AUT Journal of Civil Engineering

AUT J. Civil Eng., 4(3) (2020) 367-376
DOI: 10.22060/ajce.2019.16504.5594

Reliability and Sensitivity Analysis of Single-Layer Space Domes

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ABSTRACT: In recent years, a number of space structures have been destroyed or collapsed completely due to snow load, external wind load, sudden earthquake impacts and improper traditional design. Not considering uncertainties in materials and external loads can be the main reason in this regard. Therefore, effort has been made in the present study to examine the effects of reliability and sensitivity among random variables and performance functions on the failure probability of single-layer space domes. In order to determine the appropriate and efficient method for reliability analysis in space domes, the reliability analysis was carried out according to approximation (FORM, SORM) and simulation (Importance Sampling, Monte Carlo) methods. The modulus of elasticity, yield stress, external loads, node coordinates, and cross-section of members were considered as random variables to be used in two limit state functions (displacement and ultimate stress). Results of the FORM, SORM, MCS, and IS methods show that in space structures with many random variables, FORM yields good solutions, and sensitivity analyses of the random variables show that the results depend on the type of the limit state functions; a change in the limit state functions will also change the sensitivity. For instance, the values of the sensitivity of the cross-section random variable for the stress and displacement limit state functions are 61% and 8.6%, respectively.

Review History:

Received: 2019-06-24
Revised: 2019-08-08
Accepted: 2019-08-09
Available Online: 2019-08-25

Keywords:

space structures
reliability index
sensitivity analysis
performance functions

1. INTRODUCTION

Buildings such as industrial halls, exhibition centers, airport terminals, and shopping malls are large span structures that generally have flat or low-slope light weight steel roofs. Nowadays, domes are increasingly used compared to the other structural forms due to their beauty, appropriate behavior against external loads, low weight and cost-effectiveness. However, structural failures or even catastrophic collapses due to improper design, underestimated design loads, unexpected extreme loads, improper manufacturing, poor workmanship, and so on are reported worldwide [1–4] a reason for which is neglecting uncertainties in materials and external loads on the structure [5]. A proper structure design should first incorporate such uncertainties [6]. Although many papers have focused on the optimization, buckling characteristics, seismic behavior assessment and failure mechanisms of these structures, few studies have examined the sensitivity of the solution random variables and correlation among the responses of long-span space structures [7-8-9]. In recent years, some reliability aspects of space structures have been studied [10]. In the design of some engineering structures, some geometric and physical parameters are considered constant while most of them are not so and behave as a random variable. The sensitivity analysis is usually aimed to identify the parameters that affect the structures' probabilistic analysis; those with negligible effects on the structure response can be assumed constant to reduce the calculations volume and time. Correlation among random variables and performance functions can lead to a profound understanding of the mechanical properties of space structures and the relationships among performance functions [6]. The present study aimed to evaluate the reliability and sensitivity analysis between random variables and performance functions of single-layer space domes. First, the reliability index was calculated using four reliability methods. The modulus of elasticity, yield stress, external loads, node coordinates, and cross-sectional area of members were considered as random variables. Two limit state functions, one based on displacement and other based on stress limitation were regarded in this research. Second, the sensitivity of performance functions was investigated to identify the impact of different types of random variables and the different positions of the members.

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2. RELIABILITY

2.1. Limit state function

The limit state function indicates the boundary between the desirable and undesirable performance of the structure, represented by a mathematical function. Generally, this limit state is between the strength and load on the structure, with the following expression:

$$g(R,S) = R - S \tag{1}$$

where R is the structure strength and S is the external load. Function g divides the space into R_s (safe area) and R_f (rupture area) [11].

$$g(X)=\begin{cases} >0, X \in R_s \\ \le 0, X \in R_f \end{cases} \tag{2}$$

In the reliability context, failure does not necessarily mean the structure rupture; it may rather mean that certain constraints are achieved or passed through the structure. Accordingly, limit state functions in single-layer space domes are defined as follows.

2.2. Displacement limit state function

The displacement limit state function is:

$$g_{idispl}=1-\frac{|\delta_i(X)|}{\delta_a} \qquad i=1,2\cdots n \tag{3}$$

where $\delta_i(X)$ is the displacement of node i, n is total number of nodes in the structure, δ_a is the permissible displacement of node i (with limits δ_H = H/300 and δ_V = D/360 in the horizontal and vertical directions, respectively recommended by [12]), and D and H are the diameter and height of the space dome, respectively.

2.3. The ultimate limit state function

When the structure of the single-layer space domes is placed under the external load, the internal force, mainly a type of axial force, is created on its members. Therefore, when the axial stress on the structure members is higher than the yield stress or critical axial buckling stress, the failure of structure occurs and its limit state function is given by the following expressions.

$$g_{istress}=1-\frac{|\sigma_i(X)|}{\sigma_a} \qquad i=1,2\cdots n \tag{4}$$

$$\sigma_a=\min(\sigma_{cr},\sigma_y)$$

where σ_i is the stress in the i[th] member and σ_a is the allowable stress determined based on the AISC-ASD Code [13]. The yield stress in tensile members is found as follows:

$$\sigma_y=F_y \tag{5}$$

The allowable compressive stress is found based on two possible buckling failure modes as follows:

A: Non-elastic buckling ($\lambda_i < C_c$):

$$\sigma_{cr}=\left(1-\frac{\lambda_i^2}{2C_c^2}\right)\times F_y \tag{6}$$

B: Elastic buckling ($\lambda_i \ge C_c$):

$$\sigma_{cr}=\left(\frac{\pi^2 E}{\lambda_i^2}\right) \tag{7}$$

where E is the modulus of elasticity, F_y is the steel yield stress, $\lambda_i=\frac{kL_i}{r_i}$ is the member slenderness ratio, L_i and r_i are the length and radius of gyration of the i[th] member, respectively, Cc is the critical slenderness (for lean, intermediate and obese members), and k_i is the i[th] member's effective length coefficient (1 for truss members). According to AISC, the maximum slenderness ratio for members under tension and compression is limited to 300 and 200, respectively [13]. Since all coefficients in the reliability analyses equal 1, the failure probability (P_f) is estimated as follows:

$$P_f=P(g(X)\le 0)=\int_{g(X)\le 0}^{0} f_x(x)dx=\int_{R_f}^{0} f_x(x)dx \tag{8}$$

where $f_X(x)$ is the joint probability density function for all random variables. P_f is found by integrating $f_X(x)$ in the failure area g (X) ≤ 0; since this is a hard task requiring multiple integrations (and is sometimes impossible) in large practical problems, use should be made of approximate or simulation methods [14]. The structural reliability is an important indicator to evaluate the structural performance [15]. The reliability analysis are divided into two analytical and simulation method. The analytical methods are easy to use and their calculations have low cost in terms of time, while their accuracy is in doubt for some issues. The simulation methods have time-consuming calculations although they can be used for every structural model or limit state function.

3. SIMULATION METHODS

Simulation is often used as an effective and accurate method to assess the structure reliability and compare different related methods. While other methods cannot estimate the reliability, simulation methods well estimate the failure probability of structures with complex limit state functions and high variables if the number of samples is large enough. In the Monte Carlo method, since the number of the generated samples is theoretically unlimited (N_∞), P_f is found as follows [16]:

$$p_f=\frac{1}{N_\infty}\sum_{i=1}^{N_\infty} I(X_i) \tag{9}$$

where $I(X_i)$ is the failure index defined for each simulation as follows:

$$I(X_i)=\begin{cases} 1 & if \;\; G(X_i)\le 0 \\ 0 & if \;\; G(X_i)>0 \end{cases} \tag{10}$$

To calculate P_f, some independent random samples (N in number) are generated from each random variable. If N_H shows all the points lying in the failure zone, we will have:

$$P_f \cong \frac{N_H}{N} \tag{11}$$

According to this method, a large number of samples should be generated to achieve high accuracy, especially when the probability of failure is small. Therefore, the different methods of variance reduction are presented to reduce the high cost of computing in this method [16]. The IS (importance sampling method) is an advanced MCS (Monte Carlo Method) used to estimate P_f and find its zones. To generate and simulate samples, use is made of a novel probability density function $h_V(V)$ as the importance sampling density function instead of the principal probability distribution function of variables. Therefore, the simulation results should be used as weight functions to estimate the importance density function. Accordingly, P_f is modified as follows:

$$P_f = \int_D \cdots \int \left\{ I(V) \frac{f_x(V)}{h_V(V)} \right\} h_V(V) dx \tag{12}$$

where $I(V)$ is an index function the value of which is 1 for points in the failure zone; otherwise, it is 0. Hence, $h_V(V)$ is an appropriate importance density function. An unbiased estimate of P_f is as follows:

$$P_f = \frac{1}{N} \sum_{i=1}^{n} \left(I(V_i) \frac{f_x(V_i)}{h_V(V_i)} \right) \tag{13}$$

where V_i (random sample) is generated using importance distribution function $h_V(V_i)$, and $f_x(V_i)$ is the variables' principal probability distribution function [17].

4. ANALYTICAL APPROXIMATION RELIABILITY METHODS

One of the most used first-order methods was presented in 1974 and its most important feature is the invariance of the reliability index for different shapes of a limit state function under the same mappings for random variables [18]. In this method, random variables are transferred from the design space to the normal standard space with a zero mean and variable standard deviations, and the reliability index is obtained as the minimum geometric distance between the origin and the transferred limit state function. Hasofer-Lind [19] defined the design point as one on the limit state function (g = 0) with the least distance from the origin in the normal standard space; it is also known as the maximum failure probability point (MPP) (Fig. 1).

The distance between this point and the origin shows the reliability index and provides the possibility of estimating P_f using $P_f = \Phi(-\beta)$. Therefore, calculating the design point requires the use of an optimization algorithm as follows:

$$\beta = \sqrt{\sum_{i=1}^{n} U_i^{\,2}} \qquad g(U) = 0 \tag{14}$$

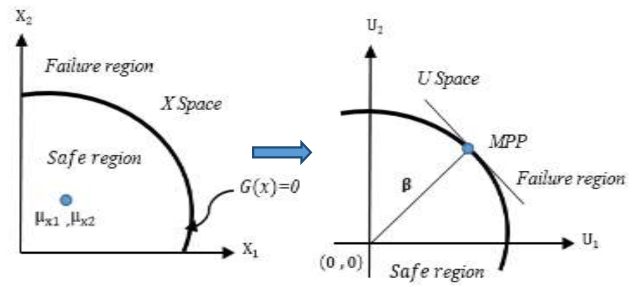

Fig. 1. Hasofer-Lind reliability index [19]

where U_i is the value of the i[th] random variable in the standard normal space and n is the number of random variables. The general optimization method or point search algorithm with the maximum P_f presented by Hasofer-Lind & Rackwitz-Fessler (HLRF) [19] is used to solve Eq. (14). Using the first-order algorithm is usually an appropriate method for linear limit state functions located near the design point. Since first-order reliability algorithms do not yield accurate safety index estimations when the limit state has great curvature, use is made of second-order reliability methods where a second curved surface is used to approximate the limit state function at the design point. When a curved surface curvature conforms to that of the limit state function, P_f in the 2[nd] order approximation method is found as follows:

$$P_f = P\{g(X) \le 0\} = \Phi(-\beta) \prod_{i=1}^{n-1} (1 + \beta k_i)^{2} \tag{15}$$

where k is the curvature of the limit state function at the maximum failure probability point (MPP) [20].

5. SENSITIVITY ANALYSIS

5.1. Sensitivity analysis with respect to the mean and standard deviation of random variables

Since numerous random variables in the reliability analyses of space structure problems increase the computational costs and time, the sensitivity analysis is a solution to identify important random variables that are effective in reliability analyses; other less important ones can be considered as deterministic variables. Hence, the sensitivity analysis can be used to estimate a structure safety with low cost. In the reliability method, P_f is related to the safety index and is found as follows:

$$P_f = 1 - \Phi(\beta) \tag{16}$$

where P_f is the failure probability and $\Phi(\beta)$ is the normal cumulative distribution function. The reliability sensitivity related to random variables' mean and standard deviation is thus found as follows [21]:

$$\frac{\partial \beta}{\partial \mu_X} = -\frac{\frac{\partial g}{\partial X}}{\left| \frac{\partial g}{\partial X} \sigma_X \right|} \quad \frac{\partial \beta}{\partial \sigma_X} = -\beta \frac{\left(\frac{\partial g}{\partial X} \right)^2}{\left| \frac{\partial g}{\partial X} \sigma_X \right|} \times \sigma_X \tag{17}$$

Accordingly, the sensitivity analysis is aimed to identify the random variables with small effects on the reliability index to reduce the calculations. If a problem is simplified by ignoring the uncertainty of x_i, the percent error obtained from the reliability index estimation is found as follows [22]:

$$error = \left[\frac{\beta_{medain} - \beta_{distribution}}{\beta_{distribution}} \right] \times 100\% = \left(1 - \left(1 - \alpha_i^{\,2} \right)^{-\frac{1}{2}} \right) \times 100\% \tag{18}$$

where $\beta_{Distribution}$ and β_{Medain} are the problem reliability indices found by assuming x_i to be once random and once deterministic at the mean point, respectively.

5.2. Sensitivity analysis of the random variables

In this method, the sensitivity of the reliability index is calculated by creating a small turmoil in random variables. In fact, first-order estimation methods present the importance factor (α_i^2) as a derivative of a linearized limit state function. These importance factors are actually the same as the conductor cosine vector in the search process which should satisfy the following equation:

$$\alpha_1^{\ 2}+\alpha_2^{\ 2}+\cdots+\alpha_i^{\ 2}=1 \tag{19}$$

$$\frac{\partial\beta}{\partial u_i}=\frac{\partial}{\partial u_i}\left(\sqrt{u^2{}_1+u^2{}_1+\cdots u^2{}_i}\right)=\frac{u_i}{\beta}=\alpha_i \tag{20}$$

Since α_i physically means the relative contribution of each random variable to P_f, the one with the highest sensitivity coefficient in the above equation has the greatest effect on the reliability index. In fact, α_i indicates the sensitivity coefficient of the reliability index at the point with the highest P_f resulted from defining the reliability index (β) as the distance of the limit state function from the MPP (g (U) = 0) to the origin in the normal standard space [23].

6. NUMERICAL RESULTS

6.1. A 24-member space dome

A 24-member single-layer space dome with 866 cm span and 82.16 cm height (Fig. 2) has been used for the reliability evaluation and sensitivity analyses [24]. Fig. 2 shows the geometric characteristics of the structure, number of nodes, number of members, concentrated loads and supporting conditions. The members of the structure are classified into three groups and the modulus of elasticity is considered $21000\,{}^{KN}\!/_{cm^2}$ and yield stress is regarded $21\,{}^{KN}\!/_{cm^2}$. This dome is carrying concentrated loads at all nodes except at supports, node 1 (-40KN), and nodes 2-7 (-20KN). Displacement is 1.40 cm in all directions and elasticity modulus, yield stress, applied loads, coordinates of nodes 1-7 in all directions, and members' section areas are random variables. Table 1 shows the variables' statistical characteristics including the probability distribution, mean and standard deviation.

6.1.1. Comparing the reliability index determination methods

Since a challenge in determining the reliability in space structures is the high number of random variables which increase the computational time, this section is aimed

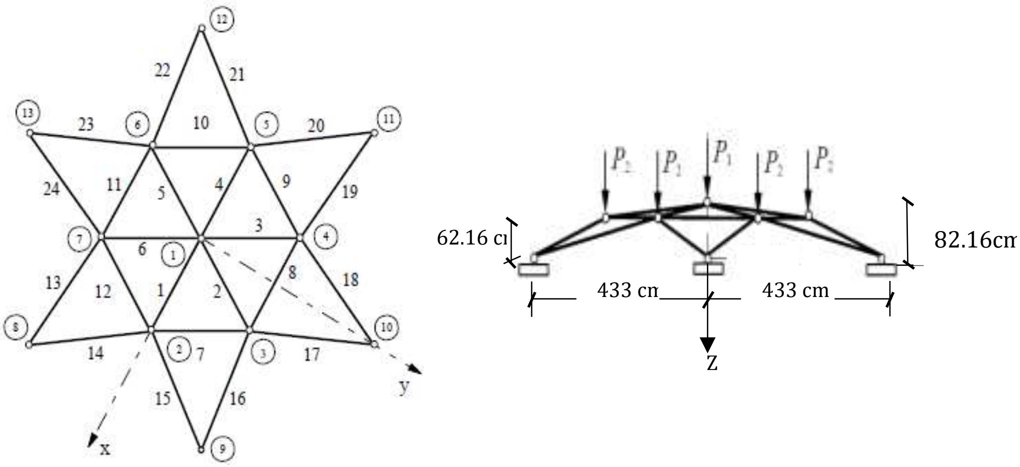

Fig. 2. The geometry and loading of the 24-member space dome [24]

Table 1. Random variables of the 24-member diamatic space dome

R. V.	Description	Distribution	Mean	Coefficient of variation
A_1 (i=1~6)	Cross-sectional area	Lognormal	19 cm^2	5%
A_2(i=7~12)	Cross-sectional area	Lognormal	13 cm^2	5%
A_3(i=13~24)	Cross-sectional area	Lognormal	14 cm^2	5%
E(i=1~24)	Young's modulus	Lognormal	21000 kN/cm^2	5%
F_y	Yield strength	Lognormal	21 kN/cm^2	5%
P	Concentrated loads	Lognormal	P_2 = 20 kN	10%
			P_1 = 40 kN	
$x(i).y(i).z(i)$ (i=1~7)	Nodal coordinates in the x, y and z directions	normal	As is	1 cm

Table 2. Comparison of several methods for determining reliability index

	FORM	SORM	ISM	MCS(1000000)
β Time (s) Error	1.4811 0.36 3.86%	1.4436 0.57 2.%	1.4376 943 0.81%	**1.4261** **3489** ----

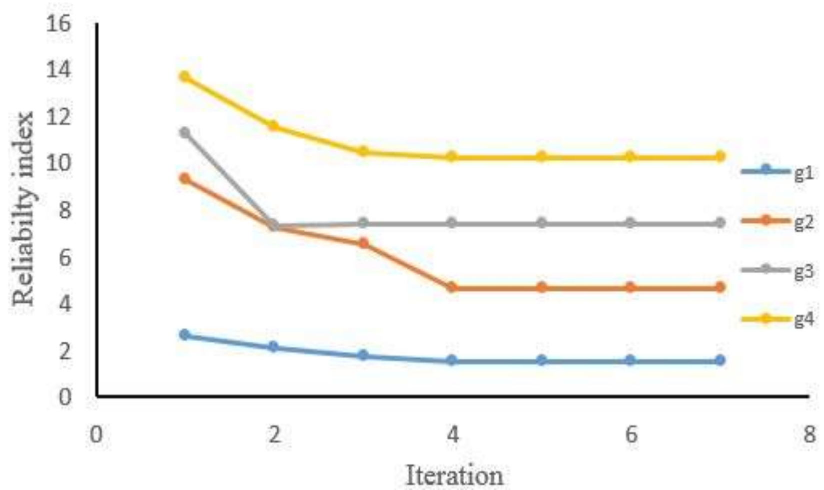

Fig. 3. Effects of limit state functions on the reliability

to select from among the FORM (first-order reliability method), SORM (second-order reliability method), ISM (importance sampling method), and MCS (Monte Carlo Sampling) the one which is the fastest and the most accurate in reliability analysis. To specify the failure mode, use should be made of such limit state functions as the fatigue, fracture, displacement, and ultimate limit state function; the one used in this research for reliability analysis is the displacement function because the space structure vertex node has the most varying displacement:

$$g_1(X) = 1 - \frac{|\delta_3(X)|}{\delta_a} \tag{21}$$

where $\delta_3(X)$ is node 1 displacement in direction z and δ_a = 1.4 cm is maximum allowable displacement of the 24-member space structure. Table 2 indicates the results. All reliability computations are implemented in a computer which configuration is as follows:

the Intel (R) Core i5 CPU M480 @2.67GHz.

As shown, FORM is the fastest among the proposed methods because it only takes 0.36 seconds to calculate the reliability index for this structure; however, its error is 3.86% compared to the MCS method. Next is SORM yields better results than FORM because it uses a parabolic surface to approximate the limit state function; however, its calculation time increases significantly with an increase in the number of random variables because it requires a second order derivative. The ISM error is considerably lower than those of the FORM and SORM although it requires much time (943 sec) to calculate the reliability index. Finally, the MCS method takes 3489 sec (more than all other methods) to determine the reliability index. It can be concluded, in general, that since simulation methods are quite time-consuming and are not cost-effective in space structures with a large number of random variables, it is preferable to use the FORM and SORM approximation methods to determine the reliability index. A comparison of the two methods can reveal that their reliability indexes are close and time consumed in FORM is less than SORM, FORM can be used to continue the work.

6.1. 2. Investigating the effect of limit state functions on the reliability index evaluation

Effort has been made in the present study to investigate the effects of different limit state functions on the reliability index of structural members. If the displacement limit state function (for the vertex node) and ultimate limit state functions for members of groups one, two, and three of the structure are g_1, g_2, g_3, and g_4, respectively, the reliability analyses results of the 24-member truss (Fig. 3) show that their reliability indices are 1.48, 4.59, 7.37, and 10.18, respectively. Since g_1 has the lowest reliability index, it is the critical limit state function because the failure probability is high. Therefore, displacement limit state function is an important function for the 24-member structure and there is no need to consider other functions.

6.1. 3. Sensitivity analysis of the 24-member dome

Since random variables involved in the reliability

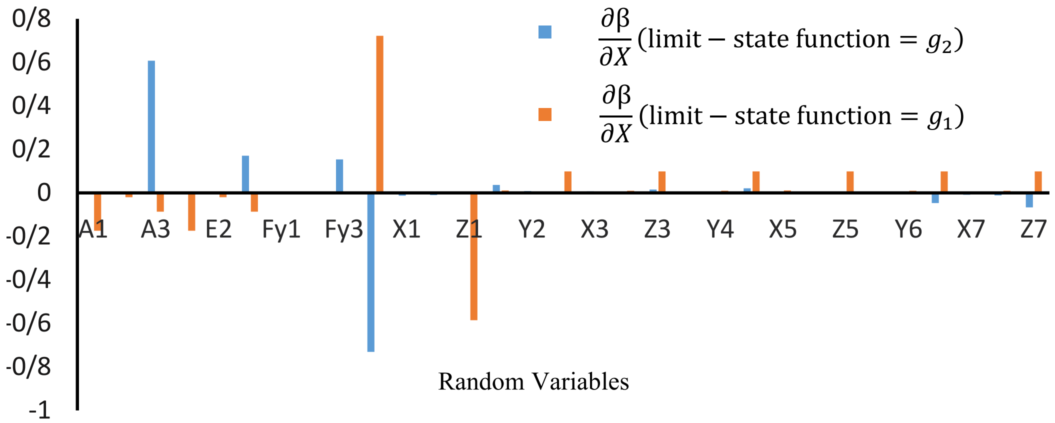

Fig. 4. Sensitivity coefficient of reliability relative to random variables

Table 3. Sensitivity analysis relative to the mean and standard deviation

R. V.	$\frac{\partial\beta}{\partial\mu_x}$	$\frac{\partial\beta}{\partial\sigma_x}$	$\text{error} = \left[\frac{\beta_{\text{Medain}} - \beta_{\text{Distribution}}}{\beta_{\text{Distribution}}}\right] \times 100\%$
A_1	1.83×10^{-1}	-4.74×10^{-2}	**1.53**
A_2	3.15×10^{-2}	-9.55×10^{-4}	**0.024**
A_3	1.23×10^{-1}	-1.57×10^{-2}	**0.37**
E_1	1.66×10^{-4}	-4.28×10^{-5}	**1.53**
E_2	1.90×10^{-5}	-5.91×10^{-7}	**0.024**
E_3	8.23×10^{-5}	-1.05×10^{-5}	**0.37**
P	2.42×10^{-1}	-2.59×10^{-1}	**47.2**
X(1)	3.6×10^{-5}	-1.88×10^{-9}	**0.00**
Y(1)	3.08×10^{-10}	0.00	**0.00**
Z(1)	5.86×10^{-1}	-5.08×10^{-1}	**23.4**
X(2)	-5.79×10^{-3}	-4.97×10^{-5}	**0.01**
Y(2)	-1.01×10^{-2}	-1.49×10^{-4}	**0.00**
Z(2)	-9.82×10^{-2}	-1.43×10^{-2}	**0.49**

evaluation of large-scale space structures are numerous, computations are time-consuming and costly. To reduce time and increase accuracy, the structure usually undergoes some sensitivity analyses prior to reliability analyses. In the sensitivity analysis, the variables with a very low impact on the reliability of the structure are considered as a deterministic parameter, leading to a reduction in the computational cost. First, we will examine the reliability sensitivity to the random variables (Fig. 4) and then discuss it based on their mean and standard deviation. Members' section areas, modulus of elasticity, yield stress, external centralized loads and nodes' coordinates in the x, y, and z analysis, the highest sensitivity will be related to loads, node 1 variations in the vertical direction,

modulus of elasticity, and members' section areas in group one, respectively. Since the maximum displacement occurs in node 1 (vertex), g_1 is more sensitive to node 1 and its connected members. If g_2 is used, the reliability will be most sensitive to loads and section areas of members of group 3 because they have larger lengths, smaller section areas, larger stresses, and more tendency to buckling than other members; its sensitivity to yield stress and modulus of elasticity in group 3 members stands next. The reliability index sensitivity is examined in relation to the variables' mean and standard deviation (Table 3). Percent error obtained from the reliability index estimation is calculated by Eq. (18) in Section 5.1 (Table 3). Results indicate that among the 27 random variables, the highest sensitivity is associated with loads and vertical coordinate variations of node 1 (dome vertex); if these are considered as deterministic variables, the reliability estimations will be quite erroneous (about 47% and 23%, respectively), and if other variables are deterministic, the error will be negligible.

6.1. 4. The effect of changing the coefficient of variation on the reliability index

Although most of the previous studies have assumed the structure node coordinates to be constant parameters, since members in space structures are numerous, variations in the coordinates of the member-connected nodes are quite possible during construction/installation and should be considered in the reliability analyses. The present study has considered the nodes coordinates as random variables and has examined the

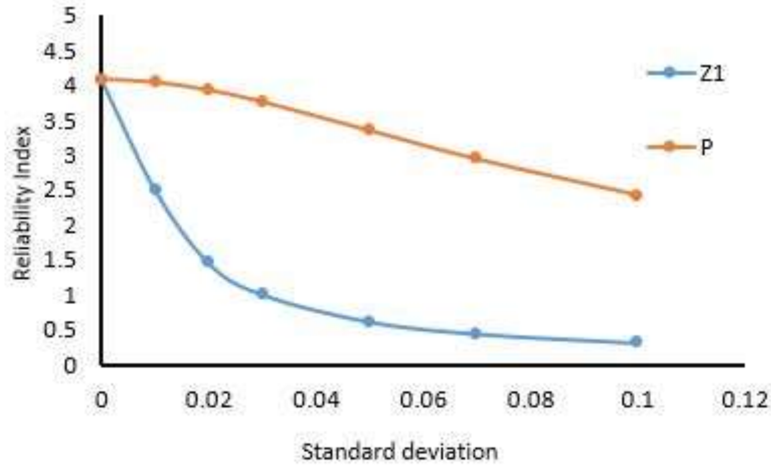

Fig. 5. Reliability index versus coefficient of variation

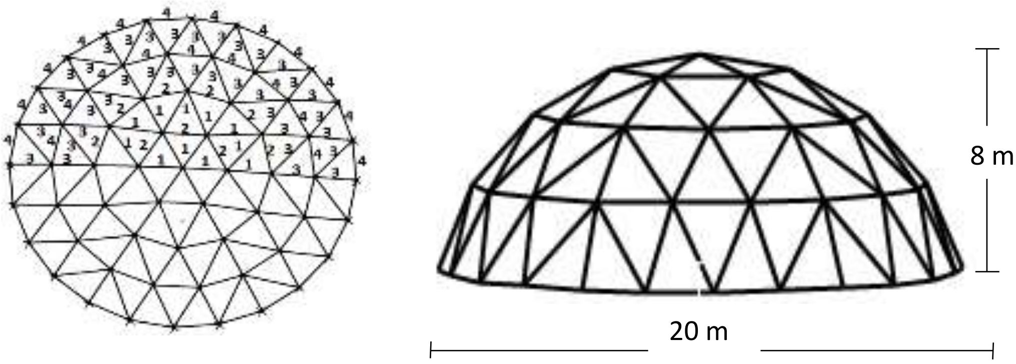

Fig. 6. The geometry and Group members domes Space

variation coefficient change effects of important structural variables on the reliability index of space structures (Fig. 2). Initially, by assuming the load and node 1 coordinates in the vertical direction as deterministic, the structure reliability index was found to be 4.09. Then, the

load was taken equal to the mean (assuming the standard deviation to be zero) and node 1 coordinate variations coefficient in the vertical direction was increased from 1 to 10% (same was done in the opposite direction). As shown in Fig. 5, an increase in this coefficient decreases the structure reliability index significantly and increases its P_f. As this coefficient reaches 10%, the reliability index becomes nearly 0.31. Therefore, a space structure's construction/installation requires more accuracy to minimize the node coordinates' standard deviation to reduce P_f.

6.2. A 156- member space dome

In this example, a 156-member single-layer space dome with 20 m span and 8 m height (Fig. 6) is used

for reliability evaluation and sensitivity analysis. The domes have been designed under 3 different load combinations as follows: The equipment load, which is concentrated and usually acts on the structure vertex vertically, is $10\,kN$ and the dead load, 0.20 kN/m^2 including the weights of the members, joints, structure cover, and snow, is 0.83 kN/m^2. The elasticity modulus, yield stress, dead and snow loads, nodes' coordinates in the Z-direction, and members' cross sections are considered as random variables. The variables' statistical specifications (probability distribution, mean, and standard deviation) are provided in Table (4). The structure consists of 61 nodes and 156 members, such that structural members are classified into four groups, and the tensile and compressive stress limitations of members are calculated in accordance with ASD-AISC regulations, in this example, for reliability analysis, the displacement limit state function is used for the head node of the space structure. The maximum

allowed displacement of nodes in all directions is 25 mm.

According to the results given in Table (5) It can be concluded, in general, that in space structures with numerous random variables, simulation methods are not cost-effective because they are quite time Consuming. Therefore, FORM and SORM approximation methods are preferable to find the reliability index. SORM yields better results than FORM because it uses a parabolic surface to approximate the limit state function; however, its calculation time increases significantly with an increase in the number of random variables because it requires a second order derivative. A comparison of the two methods can reveal that their reliability indexes are close and

Table 4. Statistical parameters of random variables

R. V.	Description	Distribution	mean	COV
P_d(kN/m^2)	Dead load	Gauss	0.20	5%
P_s(kN/m^2)	Snow load	Gauss	0.83	30%
E(kN/cm^2)	Young s modulus	Lognormal	2.059e4	5%
F_y(kN/cm^2)	Yield strength	Lognormal	23.536	5%
A_1(cm^2)	Cross-sectional area	Lognormal	9.50	5%
A_2(cm^2)	Cross-sectional area	Lognormal	6	5%
A_3(cm^2)	Cross-sectional area	Lognormal	8	5%
A_4(cm^2)	Cross-sectional area	Lognormal	5.5	5%
Z_i(cm)	Nodal coordinates in z directions	normal	---	3cm

Table 5. Reliability indexes, time consuming and error of several reliability methods

	FORM	SORM	IS	**MCS(10e6)**
β Time (s) Error	6.048 12 1.65%	6.01 28 1.06%	5.991 1285 0.69%	**5.950** **15425** **.....**

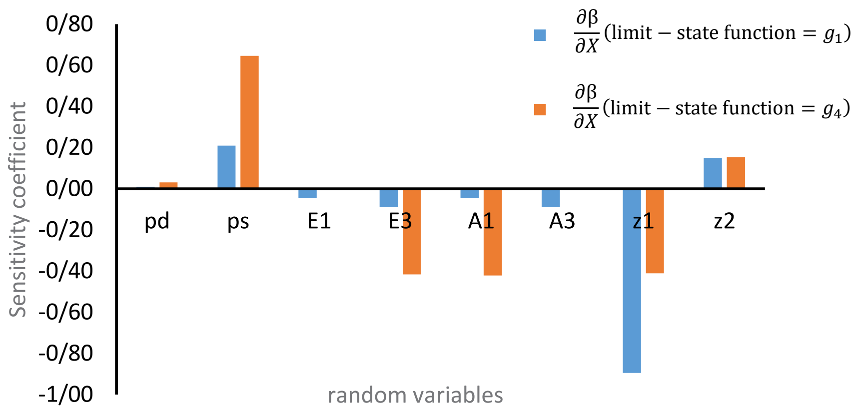

Fig. 7. Sensitivity coefficient of reliability relative to random variables

FORM can be used to continue the work.

This section investigates the effects of the limit state functions of the nodes' displacement and members' stress on the reliability of the 156-member space dome. As shown in Fig. (6), members are of four groups with a known stress limit state function: g1 (displacement limit state function in the vertex node), g2 (stress limit state function in group 1 members), g3 (stress limit state function in group 2 members), g4 (stress limit state function in group 3 members), and g5 (stress limit state function in group 4 members). Considering the mentioned five limit state functions, the reliability index for the 156-member space dome is as follows:

$$\beta = \min\left(\beta_1, \beta_2, \beta_3, \beta_4, \beta_5\right) = \left(6.048, 5.175, 12.240, 4.080, 15\right. \quad (22)$$

It can be concluded, therefore, that the critical function for this structure is g4 (stress limit state function in group 3 members) because its failure probability is more (its reliability is less) than other functions. The sensitivity analyses results of g1 and g4 relative to such random variables as the members' cross sections, elasticity modulus, yield stress, snow load, dead load, and nodes' coordinates in the vertical direction for the 156-member dome are provided in Table 6. Since the applied load and the structure shape are symmetrical, the sensitivity coefficient is equal for all nodes in a ring; hence, one node is selected from each ring to solve the space shortage problem. Z1 is node 1 (vertex node), Z2 is the node in ring 1 of the dome, Z7 is the node in ring 2, and Z19 is the node in ring 3. Results in Fig. 7 show that g1 (defined in node 1) is the most sensitive to random changes of node 1 coordinates (Z1) in the vertical direction, and then to the changes in the vertical coordinates of the nodes in ring 2 (Z2), snow load, cross section, and elasticity modulus of groups 1 and 3 members. It can be concluded, therefore, that g1 is more sensitive to physical and geometric parameters in groups 1 and 3 members. In the sensitivity analysis, if use is made of g4, the highest sensitivity would be to the snow load and to the cross-sections of group 1 and 3 members. Since the latter are longer than others, their tendency towards buckling is more; hence, the dome sensitivity to cross section is more and the reliability can be increased with an increase in the cross section.

7. CONCLUSIONS

Effort was made in the present research to study the effects of random variables and performance functions on the reliability, sensitivity and correlation, and the reliability index of a 24-member space dome was found using the FORM, SORM, MCS, and ISM. The random variables were the structure physical and geometric characteristics and two limit state functions (displacement and stress) were used for the analyses. Results are as follows:

1- In general, that in space structures with numerous random variables, simulation methods are not cost-effective because they are quite time consuming. Therefore, FORM and SORM approximation methods are preferable to find the reliability index. SORM yields better results than FORM because it uses a parabolic surface to approximate the limit state function; however, its calculation time increases significantly with an increase in the number of random variables because it requires a second order derivative. A comparison of the two methods can reveal that their reliability indexes are close and FORM can be used to continue the work.

2- With the displacement limit state function, the highest sensitivity belonged to the physical and geometric characteristics (first to the load and coordinate variations of node 1 (vertex) in the vertical direction and then to the cross-section and elasticity modulus of group 1 members). An increase in the parameter uncertainty reduced the reliability and increased the P_f.

3- With the ultimate limit state function, the highest sensitivity belonged to such random variables as the load, cross-section of group 3 members, yield stress of group 3 members, and coordinate variations of node 1 in the vertical direction, respectively. The reliability increased with a decrease in the standard deviation of these variables and with an increase in the cross-section and yield stress of group 3 members.

4- Space domes are quite sensitive to node coordinates variations; an increase in the vertical direction will reduce their reliability nonlinearly. As the displacement limit state function was used in the reliability analyses, the nodes' coordinate variations were the second most important factor after the load and the structure was most sensitive to such variations.

5- A change in the height-to-span ratio changed the performance function as well; for 24-member domes the displacement performance function is effective and 156-member domes the stress performance function is critical.

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HOW TO CITE THIS ARTICLE

N. Shabakhty, A. Bahrpeymah, M. Layegh Rafat, Reliability and Sensitivity Analysis of Single-Layer Space Domes, AUT J. Civil Eng., 4(3) (2020) 367-376.

DOI: 10.22060/ajce.2019.16504.5594