a basic result for comparing with various control strategies. Also, Fig. 6 clearly shows that the suspension outputs strongly depend on the road quality so that the body acceleration on the poor road is five times more than that on the good road. In the same

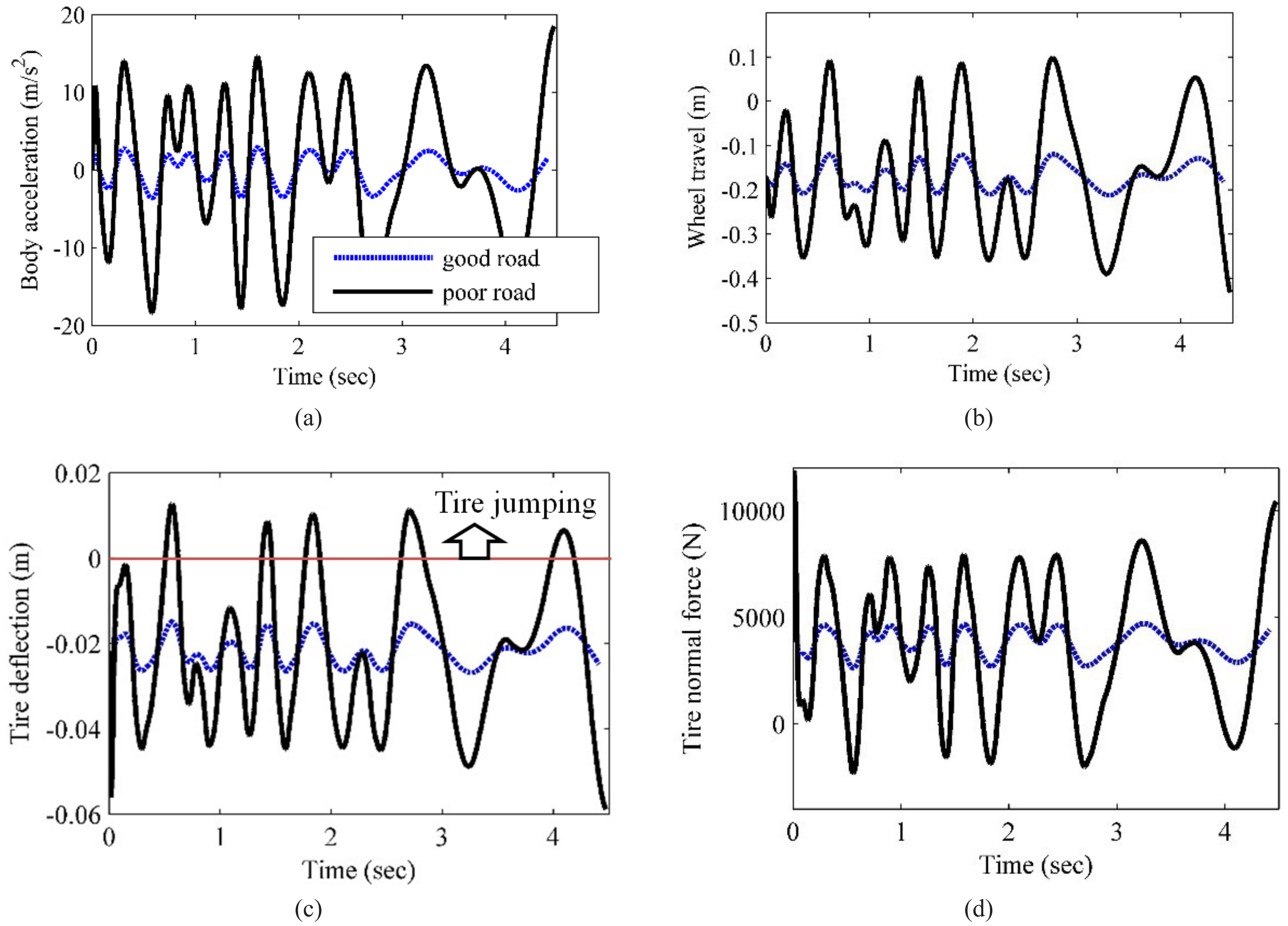

Fig. 6. Responses of passive suspension system on poor and good road profiles, (a) Body acceleration, (b) Wheel travel, (c) Tire deflection, (d) Tire normal force

manner, the tire deflection variations and the wheel travel are greatly increased on the poor road. The increment of tire deflection variation yields to the separation of the tire from the road. This phenomenon called tire jumping happens when the tire deflection is increased to positive values (Fig. 6c). The severe fluctuation of tire normal force on the poor road which results negative values in some instances indicates no contact between the tire and road (Fig. 6d).

On the other hand, the results of braking system show that the vehicle stopping distance are obtained 80.78 m and 81.88 m on the good and poor roads, respectively, by manual braking. When the vehicle is equipped only with ABS, the stopping distances are decreased about 16.72 m and 14.41m on the good and poor roads, respectively, but the suspension outputs are not changed. In fact, the ABS prevents the tire from being locked and consequently decreases the stopping distance.

For the next simulation, the performances of integrated ABS and ASS in the two modes are discussed. In the first mode, as mentioned in section 3.1.2, the ASS is set to decrease the body acceleration for ride comfort. However, in the second mode, the controller is regulated to decrease the tire deflection variations. Fig. 7 illustrates the suspension outputs on the poor road. As it is seen, by regulating the body acceleration in the first mode, the sprung mass acceleration is significantly decreased near zero. On the other hand, the tire deflection variation is more reduced in the second mode. Note that in both strategies, the tire deflection variation is reduced. However, this decrement in the first mode is not enough so that the tire jumping exist again in the system. In contrast, by direct weighting of the tire deflection variation in the second

Table 2. Stopping distance and RMS of suspension responses in different strategies on the poor road

Braking mode	Suspension mode	ASS mode	RMS of wheel travel error (mm)	RMS of Tire deflection error (mm)	RMS of body acceleration (m/s^2)	Stopping distance (m)	Stopping distance decrement (m)
manual	Passive	-	214	17.7	9.30	81.88	0
ABS	Passive	-	216	16.4	8.83	67.47	14.41 (82%)
ABS	Active	First mode	211	6.3	$\simeq 0$	64.74	17.14 (79%)
ABS	Active	Second mode	215	3.0	1.77	64.09	17.79 (78%)

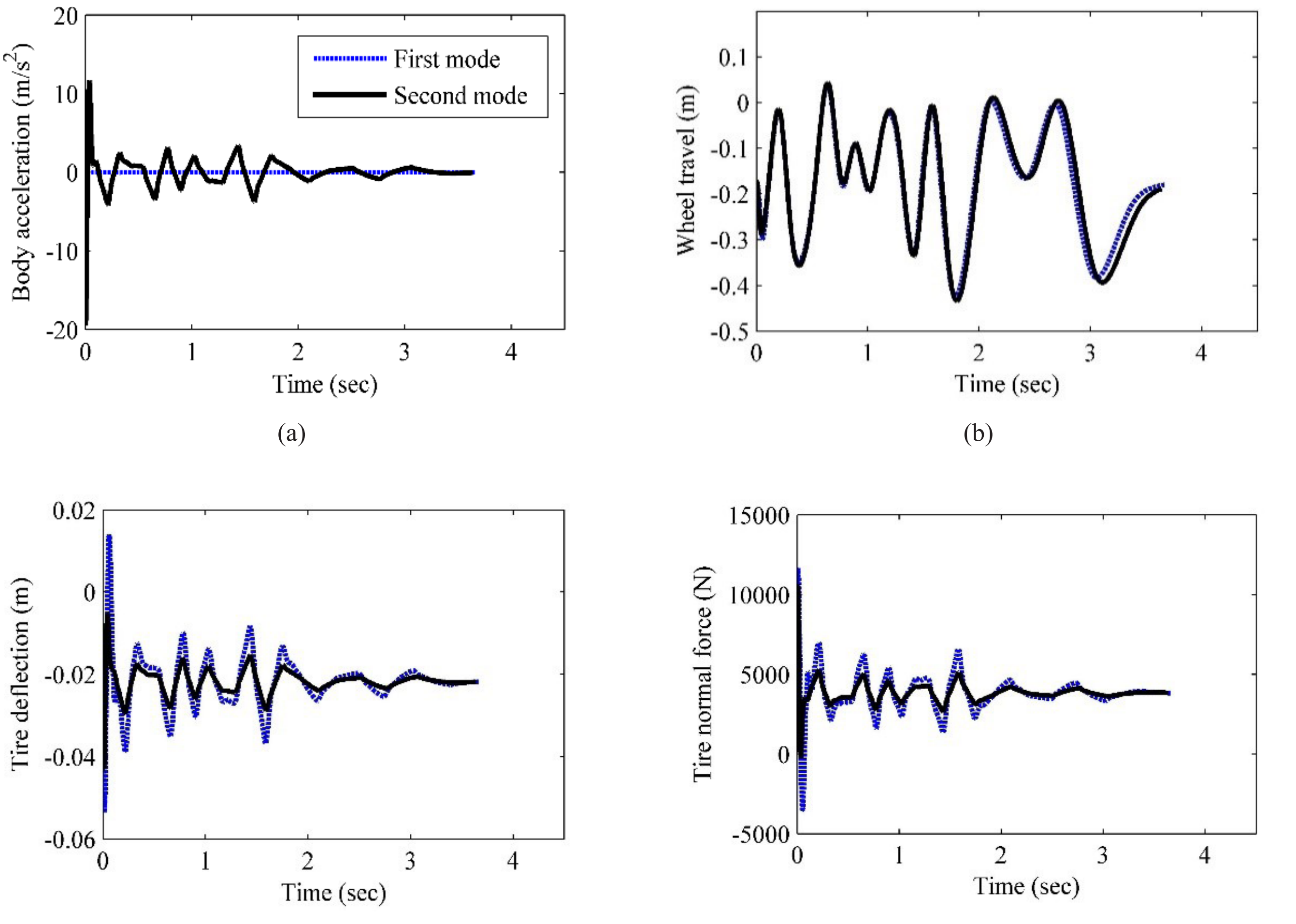

Fig. 7. Responses of suspension system on poor road in two different modes, (a) Body acceleration, (b) Wheel travel, (c) Tire deflection, (d) Tire normal force

mode, the tire jumping phenomenon is completely removed. As a result of reducing the tire deflection variation and removing the tire jumping, the stopping distance is decreased during braking of vehicle equipped with ABS.

In order to have a comprehensive comparison of different strategies in the cases of stand-alone and integrated control systems, the RMS of suspension system outputs together with the vehicle stopping distance are reported in Table 2. These results clearly show that the stopping distance has been more decreased if the ASS tries to reduce the tire deflection variations and remove the tire-road separation completely. As a result, for braking on the irregular road with poor quality, the integration of ASS and ABS improves the ABS and ASS performance simultaneously specially when the ASS tries to reduce the tire deflection variations and eliminates the tire-road separation.

Fig. 8 investigates the frequency-weighted RMS of the vertical body acceleration of the proposed strategies based on ISO 2631 [24] on the poor road. It can be seen that both “Second Mode” and “First mode” strategies deliver a lower frequency-weighted RMS body acceleration over the human body-sensitive frequency range, but the passive strategies in the first and second modes exceed the 24-hour and 8-hour working daily exposure limits.

In the final stage of simulation studies, the proposed integrated strategies are studied for braking on the road with good quality where the tire-road adhesion is provided by passive suspension. By activation of the control system on this road,

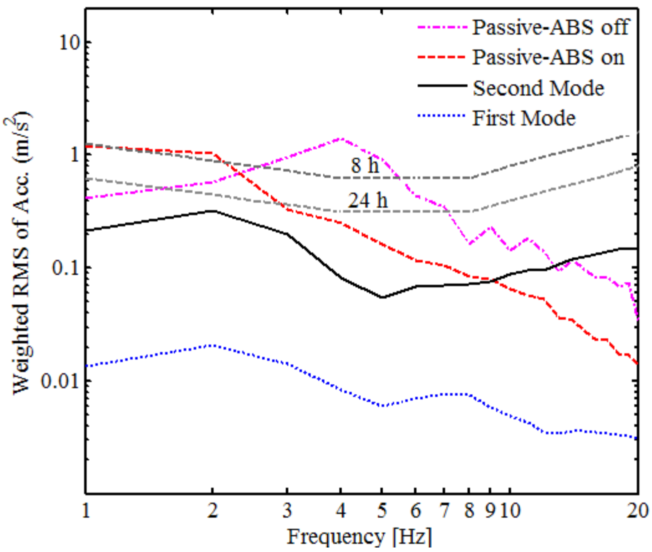

Fig. 8. Health risk assessment comparison between the proposed strategies on poor road

the effect of ASS on the ABS performance and overall vehicle behavior is investigated. The responses of vehicle suspension system with different integration strategies are compared in Fig. 9.

As it is seen, the ASS controller reduces the normal force and the tire deflection variations significantly specially in the second mode of ASS controller. Also, the ASS control is able to reduce the body acceleration which improves the vehicle ride performance. However, it is seen that on the good road where the tire jumping is not occurred, the more reduction of tire deflection variation cannot improve the ABS performance. To show this important result and in order to have a comprehensive comparison of different strategies in the cases of stand-alone and integrated control systems, the root mean square of suspension system outputs together with vehicle stopping distance are reported in Table 3. According to Table 3, the ASS controller decreases the body acceleration and tire deflection variations. The reduction of body acceleration means that the active suspension system improves the ride comfort. However, the stopping distance is not affected by different modes of ASS on the good road profile. As expected, when the ASS control is enabled, the body acceleration and tire normal force variation are decreased. The decrement of tire normal force variation yields the less stopping distance but not as much the braking on the poor road. This is for the reason that the tire jumping is not observed on the good road. Therefore, the preferred strategy for the ASS on the good road profile is its activation in the first mode by weighting on the body acceleration.

Table 3. Stopping distance and RMS of suspension responses in different strategies on the good road

Braking mode	Suspension mode	ASS mode	RMS of wheel travel error (mm)	RMS of Tire deflection error (mm)	RMS of body acceleration (m/s²)	Stopping distance (m)	Stopping distance decrement (m)
manual	Passive	-	172	3.3	1.78	80.78	0
ABS	Passive	-	173	3.5	1.86	64.06	16.72 (79%)
ABS	Active	First mode	174	1.25	≃0	63.81	16.97 (79%)
ABS	Active	Second mode	174	0.6	0.4	63.81	16.97 (79%)

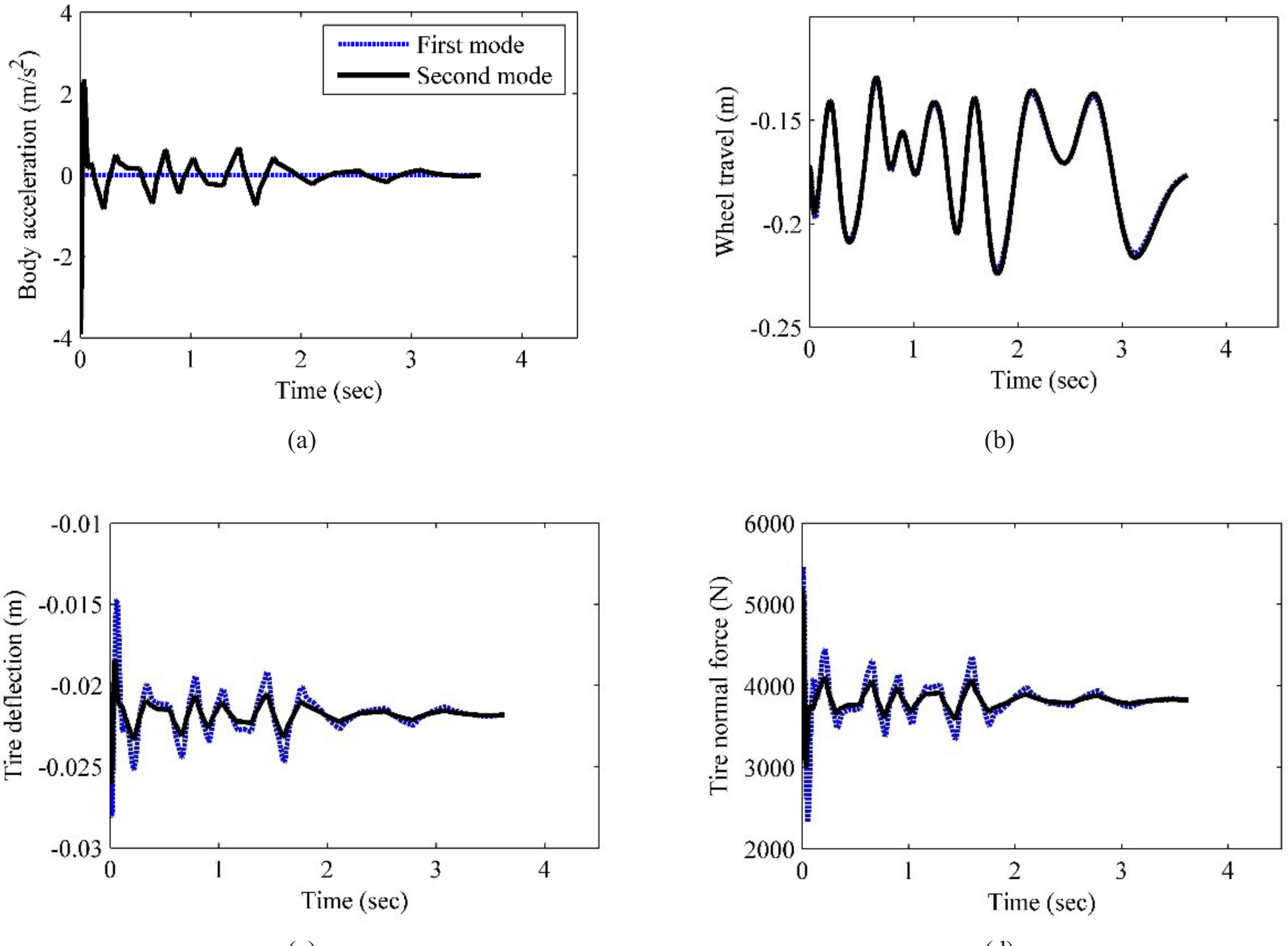

Fig. 9. Responses of suspension system on good road in two different modes, (a) Body acceleration, (b) Wheel travel, (c) Tire deflection, (d) Tire normal force

Fig. 10 presents the frequency-weighted RMS of the vertical body acceleration of the proposed strategies with respect to the threshold limit values as investigated by the ISO 2631 standard [24] on the good road. It can be seen that all proposed methods deliver a lower frequency-weighted RMS body acceleration over the human body-sensitive frequency range. However, the weighted RMS of passive system acceleration are close to 24-hour working daily exposure limit.

To evaluate the robustness of the designed controller in the presence of modeling uncertainties, the hard braking maneuver on the good road quality described in section 4.2 is considered again. Fig. 11 presents the performance of the proposed controllers in the presence of some parametric uncertainties. The parametric uncertainties are supposed to 20% changes in the sprung mass, coefficient of friction and spring coefficient from their nominal values. According to Fig. 11 the performance of the proposed controllers are preserved in the presence of uncertainties.

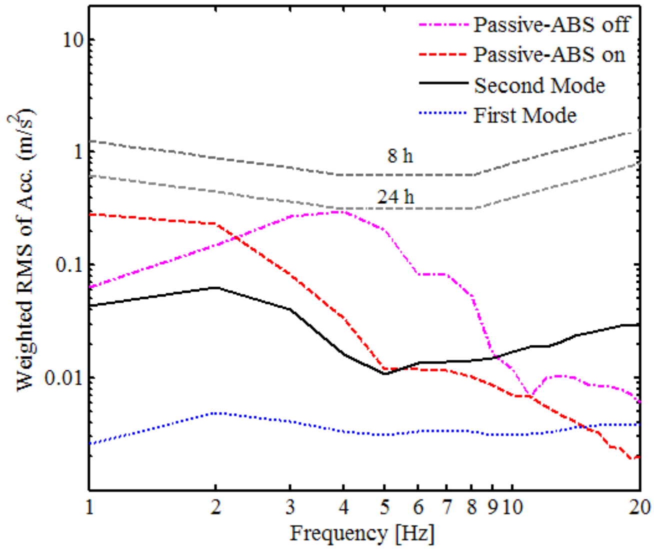

Fig. 10. Health risk assessment comparison between the proposed strategies on good road

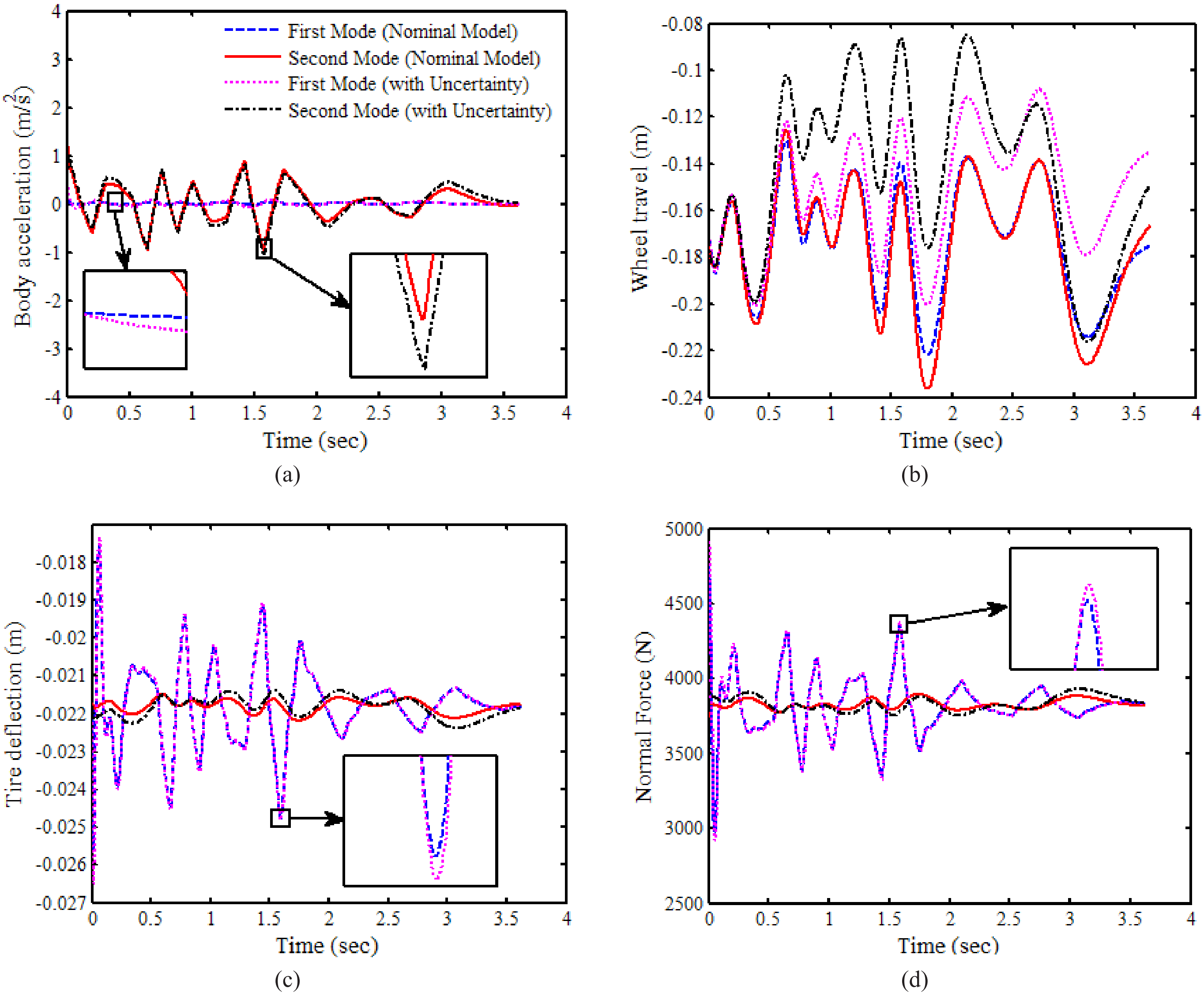

Fig. 11. Responses of suspension system on good road in two different modes with uncertainties, (a) Body acceleration, (b) Wheel travel, (c) Tire deflection, (d) Tire normal force

Table 4. Stopping distance and RMS of suspension responses in different strategies on the good road in the presence of parameter uncertainities

Braking mode	Suspension mode	ASS mode	RMS of wheel travel error (mm)	RMS of Tire deflection error (mm)	RMS of body acceleration (m/s²)	Stopping distance (m)	Stopping distance decrement (m)
manual	Passive	-	172	3.3	1.78	80.78	0
ABS	Passive	-	173	3.6	1.89	64.12	16.66
ABS	Active	First mode	174	1.28	≃0	63.87	16.91
ABS	Active	Second mode	174	0.64	0.42	63.87	16.91

Table 5. ISE and ISCI of integrated ABS and ASS controllers in different control strategies on the good road in the presence of parameter uncertainities

Braking mode	Suspension mode	ASS mode	$\int_{t=0}^{t_f} e_1^2 dt$	$\int_{t=0}^{t_f} e_2^2 dt$	$\int_{t=0}^{t_f} e_\lambda^2 dt$	$\int_{t=0}^{t_f} u^2 dt$	$\int_{t=0}^{t_f} T_b^2 dt$
manual	Passive	-	4.75	182.32	3.51	0	0
ABS	Passive	-	2.68	101.35	0.026	0	3022
ABS	Active	First mode	0.04	17.65	0.024	8295	2654
ABS	Active	Second mode	0.61	2.96	0.024	8601	2602

The root mean squares of the corresponding responses together with vehicle stopping distance are reported in Table 4. Also, the Integrated Squared Errors (ISE) and Integrated Squared Control Input (ISCI) of both ABS and ASS controllers are presented in Table 5. The results indicate that the body acceleration and longitudinal slip tracking error are relatively increased owing to modeling uncertainty, but are still acceptable. The trend of reducing the stopping distance for different strategies is the same as before. It is considered that the parameter uncertainties have no significant effect on the controller performance.

5- Conclusion

In this paper, an integrated controller is developed to control both the ABS and ASS with different aims. In this way, two optimal controllers with adjustable weighting factors are developed for the ABS and ASS based on the quarter car vehicle model. The braking on flat and irregular roads with good and poor qualities are evaluated and discussed. Different strategies for the ASS are proposed and investigated. At first, the feasibility of squeezing the tire in nonzero value for tire deflection variation to increase the tire normal force is analyzed. Then, regulating the suspension system in static situations by different weighting are presented for braking on irregular roads. It is analytically studied that the tire squeezing yields instability in the states of suspension systems. Therefore, for braking on the flat roads, there is no need to activate the ASS. It is concluded that the key point for affecting the ASS on the stopping distance of the ABS is removing the tire jumping on the irregular roads. In the absence of this phenomenon, the ASS should be focused on its first aim to improve the ride comfort. For braking on the good road with no tire jumping, the ASS improves the ride comfort greatly in the first mode and, as the side effect, can decrease the stopping distance. But, for braking on the poor road quality, the ASS avoids tire jumping in the second mode and consequently decreases the stopping distance significantly.

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