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Waveform Design using Second Order Cone Programming in Radar Systems

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ABSTRACT: Transmit waveform design is one of the most important problems in active sensing and communication systems. This problem, due to the complexity and non-convexity, has been always the main topic of many papers for the decades. However, still an optimal solution which guarantees a global minimum for this multi-variable optimization problem is not found. In this paper, we propose an attracting methodology to design transmit waveform of active sensing and communication systems with good auto-correlation properties. To this end, we tackle the non-convex optimization problem of Integrated Sidelobe Level (ISL) minimization with the unimodular constraint. Using the epigraph and Second Order Cone Programming (SOCP) approach, the in-hand non-convex optimization will resort to a Semi-Definite Programming (SDP). Then, we use Majorization- Minimization to deal with constraints and convert the obtained problem to a convex optimization problem. Finally, the obtained optimization problem is tackled using CVX toolbox. To obtain the code vectors from the extracted optimal code matrix, we use rank-one decomposition. The simulation and results indicate the powerfulness of the proposed algorithm in designing radar transmit sequences with unimodular constraint. We show the proposed algorithm can design long length sequences with a very small ISL values. The proposed framework further can be investigated for the future optimization problems, like Peak Side lobe Level (PSL) minimization.

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1. Introduction

Next generation intelligent active sensing systems, e. g., radars, further to the adaption in the receive side, should have the capability of adaptively designing transmit waveform. Waveform design was always a challenging problem in active sensing and radar systems. The well-known waveforms with good auto-correlation properties, including m-sequences, Gold, Barker codes, etc., mostly have limitation in length [1]. The goal of waveform design for every active sensing (radar, sonar, lidar) is to acquire (or preserve) the maximum amount of information from the desirable sources in the environment, where in fact, the transmit signal can be viewed as a medium that collects information [1]. Thanks to the known speed of an electromagnetic wave, the radar system can estimate the location of the target simply by measuring the time difference between the radar signal transmission and the reception of the reflected signal [2].

The target detection and estimation performance of the active sensing systems are shown to be considerably improved by a judicious design of the probing signals and processing schemes [3]. Waveform design and processing for radar has a crucial role particularly in fulfilling the above promises of adaptivity, agility and reliability: the waveform design usually deals with various measures of quality (including detection/ estimation and information-theoretic criteria), and moreover, the practical condition that the employed signals must belong to a limited signal set. Such diversity of design metrics and signal constraints lays the groundwork for many interesting research projects in waveform optimization. Indeed, efficient waveform design algorithms are instrumental in realizing the next-generation of radar systems.

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2. Background and Related Works

Research in the area of waveform design for radar systems is focused on the design and optimization of probing signals in order to improve target detection performance, as well as the target location and speed estimation. To this end, the waveform design problems are often formulated as an optimization problem with a certain metric that represents the quality objective, along with the constraint set of the transmit signals. Among others, the most widely used signal quality objectives include auto and cross correlation sidelobe metrics (see e.g. [4]– [12]), Mean-Square Error (MSE) of estimation (see e.g. [13], [14]), Signal to Noise Ratio (SNR) of the processed signals (see e.g. [13], [15]–[29]), beampattern synthesis (see e. g. [30]–[34]), information-theoretic criteria (see e.g. [35]–[37]), and excitation metrics (see e.g. [38]). The proposed methods use different optimization frameworks including, quadratic programming (QP) [39], semidefinite quadratic program (SQP) [40], alternative optimization [4], Gradient Descent (GD) [41] [42], Majorization Minimization (MM) [5], [8], [43]–[46], and Coordinate Descent (CD) [7], [9], [11], [12]. Moreover, due to implementation and technological considerations, the transmit signals should also comply with certain constraints. Other than reliability requirements, such constraints typically include (finite) energy, unimodularity (or being constant-modulus) [47], Peak-to-Average Power Ratio (PAR) (abbreviated as PAPR or PAR), and finite or *discrete-alphabet* (e.g. being integer, binary, or from m-ary constellation, also known as roots-of- unity [3], [11], [48], [49]). Among the different sets of constraints, considering constant modulus in the design stage is rather important. This constraint is considerable since the radar transmitters operate more efficiently when using a high power amplifier in the saturation region [1], [50]. Further, the emergent applications tend to be power constrained necessitating power efficient operation of the sensing modules. This constraint means that the only degree of freedom is the waveform phase; this makes the optimization problem non-convex and non-linear resulting in a significantly more complex problem than the unconstrained version [50]. In this paper, we focus on the Integrated Sidelobe Level (ISL)

optimization problem, with the non-convex constant modulus constraint at the design stage. As to the background of this paper, the new family of Cyclic Algorithm New (CAN) algorithms were recently developed to handle the design of such sequence even with relatively large sequence lengths; see [3], [21]. However, since CAN algorithms are merely intended for local optimization, the radar system can have difficulty in producing high quality sequences as the optimization landscape becomes highly multi modal (i.e. possessing many local optima)—a typical phenomenon when signal constraints are enforced or problem dimensions grow large. In [21], a computationally attractive algorithm for designing radar transmit sequence from linear combination of the different orthogonal waveforms. In [7], a CD framework is proposed to sequentially optimize code entries of a unimodular sequence to have good Peak Sidelobe Level (PSL) and/or ISL for Single Input Single Output (SISO) radar systems. In [27], a novel algorithm for designing set of Multiple Input Multiple Output (MIMO) radar sequences under PSL constraint is proposed. Finally, in [11], [51] using the Block Coordinate Descent (BCD), designing set of sequences with good correlation properties is considered. In this paper, we propose Second Order Cone Programming (SOCP) framework to design unimodular (continuous phase) transmit sequence with good ISL values, which was not addressed previously. Precisely, we introduce an optimization framework based on SOCP to minimize the ISL. We resort to the Semidefinite Programming (SDP) optimization problem and with the aim of MM we tackle the non-convex multi-variable optimization problem. Note that, the optimization technique that will be introduced in this paper to minimize the ISL is different from that of previously addressed in the literature [7]. Further, since the ISL possessing many local optima, the obtained solutions would be different as well.

3. Organization and Notation

The rest of this paper continues as follows. Problem formulation is written in Section II, where we introduce the optimization problem, as well as the non-convex constraint of constant modulus. In Section III, we introduce the SOCP optimization framework. Section IV is dedicated to the MM algorithm to deal with the obtained non-convex constraints. Finally, Section V shows some interesting results and Section VI concludes the paper. Notation: Bold uppercase letters denote matrices, bold lowercase letters are used for vectors and italics for scalars. Vector/matrix transpose denotes by $(\cdot)^T$, and the Hermitian transpose by $(\cdot)^H$. $tr(\cdot)$ is the trace of a square matrix argument. The notations $\lambda_{max}(\cdot)$ indicate the principal eigenvector of a Hermitian matrix. $\boldsymbol{diag}(\cdot)$ denotes the vector formed by collecting the diagonal entries of the matrix argument, whereas $\boldsymbol{Diag}(\cdot)$ denotes a matrix with diagonal elements of the an input vector, i. e., a matrix formed by inserting the entries of an input vector to the diagonal elements of it, keeping zero the other entries. We write $\mathbf{A} \succcurlyeq 0$ iff $\mathbf{A}$ is positive semi-definite. Finally, the Frobenius norm of a matrix $\mathbf{X}$ (denoted by $\|\boldsymbol{X}\|_2$) with entries $\{X_{k,l}\}$ is equal to matrix with diε .

4. PROBLEM FORMULATION

Typically in active sensing and radar systems, the good autocorrelation property means that the waveform has small PSL and ISL.

Detection performance in every active sensing and radar system is highly depended on the received Signal to Interference plus Noise Ratio (SINR) [52]. Assume a monostatic radar system which transmits a sequence

ε sensing and radar system iε (1)

where x_n, ?, indicates the code entries. Reflected signal from a target contaminated with noise and clutter can be written as,

(2)

with ?. The received vector $\tilde{y}$ is filtered with the weight vector ω, i. e., $\boldsymbol{\omega}^H \boldsymbol{y}$, and then the output SINR can be expressed as,

tor $\tilde{y}$ is filtered with the weigh (3)

where $\boldsymbol{R}_c$ and $\boldsymbol{R}_n$ indicate covariance matrices of clutter and noise, respectively. If there is no clutter and noise is zeromean white (i. e., ?), the matched filter $\boldsymbol{\omega} = \boldsymbol{x}$ will give the largest SNR. In [3], [53], it has been shown that incase of clutter existence, when the clutter and target have the same Doppler frequency, which is of course typical in radar systems, the output SINR is inversely related with the ISL of the transmit sequence, such that the lower ISL of the sequence the higher the SINR at the matched filter output (see [53] for more details).

The aperiodic auto-correlation function of the complex unimodular sequence $\{x_n\}_{n=1}^{N}$ is

(4)

where N is the length of the sequence. The integrated sidelobe level is defined as,

(5)

We are interested to tackle the following optimization problem,

(6)

le the following optimization probl

with . Notice that $\mathcal{P}_x$ is non-convex multi-variable optimization problem. Recently, many papers focused on the problem of PSL/ISL minimization (see [4], [7], [54], [55] and references therein). The pioneers to this problem are [4], [54], [55], where several approaches are introduced for minimizing an equivalent metric of the ISL.

5. SOCP AND ISL MINIMIZATION

In this section, we propose using SOCP to tackle Problem $\mathcal{P}_x$. First, notice that with the aim of epigraph form to the optimization Problem $\mathcal{P}_x$ [56], equivalently the ISL minimization problem can be written as

ing SOCP to tackle Problem $\mathcal{P}_x$. First, no

lem $\mathcal{P}_x$ [56], equivalently the ISL minimi

(7)

Defining $\boldsymbol{U}_k \in \mathbb{R}^{N\times N}$, $k = 0. \dots . N-1$, as the Toeplitz matrices with the k -th diagonal elements being 1 and 0 elsewhere, it is easy to show

(8)

Using trace properties [57], we can write,

$. N-1$, as the Toeplitz matrices with tl (9)

where $X = xx^H \in \mathbb{C}^{N\times N}$ is the rank-one code matrix. Therefore, the equivalent problem to $\tilde{\mathcal{P}}_x$ can be written as,

n write,

$r\{\boldsymbol{x}^H \boldsymbol{U}_k \boldsymbol{x}\} = tr\{\boldsymbol{U}_k \boldsymbol{x}\boldsymbol{x}^H\} = tr\{\boldsymbol{U}_k \boldsymbol{X}$ (10)

ink-one code matrix. Therefore, th

which is still non-convex. Note that the constraint $\boldsymbol{diag}\ (\boldsymbol{X}) = \boldsymbol{1}$ is added for designing unimodular waveforms, i. e., instead of $|x_n| = 1$. Now, using the SDP, we can replace the constraint $\sum_{k=1}^{N-1}|tr(\boldsymbol{U}_k\boldsymbol{X})|^2 \le t$ with its SDP equivalent constraint (10) [58],

ect to $\boldsymbol{diag}(\boldsymbol{X}) = \boldsymbol{1}$
$\boldsymbol{X} \succcurlyeq 0$

he constraint $\boldsymbol{diag}\ (\boldsymbol{X}) = \boldsymbol{1}$ (11)

: 1 . Now, using the SDF

Where

$\boldsymbol{X}, t$ (12)

and $\boldsymbol{I}_{N-1}$ is identity matrix with dimension of $N-1 \times N-1$. However, the rank-one constraint is non-convex and consequently the optimization problem (11) is still non-convex and cannot be solved using the convex optimization toolboxes. The idea to tackle the problem (11), is to use relaxation by neglecting the non-convex rank-one constraint, and imposing two new constraints to keep the rank-one property of the code matrix $\boldsymbol{X}$ in the same time. Precisely, we resort to the following optimization problem,

nd imposing two new constraints

ecisely, we resort to the followin

(13)

$$\underset{\boldsymbol{X}.t}{\text{minimize}} \quad t$$
$$\begin{bmatrix} t & \boldsymbol{y}^H \\ \boldsymbol{y} & \boldsymbol{I}_{N-1} \end{bmatrix} \succcurlyeq 0$$

where $\lambda_{max}(\boldsymbol{X})$ is the maximum eigen value of $\boldsymbol{X}$ and ϵ_1 and ϵ_2 are arbitrary small values greater than zero and lower than one. The idea behind imposing the two constraints $\lambda_{max}(\boldsymbol{X}) \ge N - \epsilon_1$ and $\|\boldsymbol{X}\|_2^2 \ge N - \epsilon_2$ is described in following. In this paper, we are supposed to design a code vector x^* under the non-convex constant modulus constraint. This problem is difficult to address as we propose to use SDP to tackle the problem. Using the SDP means that instead of dealing with the quadratic objective function, we deal with

a linear objective function, namely $\boldsymbol{X} = \boldsymbol{x}\boldsymbol{x}^H$. However, since the matrix $\boldsymbol{X}$ is originally made by a vector $\boldsymbol{x}\boldsymbol{x}$, it would have rank-one. The rank-one constraint is non-convex, thus we relax it in the optimization procedure, to be able to obtain the optimal code matrix $\boldsymbol{X}$. If we do not consider any additional constraint on the code matrix $\boldsymbol{X}$, then we may obtain a full rank solution, where we cannot extract a code vector $\boldsymbol{x}^*$ from that, and there is no feasible solution to extract a code vector from a full-rank code matrix. Imposing the two introduced constraints help the code matrix $\boldsymbol{X}$ to preserve its rank-one property while obtaining an optimal solution. This means that the optimal code matrix $\boldsymbol{X}^*$ would not be necessarily rank one, but it would have small rank, if we impose the proposed two constraints. The reason can be inventively interpret as follows:

• The constraint $\lambda_{max}(\boldsymbol{X}) \geq N - \epsilon_1$ means that we would like to obtain an optimal code matrix with a maximum Eigen value greater than a threshold. This forces the code matrix $\boldsymbol{X}^*$ to have small Eigen values separate from the largest one, which means the rank-one property.

• The constraint $\|\boldsymbol{X}\|_2^2 \geq N - \epsilon_2$ has a similar interpretation like the first one, but on the energy of the code matrix. This again help to preserve the rank-one property of the optimal code matrix $\boldsymbol{X}^*$.

Even-though the highlighted two constraints try to preserve the rank-one property of the optimal solution $\boldsymbol{X}^*$, there is no guarantee to obtain it after performing CVX. Thus, the randomization procedure would be beneficial to obtain the good solution, i. e., optimal code vector $\boldsymbol{x}^*$.

Since the feasible set of (13) contains the set of rank-one solutions, imposing two constraints of $\lambda_{max}(\boldsymbol{X}) \geq N - \epsilon_1$ and $\|\boldsymbol{X}\|_2^2 \geq N - \epsilon_2$ helps $\boldsymbol{X}$ not to lose its rank-one property in the relaxation. On the other hand, problem (13) is still nonconvex due to the nonlinearity of the imposed constraints. In the next section, we use MM[1] algorithm to tackle this problem and replace the mentioned constraints with the majorizers.

6. MM ALGORITHM TO TACKLE WAVEFORM DESIGN OPTIMIZATION PROBLEM

In this part, using the MM algorithm we are replacing the two non-convex constraints of Problem (13), with their equivalent majorizers.

a) constraint of $\lambda_{max}(\boldsymbol{X})$ Using the MM algorithm, we are trying to find a majorizer function for the maximum Eigen vector of $\boldsymbol{X}$. For SDP matrices we can write [58],

TACKLE WAVEFORM DESIGN OPT (14)

Considering the problem of finding a supporting hyperplane for $\lambda_{max}(\boldsymbol{X})$, for an arbitrary vector of $\boldsymbol{z}$ with $\|\boldsymbol{z}\|_2 = 1$ we have,

ng the MM algorithm, ' (15)

Therefore, if we add and subtract the term $\boldsymbol{X}_i\boldsymbol{z}\boldsymbol{z}^T$ then

$$\begin{aligned} \lambda_{max}(\boldsymbol{X}) &\geq tr(\boldsymbol{X}_i\boldsymbol{z}\boldsymbol{z}^T) - tr(\boldsymbol{X}_i\boldsymbol{z}\boldsymbol{z}^T) + tr(\boldsymbol{X}\boldsymbol{z}\boldsymbol{z}^T) \\ &= tr(\boldsymbol{X}_i\boldsymbol{z}\boldsymbol{z}^T) + tr\big((\boldsymbol{X}-\boldsymbol{X}_i)\boldsymbol{z}\boldsymbol{z}^T\big) \\ &= tr(\boldsymbol{Z}\boldsymbol{X}_i) + tr\big(\boldsymbol{Z}(\boldsymbol{X}-\boldsymbol{X}_i)\big) \\ &= \lambda_{max}(\boldsymbol{X}_i) + tr(\boldsymbol{Z}(\boldsymbol{X}-\boldsymbol{X}_i)) \end{aligned} \tag{16}$$

1-The MM algorithm consists of iteratively minimizing a surrogate that upper-bounds the objective, thus monotonically driving the objective function value downhill. One of the virtues of the MM algorithm is that it does double duty [59].

So, the matrix $\boldsymbol{Z} = \boldsymbol{z}\boldsymbol{z}^T$ is a supporting hyperplane for $\lambda_{max}(\boldsymbol{X})$.

b) constraint of $\|\boldsymbol{X}\|_2^2$ Using a similar approach, we can find $2\boldsymbol{X}$ as a supporting hyperplane for $\|\boldsymbol{X}\|_2^2$. Therefore we can write,

$$\|\boldsymbol{X}\|_2^2 \geq \|\boldsymbol{X}_0\|_2^2 + tr(2\boldsymbol{X}_0(\boldsymbol{X}-\boldsymbol{X}_0)) \tag{17}$$

Using the above majorizers, the optimal solution for problem (13) can be obtained by iteratively solving of the following SDP optimization problem.

$$\begin{aligned} &\underset{\boldsymbol{X},t}{\text{minimize}} \quad t \\ &\text{subject to} \quad \begin{bmatrix} t & \boldsymbol{y}^H \\ \boldsymbol{y} & \boldsymbol{I}_{N-1} \end{bmatrix} \succcurlyeq 0 \\ &\qquad \lambda_{max}(\boldsymbol{X}_i) + tr(\boldsymbol{Z}(\boldsymbol{X}-\boldsymbol{X}_i)) \geq N - \epsilon_1 \\ &\qquad \|\boldsymbol{X}_i\|_2^2 + tr(2\boldsymbol{X}_i(\boldsymbol{X}-\boldsymbol{X}_i)) \geq N - \epsilon_2 \\ &\qquad \boldsymbol{diag}(\boldsymbol{X}) = \boldsymbol{1} \\ &\qquad \boldsymbol{X} \succcurlyeq 0 \end{aligned} \tag{18}$$

which is convex and can be efficiently tackled using CVX matlab toolbox [58]. Notice that, in the above optimization problem, the two parameters of $\grave{O}_1$ and $\grave{O}_2$ are small enough to ensure the rank-one property of $\boldsymbol{X}$. Let $\boldsymbol{X}^*$ be an optimal solution of (18). We remark that if rank of $\boldsymbol{X}^*$ happens to be one, then the radar code design problem (18) is optimally solved and the SDP relaxation is tight. If it was not rank-one, depending on the rank of $\boldsymbol{X}^*$, there are some suboptimal solution for the problem (see [39] for more details). Finally, to obtain the optimal code vector $\boldsymbol{x}^*$, we resort to the rank-one randomization, as described in the next sub-section.

6-1- Rank-One Randomization Technique

The randomization procedure is the factorization of a matrix $\boldsymbol{X}^*$ into optimal vector $\boldsymbol{x}^*$. This procedure requires the definition of a suitable "ad-hoc" covariance matrix of the Gaussian distribution to be adopted in the randomization step. The basic criterion for selecting such a covariance matrix is that the entire randomization procedure has to lead to a feasible solution of the original problem with probability one and it has also to provide mathematical tractability in assessing the quality of the resulting solution. According to this guideline, denote by

$$\boldsymbol{d} = \sqrt{\boldsymbol{diag}(\boldsymbol{X}^*)} \tag{19}$$

And by $\boldsymbol{d}^-$

$$(\boldsymbol{d}^-)_i = \begin{cases} \frac{1}{d_i}. & if\ d_i > 0; \\ 1. & if\ d_i = 0; \end{cases} \tag{20}$$

Where $i = 1.2.\ldots.N$, indicates entries of the code vector $\boldsymbol{d}^-$. Additionally let,

Algorithm 1 Gaussian Randomization Procedure for Radar Code Design Problem

Input: optimal solution of $\boldsymbol{X}^*$

Output: a randomized approximate solution ($\boldsymbol{x}^*$)

1) define $\boldsymbol{d}, \boldsymbol{d}^-, \boldsymbol{D}, \boldsymbol{D}^-, \tilde{\boldsymbol{X}}^*$ according to the above equations

2) draw a random vector $\boldsymbol{\xi} \in \mathbb{C}^N$ from the complex normal distribution $\mathcal{N}\left(0. \boldsymbol{D}^- \tilde{\boldsymbol{X}}^* \boldsymbol{D}^-\right)$

3) let $\boldsymbol{x}_i^* = \sqrt{\boldsymbol{X}_{ii}^*} e^{j \arg \xi_i}$.

Algorithm 2 Proposed algorithm for ISL minimization

Input: initial code vector of $\boldsymbol{x}$, parameters ϵ_1, ϵ_2, and stop threshold η.

Output: optimal solution of $\boldsymbol{x}^*$.

- initial $\boldsymbol{x}$ by arbitrary unimodular sequence $\in \mathbb{C}^N$;
- set $\boldsymbol{X} = \boldsymbol{x}\boldsymbol{x}^H$;
- obtain $\boldsymbol{X}^*$ by tackling (18) using CVX;
- perform rank-one decomposition according to **Algorithm 1** and obtain $\boldsymbol{x}^*$;
- set the obtained $\boldsymbol{x}^*$ as the new initial sequence and continue the procedure until the changes in ISL metric is lower than the threshold η;

$$\boldsymbol{D} = \boldsymbol{Diag}(\boldsymbol{d}) \quad (21)$$

$$\boldsymbol{D}^- = \boldsymbol{Diag}(\boldsymbol{d}^-) \quad (22)$$

Hence, by the construction of

$$\tilde{\boldsymbol{X}}^* = \boldsymbol{X}^* + (I - \boldsymbol{D}^-\boldsymbol{D}) \quad (23)$$

a feasible solution to the optimization problem can be obtained by using $x_i^* = \sqrt{X_{ii}^*} e^{j \arg \xi_i}$ with ξ_i indicating the entries of the random vector $\boldsymbol{\imath} = [\xi_1. \xi_2 \dots . \xi_N]^T \in \mathbb{C}^N$ from the complex normal distribution $\mathcal{N}\left(0. \boldsymbol{D}^- \tilde{\boldsymbol{X}}^* \boldsymbol{D}^-\right)$, $i = 1, 2, \dots, N$. This forms the optimal solution $\boldsymbol{x}^* = [x_1^* \dots . x_N^*]^T$. The Gaussian randomization procedure for radar code design problem is written in Algorithm 1. Notice that $\boldsymbol{\imath}$ is a random vector from the complex normal distribution $\mathcal{N}\left(0. \boldsymbol{D}^- \tilde{\boldsymbol{X}}^* \boldsymbol{D}^-\right)$ which helps to obtain an optimal solution using power-method like procedures (see [60] for more details.) So, using the Gaussian rank-one randomization, optimal code vector $\boldsymbol{x}^*$ can be efficiently obtained.

6-1- Optimization Algorithm and Computational Complexity

Finally, the whole proposed optimization framework is written in **Algorithm 2**. Notice that, at the final step, Eigen decomposition can be performed to obtain an optimal solution from the rank-one optimal X^*. However, we numerically observed that the obtained solution at the final stage using the randomization is almost equivalent with that of obtained by Eigen decomposition.

The computational complexity of the proposed algorithm can be calculated according to the two main steps:

1) Solving and SDP problem with the complexity of $\mathcal{O}(N^{3.5})$.

2) Performing rank-one decomposition with the complexity of the $\mathcal{O}(N^3)$.

Indeed, the highest computational complexity per iteration would be $\mathcal{O}(N^{3.5})$.

7. NUMERICAL RESULTS

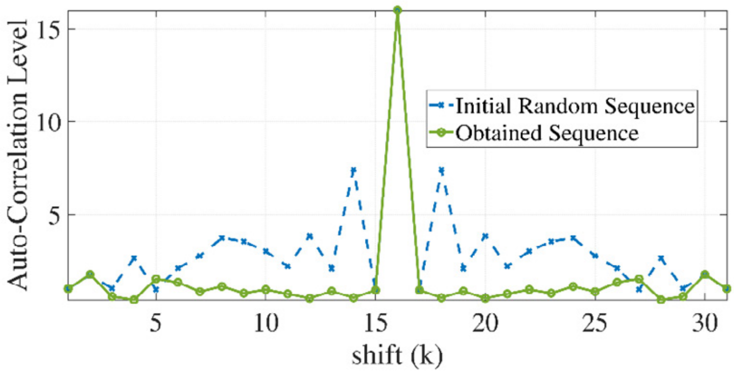
In this section, we focus on the performance of the proposed algorithm in designing radar waveforms with good ISL values. We assume $\eta = 10^{-5}$, and $\epsilon_1 = \epsilon_2 = 0.1$ in **Algorithm 2**. To proceed further, we first obtain the theoretical minimum and maximum values that can be considered for ISL as the limit values.

7-1- Bounds on the achievable ISL

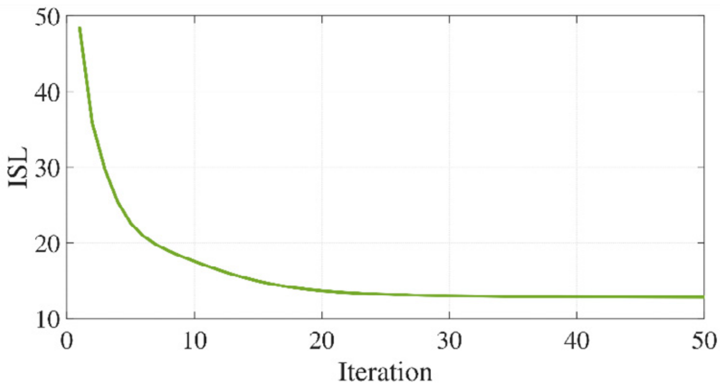
Bound on achievable ISL is known and addressed in the literature, but for designing set of sequences [61]–[63]. The aperiodic ISL lower bound for a sequence set under the total energy constraint with M as the number of set size and N as the code length can be written as [63],

$$\mathcal{B}_L = N^2 M (M - 1) \quad (24)$$

When $M = 1$ (the case we considered in this paper), according to (24) this bound is equal to 0. However, we show that for the case of designing unimodular sequences, this bound is at least greater than 1. Note that in [63], when $M \geq 2$ an analytical formula for generating set of sequences



(a) Auto-correlation level of initial and obtained sequences.



(b) ISL values of obtained sequence per iteration.

Fig. 1: Performance of the proposed algorithm in ISL minimization at length

– under the energy rather than the considered constant modulus constraint of any size that attain the ISL lower bound is derived. Also, most of the available literature that focused on ISL-oriented design problems; almost meet the lower bound, but only when designing set of sequences (not a single sequence) (see e. g., [4], [5], [9], [44], [55], [63], [64]). Looking at (5), we observe that ISL can be written as,

$$\mathrm{ISL} = \sum_{k=1}^{N-2} \left| r_k \right|^2 + \left| r_{N-1} \right|^2 \quad (25)$$

In case of designing unimodular sequences, we obtain:

$$\left| r_{N-1} \right| = \left| x_1^* x_N \right| = \left| e^{-j\phi_1} e^{j\phi_N} \right| = \left| e^{j(\phi_N - \phi_1)} \right| = 1, \quad (26)$$

Where $x_1 = e^{j\phi_1}$ and $x_N = e^{j\phi_N}$. Thus, for case of designing unimodular sequences we obtain $\mathcal{B}_L \geq 1$ (Notice that the available lower bound in (24) shows that $\mathcal{B}_L \geq 0$, but here we have shown that this bound is greater than 1 for case of designing unimodular sequences).

The upper bound for the ISL happens when all the correlation lags are equal to the energy of the signal (i. e., N, in case of unimodular sequences). This upper bound for the constant modulus sequences can be obtained by,

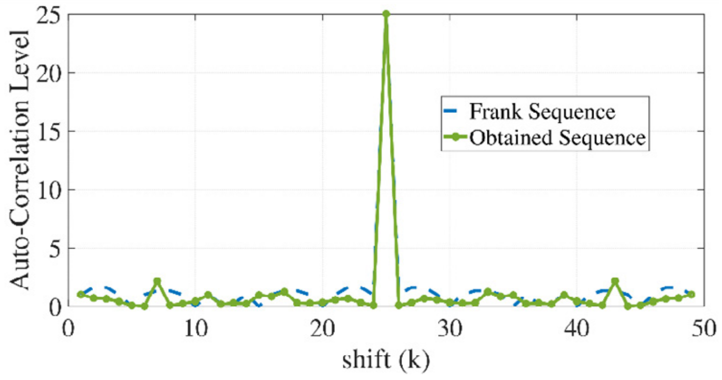
$$\mathcal{B}_U = N^2 \left(N - 1 \right) \quad (27)$$

In the following, we assess the performance of the proposed algorithm. Considering the introduced lower and upper bounds, it is easy to intuitively observe the enhancement suggested by this paper.

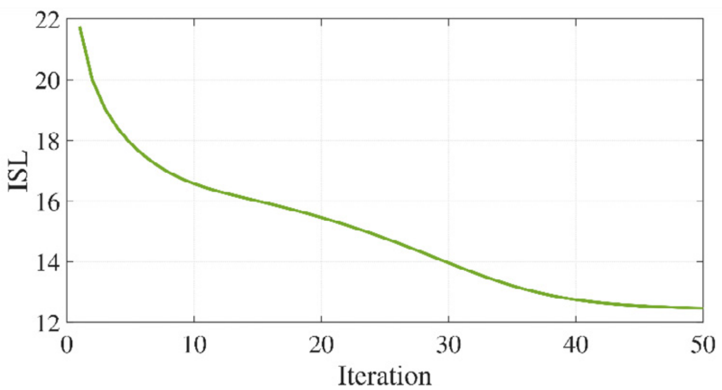
7-2- Performance Assessment

In this part we assess the performance of the proposed algorithm when it is initialized with different sequences. In Fig. 1a, we set $N=16$ and initialize the proposed algorithm with a random sequence, the depict the correlation levels of the obtained sequence, as well as its initial sequence. From Fig. 1a, observe that all the sidelobes are interestingly minimized, in comparison with the initial sequence. This Figure gives an intuitive understanding of the performance of the proposed algorithm. In Fig. 1b we show the convergence behavior of the proposed algorithm when obtaining the sequence illustrated in Fig. 1a. An improvement around 5 times is observable in the ISL values achieved by the proposed algorithm.

Next, we investigate the performance of the proposed



(a) Auto-correlation level of Frank and obtained sequences.



(b) ISL values of obtained sequence per iteration.

Fig. 2: Performance of the proposed algorithm in ISL minimization at length N = 25 initialized by Frank sequence.

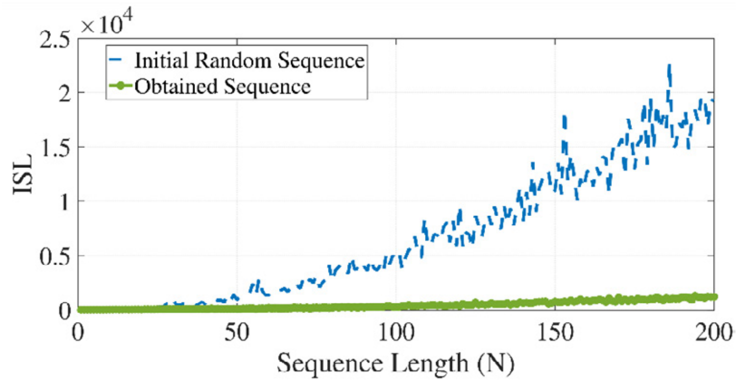
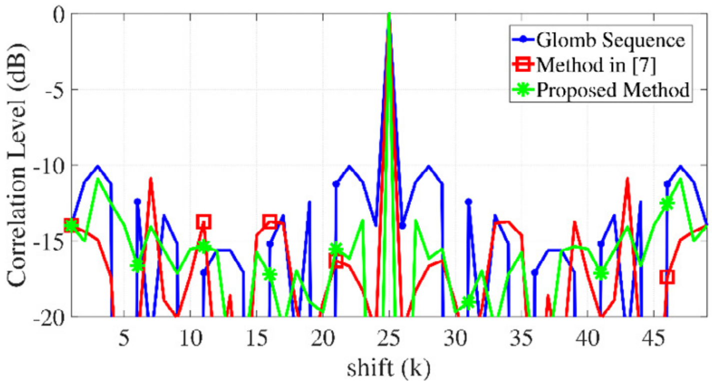


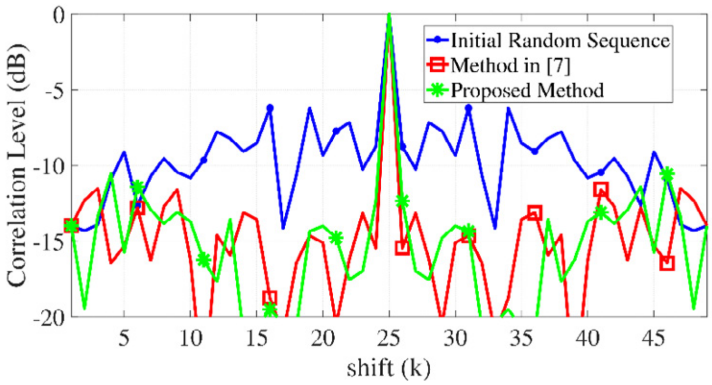
Fig. 3 : The initial and obtained ISL values for the code length.

algorithm in improving the ISL of a sequence which is well known in the literature, due to the interesting properties in its autocorrelation function. Notice that, typically random sequences does not have good ISL values. Further, the proposed algorithm can also improve the ISL value, even if we initialize it from a sequence which itself has a good ISL. In Fig. 2, we initialize the algorithm with Frank, which is known as a polyphase sequence that has a very small ISL value. Since, Frank sequences are only available when the code length is a perfect square, we $N=25$. In Fig. 2a, the auto-correlation levels of the initial Frank and obtained optimal sequence is depicted. Here, the visual goodness of the obtained sequence, may not be clearly observed. However, looking at Fig. 2b, we

observe that the obtained sequence has the ISL of 12, whereas the initial sequence has the ISL value around 22. This means that, about dB improvement happens, when initializing the algorithm by Frank sequence in length $N=25$. This example clearly shows the powerfulness of the proposed algorithm in ISL minimization. In Fig. 3, we change the code length N from $2 to 200$, and assess the obtained ISL values



(a) Correlation level of initial Golomb sequence, method in [7], and the proposed method.



(b) Correlation level of initial random sequence, method in [7], and the proposed method.

Fig. 4: Comparison between performance of the proposed method and method in [7], when both are starting from similar sequences of length N = 25.

comparing with random initial sequences. We observer that by increasing the code length, the gap between the ISL values of the initial sequence and the obtained optimal codes increases. Indeed, the longer the sequence the higher the gain can be obtained by the proposed ISL minimization framework.

7-3- Comparing the Performance with the Counterpart based on a Normalized Measure

In this part we compare the performance of the proposed method, with the method proposed in [7], which is the most recent counterpart in ISL minimization. Note that the performance of the algorithm in [7] is reported as the best among the most recent literature. So, we adopt this method to compare the performance of the proposed method. In Fig. 4, we start both the proposed algorithm and the algorithm proposed in [7]

from similar initial sequences of length N = 25. Precisely, in Fig. 4a, we start both the algorithms form Golomb sequence, while in Fig. 4b we initial both the methods with a random polyphase sequence. Further, as a normalized measure, we plot correlation level (dB) that is

$$\text{Correlation Level}(\text{dB}) = 10\log_{10}\frac{|r_k|}{N}$$

where r_k is the aperiodic auto-correlation function of the complex unimodular sequence $\{x_n\}_{n=1}^{N}$ defined by (4) with N as the code length. The Fig. 4 intuitively depicts the performance of the proposed algorithm in comparison with [7]. Some enhancements in different lengths are observable in this Figure.

In order to evaluate the performance of the proposed algorithm quantitatively and via a normalized metric, in the following we use the definition

$$\text{ISLR}(\text{dB}) = 10\log_{10}\frac{ISL}{N^2}$$

which is the ratio of integrated energy of the sidelobes to the peak energy of the mainlobe. For the comparison, we start from 5 independent random sequences at different code lengths $N = 2,3,4,\ldots,75$, and report the averaged ISLR(dB) values of the obtained sequences for all the initial, proposed, and method in [7]. A slightly enhancement at different lengths can be observed from this Figure. Precisely, in length 16 to 18, an improvement around 2dB is obtained by the proposed method in comparison with the counterpart.

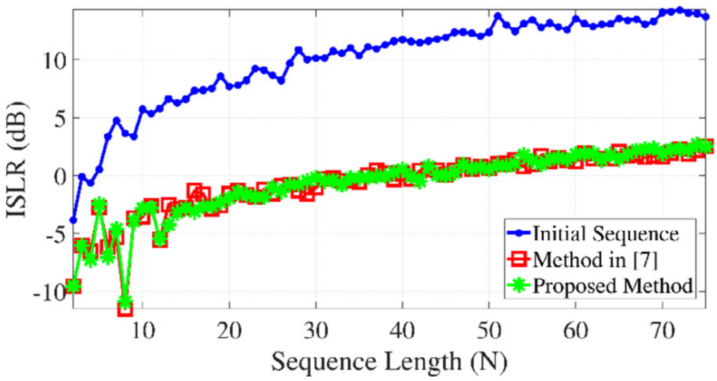


Fig. 5 : Comparison between averaged ISLR(dB) values over 5 independent trials between method in [7] and the proposed method. Both methods are initialized with same set of random sequences in each length , The averaged ISLR(dB) values of the initial sequence is also depicted.

8. CONCLUSION

In this paper, we proposed an optimization framework to design radar transmit waveform with good aperiodic autocorrelation properties. We precisely used the SOCP and SDP and with the aim of MM we converted the non-convex optimization problem to a convex version. Finally using the CVX toolbox we tackled the problem. A possible future track is to tackle the PSL minimization problem.

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