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Efficient Analysis of Plasmonic Circuits Using Differential Global Surface Impedance (DGSI) Model

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ABSTRACT: Differential global surface impedance (DGSI) model, a rigorous approach, has been applied to the analysis of three dimensional plasmonic circuits. This model gives a global relation between the tangential electric field and the equivalent surface electric current on the boundary of an object. This approach helps one bring the unknowns to the boundary surface of an object and so avoid volumetric discretization. It also eliminates the need for equivalent surface magnetic current consideration. Therefore, there is no need to evaluate the rather complex integral operator related to this current. This will result in a great reduction in computation time and memory resources. On the other hand, due to small field variations along the longitudinal direction of each boundary segment, it is suggested to use the two dimensional DGSI matrix in the analysis of a three dimensional plasmonic circuit. This leads to a much simpler formulation of the DGSI model. Besides, our numerical results verify that this simplifying assumption will not greatly affect the accuracy of the analysis. Plasmonic waveguides with different thicknesses along with a line coupler have been analyzed. The results are verified with the results of a commercial software as well as global surface impedance (GSI) model previously presented in the literature.

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1. INTRODUCTION

The study of metallic plasmonic structures enables researchers to manipulate light. The surface plasmon polariton (SPP) phenomenon is introduced as the interaction of light and metal in such structures. The electromagnetic field confinement described by this phenomenon, supports fundamentals of electromagnetic fields propagation along the metal/dielectric interface resulting in the possibility of building a plasmonic waveguide. An important building block of these kind of waveguides is a plasmonic metal strip. Such a structure can support long-range surface plasmon polaritons (LRSPPs). Hence, for a plasmonic circuit design efficient modeling of a plasmonic metal strip is of great interest.

At microwave frequencies, metal can be modeled like a perfect electric conductor, while, in the optical frequencies the electromagnetic fields penetrate inside the metal. In this frequency regime metal is modeled as a lossy dielectric material with negative dielectric constant. Due to the fast decaying nature of the electromagnetic fields inside a plasmonic metal, numerical simulation of such a structure is very challenging in this regime. To follow near-field variations correctly, very fine discretization is unavoidable. Large number of unknowns as well as high computational cost (both memory resources and processing time) will be the result of such a fine discretization. Therefore, it is quite challenging to find an appropriate numerical modeling method to simulate such problems.

Different numerical methods have already been used in the analysis of plasmonic structures. Finite element method (FEM) [1] and finite difference time domain (FDTD) [2] are among the most popular methods applied to the analysis of such structures. Using these methods, the whole solution domain of the desired problem should be discretized. The volume integral equations (VIE) [3] limits the discretization to the object at hand rather than the whole solution domain. This will decrease the number of unknowns in the simulation. However, due to the above-mentioned fast decaying nature of electromagnetic fields inside plasmonic structures, the volumetric discretization of a plasmonic circuit is an important challenge in the analysis of such structures. Thus, it is an important goal to find a method which can decrease the number of unknowns by limiting the simulation to the boundary surface of the object. Classic surface integral equation (SIE) techniques such as PMCHWT are formulated based on equivalence principle [10]. In such methods the problem is divided into two exterior and interior problems presuming zero fields either inside or outside of the object; and at the same time solving for the both unknown electric and magnetic currents on the boundary using these two sets of equations. Surface impedance models bring the electromagnetic fields variations inside the object to its boundary surface. The conventional local surface impedance

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models establish a relationship between the tangential electric field of a boundary segment of an object and the magnetic field (corresponding surface electric current) of that point [4]. Since the precision of this model deteriorates while approaching the corners or sharp edges or when the metal strip is very thin, global surface impedance model has attracted attentions. Global surface impedance (GSI) model relates the tangential electric field on each boundary segment of an object to the tangential magnetic field of all the boundary segments. This relation is established through a matrix form called global surface impedance matrix. Such a nonlocal model has been used to model skin effect of conductors in [5-7]. Plasmonic structures have also been analyzed with this model [8, 9]. In the approaches introduced in [8, 9], both surface electric and magnetic currents should be taken into account to correctly model electromagnetic fields variations. The magnetic current will then be replaced by the electric current through the mentioned GSI impedance leading to a single source formulation. These single source formulations halve the number of unknowns in comparison to the classic SIE techniques. However, evaluating the rather complex integral operator of the magnetic current in the surface integral equation is still a challenge in such an approach. Taking advantage of equivalence principle, the authors developed differential global surface impedance (DGSI) model [11]. The usage of this model will result in a problem of single surface electric current's radiation in a homogenous unbounded region, eliminating the need for magnetic current consideration. In that paper, periodic structures made of 2D dielectric cylinders have been analyzed using DGSI model. It has been observed that using this model the complexity of the problem along with the computational resources and cost will be reduced.

In this paper, 3D plasmonic circuits will be simulated using DGSI model. The challenging part of a plasmonic circuit analysis is modelling the plasmonic metal strip. To this end, the 2D cross section of the metal strip will be modeled by DGSI. This 2D analysis simplifies computation of the surface impedance. The proposed DGSI model, is applied to the interior problem of a metal strip, finding the relation between the tangential electric field and the equivalent surface electric current on the boundary surface of the metal strip. In the next step, the derived DGSI model is employed to the surface integral equation to analyze the plasmonic circuit. Then, using the method of moments (MoM) [12] to solve the achieved equation, one can obtain the unknown equivalent surface electric current distribution on the metal strip. The proposed method is applied to the analysis of a plasmonic waveguide and a line coupler. The efficiency and accuracy of the method is compared with the previously proposed GSI method.

This paper is organized as follows. In section II the formulation of the problem is presented. The numerical results are remarked in Section III following some concluding remarks in section IV.

2. FORMULATION OF THE PROBLEM

In this section, first, the DGSI formulation will be described briefly. Then, the exterior problem formulation using mixed potential integral equation (MPIE) [14] will be explained. After that, DGSI model will be employed into the mentioned MPIE and is used for the analysis of a typical plasmonic circuit as in Fig. 1.

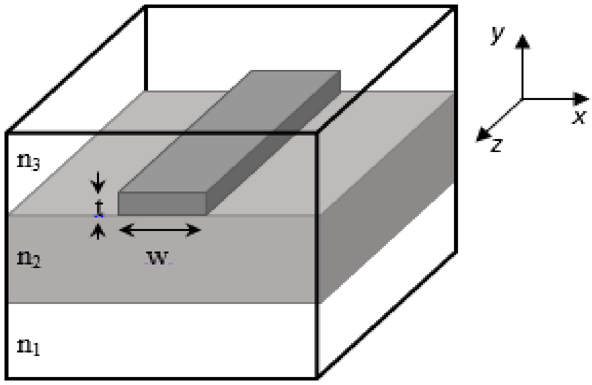

Fig. 1 . a typical plasmonic circuit

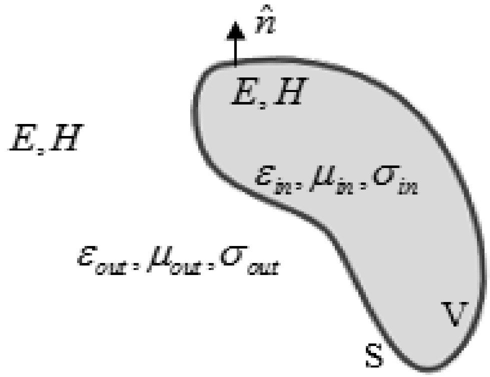

Fig. 2 . Problem understudy with an arbitrary shape

2-1 DGSI Formulation

As mentioned earlier, DGSI model gives a relation between the tangential electric field and the equivalent surface electric current on the boundary of an object. The equivalent surface electric current determines the discontinuity of the tangential magnetic field along the boundary. Therefore, one can define this current in terms of the tangential magnetic fields on either sides of the boundary. To this end, a boundary value problem for the magnetic field interior to the object boundary may be set up. Let's assume an arbitrary object as in Fig. 2 made of a linear, homogenous and isotropic medium characterized by the permittivity of ε_{in}, permeability of μ_{in} and the conductivity of σ_{in} embedded in a medium filled with a material with constitutive parameters $(\varepsilon_{out}, \mu_{out}, \sigma_{out})$. The reasoning put forward in the paper is valid in a piecewise homogenous medium (multilayered medium) while the object is wholly embedded in one layer. However, when the object is partly located in one layer and partly in another, it should be divided in two separate parts. The following equation shows the introduced boundary value problem [11]:

$$\begin{cases} \nabla \times \nabla \times \vec{H}_\xi - k_\xi^2 \vec{H}_\xi = 0 \\ \hat{n} \times \left[\nabla \times \vec{H}_\xi \right]\Big|_s = j\omega\varepsilon_\xi \hat{n} \times \vec{E}_\xi \end{cases} ;(\xi = in, out), \quad (1)$$

where $k_\xi = \sqrt{-j\omega\mu_\xi \left(j\omega\varepsilon_\xi + \sigma_\xi \right)}$ is the wave number, $\vec{H}$ the magnetic field, and the subscript ξ determines that the object volume (V in Fig. 2) is either filled with the material with the same properties of the material inside or outside of the object. $\hat{n}$ is the normal vector directed from inside of the object to the boundary. In order to solve such a boundary value problem one can obtain its corresponding Green's function as in the following formula:

$$\begin{cases} \nabla \times \nabla \times \overline{\overline{G}}_\xi (r, r') - k_\xi^2 \overline{\overline{G}}_\xi (r, r') = \overline{\overline{I}} \delta (r - r') \\ \hat{n} \times \left[\nabla \times \overline{\overline{G}}_\xi (r, r') \right]\Big|_s = 0 \end{cases} ;(\xi = in, out), \quad (2)$$

where $\vec{r}, \vec{r}'$ define field and source points positions respectively.

Using these equations the equivalent surface electric current which is the difference of the tangential magnetic fields on either sides of the boundary can be found as follows:

$$\vec{J}_s = -\hat{n} \times j\omega \left\{ \oint \left[\varepsilon_{in} \overline{\overline{G}}_{in} (\vec{r}, \vec{r}') - \varepsilon_{out} \overline{\overline{G}}_{out} (\vec{r}, \vec{r}') \right] \cdot \hat{n} \times \vec{E}_{in} (\vec{r}') ds' \right\}. \quad (3)$$

where the integration is taken over the boundary surface *S*.

This equation gives an integral relation between the equivalent surface electric current and the tangential electric field on the boundary. To find a matrix form of this relation, using appropriate test and basis functions, one can apply MoM to this equation, i.e.:

$$\left[J_s \right] = \left[Y_s^{DGSI} \right] \left[E_s \right], \quad (4)$$

where $[E_s]$ and $[J_s]$ are column vectors containing amplitudes of the electric field and surface electric current respectively. The differential global surface admittance matrix is defined by $\left[Y_s^{DGSI} \right]$. Taking the inverse of this matrix, $\left[Z_s^{DGSI} \right]$ the DGSI impedance matrix will be evaluated. This impedance matrix presents the DGSI relation between the equivalent surface electric current and the tangential electric field. That is:

$$\left[E_s \right] = \left[Z_s^{DGSI} \right] \left[J_s \right]. \quad (5)$$

For the analysis of a plasmonic circuit, DGSI matrix for the 3D plasmonic metal strip should be found. Since each segment of the discretized strip is very small, it can be assumed that there exists a slight variation along the longitudinal direction on each basis function. Hence, it is possible to assume no axial variations i.e. $\partial/\partial z = 0$ along this direction [13]. Therefore, it will be presumed that TM_z relation resides between axial components of the surface electric current and field, while, between transverse components of these quantities a TE_z relation exists. This simplifying assumption helps one to evaluate 2D DGSI matrices of an object cross section for both TM_z and TE_z cases. Then, a block-diagonal matrix $\left(\left[Z_s^{DGSI} \right] \right)$ made of these matrices can be used in the 3D problem of the plasmonic circuit. That is:

$$\begin{aligned} &\left[Z_s^{DGSI} \right] = \\ &\begin{bmatrix} \left[bdiag\left(Z_{s,TM}^{DGSI} \right) \right] & [0] \\ [0] & \left[bdiag\left(Z_{s,TE}^{DGSI} \right) \right] \end{bmatrix}; \\ &\left[bdiag\left(Z_{s,TM/TE}^{DGSI} \right) \right] = \\ &\begin{bmatrix} \left[Z_{s,TM/TE}^{DGSI} \right] & [0]\cdots & [0] \\ [0] & \ddots & [0] \\ \vdots & & \vdots \\ [0]\cdots & [0] & \left[Z_{s,TM/TE}^{DGSI} \right] \end{bmatrix}, \end{aligned} \quad (6)$$

where $\left[bdiag\left(Z_{s,TM}^{DGSI} \right) \right]$ and $\left[bdiag\left(Z_{s,TE}^{DGSI} \right) \right]$ are block-diagonal matrices made of 2D DGSI matrices for TM_z $\left(\left[Z_{s,TM}^{DGSI} \right] \right)$ and TE_z $\left(\left[Z_{s,TE}^{DGSI} \right] \right)$ polarizations, respectively. In the Appendix the calculation of TM_z DGSI matrix elements for a 2D rectangular strip is presented. In order to separate the effect of axial and transverse source points on their corresponding field points, these matrices are placed in DGSI matrix $\left(\left[Z_s^{DGSI} \right] \right)$ in the form of four submatrices (blocks). The first block of this matrix represents the effect of axial source points on axial field points, while the fourth block defines this effect for transverse source and field points. The zero matrix $[0]$ describes that axial (transverse) source points has no contribution on transverse (axial) field points. This is due to our simplifying assumption, explaining the zero coupling between axial and transverse segments. According to the explained arrangement of DGSI matrix, one should arrange the electric field and the equivalent surface electric current vectors in the same manner.

It is obvious that, in a 3D analysis of a structure the hybrid nature of TM_z and TE_z polarizations should be taken into account. Since this issue has been considered in the evaluation of the MoM coefficient matrix obtained in the next section, the mentioned simplifying assumption in the construction of DGSI matrix may not greatly affect the results accuracy. Our numerical results given in section III verifies this theory.

2-2 Exterior problem formulation

As explained in the previous section, the electromagnetic fields behavior inside the object under study is simulated using the DGSI model. The relation between surface electric field and current on the boundary surface of the object given by this model is then employed to the exterior problem formulation to solve for the unknown surface electric current. Due to uniqueness theorem, in a lossy medium an electromagnetic field is specified by the sources inside the region, the tangential component of either electric or magnetic field over the boundary or tangential component of electric field on part of the boundary and magnetic field on the remaining parts. After finding the unknown surface electric current, the tangential component of the electric field on the boundary can be calculated using (5). Therefore, having the tangential electric field at hand one can solve Helmholtz equation inside to find the fields internal to object. Hence, the correct fields inside the scatterer will be found and one can will be able to obtain the absorption property of the plasmonic material.

Applying the DGSI model to the analysis of a plasmonic circuit, it is obvious that the exterior problem will be a problem of an equivalent surface electric current's radiation in an unbounded region with the same constitutive parameters as the material of the exterior region. Thus, the conventional SIE for penetrable objects will be modified to a simple form. That is:

$$\vec{E}_s = \hat{n}\times\vec{E}^{ext} + \hat{n}\times\oint_{s^+}\overline{\overline{G}}\cdot\vec{J}_s ds', \tag{7}$$

where $\vec{E}^{ext}$ defines the excitation electric field and $\overline{\overline{G}}$ is the electric field dyadic Green's function for the exterior region. Applying MoM to this equation along with choosing appropriate test and basis functions will give the matrix form of this equation:

$$\left[E_s\right] = \left[E^{ext}\right] + \left[L_{mn}\right]\left[J_s\right], \tag{8}$$

where $\left[J_s\right]$ and $\left[E_s\right]$ are the column vectors containing the amplitudes of the equivalent surface electric field and the electric field respectively. $\left[E^{ext}\right]$ is also a column vector containing the amplitudes of the excitation electric field and $\left[L_{mn}\right]$ matrix is the MoM coefficient matrix. Substituting Es from (5) into (8), and doing some manipulations, the following equation will be obtained to evaluate the unknown vector $\left[J_s\right]$:

$$\left(\left[Z_s^{DGSI}\right] - \left[L_{mn}\right]\right)\left[J_s\right] = \left[E^{ext}\right]. \tag{9}$$

Using the Mixed Potential Integral Equation formulation, one can solve this equation. Therefore, the elements of MoM coefficient matrix will be:

$$L_{mn} = -j\omega\mu\Delta\vec{l}_m.\Delta\vec{l}_n\Psi(m,n) - \frac{1}{j\omega\varepsilon}\begin{bmatrix}\Psi(m^+,n^+) - \Psi(m^+,n^-) - \\ \Psi(m^-,n^+) + \Psi(m^-,n^-)\end{bmatrix}, \tag{10}$$

where

$$\Psi(m,n) = \frac{1}{\Delta l_m \Delta l_n}\int_{n_{th}\ segment}\frac{e^{-jkR_m}}{4\pi R_m}ds'; \quad R_m = \left|\vec{r}_m - \vec{r}'\right|, \tag{11}$$

$\vec{r}'$ is the vector pointing to the source point with the coordinates of (x', y', z') while $\vec{r}_m$ is the corresponding vector for the observation point with the coordinates of (x_m, y_m, z_m). The length of $m_{th}(n_{th})$ segment is denoted by $\Delta l_{m(n)}$ and vector $\Delta\vec{l}_{m(n)}$ shows the current direction along the segment. The charge nodes defined by $m^{\pm}(n^{\pm})$ are related to the $m_{th}(n_{th})$ current segment as explained in [12].

MoM matrix elements should be arranged in the same manner as the DGSI matrix elements. Therefore, MoM matrix should be divided into four submatrices to construct a block matrix:

$$\left[L_{mn}\right] = \begin{bmatrix}\left[L_{mn}^{AA}\right] & \left[L_{mn}^{AT}\right] \\ \left[L_{mn}^{TA}\right] & \left[L_{mn}^{TT}\right]\end{bmatrix}, \tag{12}$$

where submatrix $\left[L_{mn}^{AA}\right]\left(\left[L_{mn}^{TT}\right]\right)$ located on the main diagonal models the effect of axial (transverse) source points on axial (transverse) field points. On the other hand, the coupling effect of the axial (transverse) source points on the transverse (axial) observation points are represented in $\left[L_{mn}^{TA}\right]\left(\left[L_{mn}^{AT}\right]\right)$. The nonzero value of the off-diagonal submatrices describes that in the MoM matrix the hybrid nature of the axial and transverse polarizations has been taken into consideration.

3. NUMERICAL RESULTS

In this section a plasmonic waveguide as shown in Fig.1 is simulated. Modeling the rectangular metal strip is the most challenging part of a waveguide analysis. Therefore, without loss of generality this strip can be assumed to be located in free space i.e. $n_1 = n_2 = n_3 = 1$. Hence, the free space Green's function can be easily used to evaluate MoM matrix (equation 11) while there is no need for the calculation of a layered medium Green's function in case of existence of a substrate. In this way, the field behavior of the metal strip can be directly studied without being worried about the reflections from dielectric layers.

The first structure is assumed to be a plasmonic metal strip made of silver with the complex dielectric constant $\varepsilon_r = -19 - j0.53$ in the free space wavelength of $\lambda_0 = 633nm$. The proposed 3D rectangular structure, is illuminated by a TM_z polarized plane wave propagating normal along $-\hat{y}$ axis. The strip, terminated with PEC walls at both ends, has the width of $w = 0.2\mu m$. Its thickness is assumed to be $t = 0.1\mu m$ and its length is $l = 0.5\mu m$. The absolute value of the axial electric field ($|E_z|$) is shown in Fig. 3. The field values are evaluated along the center axial line of the top surface of the waveguide (i.e. the z axis passing through $(x = w/2, y = t)$). The results are verified with the results of CST Microwave Studio simulation. GSI model has also been used to simulate this structure showing good agreement with our numerical results.

As another example, waveguide constructed from gold with the complex dielectric constant $\varepsilon_r = -131.95 - j12.5$ in the free space wavelength of $\lambda_0 = 1.55\mu m$ is assumed. The line

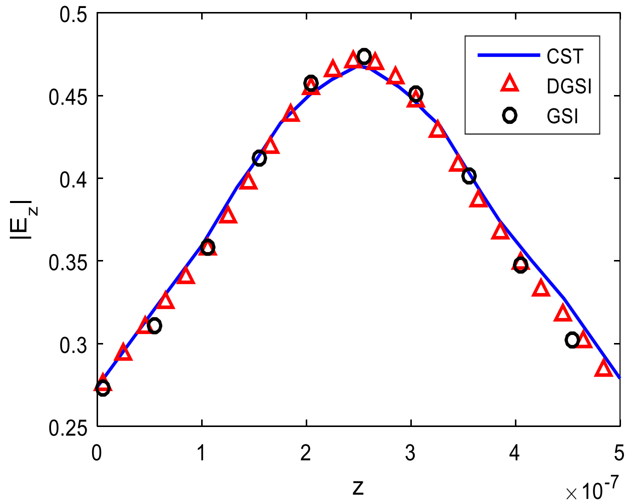

Fig. 3 . Absolute value of the axial electric field along the z axis passing through $(x = w/2, y = t)$ **.**

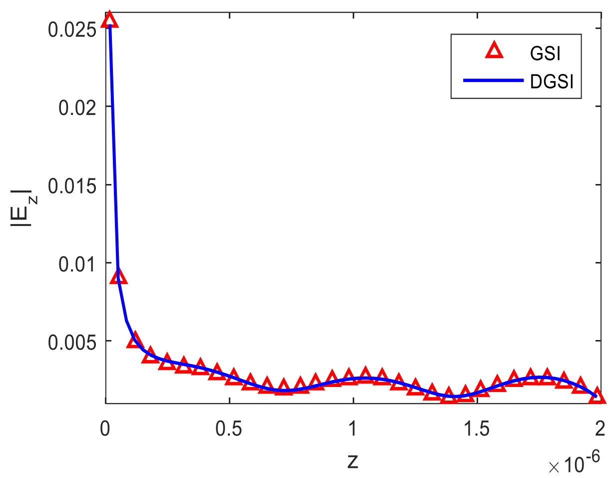

Fig. 6 . Absolute value of the axial electric field on the center line of the top surface of the strip, $t = 100nm, W = 1\mu m, l = 2\mu m$

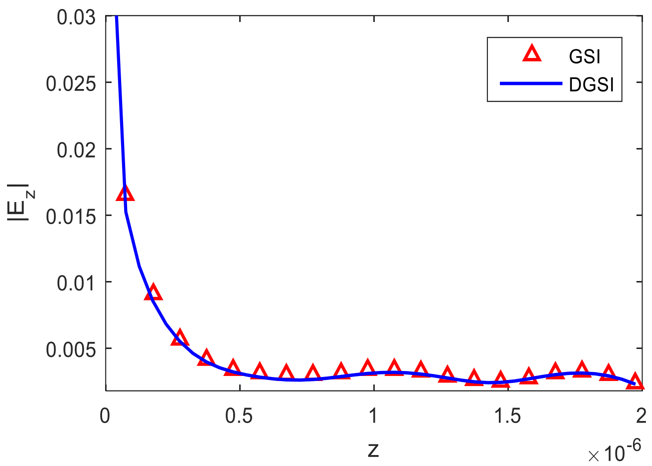

Fig. 4 . Absolute value of the axial electric field on the center line of the top surface of the strip, $t = 500nm, W = 1\mu m, l = 2\mu m$

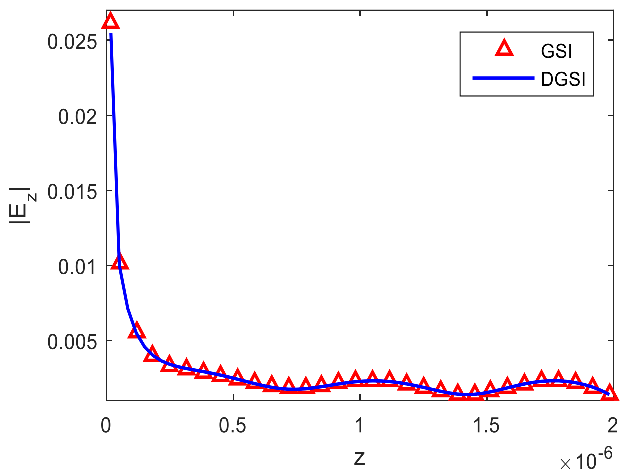

Fig. 5. Absolute value of the axial electric field on the center line of the top surface of the strip, $t = 200nm, W = 1\mu m, l = 2\mu m$

is terminated with a PEC wall at the end modeling a short circuited line. Delta gap generator is used as the excitation on one side of the waveguide. Fig. 4 shows the absolute value of the axial electric field along the center line of the top surface of the waveguide with $t = 500nm, W = 1\mu m, l = 2\mu m$. Our analysis is done on a PC Intel (R) Corei7-4790K CPU@4.00GHz, with 32 GB of RAM. In GSI method it takes about 91 seconds to fill the MoM matrix while in DGSI method this time is about 86 seconds. Since the size of MoM matrices is the same in both methods the time needed for inversion of this matrices is almost the same. Therefore, using the DGSI approach, the computation time in the analysis of this circuit is 0.95 times less than the computation time of GSI model.

This shows that elimination of the magnetic current from the exterior problem formulation, decreases the computation time, since there is no need for evaluating the complex magnetic current operator. The absolute value of the axial electric field on the center line of the top surface of the strip for different thicknesses ($t = 200nm$, $100nm$ respectively) is also shown in Figures 5 and 6.

A line coupler as typically shown in Fig. 7 is the next example investigated. This coupler is made of two similar lines made of silver with the characteristics mentioned for the first structure. Each through line has the thickness of $t = 0.3\mu m$, width of $w = 0.5\mu m$ and length of $l = 1\mu m$. The spacing between the two lines is $d = 0.2\mu m$.

Since the two through lines have the same shape and size, the DGSI model remains the same for these two strips. Hence, the DGSI matrix can be evaluated for one of the strips and used for the other one avoiding extra calculations. This means that the interior problem of the two strips is simulated evaluating DGSI matrix for one of these strips. This leads to a great reduction in the computation cost. Fig. 8 shows the absolute value of the axial electric field along the centerline of the top surface in the first strip while this strip is excited. For the non-excited strip the axial electric field is sketched

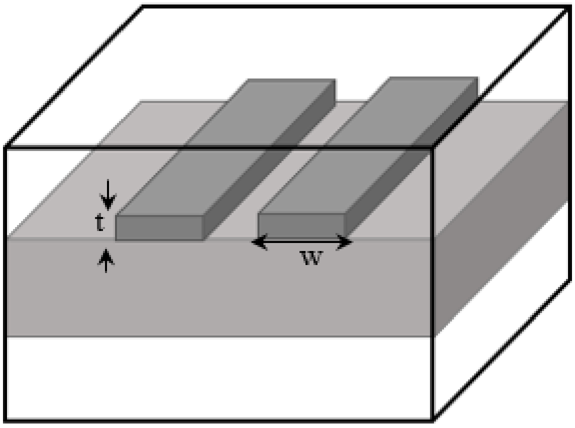

Fig. 7 . a typical plasmonic line coupler

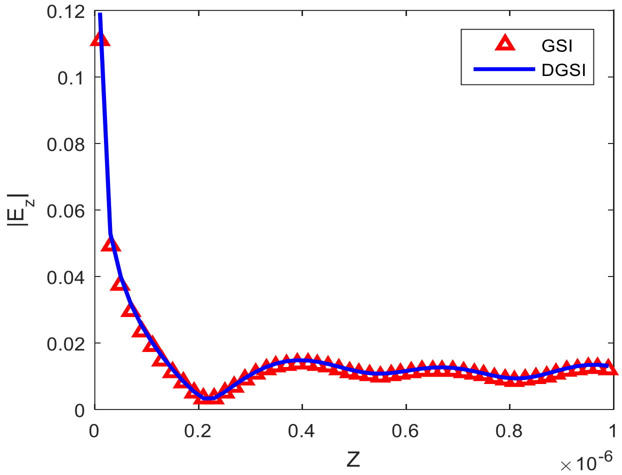

Fig. 8 . Absolute value of the axial electric field on the centerline of the top surface of the first strip (excited line)

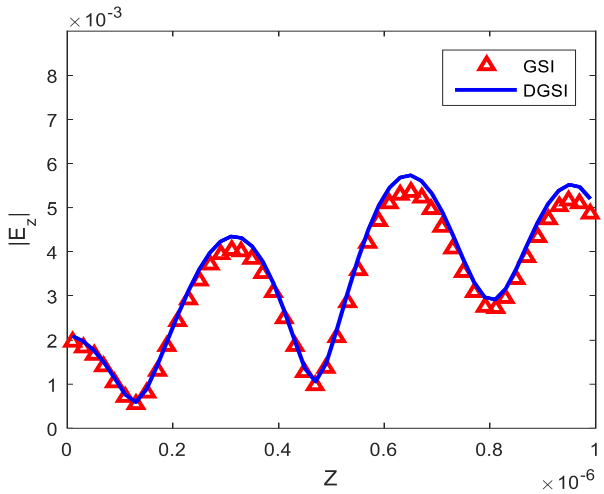

Fig. 9 . Absolute value of the axial electric field on the centerline of the top surface of the second strip (non-excited line)

in Fig. 9. A standing wave behavior is observed due to the existence of PEC wall at the end of the lines. The results are also compared with the results of GSI model and show good agreement.

4. CONCLUSION

In this paper, the previously introduced differential global surface impedance model has been successfully applied to the analysis of plasmonic circuits. This model gives a relation between the equivalent surface electric current and the tangential component of the electric field on the boundary surface of an object under study. Using this model there is no need to consider the effect of equivalent surface magnetic current as in the conventional SIE formulation for penetrable objects. It means that making use of the DGSI model, the problem can be solved as a problem of radiation of a surface electric current in an unbounded homogenous region. This will result in a much simpler and faster evaluation of the MoM matrix. It is also shown that due to the small field variations along the longitudinal direction in each segment, 2D DGSI matrix can be used in the analysis of a 3D problem without greatly sacrificing the accuracy. Moreover, this approach will lead to a much simpler formulation of the DGSI model. Hence, the complexity of the method will be reduced. The proposed method has been applied to plasmonic waveguides and a line coupler. The results are compared with the results of GSI model showing good agreement and less computation time.

Appendix: DGSI Evaluation for a rectangualr metal Strip

In a 2D structure there is no field variations along z axis (i.e. $\partial/\partial z = 0$). Therefore, the problem of finding the DGSI matrix for a 2D rectangular strip can be independently formulated for either the TM_z or the TE_z field configurations. In the TM_z case (E_z, H_x, H_y) field components exist in the structure, so the dyadic Green's function in will only have one component G_{zz} . This equation can be easily solved to obtain this Green's function as:

$$G_{zz} = \begin{cases} \sum_{l=0}^{\infty} \frac{\varepsilon_l}{t} \cos\left(\frac{l\pi}{t} y\right) \cos\left(\frac{l\pi}{t} y'\right) \frac{1}{k_{xl} \sin k_{xl} W} \cdot \begin{cases} \cos k_{xl} x & x' = W \\ \cos k_{xl} (W - x) & x' = 0 \end{cases} \\ \sum_{l=0}^{\infty} \frac{\varepsilon_l}{W} \cos\left(\frac{l\pi}{W} x\right) \cos\left(\frac{l\pi}{W} x'\right) \frac{1}{k_{yl} \sin k_{yl} t} \cdot \begin{cases} \cos k_{yl} y & y' = t \\ \cos k_{yl} (t - y) & y' = 0 \end{cases} \end{cases}, \quad (13)$$

where $\varepsilon_l = \begin{cases} 1 & l = 0 \\ 2 & l \neq 0 \end{cases}$ is the Neumann number and $k_{xl} = \sqrt{k^2 - (l\pi/t)^2}, k_{yl} = \sqrt{k^2 - (l\pi/W)^2}$. The width and thickness of the rectangular strip are represented by W and t, respectively (see Fig. 1). The elements of the DGSI admittance matrix $\left[Y_s^{DGSI}\right]$ are found by substituting this Green's function into (3), It is clear that this matrix can be written as a differential admittance $Y_{s,TM}^{DGSI} = Y_{s,TM}^{DGSI,in} - Y_{s,TM}^{DGSI,out}$, related to inside and outside admittance matrices:

$$Y_{mn,TM}^{DGSI}=\begin{cases} -j\sum_{l=0}^{\infty}\frac{2\varepsilon_l}{l\pi}\cos\left(\frac{l\pi}{t}y_m\right)\cos\left(\frac{l\pi}{t}y_n\right)\sin\left(\frac{l\pi\Delta_t}{2t}\right)\left(\frac{k_\xi}{\eta_\xi}\frac{\cos k_{xl,\xi}\left(W-x_m\right)}{k_{xl,\xi}\sin k_{xl,\xi}W}\right) & x'=0 \\ -j\sum_{l=0}^{\infty}\frac{2\varepsilon_l}{l\pi}\cos\left(\frac{l\pi}{W}x_m\right)\cos\left(\frac{l\pi}{W}x_n\right)\sin\left(\frac{l\pi\Delta_W}{2W}\right)\left(\frac{k_\xi}{\eta_\xi}\frac{\cos k_{yl,\xi}y_m}{k_{yl,\xi}\sin k_{yl,\xi}t}\right) & y'=t \\ -j\sum_{l=0}^{\infty}\frac{2\varepsilon_l}{l\pi}\cos\left(\frac{l\pi}{t}y_m\right)\cos\left(\frac{l\pi}{t}y_n\right)\sin\left(\frac{l\pi\Delta_t}{2t}\right)\left(\frac{k_\xi}{\eta_\xi}\frac{\cos k_{xl,\xi}x_m}{k_{xl,\xi}\sin k_{xl,\xi}W}\right) & x'=W \\ -j\sum_{l=0}^{\infty}\frac{2\varepsilon_l}{l\pi}\cos\left(\frac{l\pi}{W}x_m\right)\cos\left(\frac{l\pi}{W}x_n\right)\sin\left(\frac{l\pi\Delta_W}{2W}\right)\left(\frac{k_\xi}{\eta_\xi}\frac{\cos k_{yl,\xi}\left(t-y_m\right)}{k_{yl,\xi}\sin k_{yl,\xi}t}\right) & y'=0 \end{cases};(\xi=in,out), \tag{14}$$

where the length of each segment along the thickness and width of the rectangle are denoted by Δ_t and Δ_W, respectively. The medium intrinsic wave is defined by η_ξ. The DGSI impedance matrix will be obtained by taking the inverse of this admittance matrix. Moreover, the impedance matrix for the transverse polarization (the TE_z field configuration) will be obtained following the same procedure.

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