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Adopting GRASP to solve a novel model for bus timetabling problem with minimum transfer and fruitless waiting times

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ABSTRACT: This paper addresses a variant of bus timetabling problem assuming that travel times changes dynamically over the planning horizon. In addition to minimizing the transfer waiting time, another objective, namely minimizing the fruitless waiting time, is introduced in this paper as a new realistic objective. First, the problem is formulated as a mixed integer linear programming model. Then, since commercial solvers become inefficient to solve moderate and large sized instances of the problem (due to the NP-hardness), a GRASP heuristic algorithm is developed. Computational experiments over a variety of random instances verify the performance of the proposed method.

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1- Introduction

The planning process of a bus network is a complex issue, and commonly includes four basic phases, usually performed in sequence: (1) design of routes (2) development of a timetable, (3) vehicle scheduling and (4) crew scheduling [1, 2]. The focus of this paper is on the second phase in which the departure time of each bus is determined. Preparing a timetable for a bus network is a critical subject since service quality and subsequent planning phases such as vehicle and crew scheduling are dependent on it.

Bus network timetabling problem (BNTP) was first addressed by Ceder et al. [3]. They introduced the bus network planning as a systematic sequence of decisions. Later, Ceder et al. [4] defined the concept of synchronization as the simultaneous arrival of the buses of two different lines at a given transfer station, and presented a mixed integer linear programming (MILP) model to maximize the number of synchronizations at transfer stations of the network. Eranki [5] improved the study of Ceder et al. [4] by changing the definition of synchronization. With respect to the new definition, a synchronization occurs if the difference between the arrival times of the buses of two different lines at a given transfer station does not exceed the allowed waiting time. Ibarra-Rojas and Rios-Solis [6] improved the model of Ceder et al. [4] by redefining the variables to have oriented synchronization, and added a new constraint to make uniform distribution of departure times over the planning horizon. They proved that the problem is NP-hard and presented a hill climbing heuristic algorithm to solve large sized instances.

Shafahi and Khani [7] stated that the objective of minimizing the total transfer waiting time is more reasonable than other objective functions such as maximizing the number of simultaneous arrivals. Thus, they considered BNTP with the aim of minimizing the total transfer waiting time assuming that each route has a fixed headway during the planning period. They developed a mathematical model, the complexity of which was decreased due to the property of fixed headways. Thus, their model could be solved for small and medium transit networks to optimality. They also provided a Genetic Algorithm (GA) to solve large sized instances. Khani and Shafahi [8] proposed a nonlinear mathematical programming model and a GA for the BNTP. Their model is developed to decide the departure time of each trip and the slack time associated with each transfer station to make a trade-off between the operational cost and the cost of waiting time. Wu et al. [9] proposed a reliable timetable design model to transit network by taking into account the stochastic bus travel times, and with the objective of minimizing total waiting time for three types of passengers, including transferring passengers, boarding passengers and through passengers. Then, a GA was proposed to solve the problem. Fouilhoux et al. [10] proposed four classes of valid inequalities to strengthen the LP relaxation bound of existing MILP models of BNTP. Ibarra-Rojas et al. [11] considered the BNTP under a realistic assumption indicating that the planning periods are not necessarily identical for all lines. They formulated the multi-period problem as an MILP and presented efficient meta-heuristics to solve it. Liu and

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Ceder [12] integrated SBTP and vehicle scheduling simultaneously within a bi-objective bi-level model and solved it by a novel deficit function based sequential search method.
With respect to the aforementioned studies, researchers believe in two main criteria to improve a bus timetable, maximizing the number of synchronizations and minimizing the transfer waiting times. In this paper, we concentrate on BNTP with the objective of minimizing the transfer waiting time with two more realistic assumptions. First, instead of considering fixed travel times, we assume that the travel times may change dynamically throughout the planning horizon. Second, we introduce a new concept, namely fruitless waiting time that happens when there is no chance for passengers to transfer to desirable line until the end of planning horizon. This type of waiting time occurs in transfer stations at the end of trips and plays a significant role in timetabling development. Throughout the paper, we refer to this problem as extended BNTP (EBNTP). We formulate EBNTP as an MILP model and propose a greedy randomized adaptive search procedure (GRASP) to find near optimal solutions to large sized instances.
The rest of this paper is organized as follows: Section 2 describes EBNTP and formulates it as an MILP model. Section 3 presents GRASP heuristic method to solve the model. Section 4, evaluates the performance of the GRASP over different random instances. Finally, Section 5 draws conclusions and proposes directions for future research.

2- Mathematical model

2- 1- Problem description

Consider a bus transit network with the set $\mathbb{I}$ (indexed by i,j) of lines having known origin and destination points. The departure time of the trips over each line $i \in \mathbb{I}$ should be scheduled so that at least $\underline{f}_i$ and at most $\overline{f}_i$ trips are organized and the minimum and the maximum headways are respected. A transfer station is a point where passengers may transfer from one line i to another line j. The walking time between stops of two lines at a transfer station and the stopping time for vehicles at transfer stations are known in advance.
The planning horizon of each line, denoted by $[0,T]$, is divided into n_i time-slots such that:

$$[0,T) = \bigcup_{s\in\{1,2,\ldots,n_i\}} [a_{i,s-1}, a_{i,s}) \ , \ a_{i,0} = 0, a_{i,n_i} = T.$$

Where $[a_{i,s-1}, a_{i,s})$ represents the interval associated with the s$^{\text{th}}$ time-slot. See Fig. 1 for more illustration.
For each line, the travel time of a bus from the line's origin to a given transfer station along the line is dependent on the time-slot associated with the departure time of that bus. In other words, travel time is a dynamic parameter that is only dependent on trips' departure time. In each transfer station, passengers leave the station with the first vehicle of the target line in order to reduce their transfer waiting time. In addition to the transfer waiting time, a new criterion, namely fruitless waiting time is introduced in this paper. Fruitless waiting time happens when there is no chance for passengers to transfer to desirable line until the end of planning horizon.
Here, three types of cost are considered: the cost of transfer waiting time; the cost of fruitless waiting time, the cost of extra trip which is incurred for a line if the number of trips is more than the minimum frequency $\underline{f}_i$; and the cost associated with the maximum value of waiting times (transfer or fruitless). The EBNTP schedules the departure times for the trips of each line so that the total cost is minimized.

2- 2- Problem formulation

The sets, indices, and parameters of the problem are defined as follows:

Sets, indices, and parameters

$\mathbb{I}$	:	Set of network lines (indexed by i,j)
$\mathbb{K}$	:	Set of transfer stations over the network (indexed by k)
$\mathbb{K}_i^{'}$	:	A subset of $\mathbb{K}$, containing transfer stations on the line i
$\mathbb{I}_i^{'}$	:	A subset of $\mathbb{I}$ containing the lines which have at least one common station with line i
$\mathbb{K}_{i,j}^{"}$	:	A subset of $\mathbb{K}$, containing common transfer stations of the lines i,j
$\overline{f}_i$	:	Maximum frequency of line i

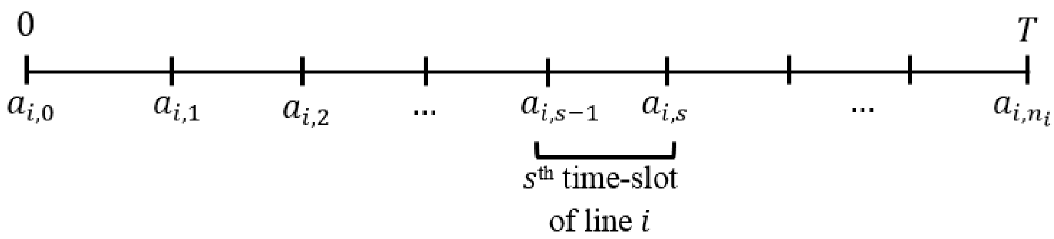

Fig. 1: Time-slots associated with line i

$\underline{f}_i$: Minimum frequency of line i

$\mathbb{T}_i = \{1, 2, \ldots, \bar{f}_i\}$: Set of trips of line i (indexed by t)

$\bar{h}_i$: Maximum headway of line i

$\underline{h}_i$: Minimum headway of line i

T : Planning horizon

n_i : Number of time-slots associated with line i

$\mathbb{S}_i$: Set of time-slots associated with line i (indexed by s)

$a_{i,s}$: Beginning time of the s th time slot of line i

$\tau_{i,k,s}$: Travel time of a vehicle from starting station of line i to transfer station k if its departure time is on the s th time slot

$\tau'_{i,j,k}$: The walking time of a passenger in transfer station k to walk off line i to line j

$\tau''_{i,k}$: Stopping time of a vehicle at transfer station k of line i

$p_{i,j,k}$: The penalty per-unit of transfer waiting time in the transfer station k to transfer from line i to line j

$p'_{i,j,k}$: The penalty per-unit of fruitless waiting time in the transfer station k to transfer line i to line j

c_i : The cost of extra trip (i.e. performing more than $\underline{f}_i$ trips) in line i

d : The cost associated with the maximum value of waiting times (transfer or fruitless)

The following decision variables are defined:

Decision variables

$x_{i,t}$: Nonnegative continuous variable indicating the departure time of the t th trip of line i ($i \in \mathbb{I}$, $t \in \mathbb{T}_i$)

$\eta_{i,t,s}$: Binary variable that is 1 if the departure time of the t th trip of line i is in the s th time-slot; otherwise 0 ($i \in \mathbb{I}$, $t \in \mathbb{T}_i$, $s \in \mathbb{S}_i$)

$\gamma_{i,t}$: Binary variable that is 1 if the t th trip of line i is utilized; otherwise 0

$w_{i,j,k,t,t'}$: Free continuous variable indicating the waiting time for passengers in the transfer station k to transfer from the t th trip of line i to the t' th trip of line j

$w'_{i,j,k,t}$: Nonnegative continuous variable indicating the waiting time for passengers in the transfer station k to transfer from the t th trip of line i to line j

$w''_{i,j,k,t}$: Nonnegative continuous variable indicating the fruitless waiting time for passengers in the transfer stations k to transfer from the t th trip of line i to line j

v : Nonnegative continuous variable indicating the maximum of the weighted transfer waiting time and the weighted fruitless waiting time

$\delta_{i,j,k,t,t'}$: Binary variable that is 1 if $w_{i,j,k,t,t'}$ takes nonnegative value; otherwise 0

$\theta_{i,j,k,t,t'}$: Binary variable that is 1 if the variable $w_{i,j,k,t,t'}$ takes the minimum nonnegative value of $\{w_{i,j,k,t,t''} : t'' \in \mathbb{T}_j\}$; otherwise 0.

$\sigma_{i,j,k,t}$: Binary variable that is 1 if the variable $w_{i,j,k,t,t'}$. takes negative value for $t' = \bar{f}_j$; otherwise 0

With respect to above notations, the problem is formulated as follows:

$$\min \sum_{i \in \mathbb{I}} \sum_{j \in \mathbb{I}'_i} \sum_{k \in \mathbb{K}''_{i,j}} \sum_{t=1}^{\bar{f}_i} \left(p_{i,j,k} w'_{i,j,k,t} + p'_{i,j,k} w''_{i,j,k,t} \right) + \sum_{i \in \mathbb{I}} \sum_{t=\underline{f}_i+1}^{\bar{f}_i} c_i \gamma_{i,t} + dv \,, \tag{1}$$

$$s.t.$$

$$x_{i,1} \le \overline{h}_i \qquad \forall i \in \mathbb{I} \tag{2}$$

$$x_{i,\overline{f}_i} \le T \qquad \forall i \in \mathbb{I} \tag{3}$$

$$\gamma_{i,t} = 1 \qquad \forall i \in \mathbb{I}, \forall t \in \mathbb{T}_i : t \le \underline{f}_i \tag{4}$$

$$\gamma_{i,t+1} \le \gamma_{i,t} \qquad \forall i \in \mathbb{I}, \forall t \in \mathbb{T}_i : \underline{f}_i \le t \le \overline{f}_i - 1 \tag{5}$$

$$\underline{h}_i \gamma_{i,t+1} \le x_{i,t+1} - x_{i,t} \le \overline{h}_i \gamma_{i,t+1} \tag{6}$$

$$x_{i,t} \ge a_{i,s-1} - M\left(1-\eta_{i,t,s}\right) \quad \forall i \in \mathbb{I}, t \in \mathbb{T}_i, s \in \mathbb{S}_i \tag{7}$$

$$x_{i,t} \le a_{i,s} - \varepsilon + M\left(1-\eta_{i,t,s}\right) \quad \forall i \in \mathbb{I}, t \in \mathbb{T}_i, s \in \mathbb{S}_i \tag{8}$$

$$\sum_{s \in \mathbb{S}_i} \eta_{i,t,s} = 1 \qquad \forall i \in \mathbb{I}, t \in \mathbb{T}_i \tag{9}$$

$$\begin{gathered}\left(x_{j,t'} + \sum_{s \in \mathbb{S}_j} \tau_{j,k,s}\, \eta_{j,t',s} + \tau''_{j,k}\right) - \left(x_{i,t} + \sum_{s \in \mathbb{S}_i} \tau_{i,k,s}\, \eta_{i,t,s}\right) - \tau'_{i,j,k} = w_{i,j,k,t,t'} \\ \forall i \in \mathbb{I}, j \in \mathbb{I}'_i, k \in \mathbb{K}''_{i,j}, t \in \mathbb{T}_i, t' \in \mathbb{T}_j\end{gathered} \tag{10}$$

$$\begin{gathered}w_{i,j,k,t,t'} \ge M\left(\delta_{i,j,k,t,t'} - 1\right), \\ \forall i \in \mathbb{I}, j \in \mathbb{I}'_i, k \in \mathbb{K}''_{i,j}, t \in \mathbb{T}_i, t' \in \mathbb{T}_j\end{gathered} \tag{11}$$

$$\begin{gathered}w_{i,j,k,t,t'} \le M\, \delta_{i,j,k,t,t'}, \\ \forall i \in \mathbb{I}, j \in \mathbb{I}'_i, k \in \mathbb{K}''_{i,j}, t \in \mathbb{T}_i, t' \in \mathbb{T}_j\end{gathered} \tag{12}$$

$$\begin{gathered}\theta_{i,j,k,t,t'} \le \delta_{i,j,k,t,t'}, \\ \forall i \in \mathbb{I}, j \in \mathbb{I}'_i, k \in \mathbb{K}''_{i,j}, t \in \mathbb{T}_i, t' \in \mathbb{T}_j\end{gathered} \tag{13}$$

$$\begin{gathered}w'_{i,j,k,t} \ge w_{i,j,k,t,t'} - M\left(2 - \theta_{i,j,k,t,t'} - \gamma_{i,t}\right), \\ \forall i \in \mathbb{I}, j \in \mathbb{I}'_i, k \in \mathbb{K}''_{i,j}, t \in \mathbb{T}_i, t' \in \mathbb{T}_j\end{gathered} \tag{14}$$

$$\begin{gathered}\sum_{t' \in \mathbb{T}_i} \theta_{i,j,k,t,t'} = \gamma_{i,t} - \sigma_{i,j,k,t}, \\ \forall i \in \mathbb{I}, j \in \mathbb{I}'_i, k \in \mathbb{K}''_{i,j}, t \in \mathbb{T}_i\end{gathered} \tag{15}$$

$$\begin{gathered}w_{i,j,k,t,\overline{f}_j} \le M\left(1 - \sigma_{i,j,k,t}\right), \\ \forall i \in \mathbb{I}, j \in \mathbb{I}'_i, k \in \mathbb{K}''_{i,j}, t \in \mathbb{T}_i\end{gathered} \tag{16}$$

$$\begin{gathered}w''_{i,j,k,t} \ge M\left(\sigma_{i,j,k,t} - 1\right) + T - \left(x_{i,t} + \sum_{s \in \mathbb{S}_i} \tau_{i,k,s} \eta_{i,t,s}\right) - \tau'_{i,j,k}, \\ \forall i \in \mathbb{I}, j \in \mathbb{I}'_i, k \in \mathbb{K}''_{i,j}, t \in \mathbb{T}_i\end{gathered} \tag{17}$$

$$v \ge p_{i,j,k} w'_{i,j,k,t}, \ \forall i \in \mathbb{I}, j \in \mathbb{I}'_i, k \in \mathbb{K}''_{i,j}, t \in \mathbb{T}_i \tag{18}$$

$$v \ge p'_{i,j,k} w''_{i,j,k,t}, \forall i \in \mathbb{I}, j \in \mathbb{I}'_i, k \in \mathbb{K}''_{i,j}, t \in \mathbb{T}_i \tag{19}$$

$$\begin{gathered}w_{i,j,k,t,t'} \ \mathit{free}, \quad w'_{i,j,k,t}, w''_{i,j,k,t} \ge 0, \\ \forall i \in \mathbb{I}, j \in \mathbb{I}'_i, k \in \mathbb{K}''_{i,j}, t \in \mathbb{T}_i, t' \in \mathbb{T}_j\end{gathered} \tag{20}$$

$$\gamma_{i,t}, \eta_{i,t,s}, \sigma_{i,j,k,t} \in \{0,1\},$$
$$\forall i \in \mathbb{I}, j \in \mathbb{I}_i^{'}, k \in \mathbb{K}_{i,j}^{"}, t \in \mathbb{T}_i, s \in \mathbb{S}_i \tag{21}$$

$$\delta_{i,j,k,t,t'}, \theta_{i,j,k,t,t'} \in \{0,1\},$$
$$\forall i \in \mathbb{I}, j \in \mathbb{I}_i^{'}, k \in \mathbb{K}_{i,j}^{"}, t \in \mathbb{T}_i, t' \in \mathbb{T}_j \tag{22}$$

Objective function (1) minimizes the total cost of transfer waiting time, fruitless waiting time, cost of extra trips, as well as the penalty of maximum value of weighted waiting times (transfer or fruitless). Constraint set (2) ensures that for each line, the departure time of the first trip does not exceed the maximum headway. Constraint set (3) guarantees that for each line, the departure time of the last trip is within the planning horizon. Constraint set (4) forces the minimum frequency for each line. Constraint set (5) indicates that trips depart consecutively with respect to index t . Constraint set (6) ensures that the difference between the departure times of consecutive trips of each line is within the minimum and maximum headways. Furthermore, as mentioned earlier, the minimum frequency of each line i is f_i , and next trips of this line are referred to as extra trips. Therefore, we have:

$$x_{i,t-1} < x_{i,t} \quad \Rightarrow \quad \gamma_{i,t} = 1$$

$$x_{i,t-1} = x_{i,t} \quad \Rightarrow \quad \gamma_{i,t} = 0$$

Above conditions are guaranteed by constraint set (6). Constraint sets (7)-(9) determine the time-slot in which the departure time is happened for a given trip of line i . See Fig. 2 for illustration.

Table 2. Timetable created for each case

	Departure times in Case 1				Departure times in Case 2				Departure times in Case 3			
Line	**1**	**2**	**3**	**4**	**1**	**2**	**3**	**4**	**1**	**2**	**3**	**4**
1	0	8	8	-	15	26	26	-	12	26	26	-
2	13	23	33	33	15	25	35	35	15	25	35	35
3	6	16	26	26	14	24	34	34	14	24	34	34
4	4	18	18	-	16	30	30	-	16	30	30	-

Table 3. Comparing results of examples in all three cases

	Summation of transfer waiting times and fruitless waiting times (in minute)	Number of trips that cannot transfer to target line	Maximum value of transfer and fruitless waiting times (in minute)
Case 1	120	7	21
Case 2	57	3	9
Case 3	57	3	6

Constraint set (10) determines the waiting time in transfer station k for passengers transferring from the t th trip of line i to the t'th trip of line j . See Fig. 3 for illustration.

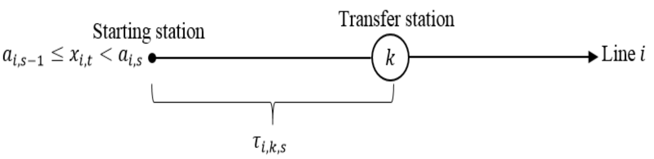

Fig. 2. Relation between trip departure time, time slot and travel time

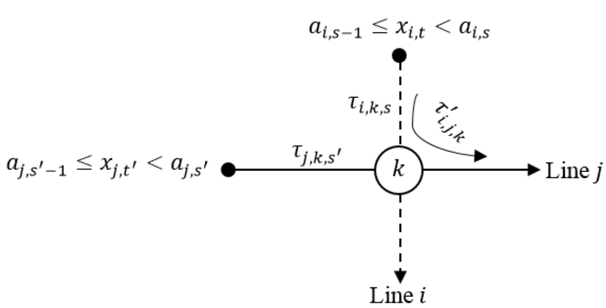

Fig. 3. Transfer from trip *t* of line *i* to trip *t′* of line *j* at transfer station *k*

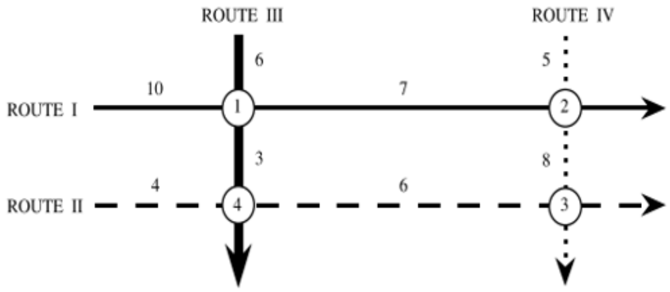

Fig. 4. A transit network [4]

Table 1. Headways and frequencies of lines of Fig. 4

Line	1	2	3	4
$\underline{h}_i$	8	10	10	14
$\overline{h}_i$	15	15	15	20
$\underline{f}_i$	2	3	3	2
$\overline{f}_i$	3	4	4	3

Constraint sets (11) and (12) determine that whether or not it is possible to transfer from the t th trip of line i to t'th trip of line j, in transfer stations k. As mentioned earlier, passengers leave the station with the first vehicle of the target line in order to reduce their transfer waiting time. Thus, the transfer waiting time for t th trip of line i to line j can be determined as follows:

$$w'_{i,j,k,t} = \min_{t' \le \overline{f}_j} \left\{ w_{i,j,k,t,t'} : w_{i,j,k,t,t'} \ge 0 \right\}$$

Constraint sets (13)-(15) determine the value of waiting time for transferring passengers in the transfer station k from the t th trip of line i to line j.

Note that $w_{i,j,k,t,t'} < 0$ means that the t'th trip of line j passes station k before arriving the t th trip of line i. In such a case, the transfer is impossible. Hence, if the value of $w_{i,j,k,t,t'}$ is negative for $t' = \overline{f}_j$, then fruitless waiting time is happened which is shown by $w''_{i,j,k,t}$, and is calculated as:

$$w''_{i,j,k,t} = \begin{cases} T - \left(x_{i,t} + \tau_{i,k,s}\right) - \tau'_{i,j,k} & w_{i,j,k,t,\overline{f}_j} < 0 \\ 0 & w_{i,j,k,t,\overline{f}_j} \ge 0 \end{cases}$$

Constraint set (16) determines whether or not the last trip of line j passes station k before arriving the t th trip of line i. Constraint set (17) calculates the value of fruitless waiting time in transfer station k for the t th trip of line i to target line j. Constraint sets (18) and (19) determine the maximum value of weighted transfer and fruitless waiting time. Constraint sets (20)-(22) determine the domain of variables.

2- 3- Simple example

To analyze the model performance under different objective functions, the following example is considered. Consider a planning horizon [0,45] (specified in minute) for the network depicted in Fig. 4. Labels on lines show travelling times (in minute) and are assumed to be fixed over the planning horizon. The walking and stopping times are assumed to be 2 and 0 minutes, respectively. The penalties associated with each unit of waiting time (in both cases transfer and fruitless) are assumed to be 1 monetery units (mu), and the cost of each extra trip is 20 mu. The headways and frequencies are shown in Table1.

Table 4. Numerical results (five instances in each row)

Data set	$\lvert\mathbb{I}\rvert$	$\left\lvert\bigcup_{i\in\mathbb{I}}\mathbb{I}'_i)\right\rvert$	B	Obj. Val. (CPLEX)	Obj. Val. (GRASP)	Time (m)	Gap%
1	4	4	8	177	177	0.1	0
2	10	20	20	5231	5384	5	3
3	10	20	56	12370	12584	5	2
4	20	30	104	20396	21242	10	8
5	20	30	626	-	13881	30	-
6	20	30	840	-	185998	30	-
7	30	40	950	-	236515	30	-
8	40	60	1892	-	498878	40	-
9	70	90	6286	-	1686020	60	-

The following three cases are considered:

Case 1: In addition to minimizing the cost of optional trip, the sum of transfer waiting times is also minimized (i.e., $p_{ijk}=1$, $p^{'}_{i,j,k}=0$, $c_i=20$, $d=0$)

Case 2: In addition to minimizing the cost of optional trip, the summation of transfer waiting times and fruitless waiting times is also minimized (i.e., $p_{ijk}=1$, $p^{'}_{i,j,k}=1$, $c_i=20$, $d=0$)

Case 3: In addition to minimizing the cost of optional trip, the summation of transfer waiting times and fruitless waiting times, as well as the maximum transfer and fruitless waiting time are also minimized (i.e., $p_{ijk}=1$, $p^{'}_{i,j,k}=1$, $c_i=20$, $d=1$).

The model is solved for three cases above and the results are summarized in Table 2. In Table 3, the results of all three cases are compared regarding the summation of the transfer waiting times and fruitless waiting times (reported in column 1), the number of trips that cannot transfer to desired line (reported in column 2), and the maximum value of transfer and fruitless waiting times (reported in column 3).

Table 3 evaluates the impact of considering the objective of minimizing the fruitless waiting time and demonstrates that using fruitless waiting time plays a significant role in making a better-distributed waiting time on transferring. Additionally, the result shows more improvement when considering a penalty for maximum weighted transfer and fruitless waiting time in case 3.

3- GRASP heuristic method

Commercial MIP solvers such as CPLEX can easily solve the model proposed in this paper for small-sized transit network. However, using CPLEX to solve this problem on larger-scale transit networks is not practical. Therefore, an algorithm based on GRASP heuristic is designed. GRASP, was first introduced by Feo and Resende [13] and successfully applied to solve different combinatorial optimization problems [14, 15, 16]. It is a multi-start procedure, each iteration of which consists of a construction step, where a greedy procedure is used to construct a solution, followed by a local search step aiming to enhance the solution found in the construction step.

Let matrix $X=\{x_{i,t}\}$ denote a timetable where i and t refer to row and column indices of X, respectively. Then, $\bar{X}$ is defined as a neighbor of X if it differs from X in only one entry. The set of all neighbors of the solution matrix X is denoted by $\mathbb{L}(X)$. In the following, the steps of the proposed GRASP method are explained. Throughout the description of the algorithm, $rand\left[r_1,r_2\right]$ refers to a random generator function generating a random integer number in the interval $\left[r_1,r_2\right]$.

Step 1: Initialization

For every $i\in\mathbb{I}$, determine the frequency of each line as $f_i=rand\left[\underline{f}_i,\bar{f}_i\right]$. Then, generate a random solution by using the following recursive sequence:

$$x_i^1=rand\left(0,\min(\bar{h}_i,\ T-(f_i-1)\underline{h}_i\right)\quad \forall i\in\mathbb{I}$$

$$x_{i,t}=x_{i,t-1}+rand\left(\underline{h}_i,\ \min(\bar{h}_i,\ T-(f_i-t)\underline{h}_i-x_{i,t-1}\right)\quad \forall t=2,\dots,f_i$$

Step 2: Mechanism of selection

Let X show the current timetable. Obtain a list of ℓ-highest weighted transfer and weighted fruitless waiting times for X. Then, select one of them randomly. Assume that $p_{i_0j_0k_0}\mathrm{w}_{i_0j_0k_0,t_0}$ or $p^{'}_{i_0j_0k_0}\mathrm{w}^{''}_{i_0j_0k_0,t_0}$ is selected, then, the vector (i_0,t_0) and j_0 are saved as desired indices.

Step 3: Allowable range and neighbor of current solution

Regarding the value of t_0, the three following cases are possible. For each case, the allowable range of changing the value of x_{i_0,t_0} is determined appropriately.

Case 1: $t_0=1$

The variable $x_{i_0,1}$ must satisfy the following headway constraints:

$$0\le x_{i_0,1}\le\bar{h}_{i_0},\underline{h}_{i_0}\le x_{i_0,2}-x_{i_0,1}\le\bar{h}_{i_0},$$

Thus, the allowable range of $x_{i_0,1}$ is as follows

$$\max\left\{0,x_{i_0,2}-\bar{h}_{i_0}\right\}\le x_{i_0,1}\le\min\left\{\bar{h}_{i_0},x_{i_0,2}-\underline{h}_{i_0}\right\}.$$

Case 2: $t_0\in\{2,3,\dots,f_{i_0}-1\}$

The variable x_{i_0,t_0} satisfies the following headway constraints:

$$\underline{h}_{i_0}\le x_{i_0,t_0}-x_{i_0,t_0-1}\le\bar{h}_{i_0},\ \underline{h}_{i_0}\le x_{i_0,t_0+1}-x_{i_0,t_0}\le\bar{h}_{i_0},$$

Then, we get

$$\max\left\{x_{i_0,t_0-1}+\underline{h}_{i_0},\, x_{i_0,t_0+1}-\overline{h}_{i_0}\right\}\le x_{i_0,t_0}\le\left\{x_{i_0,t_0-1}+\overline{h}_{i_0} x_{i_0,t_0+1}-\underline{h}_{i_0}\right\}$$

Case 3: $t_0=f_{i_0}$
The variable $x_{i_0,f_{i_0}}$ must satisfy the following constraints:

$$\underline{h}_{i_0}\le x_{i_0,f_{i_0}}-x_{i_0,f_{i_0}-1}\le\overline{h}_{i_0}\quad,\quad x_{i_0,f_{i_0}}\le T\,,$$

Consequently,

$$x_{i_0,f_{i_0}-1}+\underline{h}_{i_0}\le x_{i_0,f_{i_0}}\le\min\left\{x_{i_0,f_{i_0}-1}+\overline{h}_{i_0},T\right\}.$$

In addition, if $f_{i_0}>\underline{f}_{i_0}$, then remove last trip and create a neighbor of X. Therefore,

$$x_{i_0,f_{i_0}}=x_{i_0,f_{i_0}-1}\quad,f_{i_0}=f_{i_0}-1$$

Also, if $f_{i_0}<\overline{f}_{i_0}$ and $x_{i_0,f_{i_0}}\le T-\underline{h}_{i_0}$, then add a new trip and create neighbor of X. Departure time of new trip, $x_{i_0,f_{i_0}+1}$, must satisfy the following constraints

$$\underline{h}_{i_0}\le x_{i_0,f_{i_0}+1}-x_{i_0,f_{i_0}}\le\overline{h}_{i_0}\,,\; x_{i_0,f_{i_0}+1}\le T$$

Then, we get

$$x_{i_0,f_{i_0}}+\underline{h}_{i_0}\le x_{i_0,f_{i_0}+1}\le\min\left\{x_{i_0,f_{i_0}}+\overline{h}_{i_0},T\right\}.$$

Changing x_{i_0,t_0} to an integer in allowable range yields a new element of $\mathbb{L}(X)$.

Step 4: Restricted candidate list (RCL)

$$RCL=\left\{\overline{X}\in\mathbb{L}(X)\mid\overline{Z}\in\left[\overline{Z}_{min},\overline{Z}_{min}+\alpha\left(\overline{Z}_{max}-\overline{Z}_{min}\right)\right]\right\}$$

Where $\overline{Z}_{min}$ and $\overline{Z}_{max}$ are minimum and maximum values of objective function of elements in $\mathbb{L}(X)$, respectively.

Step 5: Update the value of parameter α
The value of $\alpha\in[0,1]$ is decreased during the algorithm by $\alpha=\alpha-\varepsilon_0$ where ε_0 is a small positive number.

Step 6: Local search algorithm
Let j_0 be what was saved before getting into "Mechanism of selection". The current solution should be improved by Hill Climbing algorithm via changing $x_{j_0,1},x_{j_0,2},\ldots,x_{j_0,f_{j_0}}$ elements in allowable range. Hill Climbing is an iterative algorithm that starts with an arbitrary solution, and then attempts to find a better solution by incrementally changing a single element of the solution vector. If the change produces a better solution, an incremental change is made to the new solution, repeating until no further improvement can be found [17].

4- Computational results

This section evaluates the performance of the proposed algorithm on different randomly generated instances. For this purpose, the best solution obtained by the proposed GRASP is compared with the one obtained by directly solving the proposed model via commercial solver CPLEX. All experiments are performed on a PC with 2.6 GHz CPU and 2 GB of RAM.
For all instances, the planning horizon is 240 minutes, which is divided into $n_i=3$ time-slots [0,80], [80,160], [160,240] for each line $i\in\mathbb{I}$. The minimum and maximum frequencies for each line are between [8, 15] and [15, 20], respectively. In addition, the minimum and maximum headways get the following values:

$$\underline{h}_i=\frac{T}{\overline{f}_i}\;,\;\overline{h}_i=\frac{T}{\underline{f}_i}$$

The cost of each extra trip is 50 mu and the penalty associated with the maximum value of transfer and fruitless waiting times is 10 mu. In addition, the penalty value of transfer and fruitless waiting time is within [1, 3]. Finally, travel times are set randomly.
The results of the proposed algorithm are summarized in Table 4. Each row of this table is associated with a given data set for which five instances are randomly generated and the averaged results are reported. In this table, the column labeled by *B* represents a measure responsible for the difficulty level of instances and is calculated as follows:

$$\mathcal{B}=\sum_{i\in\mathbb{I}}\sum_{j\in\mathbb{I}_i^{'}}\left|\mathbb{K}_{i,j}^{''}\right|$$

The column labeled by Gap represents the averaged gap between the optimal objective value obtained by CPLEX and the objective value of the best solution found by GRASP. The gap for a given instance is calculated as follows:

$$\text{Gap}=\frac{\text{GRASP objective value }-\text{CPLEX optimal objective value}}{\text{CPLEX objective value}}*100$$

The symbol “-“ in some rows of columns “Obj. Val. CPLEX” and “GAP” indicates that for some instances of that row, CPLEX could not terminate the process within a time limit of 2 hours.
With respect to the results provided in Table 4, it can be concluded that the proposed GRASP method is able to find good solutions in a reasonable amount of time.

5- Conclusion

In this paper, an MILP model was presented for the bus network timetabling problem with the aim of minimizing waiting time in transfer stations. Fruitless waiting time was introduced, as a novel concept, to increase the timetable’s quality in term of decreasing waiting time and distributing it over all stations. The results show that incorporating new features such as fruitless waiting time, and dynamic travel time to the model may lead to a high improvement in timetable quality. As a future work, the proposed model can be extended to deal with uncertainty in travel times and demands.

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