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An Exact Solution for Fluid Flow and Heat Convection through Triangular Ducts Considering the Viscous Dissipation

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ABSTRACT: Today, the study of flow and heat transfer in non-circular ducts are of increasing importance in various industries and applications such as microfluidics, where lithographic methods typically produce channels of square or triangular cross-section. Also, heat transfer in non-circular ducts is important in designing the compact heat exchangers to enhance the heat transfer. In the current study, an exact analytical solution for the convective heat transfer in conduits with equilateral triangle cross-section is presented for the first time. The effect of viscous dissipation on heat transfer and temperature distribution through the duct is investigated in detail. This effect is of great importance especially in flow of high viscous fluids in micro-channels. In order to study the effect of viscous dissipation in both cooling and heating cases, the Brinkman number is employed. The exact solution is found by calculating the particular solution which satisfies the thermal boundary conditions. Based on the finite expansion method, an exact analytical solution for temperature distribution and a correlation for dimensionless Nusselt number is obtained as functions of the Brinkman number. The maximum temperature and Nusselt number at the centroid of the conduit for the specific case of Brinkman number equal to zero is calculated equal to 5/9 and 28/9, respectively. The proposed method of solution could be used to find the exact solution for similar problems such as analysis the heat convection in non-circular geometries.

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1- Introduction

There has been an increasing interest allocated to the heat transfer of visco-laminar flows in the conduit due to its vast range of applications in industry and bioengineering fields. Taking the advantages of this valuable type of investigations in industry, it is possible to design high performance equipment in engineering, industry fields and bio related devices. Today, the convective heat transfer in non–circular ducts is of increasing importance in microfluidics, where lithographic methods typically produce channels of rectangular or triangular cross-section. These channels are also extensively used in equipment such as biological kits, (such as the kits for extraction the DeoxyriboNucleic Acid (DNA), detection of cancers cells and bacteria, blood sample preparation and glucose monitoring), fuel cells and cooling systems for small spaces. Generally, in this type of investigation, Nusslet number and friction factor are two of the most important factors that we would like to obtain, control and have domination over them for best designing and manufacturing purposes in heat exchanger related analysis. A large number of investigations in this field are carried out to analyze the possible effects of different parameters playing a role in these parameters. An interesting category of investigations are those who focused on the effect of shape and geometry of cross sections on Nusselt number. Shah [1] in his comprehensive study investigated the effect of cross section of ducts on force convective heat transfer with various shape of cross-section like isosceles triangular, rounded corner equilateral triangular, sine, rhombic, and trapezoidal. He revealed that making rounded the corners of the pipe cross-section can change and enhance the heat transfer rate of channel. In following, Shah and London [2], extend previous investigation to analyze the effect of centrifugal force on heat transfer of internal flow. They carried out a numerical investigation on the laminar fully developed forced convective heat transfer in straight and curved ducts for both constant heat flux and wall temperature boundary conditions using finite difference method to show using curved pipes can significantly increases the coefficient of heat transfer in internal flows. Recently, Erdoğan and Imrakin [3] in their interesting work analyzed the heat transfer of different cross sections. The values of the Nusselt numbers obtained are about, 3.5441771 for a duct of square cross-section; 48 /11 for a circular pipe, about 4.088184147 for a duct of semicircular cross-section and 140/17 for a parallel plate duct. More recently, Shahmardan et al. [4,5] employed analytical methods to obtain exact analytical solutions to the triangular ducts scenario. They assumed the iso-flux situation around the boundary and obtained the value of Nusselt number in this regime as 28/9. This value of the Nusselt number shows the effect of duct shape on heat transfer. The heat transfer in ducts of rectangular, equilateral triangular, right-angled isosceles triangular, and semicircular cross-sections have been studied by Marco and Han [6]. They used an analogy theory and gave the conditions under which the solution is permissible. By this analogy, first the temperature distribution is obtained and then the velocity distribution is found. Rajagopala and Sadegh [7] using boundary integral method numerically investigated on the fully developed forced convection heat transfer of vide ranges of conduit cross section including circular, elliptical, rectangular,

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and triangular. Lakshminarayanan and Haji-Sheikh [8] numerically analyzed temperature development in ducts with different triangular cross-sections using Galerkin finite element method. Zhang et al. [9] using an analytical/numerical solution analyzed the convective heat transfer in the thermal entrance and fully developed flow region of three different cross sections of square, rectangular and equilateral triangular conduits with isothermal scenario. Furthermore, Zhang [10] studied numerically the hydrodynamic fully developed flow and thermally devolving heat convection in plate-fin isosceles triangular ducts under uniform temperature condition. There, different Nusselt numbers related to both of developing and fully developed regions for various apex angles and fin conductance parameters are obtained.

Zhang and Chen [11] studied the fluid flow and convective heat transfer in a cross-corrugated triangular duct under a uniform heat flux at walls employing a numerical method and corroborated their result with experimental parts. They extended their work and presented correlations for estimation of the pressure drop and the mean Nusselt number. Ray and Misra [12] numerically investigated the effect of making rounded the corners of the ducts of square and equilateral triangular cross sections on both pressure drop and heat transfer characteristics of laminar fully-developed flow. There, they employed a dimensionless radius of curvature parameter to study the effect of curvature on pressure drop coefficient and Nusselt number. The similar numerical studies were extended this scenario to analyze the forced convection in a porous medium [13], or to study the effect of viscous dissipation on heat transfer [14] and modeling the heat convection of turbulent flow in triangular straight ducts [15]. In this among some of the researchers also suggested taking the advantages of rheological properties of viscoelastic fluids we can have domination over these parameters in heat and flow analysis [16-19].

According to the literature, most of the studies in the field of flow and heat transfer in triangular ducts are restricted to the numerical investigations which did not consider the effect of viscous dissipation and there are not a lot of analytical techniques which can be attributed to the complex form of geometry. To the best knowledge of authors, there is not any exact analytical solution available about the heat convection in straight triangular ducts in which the effect of flow dissipation is considered. In this paper, an exact analytical solution for forced convective heat transfer considering flow dissipation effect in straight pipes with equilateral triangle cross section is presented for the first time. The schematic shape of problem is shown in Fig. 1. An exact analytical solution is obtained based on the finite series expansion method for steady convective heat transfer under the constant heat flux at walls considering the effect of flow dissipation in this geometry. The closed form of dimensionless temperature distribution in Cartesian coordinate system and Nusselt number is derived analytically.

2- Governing Equations

The governing equation including conservative mass, momentum and energy equations in a duct with equilateral triangle cross section concerning with the incompressible fluid flow and heat transfer is presented as following:

$$\nabla.\tilde{V} = 0 \tag{1a}$$

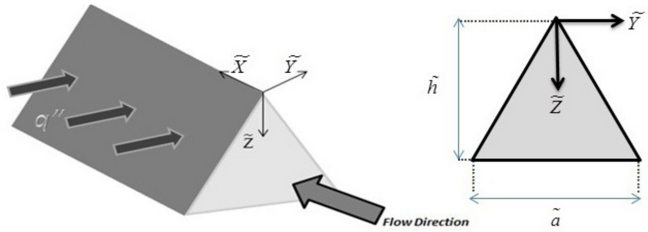


Fig. 1. The geometry of duct in current study

$$\rho\left(\frac{\partial \tilde{V}}{\partial t} + \tilde{V}.\nabla\tilde{V}\right) = -\nabla\tilde{P} + \rho\tilde{g} + \mu\nabla^2\tilde{V} \tag{1b}$$

$$\rho c_p \tilde{V}.\nabla\tilde{T} = k\nabla^2\tilde{T} + \Phi \tag{1c}$$

where $\tilde{V}$ is the velocity vector, ρ is the density, $\tilde{P}$ is the static pressure, $\tilde{t}$ is the time, μ is the viscosity, $\tilde{T}$ is the temperature of the fluid flow and Φ is flow dissipation portion of energy equation which is defined as:

$$\Phi = \tau_{ij}\frac{\partial u_i}{\partial x_j} \tag{2}$$

It is convenient to employ the non-dimensional style of governing equation in this type of investigation. The non–dimensional parameters which will be used in the current study can be expressed as following:

$$y = \frac{\tilde{y}}{d_h} \qquad z = \frac{\tilde{z}}{d_h} \qquad a = \frac{\tilde{a}}{d_h} \qquad h = \frac{\tilde{h}}{d_h}$$
$$u = \frac{\tilde{u}}{u_b} \qquad \alpha = \frac{k}{\rho c_p} \qquad T = \frac{\tilde{T} - \tilde{T}_w}{q'' d_h / k} \qquad Br = \frac{\mu u_b^{\,2}}{d_h q''} \tag{3}$$

where α is the thermal diffusivity coefficient, $\tilde{u}$ is the main flow velocity, y and z are coordinates in Cartesian coordinate system, and $\tilde{a}$ and $\tilde{h}$ are the side and the height of equilateral triangular cross section, respectively (see Fig. 1). In addition, d_h is the hydraulic diameter, u_b is the bulk velocity, and T_m is the mean temperature and Br is Brinkman number of fluid flow. Area, hydraulic radius, bulk velocity and mean temperature corresponding to the equilateral triangular surface parameters are defined as following:

$$\tilde{A} = \frac{3\sqrt{3}}{4} d_h \tag{4a}$$

$$d_h = \frac{4\tilde{A}}{\tilde{P}} = \frac{\sqrt{3}}{3}\tilde{a} \tag{4b}$$

$$u_b = \frac{1}{\tilde{A}}\int_A \tilde{u}\, d\tilde{A} \tag{4c}$$

$$\tilde{T}_m = \frac{1}{\rho c_p u_b \tilde{A}}\int_A \rho c_p \tilde{u}\tilde{T} d\tilde{A} \tag{4c}$$

Using Eqs. (3) and (4a), we can write:

$$a=\sqrt{3} \tag{5a}$$

$$h=\frac{3}{2} \tag{5b}$$

where $\tilde{A}$ and $\tilde{P}$ are the cross-sectional area and perimeter of the cross section, respectively. In order to obtain a correlation between variation of mean temperature in axial direction of conduit and flow dissipation, a thermal energy balance on a differential control volume in the axial direction is written as [20]:

$$\begin{aligned}&q''\tilde{p}d\tilde{x}=\rho\tilde{A}u_b c_p dT_m+\int_{\tilde{A}}\mu(\frac{\partial\tilde{u}}{\partial\tilde{y}}+\frac{\partial\tilde{u}}{\partial\tilde{z}}d\tilde{A})d\tilde{x}\\ &\Rightarrow\frac{d\tilde{T}_m}{d\tilde{x}}=\frac{q''\tilde{p}}{\rho\tilde{A}u_b c_p}+\frac{\mu}{\rho\tilde{A}u_b c_p}\int_{\tilde{A}}\mu(\frac{\partial\tilde{u}}{\partial\tilde{y}}+\frac{\partial\tilde{u}}{\partial\tilde{z}}d\tilde{A})=cte\end{aligned} \tag{6}$$

The fully developed thermal condition for convective heat transfer in a closed channel is generally defined as:

$$\frac{\partial}{\partial\tilde{x}}\left(\frac{\tilde{T}-\tilde{T}_w}{\tilde{T}_m-\tilde{T}_w}\right)=0 \tag{7}$$

Using Eqs. (6) and (7), and considering the constant heat flux at walls ($q''=h(\tilde{T}_w-\tilde{T}_m)$), we have:

$$\begin{aligned}&\frac{d\tilde{T}_m}{d\tilde{x}}=\frac{\partial\tilde{T}}{\partial\tilde{x}}=\frac{d\tilde{T}_w}{d\tilde{x}}=\frac{q''\tilde{p}}{\rho\tilde{A}u_b c_p}\\ &+\frac{\mu}{\rho\tilde{A}u_b c_p}\int_{\tilde{A}}\mu(\frac{\partial\tilde{u}}{\partial\tilde{y}}+\frac{\partial\tilde{u}}{\partial\tilde{z}}d\tilde{A})=cte\end{aligned} \tag{8}$$

Assuming a constant viscosity for the fluid and taking the advantages of Eqs. (3) and (8), substituting them into the Eq. (1c), after making some re-arrangements, the following dimensionless form of the heat transfer equation is obtained. It should be noted that since the velocity distribution of fully developed laminar flow is rectilinear, the transverse velocity components should be set to zero, so we have:

$$\begin{aligned}&\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}+\frac{\mu u_b^{\,2}}{q'' d_h}\left((\frac{\partial u}{\partial y})^2+(\frac{\partial u}{\partial z})^2\right)=\\ &4u(y,z)+\frac{\mu u_b^{\,2}}{q'' d_h}Cu\end{aligned} \tag{9}$$

where C is a constant and is defined as:

$$C=\frac{4\sqrt{3}}{9}\int_A(\frac{\partial u}{\partial y})^2+(\frac{\partial u}{\partial z})^2 dA \tag{10}$$

Using the non-dimensional parameters presented in Eq. (3), Eq. (9) is reduced into:

$$\nabla^2 T=(4+C\,Br)u-Br\left((\frac{\partial u}{\partial y})^2+(\frac{\partial u}{\partial z})^2\right) \tag{11}$$

Main flow velocity and flow rate of rectilinear flow in a duct with equilateral triangle cross section are [21]:

$$\tilde{u}(\tilde{y},\tilde{z})=\frac{-\frac{d\tilde{p}}{d\tilde{x}}}{2\sqrt{3}\tilde{a}\mu}\left(\tilde{z}-\frac{\tilde{a}\sqrt{3}}{2}\right)\left(3\tilde{y}^2-\tilde{z}^2\right) \tag{12a}$$

$$\tilde{Q}=\frac{\tilde{a}^4\sqrt{3}}{320\mu}\left(-\frac{d\tilde{p}}{d\tilde{x}}\right) \tag{12b}$$

Therefore, the bulk velocity ($\tilde{u}_b$) corresponding to this geometry can be obtained as:

$$\tilde{u}_b=\frac{\tilde{Q}}{\tilde{A}}=\frac{\tilde{a}^2}{80\mu}\left(-\frac{d\tilde{p}}{d\tilde{x}}\right) \tag{13}$$

Based on the Eq. (3), the dimensionless form of flow distribution can be expressed as:

$$u(y,z)=\frac{40}{9}\left(z-\frac{3}{2}\right)\left(3y^2-z^2\right) \tag{14}$$

Substituting dimensionless velocity (Eq. (14)) into the Eq. (10), the C constant is obtained as following:

$$C=\frac{80}{3} \tag{15}$$

Using the obtained value of C constant, the conservative energy equation (Eq. (11)) for a fully developed flow in a straight duct with equilateral triangular cross section is obtained as:

$$\begin{aligned}&\nabla^2 T=(3A+60hB)y^2 z-(3hA+36h^2B)y^2\\ &+(12hB-A)z^3+(hA-4h^2B)z^2-9By^4-9Bz^4\\ &-18By^2z^2\end{aligned} \tag{16}$$

where A and B are defined to simplify the equation as following:

$$A=\frac{160}{27}(3+20Br) \tag{17}$$

$$B=\left(\frac{40}{9}\right)^2 Br \tag{18}$$

3- Exact Solution and Results

Considering the non-homogenous style of non-dimensional energy equation, it can be easily confirmed that temperature solution is consisted of two parts; a general solution and a particular solution as:

$$T(y,z)=T_P(y,z)+T_G(y,z) \tag{19}$$

Noting the fact that both the boundary condition and governing equation for general solution are homogenous the general solution of temperature distribution is equal to zero. Obtaining the particular solution of this type of investigation is not generally a simple procedure. Noting the Non-homogenous part of energy equation it can be deduced that in a fully developed rectilinear flow the temperature distribution should be in form of finite expansion series. Based on this equation (Eq. (11)) and a simple applying Laplacian operator on temperature distribution, it can be easily noted that the order of results (which is equal with order of velocity) is decreased two times. Here, using the finite expansion method, a five order series with 8 unknown constants is considered to be solution of temperature distribution as:

$$T_P(y,z)=\left(z-h\right)\begin{pmatrix} C_1y^4+C_2y^2z^2+C_3y^2z+C_4z^4+ \\ C_5z^3+C_6y^4z+C_7y^2z^3+C_8z^5 \end{pmatrix} \tag{20}$$

Substituting the above equation (Eq. (20)) into Eq. (11), one can obtain the following equation:

$$\begin{aligned}\nabla^2T&=(12C_1+6C_2-12hC_6-6hC_7)y^2z\\&-((12C_1+2C_2)h-2C_3)y^2+(2C_2+20C_4-2hC_7-20hC_8)z^3\\&-((2C_2+12C_4)h-2C_3-12C_5)z^2-(2C_3+6C_5)hz+2C_6y^4\\&+(2C_7+30C_8)z^4+(12C_6+12C_7)y^2z^2\end{aligned} \tag{21}$$

The terms with order less than three is not considered in Eq. (21) because by applying the Laplacian operator on these terms, it seems that there are not any same order terms in the result of operation with the terms of the right hand side of Eq. (11). Therefore, the coefficients of these terms are obtained equal to zero. In order to find the non-zero coefficients (C_i, i=1, 2, … and 8), the Eq. (21) should be substitute in Eq. (11):

$$\begin{aligned}\nabla^2T&=(12C_1+6C_2-12hC_6-6hC_7)y^2z\\&-((12C_1+2C_2)h-2C_3)y^2\\&+(2C_2+20C_4-2hC_7-20hC_8)z^3\\&-((2C_2+12C_4)h-2C_3-12C_5)z^2\\&-(2C_3+6C_5)hz+2C_6y^4+(2C_7+30C_8)z^4\\&+(12C_6+12C_7)y^2z^2\\&=\begin{pmatrix}(3A+60hB)y^2z-(3hA+36h^2B)y^2+(12hB-A)z^3\\+(hA-4h^2B)z^2-9By^4-9Bz^4-18By^2z^2\end{pmatrix}\end{aligned} \tag{22}$$

By considering the same order terms to be equal in both sides of Eqs. (22) and (23), we have:

$$\begin{cases}12C_1+6C_2-12hC_6-6hC_7=3A+60hB\\(12C_1+2C_2)h-2C_3=3hA+36h^2B\\2C_2+20C_4-2hC_7-20hC_8=-A+12hB\\(2C_2+12C_4)h-2C_3-12C_5=4h^2B-hA\\2C_3+6C_5=0\\12C_6+12C_7=-18B\\2C_7+30C_8=-9B\\2C_6=-9B\end{cases} \tag{23}$$

By solving the above set of equation, 8 unknown coefficients are achieved as:

$$C_1=\frac{3}{16}A+\frac{15}{4}hB \tag{24a}$$

$$C_2=\frac{1}{8}A-\frac{7}{2}hB \tag{24b}$$

$$C_3=-\frac{1}{4}hA+h^2B \tag{24c}$$

$$C_4=-\frac{1}{16}A+\frac{3}{4}hB \tag{24d}$$

$$C_5=\frac{1}{12}hA-\frac{1}{3}h^2B \tag{24e}$$

$$C_6=-\frac{9}{2}B \tag{24f}$$

$$C_7=3B \tag{24g}$$

$$C_8=-\frac{1}{2}B \tag{24h}$$

In order to obtain the final solution of temperature distribution in a equilateral triangular conduit, it is needed to plug the above obtained constants (Eq. (24)) into Eq. (21) to find the temperature distribution. The non-dimensional temperature profile corresponding to the scenario is obtained as following:

$$T(y,z)=\frac{10}{27}\left(z-\frac{3}{2}\right)\begin{pmatrix}\left(9y^4+6y^2z^2-18y^2z-3z^4+6z^3\right)+\\Br(360y^4+40z^4-240y^2z^2\\-240y^4z+160y^2z^3-(\frac{80}{3})z^5)\end{pmatrix} \tag{25}$$

It can be easily proved that Eq. (25) can satisfy the boundary conditions at walls (at walls: $\tilde{T}=\tilde{T}_w$ or T=0):

$$at\quad \left.\begin{aligned}z=h=\frac{3}{2}\\z=\sqrt{3}y\\z=-\sqrt{3}y\end{aligned}\right\}\Rightarrow\quad T=0 \tag{26}$$

The convection coefficient can be found using the following relation:

$$h=\frac{q''}{\tilde{T}_w-\tilde{T}_m} \tag{27}$$

Using the above equation and the velocity distribution for fully developed laminar flow, the Nusselt number can be easily calculated employing the following equation:

$$Nu = -\frac{1}{T_m} = -\frac{1}{\frac{1}{A}\int_A uTdA} \tag{28}$$

Using the achieved temperature distribution to obtain mean temperature, the Nusselt number can be calculated as a function of Brinkman dimensionless number as following:

$$Nu = \frac{308}{9(40\mathrm{Br}+11)} \tag{29}$$

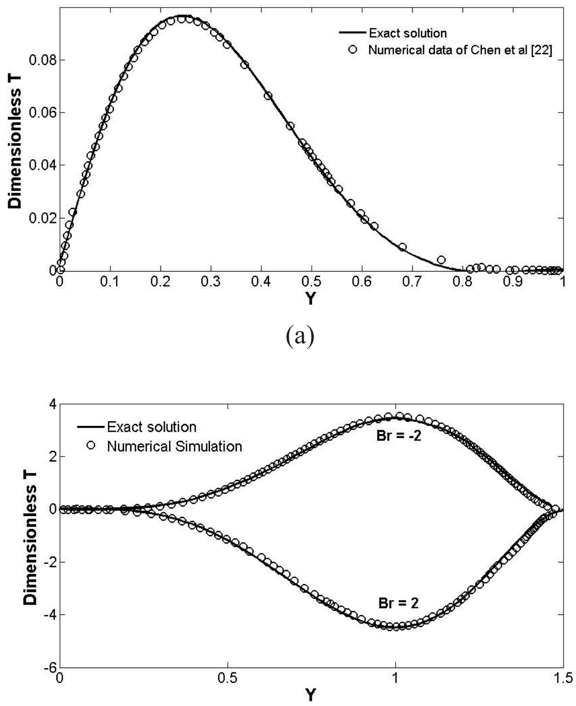
In the special case of $Br=0$ (scenario with no flow dissipation) the above equation is reduced to:

$$Nu = \frac{28}{9} \approx 3.11 \tag{30}$$

The above value is the Nusselt number of fully developed flow and heat transfer in straight pipes with equilateral triangular cross section under the constant heat flux at walls which is reported in previous numerical studies [4,5].

As the first benchmark comparison, the fully developed flow and heat transfer through a duct with isosceles triangular cross-section at $Br=0$ has been solved and compared with the results of Chen et al. [22]. Fig. 2a shows the dimensionless temperature distribution along the symmetry axis. As it can be seen in this figure, there is a good consistency between our exact solution and the numerical results of Chen et al. [22]. To validate the presented exact solution for $Br \neq 0$, numerical simulations were conducted at $Br=2, -2$. As shown in Fig. 2b, it can be observed that the numerical results are in a good agreement with the presented exact solution.

Fig. 3 shows the contour of dimensionless temperature distribution without flow dissipation in a cross section of triangular duct. Here, the isothermal lines are appeared as the round corner equilateral triangles. As it is illustrated in the cases that heat flux is applied to the boundary, the highest and lowest temperature of conduit are related to the wall and centroid of triangular pipe. As it is well-known Brinkman dimensionless number presents the level of heat generation of flow and presented the effect of viscous dissipation. The positive value of Br is representing heating scenarios, while the negative value is representing cooling situations. For better investigation over the effect of viscous dissipation on both heating and cooling regimes, Figs. 3 and 4, respectively, reveal the temperature distribution corresponding to these cases. Assuming the wall duct is subjected to the constant heat flux and the fluid is hydrodynamic and thermally developed, it can be expected that in heating cases, the wall should obtains the highest value of temperature while in cooling situation, it has the lowest value. As shown in Fig. 3, when wall heating is applied, increasing in the value of the Br number for a constant value of heat flux, increase the value of dissipation of fluid. Considering the fact that generally velocity gradient and consequently flow dissipation around the wall are large, temperature of this region is increased and obtains a value around the value of wall. Although dissipation



(a)

(b)

Fig. 2. Comparison of presented exact solution for dimensionless temperature distribution with (a) numerical results of Chen et al. [22] at *Br*=0 and (b) numerical simulations at *Br*=2, -2

increases the bulk temperature of the fluid as a heat source, less amount of heat is transferred to the fluid. Therefore, the temperature difference between the wall and the core region increase as it is revealed in Fig. 3. The temperature contours for cooling scenario is shown in Fig. 4. In this case, the value of the dissipation is increased in following of a decrement in the value of Br number. In fact, the viscous dissipation and the constant heat flux (which is imposed to the wall) have opposite effects. So, the wall cooling will overcome the effect of heat generated internally by viscous dissipation process as shown in Fig. 4.

Fig. 5 is representing the mean temperature of fluid passing through an equilateral triangular conduit. As it could easily expected the mean temperature of the fluid is increased in following of an increment in Brinkman Number. The filled square in this figure is related to the critical Brinkman number ($Br_{critical}$=-0.275) where the mean temperature of fluid is equal to zero.

Figs. 6 and 7 show the variation of dimensionless temperature at vertical axis of the cross section (at $y=0$) and horizontal axis of centroid (at $z=2h/3=1$), respectively, with different positive and negative heating effects. According to these Figures, the minimum and maximum values of this temperature is related to the centroid of the triangle ($y=0$ & $z=2h/3=1$). In special case of Brinkman number equal to zero, maximum absolute value of dimensionless temperature is calculated as:

$$\left|T\right|_{\max} = \frac{5}{9} \tag{31}$$

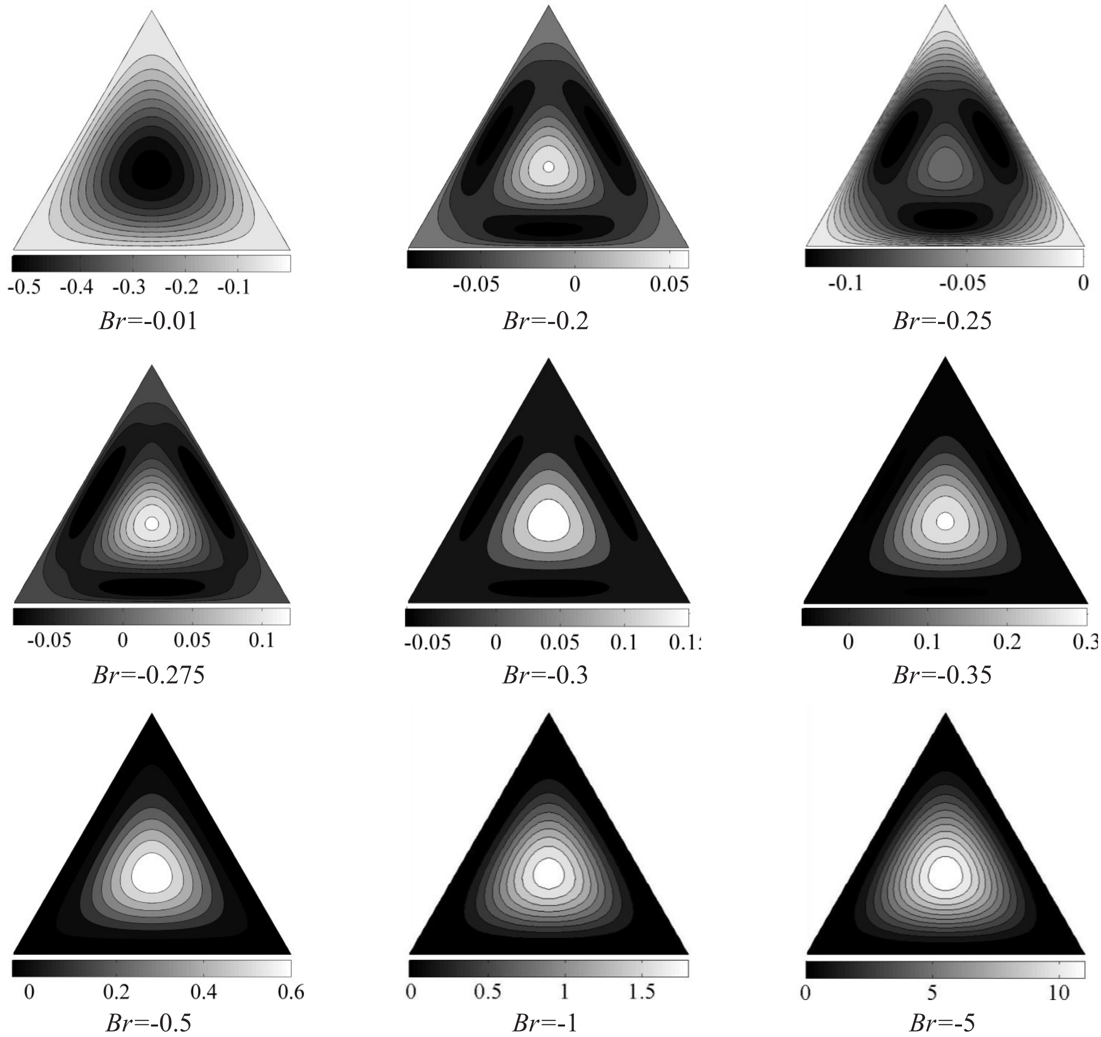


Fig. 3. Dimensionless contours of temperature for flow in triangular ducts (Cooling).

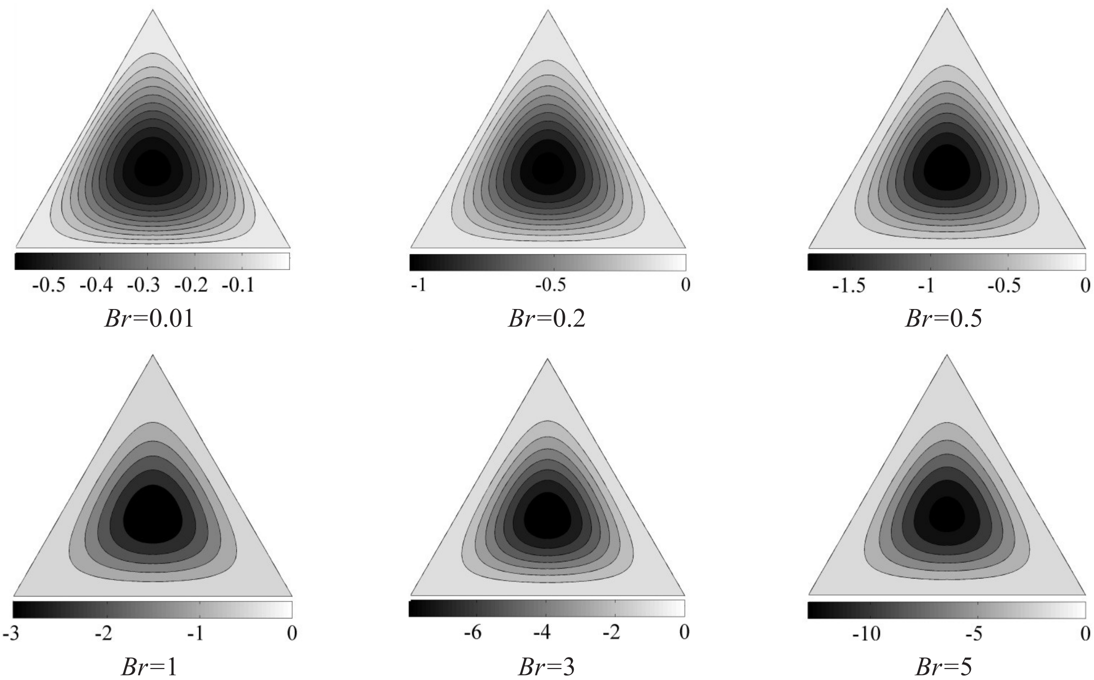


Fig. 4. Dimensionless contours of temperature for flow in triangular ducts (Heating).

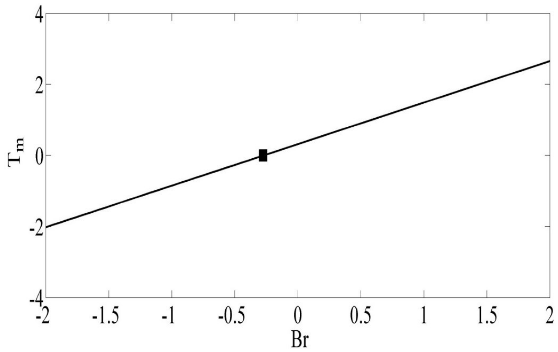


Fig. 5. Diagram of mean temperature versus Brinkman number

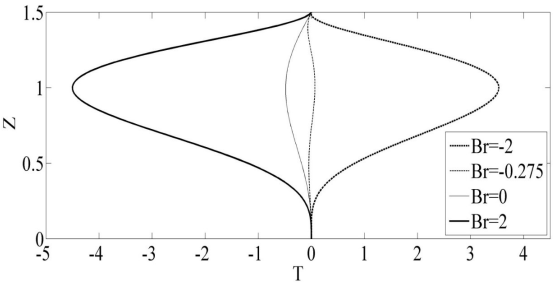


Fig. 6. Profile of absolute value of dimensionless temperature along the symmetry axis of cross section

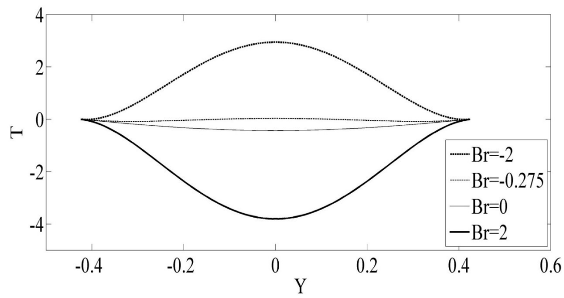


Fig. 7. Profile of absolute value of dimensionless temperature along the horizontal axis of centroid

Variations of these critical points are plotted in Fig. 8. Interesting results are related to the cases with critical values around $Br_{critical}$=-0.275 in which the maximum value of temperature has a small value around zero and is not related to the centroid of cross section and shift away toward walls. The maximum temperature in this situation is equal to zero at the centroid of pipe. As it is presented previously in Fig. 5 in this situation, the meant value of fluid temperature is also obtained as zero. In this situation, as it can be concluded from Eq. (29), the value of Nusselt number tends to infinity. Variation of Nusselt number with Brinkman number is presented in Fig. 9. It can be easily observed that as absolute value of Brinkman number tends to infinity, Nusselt number tends to zero (this result can be also obtained from Eq. (29)). In special case of Brinkman number equal to -11/40, the Nusselt number is shown to tend to infinity. In lower values, the Nusselt number obtains a negative value while in higher values Nusselt number is positive.

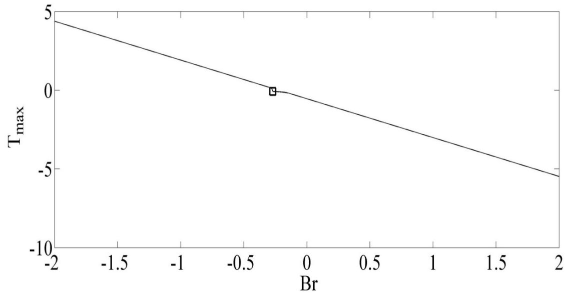


Fig. 8. Diagram of maximum temperature versus Brinkman number

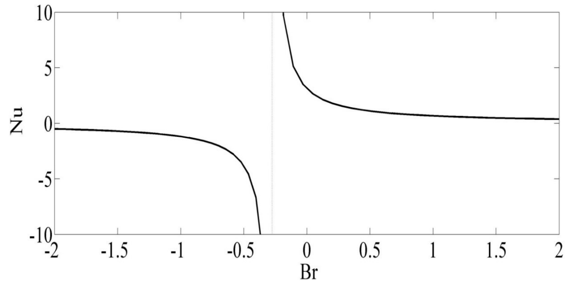


Fig. 9. Diagram of Nusselt number in terms of Brinkman number

4- Conclusions

In this study exact analytical solutions are presented for the temperature distribution and Nusselt number as a function of Brinkman number for the convective heat transfer with flow dissipation of viscous fluids passing through the rectilinear triangular conduits. Presented analytical solution is obtained taking advantages of finite series expansion method based on the finding the possible orders of solution. In special case of Brinkman number equal to zero (flow dissipation is neglected), the Nusselt number is obtained as 28/9 and maximum temperature which is concerned with the centroid of cross section is calculated as 5/9. A critical Brinkman number in this scenario is calculated as -0.275 where Nusslet number tends to infinity and mean temperature is equal to zero. The solution shows that in following of an increment in Brinkman number, the mean temperature is increased. The results of present study could be useful in heat transfer analysis the microfluidics and compact heat exchangers. The authors believe that the proposed method of solution could be used to find the exact solution for similar problems such as analysis the heat convection in non-equilateral geometry, finding the solution for other thermal boundary conditions and modeling the effect of heat dissipation in triangular ducts.

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