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Kinematic Reliability Analysis of 3-PSS manipulator based on the explicit solution and design of experiment method

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ABSTRACT: This paper aims at the kinematic reliability analysis of the 3-PSS parallel robot. Parallel manipulators bear many advantages like higher stiffness, more accuracy, and speed compared to the serial counterparts. Because of several uncertainty factors such as actuators error, links flexibility, etc. a robot moving platform cannot follow the desired trajectory without an error. In this study, at first, eight, and next twelve uncertainties that seem to affect the kinematics of robot are selected. Next, the probability distribution of the moving platform position is conducted using the closed-form kinematic relation of a robot and the Monte Carlo Simulation, then, the kinematic reliability is calculated for different levels of accuracy. As the closed-form kinematic relation between the actuators rate and the moving platform position cannot be obtained, a polynomial algebraic equation is fitted via the design of experiments method. Using fitted polynomial kinematic equation at hand, the reliability analysis is conducted and evaluated for different levels of accuracy. Also, an experimental reliability analysis is performed to evaluate the results which are obtained numerically. The results show that the difference between the reliability values obtained by the design of experiments and the Monte Carlo Simulation methods is %4.7, and between the design of experiments method and experimental data is %7.7. In the end, a sensitivity analysis is conducted to determine the influence of each uncertainty on the accuracy.

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1. INTRODUCTION

Robot Manipulators (RM) are widely used in industries for different purposes such as assembling, welding, packaging, etc. Generally, manipulators are in two types, parallel and serial [1, 2]. Serial Manipulators (SM) are formed by the concatenation of links and joints [3, 4]. However, parallel counterparts are formed by connecting some serial chains in parallel [2, 5]. Recently, Parallel Manipulators (PM) have been used in different industries and for distinct processes such as CNC machining, and 3D printing. The main advantages of PM are higher stiffness, speed, and accuracy compared to SM [6-9], which results in higher reliability [7, 10-12]. However, because of the non-linear relation between inputs and outputs, the complex kinematics and calibration are the disadvantages [13-15]. However, the accuracy of parallel robots can be improved via analysis of kinematic accuracy at the embodiment design stage or with calibration after manufacturing the robot [16, 17]. To conduct a kinematic reliability analysis, forward and inverse kinematics of the robot should be properly developed. Rao et al. analyzed the reliability of a serial manipulator by using a statistical method and presented a relation between geometrical parameters, tolerances, and reliability [18]. Liu et al. presented a kinematic reliability analysis of two links serial manipulator [19]. They calculated the kinematic accuracy of the robot at each position by incorporating the mean value and deviation quantities. Shi introduced the kinematic reliability analysis of the Stanford robot by Denavit–Hardenberg (D-H) parameters [20]; also, they derived a closed-form equation for the kinematic accuracy. Carrerasa and Walker [21] presented a reliability analysis of a serial robot by resorting to the interval method. Cui et al. [7] introduced influential parameters on the kinematics of Stewart's platform. Pan et al. [22] analyzed the reliability of an exoskeleton robot, and the reliability contour plots at the various posture of the robot were presented. In recent work, Fu et al. [23] analyzed the kinematic accuracy of a novel six degrees of freedom parallel robot with three legs. Xu [24] conducted kinematic reliability and sensitivity analyses for a modified parallel robot. He depicted the effect of some uncertainties on the output errors of the robot using the surface method.

The current work aims at the kinematic reliability analysis of a delta parallel robot with 3-PSS architecture [25]. A prototype of this robot has been manufactured at the Amirkabir University of Technology (Tehran Polytechnic) to be used as a parallel robot 3D printer (Fig. 1). First of all, an input-output relation between the actuator rates and the Moving Platform (MP) position is derived. Next, some important uncertainty sources which are predicted to have significant effects on the robot kinematic are assumed; their

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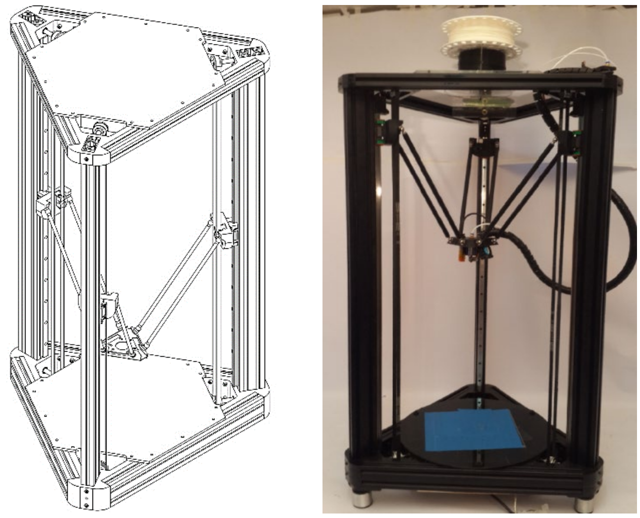


Fig. 1. The Delta 3D printer prototyped at the Amirkabir University of Technology (Tehran Polytechnic)

limit values are associated based on the manufacturing Process Standard Deviation (PSD) or company datasheets. Because of the complexity of kinematic relation in this robot, if the number of uncertainty sources is increased, a closed-form relation between the actuator rates and the MP position cannot be readily obtained. Therefore, in the first try, only eight uncertainties are assumed. Afterward, by resorting to the Mont-Carlo method (MCS), the probability distribution of the MP position of the robot is calculated, and the kinematic reliability is calculated based on the assumed errors. In the second try, the number of uncertain parameters is increased to 12. In this case, a closed-form kinematic input-output relation cannot be obtained, and hence, the approximated relation is derived using the Design Of Experiments (DOE). In this method, the kinematic equation is fitted by performing some numerical trials in the neighborhood of a desired point. Then, the kinematic reliability analysis is conducted based on the fitted relation. The numerical results are experimentally validated on the robot. Finally, a sensitivity analysis is performed to determine those affecting uncertainties on the MP positioning accuracy.

2. PRELIMINARIES

2.1. Accuracy and reliability

Generally, a manipulator is demanded to follow the desired trajectory or positioned in an accurate demanded position. In this study, accuracy is defined as the difference between performed position and the desired positions. Because of several uncertainties such as actuator errors, joint clearances, link flexibilities, and unknown factors, the robot MP cannot be placed at a desired position or trajectory without an error [10, 26, 27]. Therefore, the kinematic reliability of a manipulator is defined as the probability of MP to be in an equilateral interval about the desired position. It is very important to evaluate the kinematic reliability of a manipulator to perform a task with a certain level of accuracy [28, 29].

2.2. Fundamentals of reliability

Generally, reliability is defined as the probability of success under certain working conditions [30, 31]. There are two types of reliability analysis; in the first type, all of the variable tolerances are calculated based on the desired reliability of the system [27], and in the second type, the reliability of a manufactured machine is calculated according to the known parts tolerances.

In general, if reliability analysis is performed for the manufacturing of a product, such as a robot, any design parameter should be assumed as a variate (random variable). However, if the analysis is applied on an existing robot, only parameters that can vary randomly, such as the actuator's input, are regarded as variates. As it was mentioned, reliability means the probability of success. Hence, by considering an allowable error, the probability of the MP position within its upper and lower limits is calculated as follows [32, 33],

$$\text{Reliability} = \phi\left(\frac{x - x_{0^+}}{\sigma}\right) - \phi\left(\frac{x - x_{0^-}}{\sigma}\right) \tag{1}$$

where x and x_0 are the targeted position and the allowable error limits, respectively. ϕ stands for the cumulative probability using the standard normal distribution for x_0s, and σ for the standard deviation. For example, the probability of success for the MP position with the allowable error of ±0.35 mm is

$$\text{Reliability} = \phi\left(\frac{x - x_{+0.35}}{\sigma}\right) - \phi\left(\frac{x - x_{-0.35}}{\sigma}\right) \tag{2}$$

In a design approach, variables are quantitatively modeled by parameters [27]. Variables have different magnitudes with a degree of uncertainty, hence, a continuous variate x is defined with mean value (μ) and standard deviations (σ) [34-37], as follows:

$$\mu_x = \int_{-\infty}^{+\infty} \phi(x) f(x) dx = \sum_{i=1}^{\infty} x_i . f(x_i) \tag{3}$$

$$\sigma_x = \sqrt{\sum_{i=1}^{\infty} [x_i - E(x)]^2 . f_i} \tag{4}$$

If there exists a multi-variable function $z = \varphi(x, y, \ldots)$ the mean value is calculated as,

$$\mu_z = E\left[\varphi(x, y, \ldots)\right] \tag{5}$$

$$\mu_z = E\left[\varphi(x, y, \ldots)\right] = \int_{-\infty}^{+\infty} \ldots \int_{-\infty}^{+\infty} \varphi(x, y, \ldots) f(x, y, \ldots) dxdy \ldots \tag{6}$$

where *f(x, y,...)* is the joint probability distribution function of variates.

The mean and standard deviation of a multi-variable function defined as $z=\varphi(x,y,\ldots)=x\pm y\pm\ldots$ can be calculated as follows,

$$\mu_x = E[\phi(x,y,\ldots)] = \mu_x \pm \mu_y \pm \ldots \\ \sigma_Z = \sqrt{\sigma_x^{\,2} + \sigma_y^{\,2} + \ldots} \tag{7}$$

2.3. Forward kinematics of 3-PSS manipulator

In this part, the forward kinematic analysis of the robot is reviewed. In this analysis, the inputs which are the actuators' rates are given and the position of MP is desired. In the sequel, the position vector of MP is obtained through each kinematic chain (Fig. 2), namely,

$$r = \boldsymbol{b}_i + \boldsymbol{q}_i e + l\boldsymbol{w}_i - \boldsymbol{a}_i (i=1,2,3) \tag{8}$$

in which

$$\boldsymbol{a}_i = [a\cos(\beta_i) \quad a\sin(\beta_i) \quad 0]^T \tag{9}$$

$$\boldsymbol{b}_i = [S\cos(\beta_i) \quad -S\sin(\beta_i) \quad 0]^T \tag{10}$$

$$\boldsymbol{e} = [0 \quad 0 \quad 1]^T \tag{11}$$

Eq. 8 can be formed in the following format

$$\boldsymbol{r} - (\boldsymbol{b}_i - \boldsymbol{a}_i) - q_i \boldsymbol{e} = l\boldsymbol{w}_i \tag{12}$$

where all the vectors are defined in Table. 1.

For the sake of brevity vector r is written in terms of the Cartesian components,

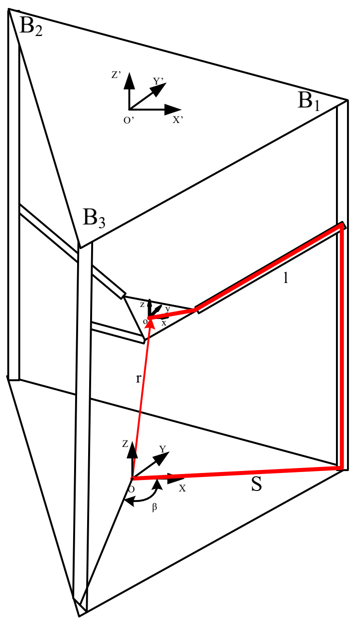


Fig. 2. Schematic of the robot with a vector loop

Table 1. The parameters

Parameter	Description
S	The radius of the base platform
β_i	Angle between columns
e	The direction of columns
b_i	The position of columns in the base plate
a	The radius of the end effector platform
q_i	Translation movement
l	Links length
w_i	Direction of links

$$\boldsymbol{r_{0i}} = \boldsymbol{b}_i - \boldsymbol{a}_i = \begin{pmatrix} x_{0i} & y_{0i} & z_{0i} \end{pmatrix} \tag{14}$$

By taking the norm of both sides of Eq. 12. it yields

$$(x-x_{01})^2 + (y-y_{01})^2 + (z-q_1)^2 - l^2 = 0 \tag{15}$$

$$(x-x_{02})^2 + (y-y_{02})^2 + (z-q_2)^2 - l^2 = 0 \tag{16}$$

$$(x-x_{03})^2 + (y-y_{03})^2 + (z-q_3)^2 - l^2 = 0 \tag{17}$$

In fact, the solution of algebraic Eqs. (7-9) gives a closed-form relation between the input variables, which are the actuator angles, and the output, which is the MP position. For a set of input variables, the foregoing algebraic equations attain two solutions, which means the manipulator can reach the desired position via two different configurations that are depicted in Fig. 3. Here in this study, the second configuration is selected.

3. KINEMATIC RELIABILITY EVALUATION

The kinematic reliability of a manipulator can be calculated by the cumulative probability of allowable MP positions. It means that, based on the forward kinematics, the mean value and standard deviation of MP position are calculated, and then, the probability is obtained [30, 38]. In this regard, the kinematic input-output relation should be obtained, namely, the relation between the actuator kinematic variables and the MP position. Next, a couple of important uncertainties should be predicted, and for each of them, an error limit is associated i.e., the motor input errors, the linkage dimensional, and geometrical tolerances. Usually, the values of uncertainties are obtained from the manufacturing process which is introduced in PSD [39] or the manufacturer datasheets.

The common method of evaluation of standard deviation considers 6σ [37, 40], for the tolerance value. For example, if a stepper motor positioning error is $\pm 1.8^{\circ}$ [41], the standard deviation of the error is considered 0.6°.

Here in this study, for a $\varnothing 340\times 500\,\text{mm}$ workspace, eight uncertainties which are depicted in Fig. 2, are defined, namely,

- The length of the connecting rods (l)
- The actuator inputs (q_i)
- The angle between columns (β)

The probability distribution of the MP position can be calculated using MCS. This is a common method to obtain a probability distribution. In this method, by generating some

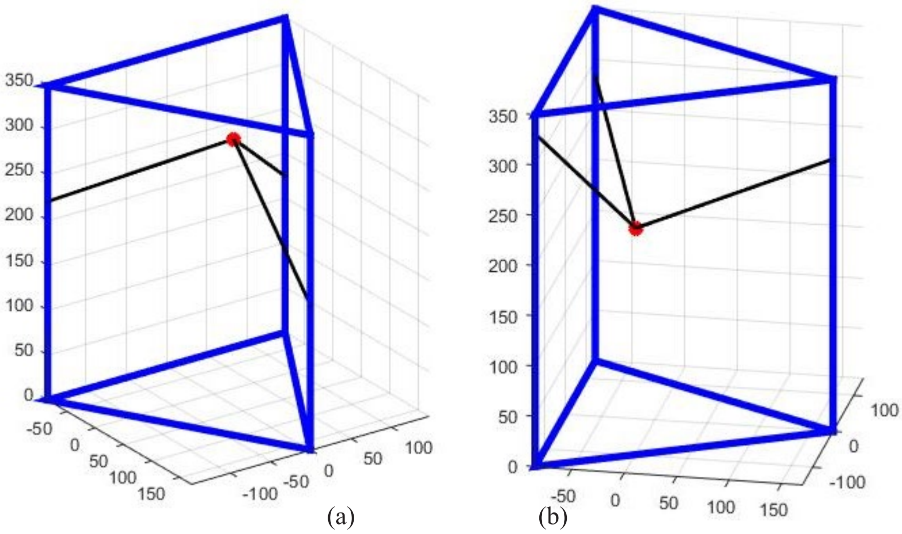
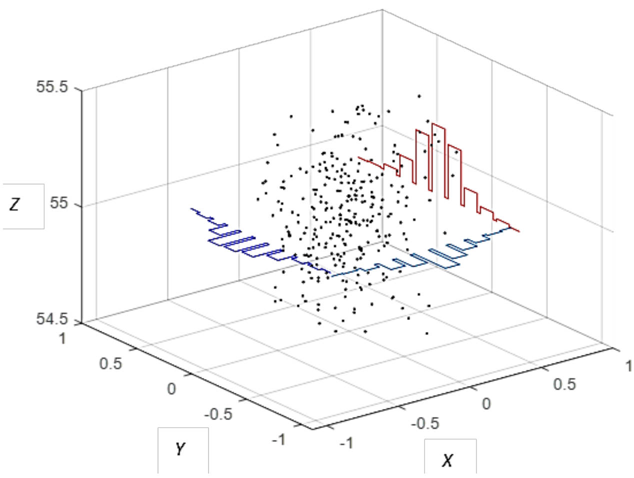


Fig. 3. Two possible configurations of the delta parallel robot to reach a position (a) Links are connected upward (b) Links are connected downward



Direction	Mean value (mm)	Standard deviation
X	0	0.2536
Y	0	0.2536
Z	55	0.2684

Fig. 4. Distribution of positioning [0,0,55]

random numbers, with the pre-calculated mean value and the standard deviation, the probability distributions of variates are generated and statistical parameters of their function is calculated [42, 43]. For example, if the MP should locate at $r=[0,0,55]$ mm, because of the system uncertainties, every time the MP may acquire different position. As a result, the distribution of the MP position in each direction is obtained by MCS which is shown in Fig. 4.

For another position of the MP, for example, $r=[55,55,55]$, the probability distribution in the X, Y, and Z directions are calculated and depicted in Fig. 5.

Accordingly, the probability distribution can be calculated at each position of a trajectory within the manipulator workspace. Fig. 6 shows the probability distribution along a line that passes from the following points (data series),

$$L:[20\,20\,20],[25\,25\,25],[30\,30\,30],[40\,40\,40],[45\,45\,45]$$

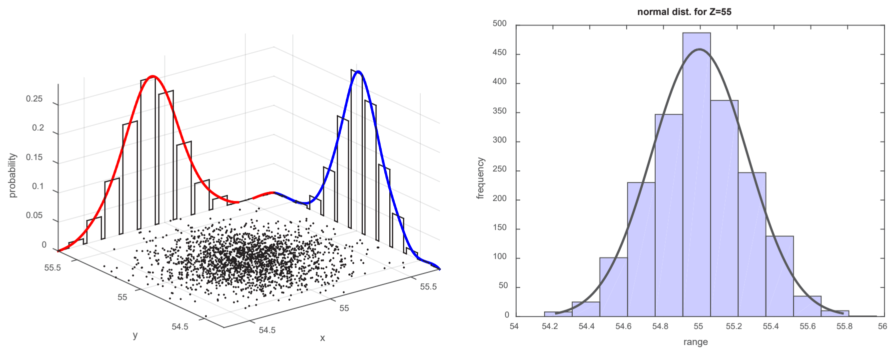
3.1. Reliability calculation of 3-PSS manipulator

Based on the foregoing discussions, now, the reliability can be evaluated for a certain level of accuracy. For example, in the case of the 3-PSS manipulator, the allowable error values are defined in the order of a 3D printer's accuracy, which are,

$$error\ value = 0.05, 0.1, 0.2\ , 0.25, 0.35, 0.4, 0.45, 0.5$$

As it was discussed, reliability explains the probability of a position error that falls within the defined limit. The highlighted region in Fig. 7 shows the probability of the acceptable region for different allowable position errors.

As it is apparent, for a higher level of accuracy, a smaller error limit should be assumed, and hence, lower reliability can be achieved. This could be shown by Fig. 8 for the variation of the kinematic reliability versus acceptable error limit.



(a) (b)

Direction	Mean value (mm)	Standard deviation
X	0	0.2536
Y	0	0.2536
Z	55	0.2684

Fig. 5. Joint probability distribution of the position vector $r=[55,55,55]$ (a) Probability distribution in the *X-Y* direction (b) Frequency distribution in the *Z* direction

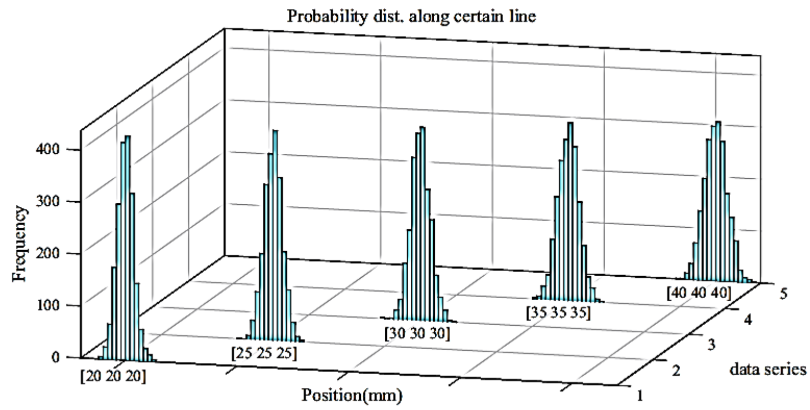
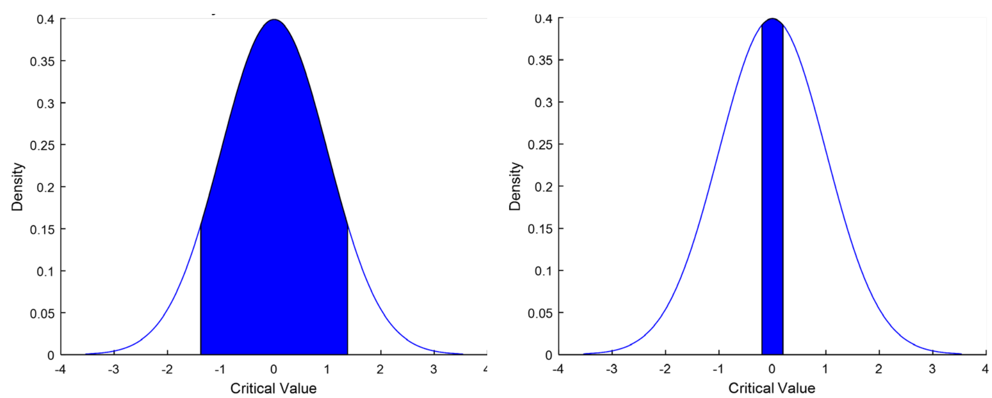


Fig. 6 Probability density distribution along a line which passes from given points (data series)



(a) (b)

Fig. 7. Coverage of data based on the defined error 0.35 and 0.05 mm, respectively (a) Acceptable region in 0.35 mm error (b) Acceptable region in 0.05 mm error

4. RELIABILITY MODELING WITH THE DOE METHOD

Generally, the forward kinematic analysis of a parallel robot is complicated, and obtaining a closed-form relation between several desired variables and the MP position is almost impossible. As a result, an input-output relation can be approximated by resorting to numerical methodologies. The DOE method is one of the common procedures which is used to fit an approximated function on numerical data. In the DOE method, by defining some factors (variables) and performing the experiments at some level combinations of factors, a polynomial algebraic function could be obtained

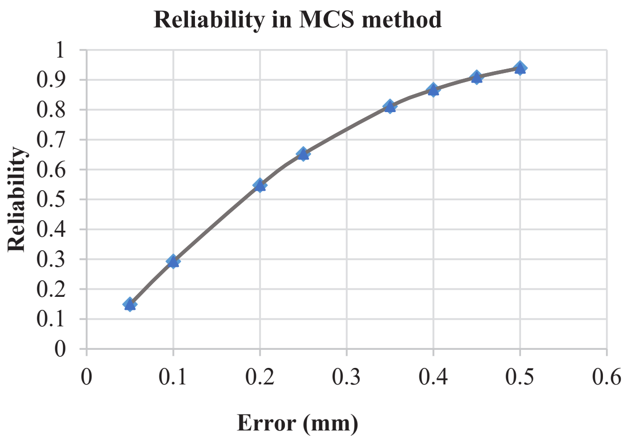


Fig. 8. Reliability versus selecting error

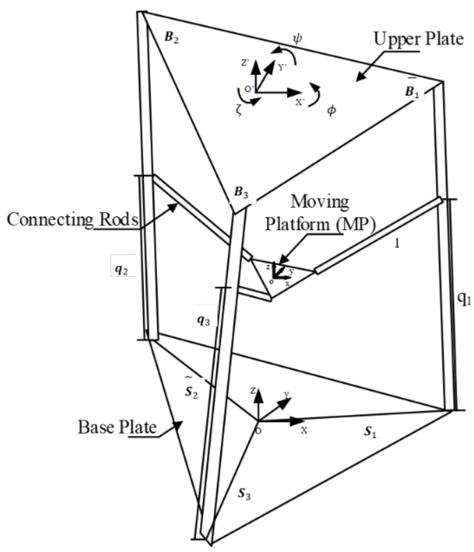


Fig. 9. Mechanism configuration with considering more error

in terms of defined variables. A defined factor should have at least two levels, i.e. upper and lower values. The number of runs is also determined by the number of variables and the design of experiment. There are several design methods such as full factorial, fractional factorial, Central Composite Design (CCD), and Taguchi [32, 37]. Each method needs a certain number of experimental runs, which can provide the desired confidence level. Among the design methods, the CCD hires a higher number of runs, and as a result, it yields a better approximating algebraic polynomial fit [32].

In this study, in the second try, twelve variables are selected to be studied, which are defined below,

- The three rotation angles of the upper plate about the non-rotating *X*', *Y*', and *Z*' axes of the coordinate frame which is located at *O*'; these angles are denoted by (ϕ, ψ, ζ), respectively (Fig. 10).
- The length of the connecting rods (l).
- The length of a line in the base plate (S_i).
- The three actuator inputs (q_i).

According to the CCD method, each variable needs lower and upper limits, which are defined in Table 3. It is noteworthy that the limits should be defined carefully and with a similar relative weight. If this rule is not respected, some of the effective factors may be considered ineffective, and hence, they are overlooked from the final polynomial function. As a case study, the governing algebraic polynomial function for the defined factors, when the MP is at $r = [0055]$ mm. In this case, the variable limits are shown in Table 3.

The corresponding Analysis of Variance (ANOVA) table is obtained (shown in Appendix 1), and the effective variables are identified; therefore, the algebraic polynomial equation can be achieved in terms of the effective variables (shown in Appendix 1). To evaluate the accuracy of an obtained polynomial equation, the goodness-of-fit is defined based on two common criteria, R-Squared, and adjusted R-Squared. These criteria are statistical tools that measure the closeness of data points from the fitted regression hyper-plane which are defined below

$$R^2 = \text{response variable variation} \Big/ \text{Total variation}$$
$$R^2_{adjusted} = 1 - \left(1 - R^2\right)\frac{n-1}{n-p-1} \tag{9}$$

where n is the number of samples and p is the number of independent regressors, i.e. the number of factors in the model. These criteria are also known as the coefficient of

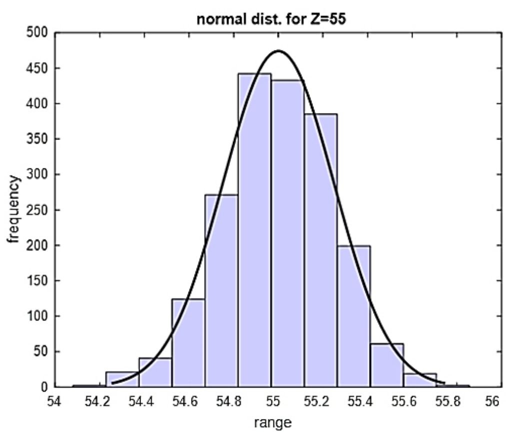


Table of Statistics	
Mean	55
Std. Dev	0.2664

Fig. 10. Probability distribution at point $r = [0055]$ and is calculated by CCD method

determination, or the coefficient of multiple determination for multiple regression [32, 35, 37, 44-47]. In our case study and at a desired MP position, these two criteria are calculated and shown in Table 3.

With the fitted kinematic equation at hand, the kinematic reliability analysis can now be conducted. shows the probability distribution at the desired MP position.

Moreover, the kinematic reliability versus different allowable errors is plotted in Fig. 11.

5. EXPERIMENTAL RELIABILITY ANALYSIS

In this section, an experimental reliability analysis validates the results that were calculated numerically in the previous section. To this end, an accurate proximity sensor (Contrinex; DW - A - 519 - M30) is incorporated to measure the position of MP (Fig. 13(a)). Due to the equipment limitation, the measurement is only conducted along the Z direction.

In this experiment, the robots' MP is repetitively placed at a desired position, for example at $=[0,0,55]$, and then it comes back to the home position. The number of repetitions is calculated via the following equation,

$$n=\left(\frac{Z_{\frac{\alpha}{2}}\sigma}{d}\right)^2 \tag{18}$$

in which d is accuracy, $Z_{\alpha/2}$ is Critical Normal Deviate which refers to a confidence level, and σ is the standard deviation. The number of repetitions based on the calculated standard deviation at $r=[0,0,55]$, with an error of 0.05 mm, and confidence level of 90% equals

$$n=\left(\frac{1.645\times 0.2533}{0.05}\right)^2=69.7\sim 70 \tag{19}$$

Therefore, to have a %90 confidence level, the robot's MP

Table 2. The assumed errors for the parameter

Error name	Method	Mean value	Error	Standard deviation
Actuator inputs (q_i) (Stepper motor)	On the shelf	302.2	$\pm 1.8^{\circ}$[41]	$\pm 0.6^{\circ}$
Connecting links (l)	Manual turning	300	0.027 mm[39]	0.009 mm
The angle between columns (β)	CNC milling	120º	$\pm 3^{\circ}$ [39]	± 1

Table 3. Variable names and their limits $i=1,2,3$

Variable name	l_i Connecting links	S_i Base Plate length	q_i Actuators input	φ Rotation about X' axis	ψ Rotation about Y' axis	ζ Rotation about Z' axis
Nominal value	30	17	30.22	0	0	0
Upper level	34.5	19.55	34.75	+0.1	+0.1	+0.1
Lower level	25.5	14.45	25.69	-0.1	-0.1	-0.1

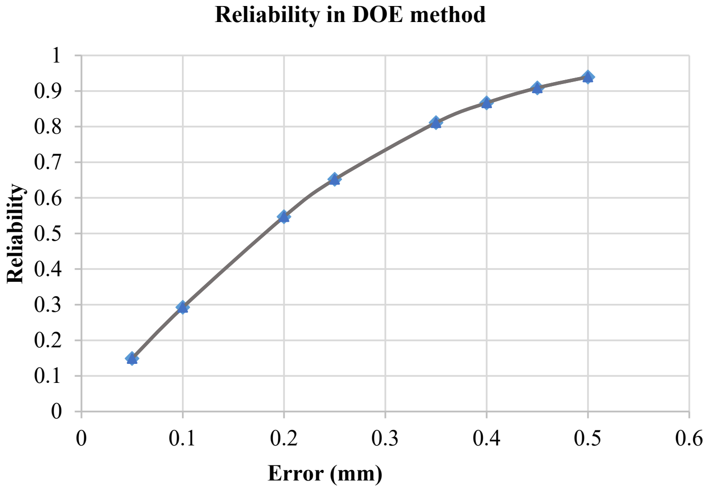
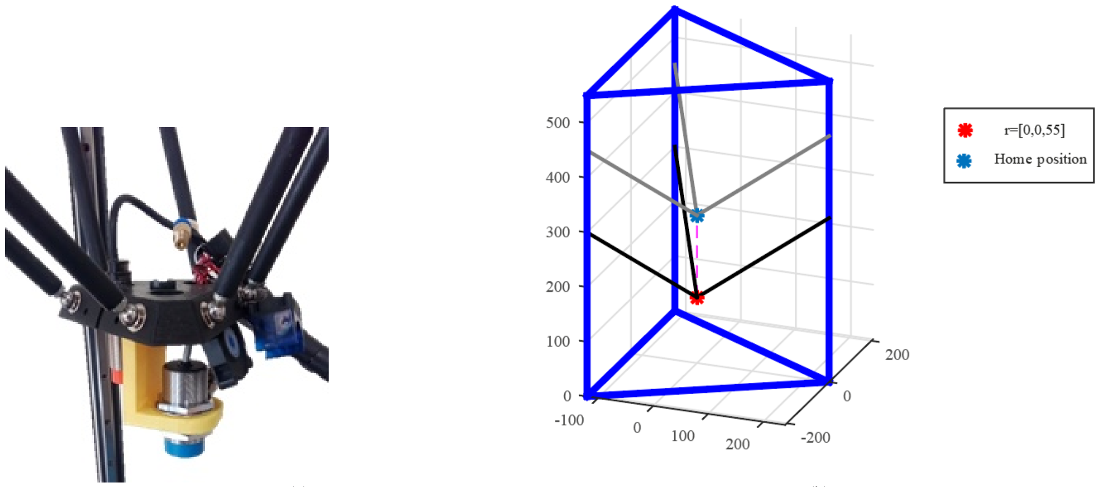
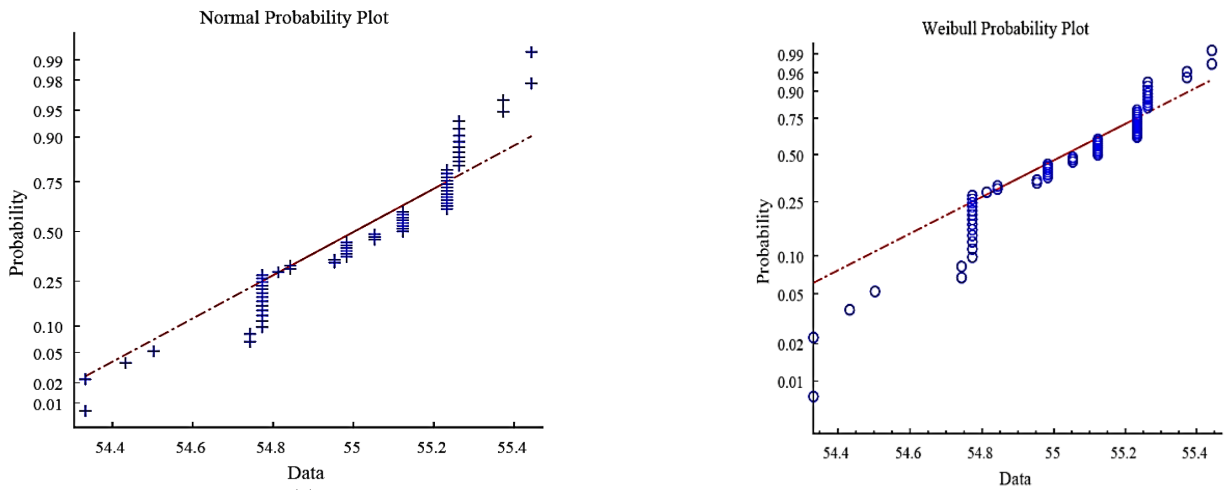


Fig. 11. Reliability variation versus acceptable error yielded by the CCD method



(a) (b)

Fig. 12. Measurement setup (a) Connecting sensor to the MP's robot (b) The experiment measurement route



(a) (b)

Fig. 13. Analysis of goodness of fit for each probability distribution (a) Normal Probability distribution (b)

Table 4. Quality of the model criteria

Criterion	Value
R-Squared	0.9812
Adjusted R-Squared	0.9792

Table 5. Comparison between experimental results using Anderson-Darlin criterion (Adj)

Method	Value
Weibull	1.823
Normal	2.484

should be placed 70 times at the desired position. Next, to choose a suitable probability distribution, the stored data can be plotted based on common probability distribution methods such as Weibull or normal distribution. Here, both methods are incorporated, and the results are depicted in Figs. 13 (a, b). To recognize the best distribution method, the results are compared in Table 5 with the Anderson-Darlin (AD) criterion [35, 48]. As it is apparent, the Weibull method provides a better distribution. However, the DOE method estimates the model using normal distribution, thus, the normal probability distribution is selected.

Based on the recognized distribution, the probability distribution of the MP position is obtained and shown in Fig. 14.

Moreover, the plots of kinematic reliability versus the assumed error, obtained from the DOE, MCS, and the experiment are shown in Fig. 15.

As it is shown in Fig. 15, when the desired error is restricted, the kinematic reliability decreases. Also, when the error is greater than 0.5 mm, the kinematic reliability becomes more than %90. By comparing the results, each method of reliability analysis attains different reliability value for a certain level of accuracy, which is because of different

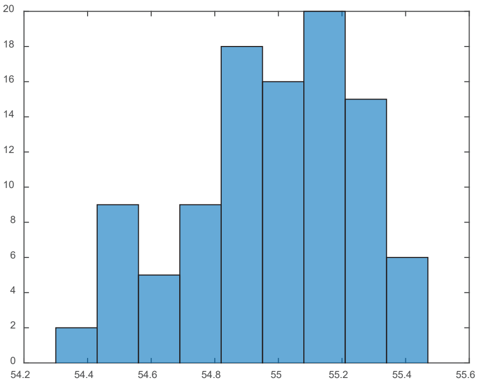


Table of Statistics	
Mean	55
Std. Dev	0.2800
Shape	251.20
Scale	55.13

Fig. 14. Weibull plot of experimental data

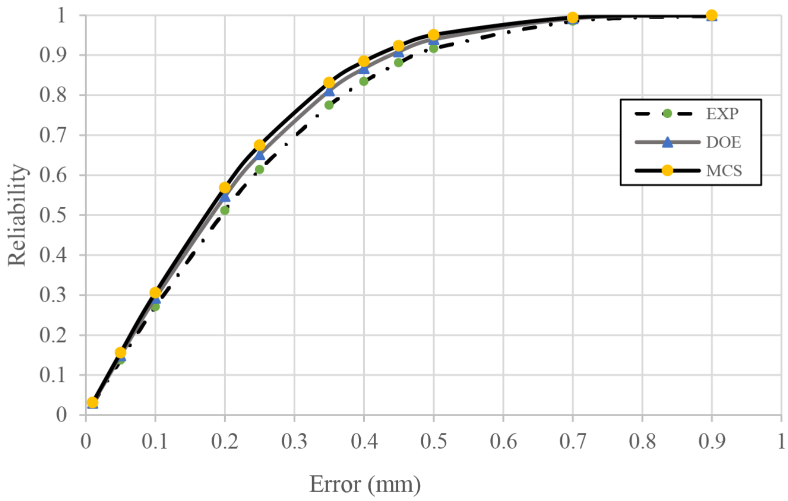


Fig. 15. The variation of the kinematic reliability versus assumed error

number of assumed variables. It must be mentioned that in the worst case, the difference between DOE and MCS method is %4.7 and between DOE method and experimental data is %7.7. Also, in the more reliable region, which the reliability is more than %90, the difference between DOE and MCS methods is under %2 and the difference between DOE and experimental data analysis is about %2.5. As it is depicted in Fig. 15, due to complexities in the design of parallel robots, even 12 kinematic uncertainties are not enough, and hence, the reliability estimated by DOE and MCS methods are still higher than the one which is obtained by the experiment. Some of the important variables that can be added to the numerical models are friction, MP speed, joints clearance, and sensor read error. Moreover, due to the closed-loop mechanism architecture of the 3-PSS parallel robot, the flexibility of components can improve the numerical reliability analysis.

6. SENSITIVITY ANALYSIS

In this section, the sensitivity analysis is conducted to

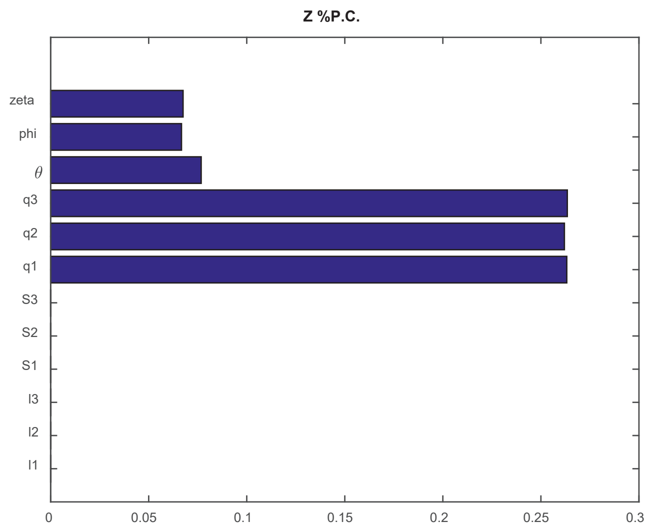


Fig.16. The percent of contribution of each parameter

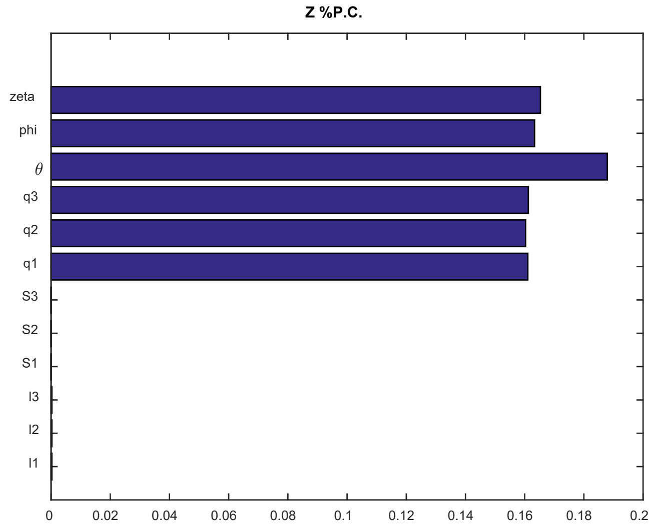


Fig. 17. The percent of the contribution of parameters when motor drivers with 1/16 step are added to the system

identify the effective factors on the kinematic reliability of manipulator under study. To this end, the Percent of Contribution (PC) is defined which shows the effect of each factor, namely [39, 49, 50],

$$PC = \frac{(\sigma_c)^2}{(\sigma_{total})^2} \tag{20}$$

where $\sigma_{total}{}^2$ is the variance of whole uncertainty and $\sigma_c{}^2$ is the variance of each uncertain factor.

The contribution of parameters at the MP position in the Z direction and at the position vector $r=[0,0,55]$ is depicted in Fig. 16.

In this regard, the actuator's input has the most effect and connecting rods (l) and radius of the base platform (S) have the least effect on the total kinematic accuracy. By incorporating an electronic driver and converted actuators, the motor accuracy was improved to the order of 1/16 micro-stepping. The new PC parameters at the MP position in the Z direction and the position vector $r=[0,0,55]$ are re-evaluated and depicted in Fig. 17.

As it is apparent, in the design of a Delta 3D printer with the foregoing architecture, the main attention must be paid to the actuator's input, and next, to the geometrical rotation tolerances (ϕ ,θ *and* ζ). If the geometrical rotation tolerances are restricted, the actuator's input can be used with higher accuracy which results in higher total accuracy and reliability. For example, as it is shown in Fig. 17, with 1/16 micro-step driver, the percentage of contribution of actuators' input variables are very close to the ones for angular tolerances. Therefore, increasing the driver accuracy more than 1/16 micro-step cannot necessarily improve the total accuracy as the angular tolerances constrain the robot performance.

7. CONCLUSION

In the present study, the kinematic reliability analysis of a Delta 3D printer with 3-PSS parallel architecture is conducted. In the first part, eight uncertainties were assumed, and based on the obtained closed-form relation between the desired variables and the MP position, the kinematic reliability was evaluated for different levels of accuracy. In this case, the probability distribution of the MP position is obtained using Monte Carlo Simulation (MCS). In the second part, the number of uncertain variables was added to 12, where a closed-form relation cannot be obtained from the kinematic equations. Hence, by resorting to the DOE method, a polynomial function was fitted to the numerical data, and the kinematic reliability analysis was conducted thereafter. The reliability results which were obtained numerically were also validated experimentally. Comparison of results showed that, for a certain level of accuracy, different methods attain different reliability values, which is due to the different number of assumed variables. In the worst case, the difference between DOE and MCS method was %4.7, and between DOE method and experimental data was %7.7. Also, when the reliability was considered more than %90, DOE and MCS methods showed less than %2 of difference, while the difference between DOE and experimental data analysis was almost %2.5. Based on the results, it revealed that due to design complexities of parallel robots, even 12 kinematic uncertainties are insufficient, and the reliability estimated by DOE is still higher than the one which was obtained experimentally. Finally, to determine the influential parameters, a sensitivity analysis was conducted based on the percentage of contribution (PC) parameter. The sensitivity analysis showed that the actuator's inputs and the geometrical rotation tolerances can affect the kinematic reliability more than other considered variables. Moreover, it was concluded that if the electronic driver and converted actuators are utilized to increase the accuracy of the motor to the order of 1/16 micro-step, the kinematic accuracy of the robot could be elevated. However, it revealed that, in the 3-PSS parallel robot, this improvement can be limited to 1/16 micro-stepping as the percentage of contribution of angular tolerances were raised, and as a result, they can dominantly affect the total accuracy.

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APPENDIX 1

The ANOVA table of the polynomial algebraic equation

ANOVA for Response Surface Reduced Quadratic model
Analysis of variance table [Partial sum of squares - Type III]

Source	Sum of Squares	df	Mean Square	F Value	p-value Prob > F	
Model	11730.24	50	234.60	507.78	< 0.0001	significant
A-l1	1709.15	1	1709.15	3699.28	< 0.0001	
B-l2	1708.75	1	1708.75	3698.42	< 0.0001	

C-l3	1708.82	1	1708.82	3698.55	< 0.0001
D-S1	67.56	1	67.56	146.23	< 0.0001
E-S2	67.46	1	67.46	146.02	< 0.0001
F-S3	67.10	1	67.10	145.24	< 0.0001
G-q1	1123.83	1	1123.83	2432.40	< 0.0001
H-q2	1120.07	1	1120.07	2424.27	< 0.0001
J-q3	1123.31	1	1123.31	2431.29	< 0.0001
AB	92.50	1	92.50	200.22	< 0.0001
AC	91.32	1	91.32	197.65	< 0.0001
AD	32.81	1	32.81	71.01	< 0.0001
AG	364.15	1	364.15	788.17	< 0.0001
AH	89.19	1	89.19	193.04	< 0.0001
AJ	92.10	1	92.10	199.34	< 0.0001
AL	101.84	1	101.84	220.41	< 0.0001
BC	99.63	1	99.63	215.64	< 0.0001
BE	32.96	1	32.96	71.33	< 0.0001
BG	89.93	1	89.93	194.64	< 0.0001
BH	358.89	1	358.89	776.77	< 0.0001
BJ	90.60	1	90.60	196.08	< 0.0001
BK	76.85	1	76.85	166.33	< 0.0001
BL	24.35	1	24.35	52.71	< 0.0001
CF	32.73	1	32.73	70.83	< 0.0001
CG	89.92	1	89.92	194.63	< 0.0001
CH	94.34	1	94.34	204.19	< 0.0001
CJ	365.79	1	365.79	791.72	< 0.0001
CK	75.57	1	75.57	163.56	< 0.0001
CL	23.06	1	23.06	49.92	< 0.0001
DG	8.32	1	8.32	18.02	< 0.0001
DL	11.17	1	11.17	24.18	< 0.0001
EH	8.57	1	8.57	18.54	< 0.0001
EK	8.39	1	8.39	18.15	< 0.0001
FJ	8.41	1	8.41	18.21	< 0.0001
FK	8.39	1	8.39	18.17	< 0.0001
FL	3.50	1	3.50	7.58	0.0061
GH	81.82	1	81.82	177.09	< 0.0001
GJ	82.00	1	82.00	177.48	< 0.0001
GL	89.87	1	89.87	194.52	< 0.0001
HJ	81.99	1	81.99	177.45	< 0.0001
HK	68.69	1	68.69	148.68	< 0.0001
HL	22.72	1	22.72	49.17	< 0.0001
JK	66.95	1	66.95	144.90	< 0.0001
JL	22.73	1	22.73	49.20	< 0.0001
A^2	8.35	1	8.35	18.07	< 0.0001
B^2	8.35	1	8.35	18.07	< 0.0001
C^2	8.35	1	8.35	18.07	< 0.0001
G^2	6.52	1	6.52	14.11	0.0002
H^2	6.52	1	6.52	14.11	0.0002
J^2	6.52	1	6.52	14.11	0.0002
Residual	224.54	486	0.46		
Cor Total	11954.78	536			

Also, the fitted polynomial kinematic equation is,

$$\begin{aligned}
&Z=+17.10-0.40\times l1-0.38\times l2-0.39\times l3+\\
&0.46\times S1+0.46\times S2+0.46*S3-0.11\times\\
&q1-0.14\times q2-0.12\times q3-0.02\times l1\times l2-\\
&0.02\times l1\times l3-0.02\times l1\times S1-0.04\times\\
&l1\times q1+0.02\times l1\times q2+0.02\times l1\times q3+\\
&1.003\times l1\times f-0.02\times l2\times l3-0.02\times l2\times\\
&S2+0.02\times l2\times q1-0.04\times l2\times q2+0.02\times\\
&l2\times q3-0.86\times l2\times t-0.47\times l2\times f-0.02\times\\
&l3\times S3+0.02\times l3\times q1+0.02\times l3\times q2-0.04\times\\
&l3\times q3+0.85\times l3\times t-0.45\times l3\times f+0.01\times\\
&S1\times q1-0.55\times S1\times f+0.01\times S2\times q2+\\
&0.50\times S2\times t+0.01\times S3\times q3-0.50\times\\
&S3\times t+0.34\times S3\times f-0.01\times q1\times\\
&q2-0.01\times q1\times q3-0.91\times q1\times f-\\
&0.01\times q2\times q3+0.80\times q2\times t+0.47\times\\
&q2\times f-0.79\times q3\times t+0.47\times q3\times f+\\
&0.02\times l1^2+0.02\times l2^2+0.02\times l3^2+\\
&0.02\times q1^2+0.02\times q2^2+0.02\times q3^2
\end{aligned}$$

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