AUT Journal of Mechanical Engineering

AUT J. Mech. Eng., 4(2) (2020) 229-240
DOI: 10.22060/ajme.2019.16034.5798

Analysis of Workpiece Locating Error Using Geometric Fixture Model: A Theoretical and Experimental Study

H. Parvaz[1*], M. Bodaghy Aleny[2]

[1] Faculty of Mechanical and Mechatronics Engineering, Shahrood University of Technology, Shahrood, Iran.
[2] Mechanical Engineering Department, Tarbiat Modares University, Tehran, Iran.

ABSTRACT: Investigation of the robustness of the locating layout is an important analysis which is usually conducted in the verification stage of the fixture design procedure. In such an analysis, workpiece locating error is modeled by considering its sources in the workpiece and locating elements. The main focus of the present study is to investigate the robustness of the locating layout which is designed for the workpiece in the machining fixture, both theoretically and experimentally. Geometric model of the fixture is employed for theoretical analysis of errors in locating a workpiece using the well-known 3-2-1 locating principle. For validation, computer-aided assembly model is designed for calculating the workpiece locating error and comparing the results to the theoretical predictions. Experiments are also designed and conducted for validation of the theoretical predictions. Two types of the machining workpiece are incorporated as case studies for the validation process. Maximum error values equal to 3.6% and 6.1% are obtained between the theoretical predictions for the workpiece locating error and results of the computer-aided design model and experiments, respectively. Results of the present study confirmed that the locating layout with the maximum distance between its locators provided the minimum locating error value for the workpiece. Agreement between the theoretical predictions and results of the computer-aided model and experiments confirmed the applicability of the geometric fixture model and credibility of its results.

Review History:

Received: 18 Mar. 2019
Revised: 25 May. 2019
Accepted: 8 Jul. 2019
Available Online: 12 Jul. 2019

Keywords:

Experiment
Fixture design
Geometric fixture model
Jacobian
Locating error

1- Introduction

The fixture design process consists of four main stages including setup planning, fixture planning, unit design, and verification. In verification stage, fixturing characteristics are further evaluated through several analyses to ensure that the designed fixture and its elements meet the fixturing requirements. In fixture verification, several analyses are conducted including locating/clamping layout analysis, tolerance analysis, stability analysis and investigation of the fixture accessibility and affordability. In locating layout analysis, which can be considered as the most important step in the verification process, the designed locating system is verified to investigate whether it meets the design requirements. Two tests are performed in locating layout analysis including evaluation of deterministic locating conditions and investigation of the robustness of the locating layout. For studying the deterministic locating condition, locating matrix is constructed using locating wrenches and its full rank is introduced as the main requirement of the deterministic locating condition. On the other hand, the robustness of a fixturing plan can be defined as its capability to keep the workpiece position and orientation errors at the minimum level by the application of disturbances at the locating points. Analysis of robustness is important in verification stage of fixture design, since inappropriate locating layout may result to the high levels of deviations at workpiece position and orientation which may lead to the defectiv or even scrap final part. These unwanted disturbances may be emerged due to the different conditions such as changing positions of the contact points due to the locator deformation in both normal and tangential directions, abrasion of locating elements and penetration of the locating elements into the workpiece surface due to the high pressure at the contact points. Geometric model of the fixture is usually used for mathematical modeling of workpiece locating errors. Such errors are induced by displacements that may occur at the locating points. In this paper, the geometric fixture model is first adjusted, programmed and incorporated for calculation of the mentioned errors in the workpiece position and orientation. Computer-aided assembly model is also prepared for measurement of these errors. The experimental setup is fabricated for validation of the theoretical predictions. Fixture verification is performed in four main stages including investigation of the deterministic condition in locating system and its robustness, tolerance analysis, workpiece stability analysis and investigation of accessibility and affordability of fixturing plan. These stages are reduced to three steps in computer-aided fixture design process which can be named as problem definition, fixture synthesis and fixture analysis [1]. Since the present study is focused on theoretical modeling and experimental investigation of the robustness of the locating system, researches that have been reported in the field of geometric fixture model are going to be reviewed in further details. Asada and By [2] introduced locating matrix that was constructed by six wrenches corresponding to the six

*Corresponding author's email: h.parvaz@shahroodut.ac.ir

locating elements. By assuming that an infinitesimal error occurred in the workpiece loci, the Jacobian matrix was constructed and defined based on the partial derivatives of the workpiece surface equation. Full rank of the Jacobian matrix was introduced as the main requirement of the deterministic locating condition. Wang [3] proposed a fixture synthesis method based on optimization of locators' positions on the workpiece surface. The objective function was defined as the minimization of the workpiece locating error. Problem was solved through the application of perturbation to the workpiece at the locating points. Carlson [4] proposed a non-linear sensitivity equation for determination of workpiece locating error which was induced by errors in the locating elements. The second order Taylor expansion was incorporated to calculate the resultant error in the quadratic form. The main contributions of the proposed model were its capabilities in consideration of contact between the fixturing elements and workpiece surfaces in non-prismatic parts. Wang [5] incorporated geometric fixture model to investigate effects of errors in locators' position, geometric tolerances of datum surfaces and locators' geometrical tolerances on two parameters: workpiece localization error and geometric tolerances of machining features. In a comprehensive study, geometric and kinematic models were developed for the computer-aided fixture design verification purposes [6]. In geometric fixture model, errors in locator space were mapped to the workpiece localization space through a linear model and specific Jacobian matrix. The locating layout performance was quantified by definition of the Performance Index (LPI) which was calculated by gramian of the Jacobian matrix. Using fixture kinematic model, Kang et al. [7] proposed a straightforward model for the analysis of workpiece stability in the machining fixture. Fixture stiffness matrix was calculated through off-line connection to Finite Element Method (FEM) software. By using the fact that each individual contact remains stable when its reaction force coincides inside its corresponding friction cone, CSI[1] parameter was derived as the workpiece stability index. Raghu and Melkote [8] proposed a mathematical model for predicting workpiece localization error by considering the fixture geometric error. A model was also suggested for predicting the fixture-workpiece compliance by taking into account the contact compliance and application of clamping forces to the workpiece. Geometric fixture model was used to predict the workpiece localization errors due to the fixture geometric error by using the Jacobian matrix. Song and Rong [9] evaluated the deterministic locating condition for the locating layout by checking the rank of the locating matrix. The study focused on the under-constrained and over-constrained conditions for the locating layouts and an algorithm was suggested to assist the fixture designer in the determination of the workpiece un-constrained motions. Qin et al. [10] classified the workpiece localization source errors into three categories, e.g. locators' setup errors, error in the manufacturing of locators and error in the manufacturing of the workpiece. A mathematical model was developed by taking into account the effects of the localization source errors using a Jacobian matrix for the transformation of the locators' setup error to the workpiece locating errors. Vishnupriyan et al. [11] proposed a Genetic Algorithm (GA) based method for locator layout optimization by considering the locator's geometric error and taking into account the satisfaction of the machining tolerances for the most critical features existed in the workpiece. Workpiece locating error was calculated through the geometric fixture model by considering the source errors in the locators. He et al. [12] suggested a mathematical model for investigating the effects of fixing errors on the machining accuracy of the features pattern. Several sources of errors were considered in calculating the fixing errors including errors in the locators, geometrical errors of the locating surfaces, and errors caused by clamping forces and machining loads. The proposed approach was tested on a rectangular block with a four-hole pattern to verify the predictions of the proposed model on the accurate calculation of the composite positional error of features pattern. In reference [13], a linear model was established between the errors in the locating points and workpiece pose errors. In calculating the errors in the locating points, two formulas were developed for expressing these errors in the surface to surface contact and pin-hole contact configuration. Transformation matrices were derived to express the tolerances for planar, cylindrical and freeform surface features. Theoretical predictions were compared to the Computer Aided Design (CAD) simulation results through two case studies. Khodaygan [14] proposed an optimization model for reducing the workpiece locating error by adjustment of the locators at the contact points. Geometric Capability Ratio (GCR) was introduced for quantifying the capability of the manufacturing process in producing the key characteristics of the workpiece within the design requirements. Uncertainty analysis was also performed based on three approaches including the direct, worst case, and statistical methods. In reference [15], a method was developed for modeling the fixture-workpiece system with the aim of establishing a relationship between the workpiece locating error and its sources. Errors in locators and workpiece tolerances were considered as the main sources of errors in the development of the suggested model. A compensation procedure was also proposed for reducing the workpiece locating error by adjustment of the length of the locators. Error analysis was conducted based on the worst case and statistical approaches. In an attempt to optimize the locating layout with the aim of reducing the workpiece pose error, Wan et al. [16] proposed a multi-objective optimization technique based on the singular values of the location matrix. Four main parameters were introduced as the design variables, as far as volume of error ellipsoid, maximal error amplification factor, condition number (or relative error amplification factor), and locating stability are involved. Predictions of the implementation were compared to the experimental results. By considering the literature survey, the well-established geometric fixture model has been incorporated in different applications for various purposes such as calculation of the workpiece locating error, tolerances analysis in the machined workpiece and locating layout design. Despite the existence of several types of research which have been published by incorporation of the geometric fixture model, no specific research has been published for experimental verification of the workpiece locating errors predicted by these models. Since simplifying assumptions are usually incorporated in mathematical modeling of the workpiece locating errors, validation of the theoretical predictions through the experimental tests seems

1 Contact Stability Index

to be necessary to ensure the accuracy of the model predictions. This can be considered as a vacancy in the literature which is not fulfilled by the previous studies. In this paper, the geometric fixture model is employed for mathematical calculation of the workpiece locating error induced by errors in the locators. Two experimental setups are also designed and fabricated with the adjustable 3-2-1 locating layouts to validate the theoretical predictions on the locating errors of the workpieces. So, the main novelty of the present study can be elaborated as the experimental verification of the theoretical predictions on workpiece locating errors predicted by geometric fixture model. The rest of the paper is organized as follows. In section 2, the theoretical foundation is elaborated for the theoretical calculation of the workpiece locating error due to the source errors in the locating elements. The procedure of the experiments and data gathering techniques are presented for two case studies in section 3. Results and discussions are elaborated in section 4 and finally, the main conclusions of the paper are described in section 5.

2- Theoretical Foundation

Geometric fixture model is widely used for calculation of the workpiece locating errors due to the source errors. The main assumptions that have been made in the derivation of this model are as follows; the workpiece is assumed to be rigid. So, the effects of the deformations are assumed to be negligible. Secondly, the locating elements and surfaces are assumed to be rigid. Thirdly, it is assumed that six contacts are established between the workpiece and locators based on the 3-2-1 locating principle. Moreover, point contact is assumed to be established between the workpiece and locators. It means that the spherical head locators are employed for locating the workpiece inside the fixture. Finally, Effects of the workpiece tolerances are not considered in derivation of the mathematical model. By these assumptions, the geometric fixture model can be demonstrated as follows [7]:

$$\begin{aligned} \{\Delta d\} &= [J]\{\Delta q\} \\ \{\Delta q\} &= [J]^{-1}\{\Delta d\} \end{aligned} \tag{1}$$

where, $\{\Delta d\} = \{\Delta d_1, \Delta d_2, \Delta d_3, \Delta d_4, \Delta d_5, \Delta d_6\}$ is the locator displacement in the normal-to-surface direction, $\{\Delta q\} = \{\Delta x, \Delta y, \Delta z, \Delta\alpha, \Delta\beta, \Delta\gamma\}$ is the workpiece locating error and $[J]$ is the Jacobian matrix which maps the errors from locators' space (local coordinate systems) to the workpiece space (global coordinate system). Jacobian matrix can be calculated as [10]:

$$J = [J_1, J_2, ..., J_6]$$

$$\begin{aligned} J_i = [&-n_{ix}, -n_{iy}, -n_{iz}, n_{iz} y_i - \\ &n_{iy} z_i, n_{ix} z_i - n_{iz} x_i, n_{iy} x_i \\ &-n_{ix} y_i] \end{aligned} \tag{2}$$

In which, n_{ix}, n_{iy} and n_{iz} are three components of the normal vector to the workpiece surface at the *i*th contact point. Also, x_i, y_i and z_i are the coordinates of the *i*th contact point. Disturbance can be applied in two ways to the locating system; changing the positions of locators in the 3-2-1 locating system and applying displacements to the locator in the normal-to-surface direction. By application of disturbances, workpiece deviates from the ideal position and orientation. The values of this deviation can be calculated

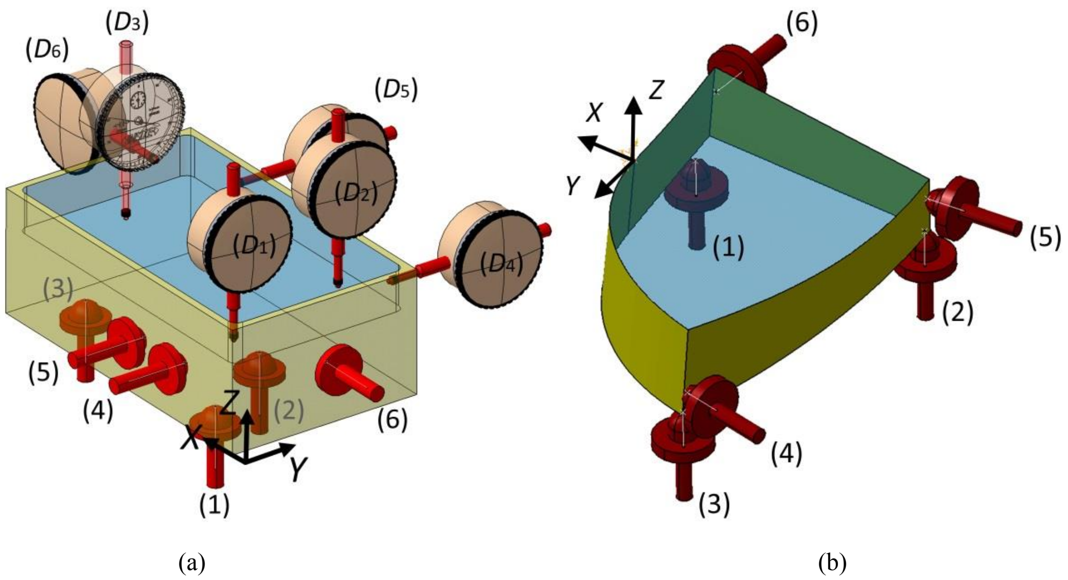


Fig. 1. Models chosen as case studies (a) polyhedral workpiece (b) freeform workpiece.

from Eq. (1), mathematically. A quantified value seems to be necessary to evaluate the efficiency of the locating layout. For this purpose, variable of locating performance index (LPI) is defined and calculated by gramian of the Jacobian matrix [6]:

$$LPI = \sqrt{\text{gram}([J])} = \sqrt{\left\| [J]^{\mathrm{T}} . [J] \right\|} \tag{3}$$

LPI can be used as an efficient and easy-to-calculate criterion to compare the locating layouts. High values of LPI indicate the better performance of the corresponding locating layout compared to its low values. This index is used for scoring the locating systems in the experimental tests.

Two workpiece models are chosen as case studies to investigate the applicability of geometric fixture model and the reliability of its results. These models include a polyhedral workpiece (Fig. 1(a)) and a workpiece with freeform surfaces (Fig. 1(b)). Initial locating layout for the polyhedral workpiece is designed based on the traditional 3-2-1 locating principle. Three base locators are applied to the workpiece with the maximum available distances. Due to the space limitations and application of the dial indicators to the workpiece, side locators are not positioned with the maximum distance on the secondary locating surface of the workpiece.

The sixth locator is applied to the workpiece at the center point of the tertiary locating surface in the normal direction. The left-hand coordinate system is located at the represented position of Fig. 1(a).

The second case study is a workpiece model which is comprised of the freeform surfaces. Similarly, locating layout is designed for this model based on the 3-2-1 locating principle. Three base locators are applied to the base locating surface with the maximum possible distances. Side locators are applied to the secondary locating surface in the normal direction with the maximum possible distance to each other. Stop locator is also applied to the tertiary locating surface with considering its maximum distance to the base and side locators. The global coordinate system is defined in the position that is represented in Fig. 1(b). The workpiece locating error is going to be calculated in this point through the experiments.

3- Experiments

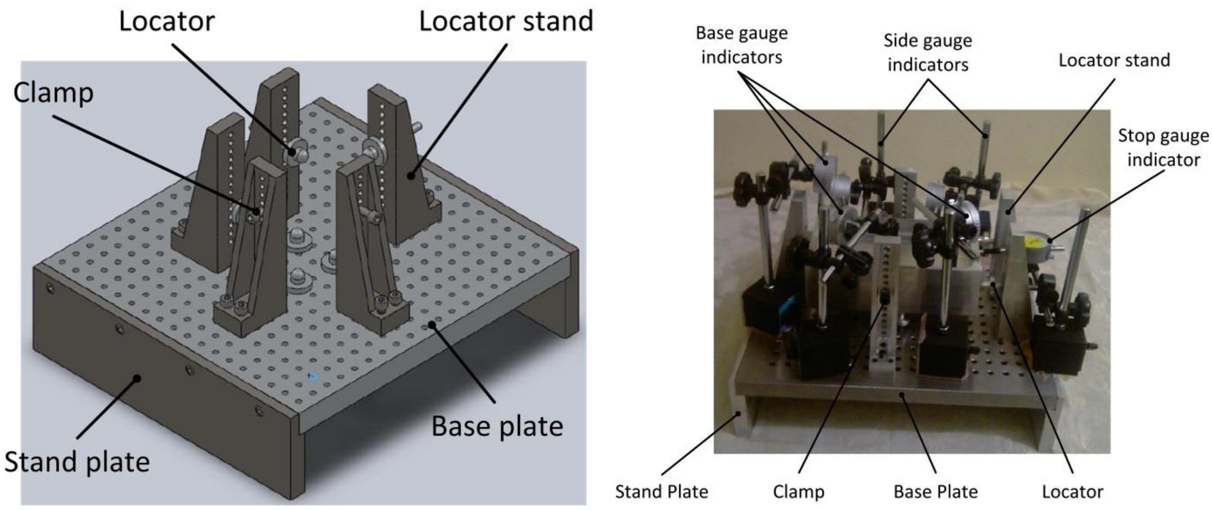
Two workpieces were chosen to investigate the validity of results predicted by the theoretical model. The procedure of the experimental tests is going to be described in the next sections.

3- 1- Case study 1

A special experimental setup is designed for evaluating the robustness of the designed locating layout for the first workpiece through several tests. Assembled model of the setup is represented in Fig.2.A $370\,\text{mm} \times 400\,\text{mm}$ steel rectangular base plate was fabricated and machined to the required dimensions. Total of 272 holes were drilled at the base plate in a 16×17 rectangular pattern. Two stand plates (with height equal to 100 mm) were also fabricated from steel material and fixed to the base plate by four M6 screws. Six head-sphered locators were installed and fastened on the base plate and locator stands through bolts and nuts. Locator stands were also bolted on the base plate with two M8 screws. Clamping forces were also applied by means of the M8 screws which were fastened on two stands. Six dial indicators were installed and fixed at six distinct points around the workpiece. Aluminum was selected as the workpiece material in order to ease the machining processes. Facing and pocket milling operations were conducted on the workpiece raw material for sizing and preparing the final part with the required dimensions. Grinding was also applied on all of the surfaces of the base plate. Experiments were designed based on changing the position of the locators and measuring the locating point displacement through six dial indicators. Direct measurement of displacements at the locating points was impossible by using the dial indicators. So, indicators were installed at the nearest available positions (in normal-to-surface direction) around the workpiece locating surfaces. Position and orientation of the dial indicators are demonstrated in Table 1. These poses remained constant during the experiments. Workpiece locating error was then calculated through geometric fixture model. Also, the quality of the locating layout is quantified through the LPI parameter. For the design of experiments, five different locating layouts were designed for workpiece by changing the position of six locators in the 3-2-1 locating system. These layouts are represented in Fig. 3. Dial indicators were installed at the fixed positions that are represented in Fig. 1(a) with coordinates that are mentioned in Table 1. Since measurements of the dial indicators are going to be used for calculation of the workpiece locating errors, they have been applied to the workpiece with the maximum possible distances according to the 3-2-1 locating principle. Displacement equal to 2 mm was applied to locator No. 3 in the normal-to-surface direction for all experiments. Measurement of Δd was impossible at the contact points. So, displacement vector was measured through six dial indicators which were installed at six points on the workpiece surfaces. Clearly, these six points were different from the locating points. Jacobian matrix was then calculated for dial indicators based on the position and

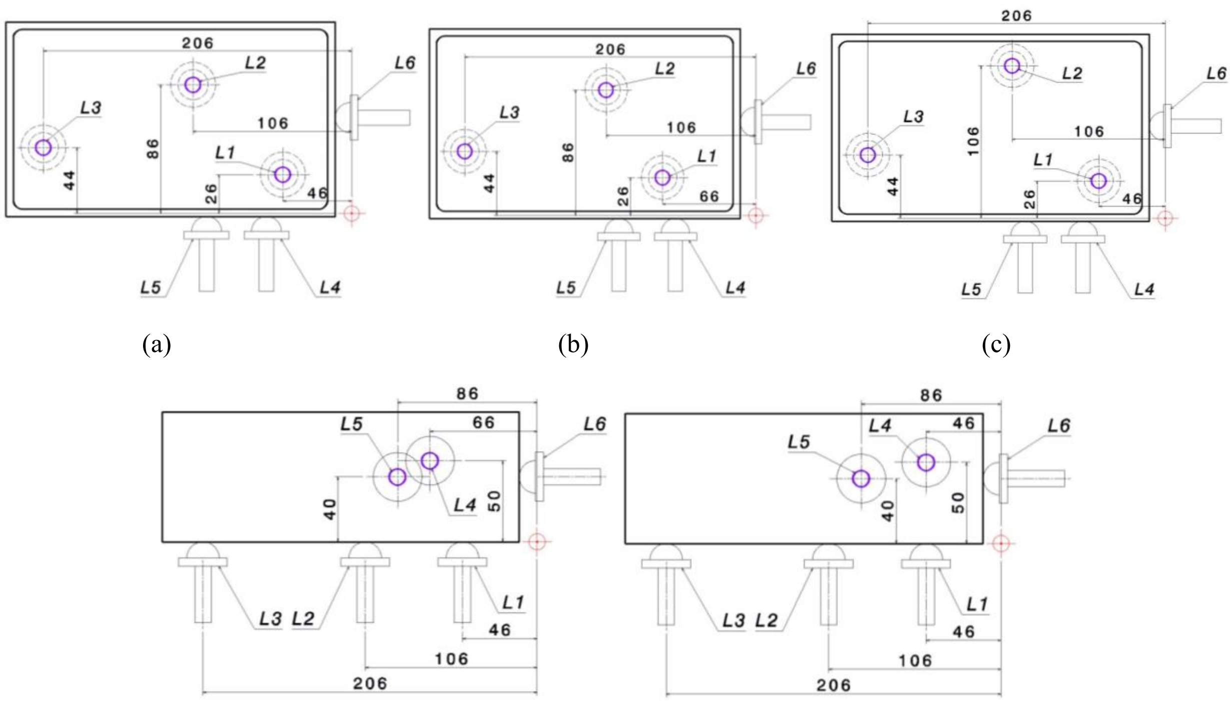
Table 1. Position and orientation of dial indicators for the first case study (D_1 to D_6 in Fig. 1(a))

r_i (mm)	n_i
(40, 20, 54)	(0, 0, -1)
(60, 108, 54)	(0, 0, -1)
(200, 60, 54)	(0, 0, -1)
(40, 128, 60)	(0, -1, 0)
(190, 128, 60)	(0, -1, 0)
(231, 105, 65)	(-1, 0, 0)



(a) (b)

Fig. 2. Experimental setup for the first case study (a) assembled CAD model, and (b) the fabricated setup.



(a) (b) (c)

(d) (e)

Fig. 3. Five locating layouts for the first case study.

orientation of Table 1 as follows:

$$J = \begin{bmatrix} 0 & 0 & 1 & 20 & -40 & 0 \\ 0 & 0 & 1 & 108 & -60 & 0 \\ 0 & 0 & 1 & 60 & -200 & 0 \\ 0 & 1 & 0 & -60 & 0 & 40 \\ 0 & 1 & 0 & -60 & 0 & 190 \\ 1 & 0 & 0 & 0 & 65 & -105 \end{bmatrix} \tag{4}$$

Workpiece locating error was calculated based on Eq. (1) by substitution of the Jacobian matrix and displacements that were measured by dial indicators. By applying the mentioned disturbance to the locator No. 3, $\{\Delta d\}$ was measured using dial indicators at each of the five locating layouts. Workpiece locating error was then predicted through Eq. (1) for each locating layout. Each test was repeated for three times and average values of measurements were used for calculation of the results.

3- 2- Case study 2

The second case study includes workpiece with freeform surfaces. Locating layout for this model has been represented in Fig. 1(b). Coordinates of the locating points are demonstrated in Table 2 [17]. The workpiece has two planar and four freeform surfaces. It was selected because its locating surfaces have curvature in the longitudinal direction, only. This geometry is important because locators should be applied in the normal-to-surface directions and their stands can only rotate along the axis that is perpendicular to the base plate. So, the stand can rotate such that the locator is applied to the locating surfaces in the perpendicular direction. Fig. 4 represents the experimental setup which is designed for the second case study. Similar to the first case study, direct

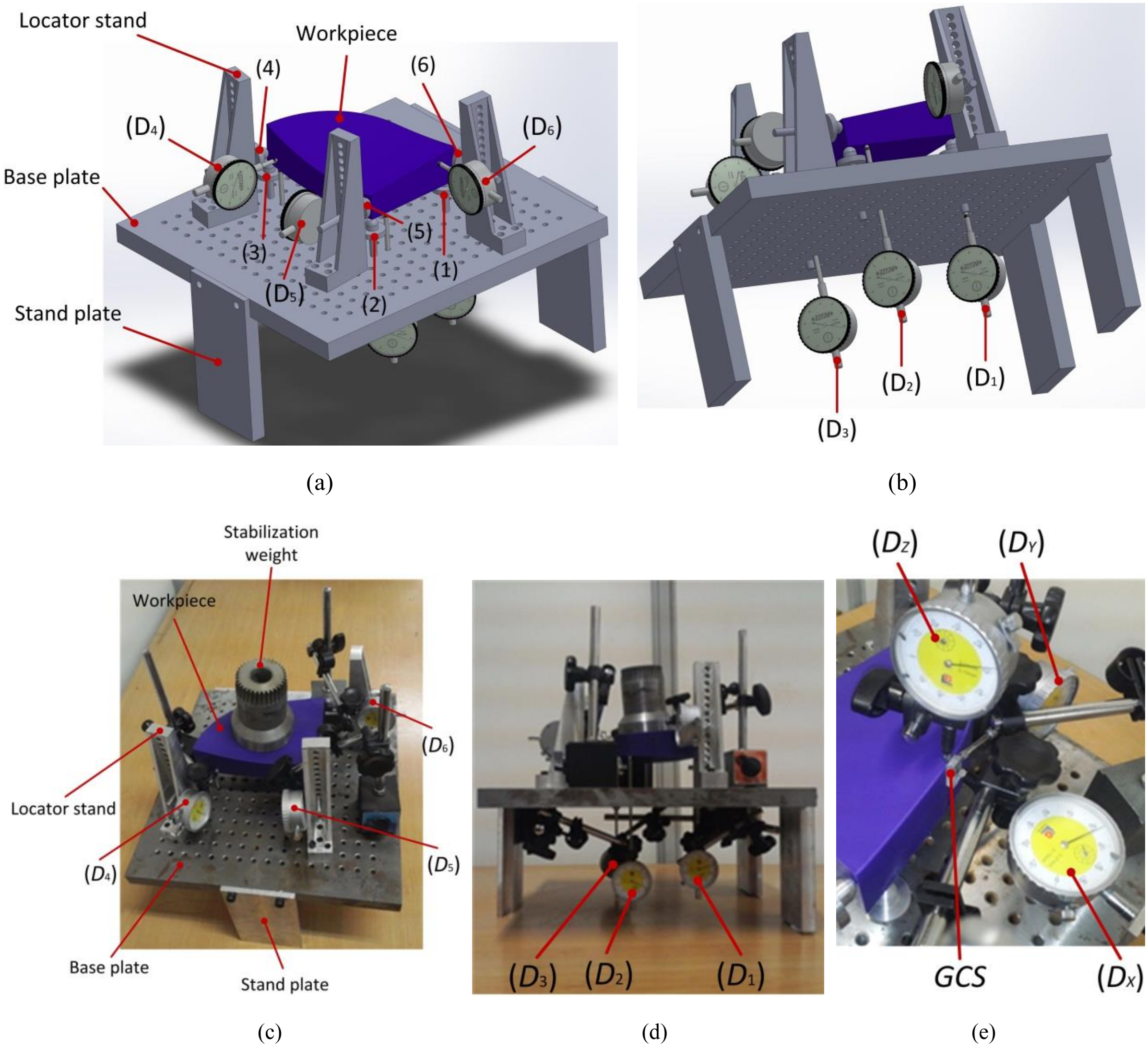


Fig. 4. The experimental setup designed for the second case study (a) and (b) setup model (c) and (d) the fabricated setup (e) experimental setup for measurement of workpiece locating error {Δq} at Global Coordinate System (GCS).

Table 2. Coordinates of the locating points for the second case study.

No.	1	2	3	4	5	6
r_i (mm)	12.83	-107.52	-101.8	-103.96	-109.84	14.87
	-63.73	-81.44	99.2	98.58	-81.46	-83.44
	-43.68	-43.68	-43.68	-31.76	-23.68	-16.28

Table 3. Position and orientation of the dial indicators for the second case study.

No.	1	2	3	4	5	6
D_i (mm)	12.83	-87.52	-101.8	-109.69	-113.51	-8.1
	-63.73	-81.44	79.2	70.49	-55	-83.44
	-43.68	-43.68	-43.68	-31.68	-21.67	-20.26
n_i	0.00	0.00	0.00	0.98	0.99	0.00
	0.00	0.00	0.00	-0.17	0.11	1.00
	1.00	1.00	1.00	0.00	0.00	0.00

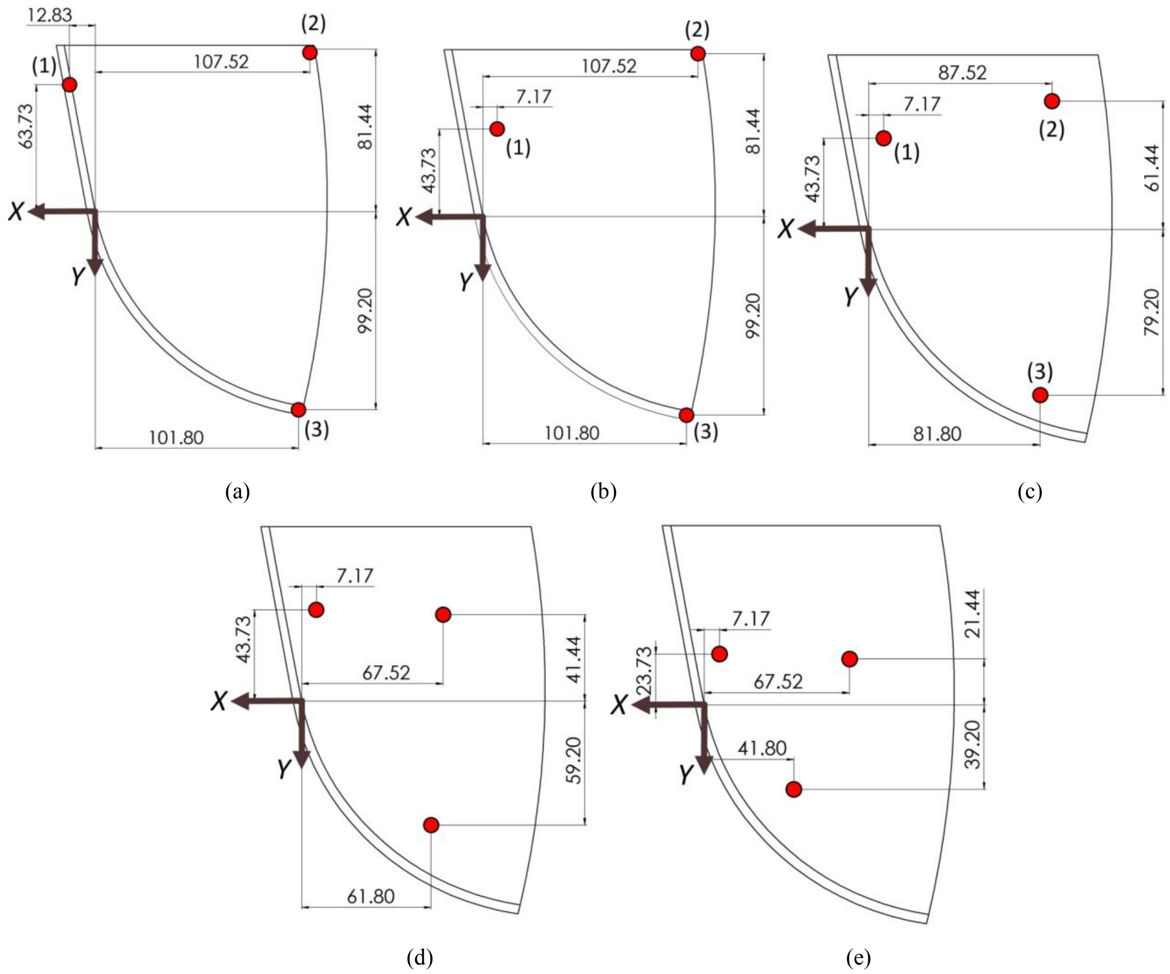


Fig. 5. Five locating layouts for the second case study.

measurements of displacements at the locating points were impossible for this model due to the accessibility problem to the contact point between the workpiece and locating elements. So, measurements were performed with small distances to the locating points through six dial indicators. According to Figs. 4(a) to 4(d), dial indicators were installed and fixed at the nearest position to the locating points. In the second case study, each test was also repeated for three times and average values were used as the experimental results. The same base plate was used for the second case study. It was fixed and leveled using three stand plates. Workpiece model was manufactured using Fused Deposition Modeling (FDM[1]) additive manufacturing process with the density equal to 30%. Since the final workpiece was a light-weight part, a stabilization weight was used to keep the contacts between the workpiece and locating elements. Stands were rotated on the base plate such that the locators were applied to the locating surfaces in the perpendicular direction. Six dial indicators (D_1 to D_6. in Fig. 4) were installed on the nearest position to the locating points to measure the displacement vector $\{\Delta d\}$. Position and orientation of these indicators are described in Table 3. Elongation bars were used at tips of the dial indicators (D_1 to D_3) to make the measurements possible from the underneath side of the base plate (Fig. 4(d)). Workpiece locating error $\{\Delta q\}$ was measured using three dial indicators (D_X, D_Y and D_Z in Fig. 4(e)) at the point where the global coordinate system was defined. According to Fig. 4(e), these indicators were installed in the mutually perpendicular pattern to measure the displacement components of the workpiece locating error. For this purpose, a small piece of plexiglass was adhered to the workpiece at GCS point to make the measurement of displacement possible in the X, Y and Z directions. Five locating layouts were designed for the second case study by alternating positions of the base locating points. These layouts are represented in Fig. 5. For all of the experiments, the disturbance was applied to the locating point No. 1 as displacement equal to 2 mm along the surface normal direction. Displacements at locating points were measured through the dial indicators. Jacobian matrix was then constructed and the workpiece locating error was calculated from the geometric fixture model. The theoretical predictions were then compared to the experimental results that were measured through dial indicators D_X, D_Y and D_Z. An assembly model was constructed for the second case study using CATIA software. According to Fig. 6, workpiece locating error was measured at the origin point between the fixed global and displaced local coordinate systems by application of disturbance at locating point No. 1. Results were then compared to the theoretical predictions and experimental results.

1 Fused Deposition Modeling

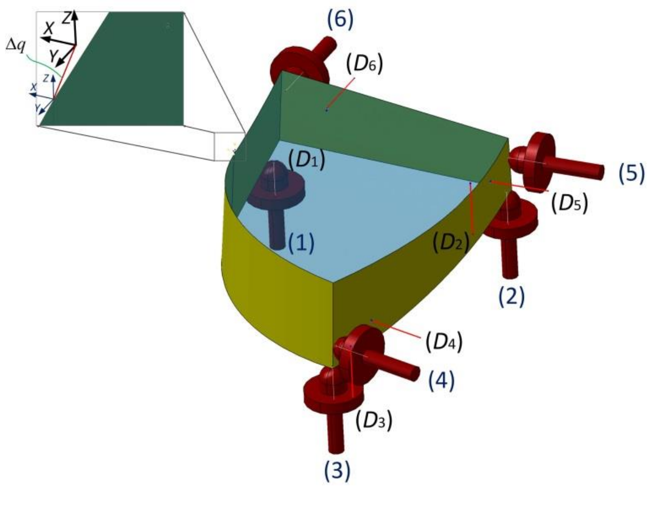


Fig. 6. Five locating layouts for the second case study.

4- Results and Discussion

For the first case study, workpiece locating error $\{\Delta q\}$ was calculated from Eq. (1) by substitution of $\{\Delta d\}$ which was measured by six dial indicators at six points of the workpiece. The locating performance index was also calculated using Eq. (2) for each locating layout. Analysis of error can be performed using two scenarios; worst case and statistical methods. The worst case scenario assumes that the effective parameters in the mathematical model are in the maximum or minimum threshold of their corresponding tolerance spans. In the case of calculating the workpiece locating error, it can be concluded that this method considers the highest values of the source errors in the locators to obtain the maximum possible value of the workpiece locating error. Compared to the statistical method, which calculates the root sum square of the product of Jacobian matrix and vector of locators' errors, more accurate results can be obtained from the direct method. Since signed values of errors and elements are incorporated in the Jacobian matrix of the mathematical model, the worst case method is employed with the direct approach for the error analysis. Table 4 depicts the results of experimental measurement of $\{\Delta d\}$ and mathematical prediction of $\{\Delta q\}$ from Eq. (1) for each of the five locating systems in the first case study. Results indicate that the LPI value is increased by moving from layout No. 1 to layout No. 3. It means that the layout No. 2 and especially, layout No. 3 can be considered as better layouts in comparison to layout No. 1. This fact is further confirmed by paying attention to the 3-2-1 locating principle which states that an appropriate locating layout consists of locating elements which have the maximum possible distances to each other. It can be observed in Fig. 3 that the distances between the base locators are increased by moving from layout No. 1 to layouts No. 2 and 3. LPI value is decreased for locating layouts No. 4 and 5 by 28% and 64% in comparison to locating layout No. 3. It indicates that the robustness is decreased for these layouts in comparison to layouts No. 3. This result can be confirmed by considering the positions of the side locators in these layouts (Figs. 3(d) and 3(e)). It can also be observed that the locating error has a reverse relationship with the LPI value. Results of Table 4 indicate that the norm of vector $\{\Delta q\}$ is decreased for locating layouts No. 2 and 3 by almost 29% and 43% in comparison to layout No. 1. Increasing trend of $\{\Delta q\}$ is also observable for locating layouts No. 4 and 5. It can be concluded that the locating layout No. 3 (with maximum LPI and minimum $\{\Delta q\}$ values) is the most robust locating plan between the locating layouts of the first case study. For the second case study, workpiece locating error

Table 4. Measured average values of {Δd} and calculated values of {Δq} from geometric fixture model for the first case study.

	Locating layout No.				
	1	2	3	4	5
$\{\Delta d\}$ (mm)	-0.19	-0.05	-0.03	-0.03	-0.07
	-0.76	-0.70	-0.55	-1.04	-1.15
	1.96	1.70	1.48	1.73	1.92
	0.16	0.16	0.16	0.14	0.15
	0.23	0.24	0.22	0.51	0.62
	-0.27	-0.27	-0.24	-0.40	-0.52
$\{\Delta q\}$ (mm)	0.8167	0.6670	0.5542	0.8148	0.8776
	-0.4650	-0.4893	-0.3683	-0.8478	-0.9359
	-0.6264	-0.3828	-0.3221	-0.3218	-0.4074
	-0.0101	-0.0105	-0.0085	-0.0148	-0.0160
	-0.0160	-0.0136	-0.0116	-0.0147	-0.0164
	0.0005	0.0005	0.0004	0.0025	0.0031
LPI	304000	336000	464000	336000	166000
$\|\{\Delta q\}\|$ (mm)	1.280	0.910	0.730	1.218	1.345

Table 5. Average values of {Δd} measured from experiments and their corresponding values of {Δq} measured from the CAD model and the geometric fixture model for the second case study

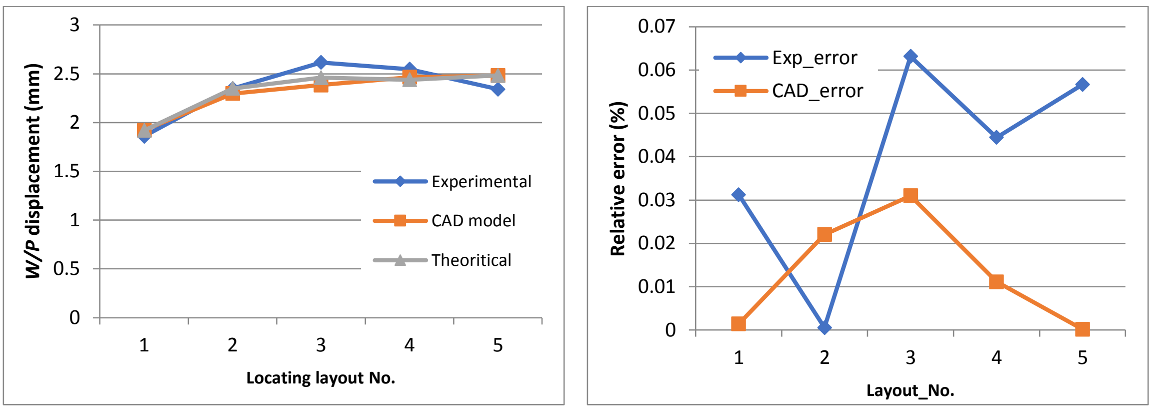
	Locating layout No.				
	1	2	3	4	5
$\{\Delta d\}$ (mm)	2.00	2.000	2.00	2.00	2.00
	0.28	0.410	0.02	-0.31	0.01
	-0.01	0.000	-0.56	-1.52	-2.30
	-0.20	-0.21	-0.28	-0.26	-0.15
	-0.41	-0.72	-0.81	-0.80	-0.99
	0.01	-0.10	-0.08	0.04	0.10
$\{\Delta q\}$ (mm) (Theoretical model)	0.764	1.026	1.240	1.370	1.488
	0.013	0.105	0.071	0.121	-0.285
	-1.760	-2.111	-2.123	2.013	-1.965
	0.003	0.008	0.001	0.004	0.011
	0.0172	0.020	0.025	0.031	0.033
	0.0004	0.003	-0.002	0.002	-0.005
$\|\{\Delta q\}\|$ (mm)	1.919	2.349	2.460	2.438	2.482
$\{\Delta q\}$ (mm) (CAD model)	0.749	0.849	1.070	1.363	1.407
	0.000	-0.001	-0.027	-0.133	-0.469
	-1.770	-2.135	-2.130	-2.050	-1.989
	0.003	0.004	0.006	0.010	0.017
	0.017	0.020	0.020	0.032	0.033
	0.0005	0.0003	-0.020	0.000	-0.006
$\|\{\Delta q\}\|$ (mm)	1.922	2.298	2.383	2.465	2.482
$\{\Delta q\}$ (mm) (Experiment)	0.55	0.95	1.01	1.25	1.24
	-0.25	-0.31	-0.32	-0.39	-0.51
	-1.76	-2.13	-2.39	-2.19	-1.92
	N.A.	N.A.	N.A.	N.A.	N.A.
	N.A.	N.A.	N.A.	N.A.	N.A.
	N.A.	N.A.	N.A.	N.A.	N.A.
$\|\{\Delta q\}\|$ (mm)	1.860	2.348	2.615	2.546	2.341
LPI	2590000	2140000	1340000	729000	590000

(N.A.: Not applicable)

was calculated for each of the five locating systems using Eq. (1). $\{\Delta q\}$ was also measured using a CAD assembly model by application of the measured $\{\Delta d\}$ values at the mentioned six points. Finally, workpiece locating error was measured using three mutually perpendicular dial indicators. Predictions of the theoretical model were then compared to the results of the CAD model and experiments. Table 5 represents the results. Angular components of workpiece locating error couldn't be measured at the experiments due to their low values which were beyond the resolution of the available commercial accelerometer sensors. For the five locating layouts of the second case study, distances between the base locators were decreased by moving from the first to the fifth locating layouts (Figs. 5(a) to 5(e)). By application of disturbance to locator No. 1, it is expected that the workpiece locating error is increased by moving from the first locating layout to the fifth one. This trend can be observed in Table 5 for workpiece locating error. By measuring the $\{\Delta d\}$ values through six dial indicators and substituting the results in Eq. (1), workpiece locating error was calculated and depicted in Table 5. Since the robustness of the locating system is reduced by moving from locating layout No. 1 to No. 5, the calculated values of $\{\Delta q\}$ values from the theoretical model are increased; however, a slight decrease in $\{\Delta q\}$ value can be observed for locating layout No. 4 which can be related to the errors in measuring the $\{\Delta d\}$ values from the experiments.The increasing trend of workpiece locating error was also observed at the results which were obtained from the CAD assembly model. The same trend is observable in the experimental results for the first to the third locating systems; however, $\{\Delta q\}$ values are slightly decreased for locating layouts No. 4 and No. 5 which were expected to increase according to the previous trend. This decrease is related to the fact that the angular components of $\{\Delta q\}$, which couldn't be measured through the experiments, construct the larger portion of $\{\Delta q\}$ in locating layout No. 4 and No. 5 in comparison to the locating layouts No. 1 to No. 3. This can be further confirmed by paying attention to the fact that $\Delta\alpha, \Delta\beta, \Delta\gamma$ components increase by worsening the conditions of the locating system. Fig. 7a represents the diagrams of workpiece locating error values which are calculated from the theoretical model, CAD assembly model and experiments. It can be observed from Fig. 7(a) that $\{\Delta q\}$ values are increased by moving from locating layout No. 1 to No. 5 for both theoretical predictions and CAD assembly model. A decrease in the magnitude of $\{\Delta q\}$ measured from the experiments can be observed in locating layouts No. 4 and No. 5 in this figure. By assuming that the calculated values for $\{\Delta q\}$ is accurate for the theoretical model, the relative error values are calculated and graphed for measurements of CAD model and experiments. The maximum relative errors in measurement of the magnitude of $\{\Delta q\}$ vector are obtained as 3.1% and 6.3% for CAD model and experiments, respectively. Also, the corresponding maximum absolute error values are obtained as 0.08 mm and 0.15 mm for the CAD model and experiments, respectively. All of these error values are obtained for the third locating layout. These values of errors confirm the accuracy of the CAD model and experiments. LPI values are also decreased by moving from the first locating layout to the last one. It represents that the robustness of the locating layouts against the disturbances is decreased by moving from locating layout No. 1 to No. 5. The main limitation of the present study to be applicable on the machining fixtures includes the incapability of the fabricated setup in measurement of the angular component of the workpiece locating error. It may cause errors in the experimental results. Also, effects of the workpiece tolerances are not considered in the present study which may lead to error in measurement of the workpiece locating error.

5- Conclusion

In the present study, geometric fixture model was used for investigation of the locating layout robustness in fixturing of polyhedral and freeform workpieces using 3-2-1 locating



(a) (b)

Fig. 7. (a) Workpiece locating error with regards to the locating layouts calculated from the theoretical model, CAD assembly model and experiments for the second case study (b) relative error in measurement of {Δq} from CAD assembly model and experiments.

principle. Predictions of the mathematical model were validated through CAD assembly model and experiments. The main conclusions can be elaborated as follows:

• For the first case study, the LPI value was increased and reached to the maximum value for locating layout No. 3. This locating layout had the maximum distance between locators in comparison to the other locating layouts.

• It was observed that the LPI value was reduced by decreasing distance between the locators.

• The magnitude of $\{\Delta q\}$ was decreased and reached to the minimum value (0.730 mm) for locating layout No. 3. By investigation of the results, it was concluded that the locating layout with the maximum distance was the most robust plan with the maximum LPI and minimum $\{\Delta q\}$ values.

• Maximum error equal to 3.1% and 6.3% were obtained in measurement of the magnitude of workpiece locating error from CAD model and experiments, respectively.

• The decreasing values of LPI and increasing values of $\{\Delta q\}$ values confirmed the maximum robustness of the first locating layout for workpieces.

• It can be generally concluded that the predictions of workpiece locating error from the geometric fixture model can be considered as accurate results for workpiece with polyhedral and freeform surfaces.

As further studies, the experimental measurement of the angular components of the workpiece locating error is going to be performed by authors using laser measurement techniques.

References

[1] S. Pehlivan, J.D. Summers, A review of computer-aided fixture design with respect to information support requirements, International Journal of Production Research, 46(4) (2008) 929-947.

[2] H. Asada, A.B. By, Kinematic analysis of workpart fixturing for flexible assembly with automatically reconfigurable fixtures, IEEE Journal on Robotics and Automation, 1(2) (1985) 86–94.

[3] M.Y. Wang, An optimum design for 3-D fixture synthesis in a point set domain, IEEE Transactions on Robotics and Automation, 16(6) (2000) 839-846.

[4] J.S. Carlson, Quadratic sensitivity analysis of fixtures and locating schemes for rigid parts, Journal of Manufacturing Science and Engineering, 123(3) (2001) 462-472.

[5] M.Y. Wang, Tolerance analysis for fixture layout design, Assembly Automation, 22(2) (2002) 153-162.

[6] Y. Kang, Y., Rong, J.-C. Yang, Computer-aided fixture design verification: Part 1, the framework and modeling, International Journal of Advanced Manufacturing Technology, 21(10-11) (2003) 827-849.

[7] Y. Kang, Y. Rong, J.C. Yang, Computer-aided fixture design verification. Part 3. Stability analysis, International Journal of Advanced Manufacturing Technology, 21(10-11) (2003) 842-849.

[8] A. Raghu, S.N. Melkote, Modeling of workpiece location error due to fixture geometric error and fixture-workpiece compliance, Journal of Manufacturing Science and Engineering, 127(1) (2005) 75-83.

[9] H. Song, Y. Rong, Locating completeness evaluation and revision in fixture plan, Robotics and Computer Integrated Manufacturing, 21(4-5) (2005) 368-378.

[10] G.H. Qin, W.H. Zhang, M. Wan, A mathematical approach to analysis and optimal design of a fixture locating scheme, International Journal of Advanced Manufacturing Technology, 29(3-4) (2006) 349-359.

[11] S. Vishnupriyan, M. Majumder, K. Ramachandran, Optimization of machining fixture layout for tolerance requirements under the influence of locating errors, International Journal of Engineering, Science and Technology, 2(1) (2010) 152-161.

[12] G. He, L. Guo, W. Tian, X. Zhao, The prediction of fixture comprehensive influence on the machining error of features pattern, in: ASME 2015 International Manufacturing Science and Engineering Conference, American Society of Mechanical Engineers, 2015, pp. 26-34.

[13] W. Tang, Y. Li, J. Yu, J. Zhang, L. Yu, Locating error analysis for workpieces with general fixture layouts and parameterized tolerances, Proceedings of the Institution of Mechanical Engineers, Part B: Journal of Engineering Manufacture, 230(3) (2016) 416-427.

[14] S. Khodaygan, A method for optimal reduction of locating error with the minimum adjustments of locators based on the geometric capability ratio of process, International Journal of Advanced Manufacturing Technology, 94(9-12) (2018) 3963-3978.

[15] S. Khodaygan, A method for locator errors compensation in the fixture-workpiece system, SAE International Journal of Materials and Manufacturing, 6(3) (2013) 494-501.

[16] X.J. Wan, J. Yang, H. Zhang, Z.Y. Feng, Z. Xu, Optimization of fixture layout based on error amplification factors, Journal of Computing and Information Science in Engineering, 18(4) (2018) 1-12.

[17] H. Parvaz, M.J. Nategh, Development of locating system design module for freeform workpieces in computer-aided fixture design platform, Computer Aided Design, 104(1) (2018) 1-14.