



## Optimization of ankle stiffness using genetic algorithm in gait modeling

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**ABSTRACT:** In the human gait modeling, it is common to employ 2D models that consist of a chain of rigid links joined together by frictionless hinge joints. Although Newton's method is usually used to obtain equations of motion in the previous studies, in this research, the constrained Lagrange's method was employed for this purpose. This method has some advantages over the previous one, such as the solution process is independent of the coordinate system and there is no necessity to know the ground reaction force beforehand. In this work, optimization was also performed by genetic algorithm so that the moment of each joint was estimated by tracking the kinematic data. Moreover, by solving the inverse dynamics and by applying Lagrange multipliers, the distribution of ground reaction force under both feet in the double support mode was calculated and compared with the experimental data to verify the effectiveness of the proposed method. Finally, as one of the applications of dynamic modeling of the human gait, the optimal value of passive stiffness in the ankle joint was obtained to provide a better design of the orthoses used for patients with motor impairment. The results show compatibility between the simulations and experiments for normalized joint moments as well as reaction forces. The optimal joint stiffness is also in the range reported by available experimental data. In conclusion, the methodology can be used for modelling human movements and can be considered as an optimal approach in designing assistive devices especially passive exoskeletons.

### Review History:

Received: May, 14, 2020

Revised: Jul. 21, 2020

Accepted: Aug. 03, 2020

Available Online: Dec. 01, 2020

### Keywords:

dynamic

gait modeling

genetic algorithm

Lagrange's method

stiffness

## 1- INTRODUCTION

One of the most important applications of dynamic modeling of the human gait is to improve and correct the gait of injured patients, via designing improved assistive devices. For instance, people who are injured in traffic or work related accidents, 10% to 20% of those who have survived a stroke, and most patients with cerebral injury have motor defects in their lower limbs. One example of these defects is an abnormal plantar-flexed position of the foot which reduces the neutral angle of the ankle [1-3]. One of the causes of these motor deficits is severe muscle spasms resulting from previously mentioned injuries. This will increase the passive stiffness of the ankle joint, thereby reducing the range of motion of that joint [4]. Passive ankle foot orthoses (AFO) are usually utilized to correct the gait pattern of these patients and to reposition the foot relative to the leg in a correct manner [5-9]. Therefore, calculating the optimum stiffness of these orthoses in the ankle joint is an important parameter in their design. This optimization is based on the energy consumed by the person during walking.

Therefore, to calculate the optimal parameter for the ankle joint stiffness, dynamic gait modeling was implemented in this paper. Walking is the most important way of human motion, so the importance of its analysis and evaluation

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is undeniable. Evaluation of human gait consists of the measurement and analysis of biomechanical parameters, all events that occur during walking and position changes of the limbs with respect to each other. In the literature, researchers have repeatedly used the inverse pendulum (IP) model for the gait stance phase which has been the basis for recent research on the gait dynamic analysis [10]. Although in earlier works, such assumption was made [11, 12], Cavagna [13] for the first time proposed IP for stance phase modeling. This research [13] was the basis for the development of research by McMahon's [14] and McGeer's [15, 16], which later elucidated the IP dynamic modeling of gait. Recent works by Donelan's group [17-19] also focused on the transition phase from one step to another. In the recent study, the IP model has been expanded to include springs, dampers, telescopic actuators, and additional segments and joints [20-23].

Typically, most of the developed models consist of several rigid links joined together by frictionless joints which ultimately form a chain that represents the limbs of a person. To provide a person's forward dynamic simulation, the equation of motion (EOM) should be derived. These equations have already been established in Pandy's articles [24, 25] by the Newtonian approach whose advantage is the reduction in computational time at a large number of degrees of freedom (DOF). Recently, McGrath [26] employed



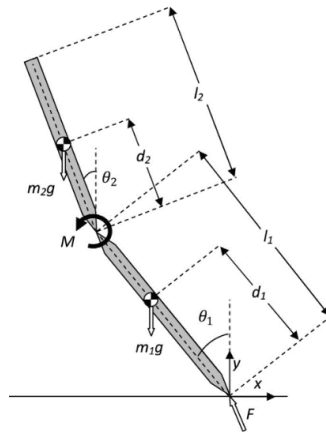


Fig 1. Free body diagram for the IP model [10]

Lagrange’s method in order to derive EOM, like what was followed in this research. Using this strategy, the solution would be independent of the coordinate frame, and there is no need to know the ground reaction force (GRF) in advance; since, Lagrange’s method uses virtual work and energy calculations instead of forces.

Optimization is used to solve the governing equations needed to simulate the human gait, as in most complex models such as what was studied in [27]. The optimization techniques are especially valuable when the number of unknowns exceeds the order of system of equations as it is the case for human gait model. In the present study, the similar approach was used, except that genetic algorithm (GA) was selected for optimization. GA is very effective in saving time during problem solving. Genetic algorithm searches for different possible solutions of the system in order to find the best answer. According to the natural selection theory, the only answers with better cost function remain in the competition. Finally, the best answer is introduced based on the considered cost function and convergence criterion [28, 29].

The contributions can be listed as follows:

- Genetic algorithm is used to solve the human gait dynamics.
- The results of GA gives both joint moments and reaction forces in single support and double support phase.
- An optimal joint stiffness is obtained to minimize the joint moments.
- The proposed method can be generalized to simulate the dynamics of other human motions.

In this study, first, the EOM was obtained by using the Lagrange method. As mentioned before, genetic algorithm was used as the optimization method. Then, as one of the applications of this modeling, the optimum passive stiffness for the ankle joint was determined. Designing orthosis based on the calculated optimal parameter can improve orthopedic movement pattern of an injured patient.

## 2- METHOD

### 2-1- Obtaining EOM

The Lagrange equation which employed in obtaining EOM in an open-loop chain mechanism with no external

force and torque is as follows [30]:

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i} = 0 \quad (1)$$

Where  $L$  is the Lagrangian function which is the difference between kinetic and potential energies and  $q_i$  is the generalized coordinate of the  $i^{\text{th}}$  link in the chain. In the case that there is no external force and torque, the right side of this equation will be equal to zero. Otherwise, the relationship is as follows:

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i} = Q_i \quad (2)$$

where  $Q_i$  represents the generalized characteristic of forces derived from virtual work  $\delta w$  :

$$\delta w = \sum_i Q_i \delta q_i = \sum_i (M_{i+1} - M_i) \theta_i \quad (3)$$

where  $M_i$  is the torque applied to the distal joint of the  $i^{\text{th}}$  link of the chain and  $\theta_i$  is the  $i^{\text{th}}$  link angle with respect to the vertical axis.

These derivations are expressed for an open-loop chain with  $n$  rigid links. The ground is considered as a constraint with zero overall work at one end of the chain, while the other end of the chain is free.  $l_i$  represents the length of each link, mass  $m_i$  and moment of inertia around the center of mass of each link  $I_i$  were considered for each link  $i$ . Position of the center of mass was determined by  $d_i$  on each link. Also,  $g$  represents the gravitational acceleration. Therefore, the position of each link was determined by (4), (5):

$$x_i = \sum_{h=1}^{i-1} (-l_h \sin \theta_h) - d_i \sin \theta_i \quad (4)$$

$$y_i = \sum_{h=1}^{i-1} (l_h \cos \theta_h) + d_i \cos \theta_i \quad (5)$$

$$v_i = \sqrt{\dot{x}_i^2 + \dot{y}_i^2} \quad (6)$$

Equation (6) represents the linear velocity of the center of mass for each link. Kinetic and potential energies were calculated by using the following equations:

$$T = \sum_{h=1}^n \left( \frac{1}{2} m_h v_h^2 + \frac{1}{2} I_h \dot{\theta}_h^2 \right) \quad (7)$$

$$V = \sum_{i=1}^n \left( m_i \left( \sum_{h=1}^{i-1} (l_h g \cos \theta_h) + d_i \cos \theta_i \right) \right) \quad (8)$$

Consequently, general equations were derived from (2), (3). and by calculating the above equations, EOM can be written in the following matrix form:

$$B \ddot{\theta} = C \begin{bmatrix} b_{1,1} & \cdots & b_{1,n} \\ \vdots & \ddots & \vdots \\ b_{n,1} & \cdots & b_{n,n} \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \vdots \\ \ddot{\theta}_n \end{bmatrix} = \begin{bmatrix} c_1 \\ \vdots \\ c_n \end{bmatrix} \quad (9)$$

Where

$$b_{p,q} = \begin{cases} \left( m_p d_p^2 + \left( \sum_{j=p}^n (m_{j+1}) l_p^2 + I_p \right) \right) & \text{if } p = q \\ \left( m_p d_p + \left( \sum_{j=p}^n (m_{j+1}) l_p \right) \right) l_q \cos(\theta_q - \theta_p) & \text{if } p > q \\ \left( m_q d_q + \left( \sum_{j=q}^n (m_{j+1}) l_q \right) \right) l_p \cos(\theta_p - \theta_q) & \text{if } p < q \end{cases} \quad (10)$$

And

$$c_p = \sum_{h=1}^{\{n|p \neq h\}} \left( \begin{matrix} \left( m_p d_p + \sum_{j=p}^n (m_{j+1}) l_p \right) l_h \sin(\theta_h - \theta_p) & \text{if } h < p \\ - \left( m_h d_h + \sum_{j=h}^n (m_{j+1}) l_h \right) l_p \sin(\theta_p - \theta_h) & \text{Otherwise} \end{matrix} \right) \theta_h^2 \quad (11)$$

$$+ \left( m_p d_p + \left( \sum_{j=p}^n m_{j+1} \right) l_p \right) g \sin \theta_p + M_{p+1} - M_p$$

$\sum_{h=1}^{\{n|p \neq h\}}$  indicates that the value of  $h$  covers all values from 1 to  $n$  except  $p$ . Equation (9) is for open loop chain mechanisms; however, there is a set of constraints for a close

loop mechanism. Taking advantage of Lagrange's method, these constraints can be easily entered in the equations by employing Lagrange multipliers. The expression of  $j^{\text{th}}$  constraint is  $f_j = 0$ .

The Lagrange equations will be rewritten as follows:

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i} - \sum_{j=1}^r \left( \lambda_j \frac{\partial f_j}{\partial q_i} \right) = Q_i \quad (12)$$

Where  $\lambda_j$  is the Lagrange multiplier for the  $j^{\text{th}}$  constraint. And  $r$  is the number of constraints of the mechanism. If constraints are merely functions of position, by twice differentiating these relations, new unknown variables or  $\lambda_j$  can be solved with  $q_i$  simultaneously. In other words, to apply positional constraints into the EOM, they were differentiated twice according to (13) and the matrix (9) was rewritten in the form of matrix (14).

$$\frac{d^2 f_j}{dt^2} (q_i, \dot{q}_i, q_i, t) = g_j (q_i, t) + h_j (\dot{q}_i, q_i, t) = 0 \quad (13)$$

$$\begin{bmatrix} b_{i,i} & -\frac{\partial f_j}{\partial q_i} \\ g_j (q_i, t) & 0 \\ q_i & \end{bmatrix} \begin{bmatrix} q_i \\ \lambda_j \end{bmatrix} = \begin{bmatrix} c_i \\ -h_j (\dot{q}_i, q_i, t) \end{bmatrix} \quad (14)$$

It should be mentioned that for a chain with  $n$  degrees of freedom and  $r$  constraints,  $n-r$  independent variables exist. Therefore, all  $q_i$  are not independent, which means that by knowing  $q_i$  for the first  $n-r$  links in the chain, one can obtain  $q_i$  for the last  $r$  links by employing the constraints equations.

### 2-2- Ground reaction force calculation

Another advantage of the Lagrange's method appears during computing the ground reaction force (GRF) in a double support situation. Lagrange multipliers determine how GRF is distributed beneath the legs. In other words, the force required to remain the trailing foot fixed on the ground is considered as the GRF under that leg. The inverse dynamics can also be used to calculate the total GRF, and to obtain the GRF under the leading foot by simple subtraction.

$$F_{q_i} = \lambda \frac{\partial f}{\partial q_i} \quad (15)$$

$$F_x = \lambda_{f_1} \sum_{i=1}^n \left( \frac{\partial f_1}{\partial \theta_i} \frac{\partial \theta_i}{\partial x} \right) = \lambda_{f_1} \sum_{i=1}^n \left( -l_i \cos \theta_i \frac{1}{-l_i \cos \theta_i} \right) = \lambda_{f_1} \quad (16)$$

$$F_y = \lambda_{f_2} \sum_{i=1}^n \left( \frac{\partial f_2}{\partial \theta_i} \frac{\partial \theta_i}{\partial y} \right) = \lambda_{f_2} \sum_{i=1}^n \left( -l_i \sin \theta_i \frac{1}{-l_i \sin \theta_i} \right) = \lambda_{f_2} \quad (17)$$

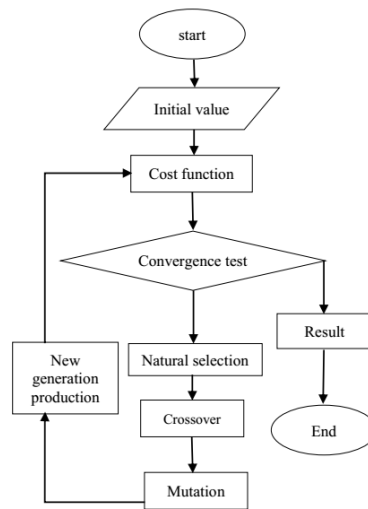


Fig. 2. The stages of GA

$$GRF_x = ma = \sum_{i=1}^n m_i x_i \quad (18)$$

$$GRF_y - mg = \sum_{i=1}^n m_i y_i \quad (19)$$

Equations(16) and(17) are referred to the horizontal and the vertical GRF beneath the trailing foot, so by subtracting these from(18) and (19) which are the total GRFs in x and y directions, respectively, the GRF under the leading foot in both directions is obtained.

### 2-3- Optimization

In this work, the genetic algorithm (GA) optimization method was performed to estimate the moments applied to each link. This estimation is based on a comparison between the kinematic data obtained from EOM and the experimental data. In this study, Winter's data [31] as laboratory data was used.

The cost function for estimating the torque on each link was defined by tracking the kinematic data as follows [26]:

$$F_c = \sum \left( \left( |\theta_s - \theta_l| \right)^2 + \left( |\dot{\theta}_s - \dot{\theta}_l| \right)^2 \right) \quad (20)$$

where  $\theta_s$ ,  $\dot{\theta}_s$  were calculated in the simulations and  $\theta_l$ ,  $\dot{\theta}_l$  were the experimental data. The stages of GA are in accordance with the flowchart in Fig. 2. Therefore, we aimed to find the minimum value of the cost function defined in (20). For this reason, the cost function was calculated for the initial values, and based on the concept of natural selection, the values resulted in higher cost function was eliminated

from the competition. Then, through the next stages, a new generation was produced using the best of the previous generations.

### 3- RESULTS AND DISCUSSION

The motion simulation includes both single support and double support modes. The genetic algorithm was used to run the simulations for each of these modes. As mentioned before, by comparing the angular position and velocity of each segment gained from experimental data and simulations results, the torque on each link was obtained as the optimization output.

The model involved a link chain with 6 DOFs, and gait of a person with 165 cm height and 65 kg weight was considered. All the anthropometry information required was taken from Winter's work [31].

Heel strike of the leading foot was considered to be the beginning of gait simulation. Therefore, the first 10% of the gait was in the double support phase, and then it switched to the single support phase until half of the gait cycle. In the second half of the gait cycle, the same phase changes will occur, symmetrically. The results of the motion simulation based on genetic algorithm are shown in Fig. 3. These results are also compared to the experimental data [31].

The simulation and experimental results are compatible with an acceptable range of error. Since in this model the foot segment and also the variation of the center of pressure for implementing the GRF have not been considered, the simulated moments at the joints differ from the experiment. Another source of error in this calculation comes from the genetic algorithm which tries to track the desired joint angles, so the errors in this tracking would be augmented in joint moments.

Using the Lagrange multipliers, the ground reaction forces beneath the feet were calculated. These calculations

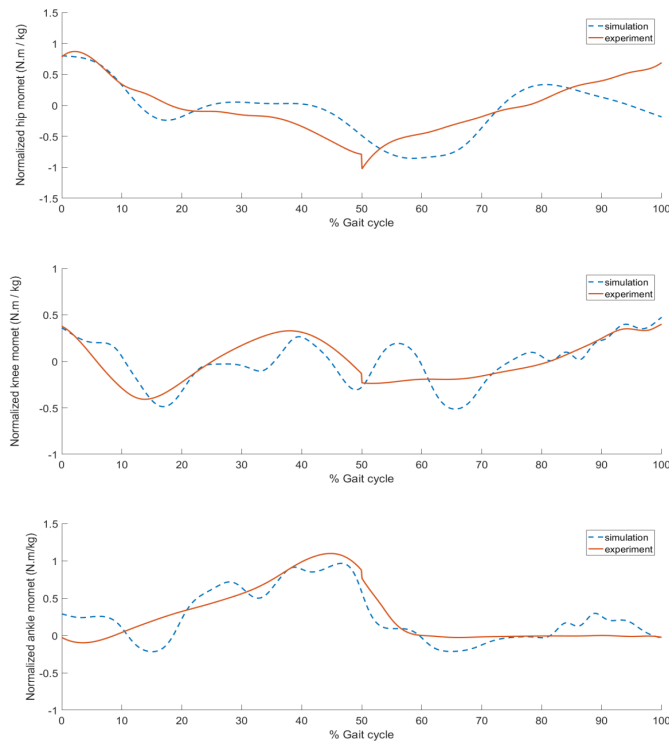


Fig. 3. Comparison of simulation results for torque values with their experimental data. All moments are normalized to the body mass.

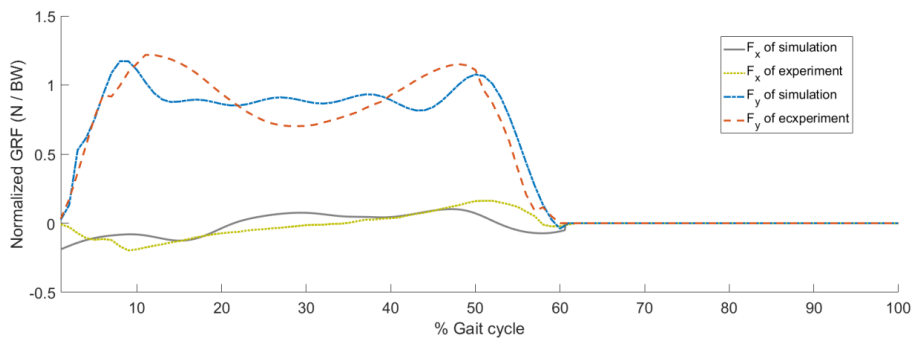


Fig. 4. The obtained GRFs based on Lagrange multipliers compared to the experimental data. GRF is normalized to the body weight (BW)

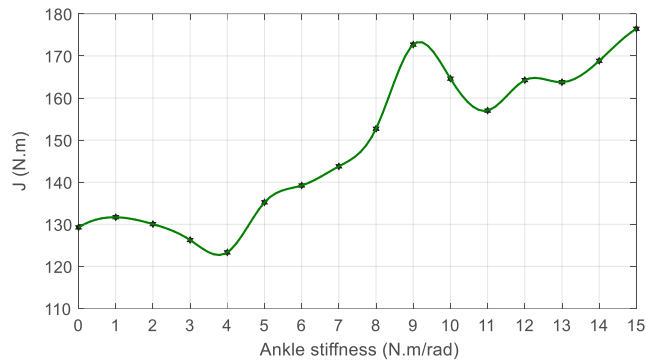
were in both vertical and horizontal directions, because the mechanism constraints were defined in these two directions. Then, the results of these computations for the leading foot, during the first half of the gait cycle were compared to the experimental data in Fig. 4. In the final section, by studying the behavior of the ankle joint stiffness parameter over the whole body motion considering all joints moments, an optimal value for this parameter was determined. To this end, first, the spring energy equation (21) must be added to the potential energy term in the Lagrangian function (8).

$$V_{spring} = \frac{1}{2} K_{ankle} \phi_{ankle}^2 \quad (21)$$

where  $K_{ankle}$  is the stiffness of the equivalent spring at the ankle, and  $\phi$  is the relative angle of two links connected at the ankle joint. Then, the effect of this spring stiffness can be easily studied by varying the value of  $K_{ankle}$  in (21). To study how this parameter affects all joints moments and determine the optimal value for it, a norm function was defined as follows:

$$J = \sum_{i=1}^n \int_{t_0}^{t_f} |M_i| dt \quad (22)$$

In other words, the idea of defining this function was based on minimizing the area under the torque-time curve



**Fig. 5. Norm function variation in the range of different values of ankle joint stiffness.**

for the members during the gait. This area is correlated to the person's energy consumption. Finally, the minimum value for this function was suggested as the optimal value. The results of the norm function, for different values of the spring stiffness, are shown in Fig. 5. The range of variation of the  $K_{ankle}$  was taken from [32]. As it can be seen, there is a global minimum in the neighborhood of point  $K_{ankle} = 4(N.m / rad)$ . The ankle stiffness obtained by experiments depend on the motion speed and subject's characteristics with the range of 0.1-0.3 Nm/deg (for plantar flexion) which is compatible with the result obtained by the proposed approach (.07 Nm/rad). So, as mentioned earlier, now that an optimal value for the ankle joint stiffness has been found, better design of the AFO orthoses will be possible. As a result, less moments will be applied to the limbs of the patients who use these optimized orthoses, and hence they will consume less energy while walking.

The proposed approach in this study can be used for simulation and optimal design of assistive devices. Usually the design of assistive devices (walkers, canes, prosthetic foot, etc.) that involve walking, do not consider the dynamics of the user. By having a validated model of the user such as the one proposed in this study, the design parameters can be varied to obtain optimal design.

#### 4- CONCLUSION

The human gait modeling was implemented as a chain consisting of rigid links and frictionless joints in both open-loop and closed-loop modes. The equation of motion for these modes was derived, which were single support and double supports, respectively. The Lagrange's method, which has several advantages over other methods, was chosen for this goal. Independency of the solutions of coordinate system and less need for prior knowledge of kinetic data can be mentioned as some of these advantages.

As mentioned above, there is no need for ground reaction force (GRF) to calculate forward dynamics, but the moment applied to each link should be estimated. For a similar purpose, an approach was used by Anderson and Pandy, where optimization was performed to estimate muscle activity in a complex model. In the present work, this estimation of

moment on each link was carried out by genetic algorithm (GA).

Another advantage of the Lagrange's method in the present modeling was in GRF calculation; where Lagrange multipliers was used to determine the distribution of GRF beneath each foot in the double support mode. The force required to remain the mechanism constraints was introduced to be proportional to the GRF under the trailing foot. Also the total GRF was calculated from the inverse dynamics. Finally, by subtracting these values, the GRF under the leading foot was obtained. As one of the important applications of human gait dynamic modeling, the optimal value for passive ankle joint stiffness was found by examining the effect of this stiffness on the human motions.

As a future work of this study, the constrained Lagrange equation is used to optimize the design of an assistive passive exoskeleton and a cane.

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#### HOW TO CITE THIS ARTICLE

A. Hashemnezhad, S. Taghvaei, K. Hashemnia S. A. Haghpahan, *Optimization of ankle stiffness using genetic algorithm in gait modeling*, *AUT J. Model. Simul.*, 52(2) (2020) 251-258.

DOI: [10.22060/miscj.2020.18427.5211](https://doi.org/10.22060/miscj.2020.18427.5211)



