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A New Load-Flow Method in Distribution Networks based on an Approximation Voltage-Dependent Load model in Extensive Presence of Distributed Generation Sources

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ABSTRACT: The power-flow (PF) solution is a basic and powerful tool in power system analysis. Distribution networks (DNs), compared to transmission systems, have many fundamental distinctions that cause the conventional PF to be ineffective on these networks. This paper presents a new fast and efficient PF method which provides all different models of Distributed Generations (DGs) and their operational modes (P-V and P-Q nodes) in DNs. This study uses voltage-dependent load model instead of traditional load model (constant P-Q) which is modelled as the combination of a current source in parallel with a constant admittance. This kind of load model is closer to reality and makes the power-flow equations closer to linear conditions. To calculate the angles of the P-V buses, the numerical method (Newton-Raphson method) is applied by separating the PF equations for P-V and P-Q buses. Considering a series of approximations on the angles of these buses, the non-diagonal elements of the Jacobin matrix in Newton-Raphson method are fixed. Hence, the proposed numerical method converges toward an appropriate response in a very low number of iterations and high speed. The voltages of other buses (P-Q buses) are calculated linearly without needing any numerical methods. The presented method proves to be robust and reliable against reconfigured structures and meshed networks. Simulations have been carried out on 14-, 33- and 70-node IEEE test systems and large scale networks such as 6118-buses. The results show that the proposed method is at least 100 and 10 times faster than Gauss-Seidel and Newton-Raphson methods, respectively.

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1. Introduction

Considering the increasing consumer demand for electrical power, there is a higher need for extending the power system structure and energy. Nowadays, tendency towards DGs is increasing due to the existing problems and disadvantages of installing centralized power plants and long transmission lines. Power generation based on renewable energy sources is expanding rapidly as a solution for decreasing fossil sources consumption and air pollution emission caused by conventional energy sources such as fossil fuel. Various research institutions such as CIRED [1] and CIGRE [2] have addressed several issues of extending DG. The extend of penetration of DG sources is significant due to the benefits that it has in the distribution networks, which makes the analysis of them more complicated.

In addition to the mentioned explanations, there are some distinctions between transmission system level and distribution system level that cause the ineffectiveness of conventional PF methods such as Gauss-Seidel [3], Newton-Raphson [4] and fast decoupled [5] on distribution networks. These distinctions include unbalanced phase currents, radial or weakly meshed structure, transposed lines in transmission systems and not in DNs and etc. Thus, many efforts are made to develop and offer PF solutions on distribution networks.

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Also, in this case, a famous PF solution method is backward/forward sweep (BFS) [6]. Regarding the advantages of this iterative method for PF calculation in the distribution networks, simple implementation and better convergence can be mentioned. Backward sweep is used to make the relationship between branch currents and bus current injections of a distribution system, and then forward sweep uses the aforementioned relationship to calculate bus voltages [7]. In [8], an improvement of BFS power flow solution used for radial distribution networks is illustrated. This method is based on BFS which uses breadth-first search to create the modified incidence matrix. In reference [9], an improved backward/forward sweep method is presented that is composed of two steps: 1) the backward sweep used for calculating each upstream bus voltage of a line or a transformer branch by means of Kirchhoff's voltage and current law, and 2) the forward sweep which uses the linear proportional principle to update each downstream bus voltage. The other

PF method is the ladder network theory [10]. In this method, it is assumed that the distribution network is linear and therefore, the impedance of loads and lines are considered constant. Load-flow problem is also solved with Z-bus method in three-phase distribution systems in which different types of loads such as constant-power, constant-current and constant-impedance are considered [11]. A direct approach is presented for unbalanced three phased DNs to solve PF problem using two developed matrices, i.e. Bus injection to branch current (BIBC) matrix and Branch current to bus voltage (BCBV) matrix [12]. The benefits of this approach are needing no Y matrix of the network and solving the PF problem without any complex iterative methods such as the Newton-Raphson method. Recently, some current injection (CI) based PF methods have been published in literatures. In [13], a three-phase current injection method (TCIM) has been proposed to solve CI equations using Newton method that is in rectangular coordinates. PF equations have been formulated considering grounding wire (neutral wire) and it is known as four-conductor current injection method (FCIM) [14]. In all of the aforementioned references, the generic load model (constant P-Q) is used. The voltage-dependent load model is proposed in [15], and a linear power-flow (LPF) approach is presented to solve the PF problem without using common iterative methods. In the mentioned paper, the load has been modelled by a current source in parallel with specific impedance. The LPF is developed for three-phase unbalanced networks in [16]. Although this method is efficient considering DG connected to bus as P-Q node, it is not applicable in case of DGs connected as P-V node.

In [17] a new load flow approach is presented by Data clustering incorporated in Monte Carlo Simulation in Probabilistic Load Flow of the power grids under uncertainty of renewable energy resources. High complexity and long processing time are the disadvantages of this method.

[18] Illustrates a survey and summary of recent development in PF methods. In this paper two major PF methods used for power loss identification and distributed generation (DG) allocation in DNs, are forward and backward sweep and direct approach. The PF method that presented in [19] is composed of two forward sweep steps; unlike conventional backward forward sweep approach. This PF formulation is presented for balanced and unbalanced radial networks. But in [18] and [19], the different modes of DGs (PV and PQ modes) are not considered.

[20] Proposed a new load flow problem formulation using the impedance matrix of the power distribution network as the admittance matrix of the system will generally be sparse in nature. Next, by using Pade approximant solved this holomorphic embedding load flow functions. The complexity and non-consideration of different modes of DGs are the disadvantages of this method.

In this paper, a new PF approach is introduced by considering a voltage-dependent load model that includes all operational modes of DGs. Numerical methods are only used for the calculation of bus angles whose voltage are controlled by the DGs connected to them (bus operates as P-V node).

DG sources include various structures, technologies and operation modes [21-22]. They are also divided to four classes of large, medium, small and very small perspective sizing [22]. Large units are a type of synchronous machines that have the capability of reactive power generation and voltage control. Whereas, medium and small units are often types of asynchronous machines which cannot provide reactive power. Eventually, very small DG units such as fuel cell, battery, hydro turbine and other sources are connected to the network by power electronic equipment (PEE) [23]. Some of the PEEs can control the DG output voltage [22]. Regarding the DGs implementation [24] in the DNs, DGs operate in one of the following modes according to contracts and control condition:

- Producing constant active and reactive power.
- Producing specific active power value under constant power factor.
- Producing specific active power value under constant voltage magnitude.

Developing a PF approach that is applicable to DNs with high penetration of DG is necessary. The proposed method is rapid, reliable and robust against meshed networks while having other mentioned advantages.

The rest of this paper is organized as follows: Load model is described in section II. In section III, the proposed approach is presented. In section IV, simulations have been tested on several distribution networks and results and discussions are presented. Finally, the conclusion is presented. This paper has an Appendix too.

2. Load Model

The loads behavior of the distribution networks varies with network voltage variation. The sensitivity of loads to voltage variation can be low or high depending on the type of load [22]. In the studies of the loads of distribution networks, it should be noted that the dependence of loads on the voltage cannot be ignored. The load model used in this paper is obtained from [15] which represent the voltage-dependent load model.

In [15], it is stated that the presence of constant term in load model as a ZIP has caused to be a factor for becoming the power flow in non-linear. In this reference, it has been proven that ignoring this part in the power flow modeling creates a small amount of error in how the load behaves against voltage changes.

Based on this model, the relationship between the active and reactive power of load and voltage is explained as follows:

$$\frac{P(V)}{P_0} = C_Z\left(\frac{V}{V_0}\right)^2 + C_I\left(\frac{V}{V_0}\right) \tag{1}$$

$$\frac{Q(V)}{Q_0} = C_Z{}'\left(\frac{V}{V_0}\right)^2 + C_I{}'\left(\frac{V}{V_0}\right) \tag{2}$$

Also, the parameters C_Z , C_I , $C_Z{}'$ and $C_I{}'$ are specific

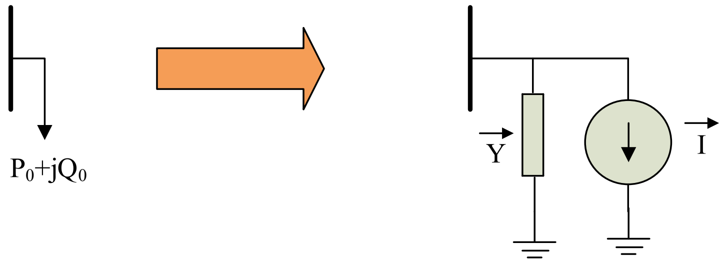


Fig. 1. Load model

and constant values for every load which were calculated using the curve-fitting procedure. The fitting objective function for the active and reactive power is to minimize the difference between the fitted approximation and the measured data for a finite number of voltage points within an acceptable range (e.g., ±10%). It should be noted in (1) and (2), there is only one independent variable resulting from the following equations.

$$C_z + C_I = 1 \tag{3}$$

$$C_z' + C_I' = 1 \tag{4}$$

By considering the above equations and by applying reasonable approximations on the imaginary part of the voltage, the load is modelled as a combination of a constant admittance in parallel with a fixed current source in Fig. 1[15].

Admittance and current source are expressed as follows:

$$Y = G + jB = \left(\frac{P_0 C_Z}{V_0^{\,2}}\right) + j\left(-\frac{Q_0 C_Z'}{V_0^{\,2}}\right) \tag{5}$$

$$I = I_p + jI_q = \left(\frac{P_0 C_I}{V_0}\right) + j\left(-\frac{Q_0 C_I'}{V_0}\right) \tag{6}$$

The subscript 0 indicates nominal value in all of equations. As shown in [15], with respect to changes in the active and reactive power of the distribution network loads due to voltage variations, the introduced load model exhibits a more accurate behavior of loads in comparison with the traditional P-Q load model. Also, the proposed model is more accurate than two exponential and polynomial models for loads that are more dependent on voltage.

This is noteworthy, a production source behaves as a P-V bus as long as its reactive power is within its allowable range. By crossing this range, this bus behaves as a P-Q bus. This case is similar to case which one bus be connected to a constant P-Q characteristic. In this case, considering that there are other loads in each bus, all of them can be considered as a voltage-dependent load model.

3. Proposed Approach

The load model used in the literatures is commonly based on constant P-Q model. It causes the PF equations to be nonlinear and forces us to solve them by means of iterative and numerical methods. Considering the mentioned load model, it is possible to re-formulate the problem of PF in such a way that the obtained equations are compatible with the linear condition.

According to contractual commitments, DG sources include different operational modes. If the DG connected to a node has the ability to produce a certain value of active and reactive power, it can be modelled as a constant P-Q load with negative values. Although other various loads can be connected to this node, but eventually, sum of the connected loads and DGs to this node can be modelled as a voltage-dependent load.

But, DG sources that behave as a P-V bus cannot be modelled to this way. Considering a distribution network with N buses, it is assumed that m-1 numbers of DG-connected buses (regardless of the main feeder) behave as P-V buses (voltage control). For the sake of simplicity in calculations, the P-V buses are counted at first. Fig. 2(a) shows the overall schematic of the mentioned network with considering the introduced load model. Fig. 2(b) is obtained by modifying the network admittance matrix by taking the constant part of the admittance of the loads model.

The below equation is obtained by means of Kirchhoff's current law:

$$\begin{bmatrix} Y_{AA} & Y_{AB} \\ Y_{BA} & Y_{BB} \end{bmatrix}\begin{bmatrix} V_A \\ V_B \end{bmatrix} = \begin{bmatrix} I_A \\ I_B \end{bmatrix} \tag{7}$$

Where the subscripts A and B are shown in Fig. 2.

V_A : is the voltage vector of the P-V buses that consists of known magnitudes of voltage and unknown angles (including main feeder).

V_B : is the vector of the unknown voltages of the load buses (part B in Fig. 2).

I_A : is the vector of the current injected to the P-V buses with unknown values (including main feeder).

I_B : is the known vector of the current injected to the load buses (the modelled current source in the introduced load model).

Y_{AA} , Y_{AB} , Y_{BA} and Y_{BB} are four sub matrices from the network's modified admittance matrix. Equations (8) and (9) are derived from (7).

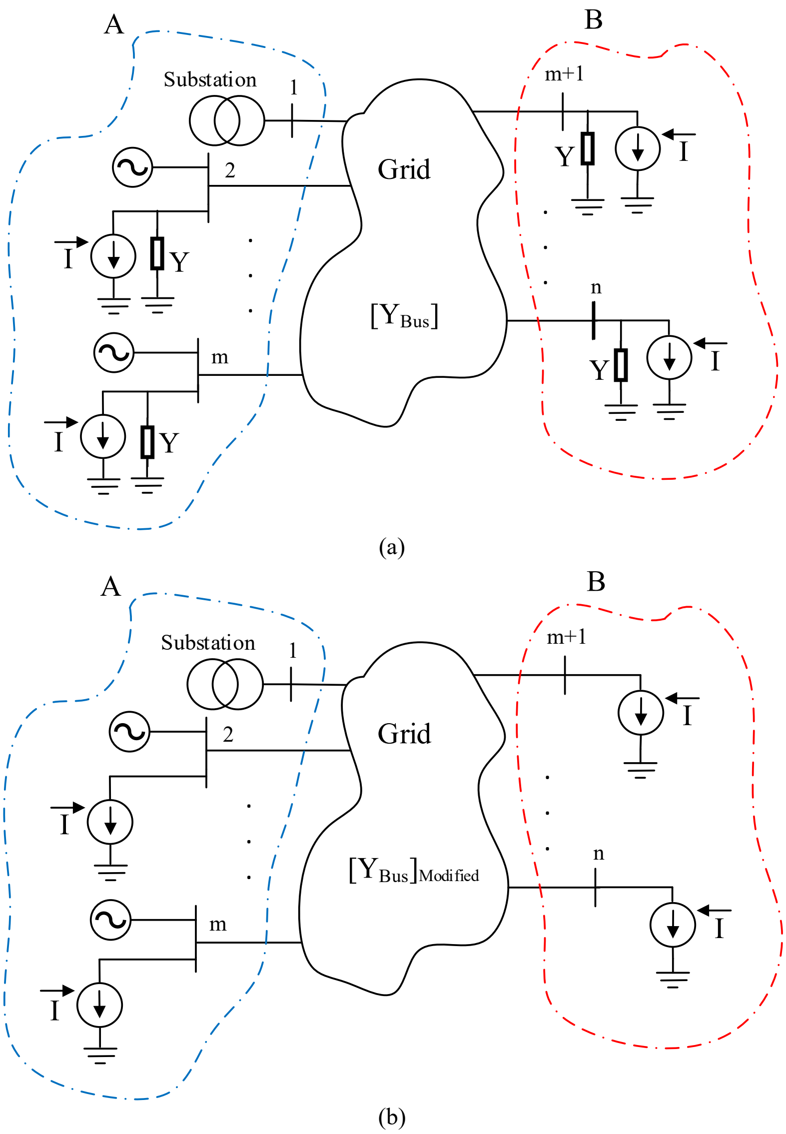


Fig. 2. Overall schemes of distribution network including DG sources with two sets of A (P-V buses) and B (P-Q buses) on the sides of the grid. (a) Voltage-dependent load model as a current source in parallel with admittance. (b) Network's admittance matrix modified by the constant admittance of loads

$$I_A = Y_{AA}V_A + Y_{AB}V_B \quad (8)$$

$$I_B = Y_{BA}V_A + Y_{BB}V_B \quad (9)$$

By simplification, (10) can be obtained

$$I_A = \left(Y_{AA} - Y_{AB}Y_{BB}^{-1}Y_{BA}\right)V_A + Y_{AB}Y_{BB}^{-1}I_B \quad (10)$$

$$Y = Y_{AA} - Y_{AB}Y_{BB}^{-1}Y_{BA} \quad (11)$$

$$H = Y_{AB}Y_{BB}^{-1}I_B \quad (12)$$

$$I_A = YV_A + H \quad (13)$$

Y and H are a $m \times m$ matrix and a $m \times 1$ vector with known elements, respectively. Equation (13) can be extended for ith bus as follows

$$I_i = \sum_{k=1}^{m} Y_{ik}V_k + H_i \quad i = 2,3,\ldots,m \quad (14)$$

By considering Fig. 3, (15) is obtained as follow

$$\vec{I}_i = \vec{I}_{gi} - \vec{I}_{di} \quad (15)$$

Equation (16) is achieved by substituting (15) in (14).

$$I_{gi} = \sum_{k=1}^{m} Y_{ik}V_k + H_i + I_{di} \quad (16)$$

The current generated by DG connected to ith bus is calculated as follows

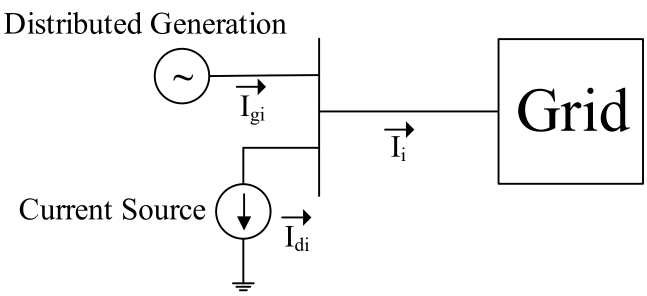


Fig. 3. Overall schematic of *ith* bus in Fig. 2 from set "A"

$$I_{gi} = \frac{P_{gi} - jQ_{gi}}{V_i^*} \tag{17}$$

Assuming that:

$$\overrightarrow{K}_i = \overrightarrow{H}_i + \overrightarrow{I}_{di} \tag{18}$$

With applying (17) in (16) and by means of (18), (19) is achieved.

$$P_{gi} - jQ_{gi} = V_i^* \sum_{k=1}^{m} Y_{ik} V_k + V_i^* K_i \tag{19}$$

By considering $\overrightarrow{V}_i$ and $\overrightarrow{K}_i$ to polarization form in (20) and (22) and the admittance matrix Y to Cartesian form in (21), (23) is achieved.

$$\overrightarrow{V}_i = |V_i| \measuredangle \delta_i \tag{20}$$

$$Y_{ik} = G_{ik} + jB_{ik} \tag{21}$$

$$\overrightarrow{K}_i = |K_i| \measuredangle k_i \tag{22}$$

Calculating the real part of (19) is enough since it's the part that determines the active power of each DG which have certain values. Considering (19) to (22), (23) is derived as follows:

$$P_{gi} = \sum_{k=1}^{m} |V_i||V_k| \{G_{ik} \cos(\delta_i - \delta_k) + B_{ik} \sin(\delta_i - \delta_k)\} + |V_i||K_i| \cos(\delta_i - k_i) \tag{23}$$

Since the difference between the voltage angles of the buses is neglectable, the approximations can be used as follows:

$$\sin(\delta_i - \delta_k) \simeq \delta_i - \delta_k \tag{24}$$

$$\cos(\delta_i - \delta_k) \simeq 1 \tag{25}$$

Relation (23) can be modified using (24) and (25) as follows

$$P_{gi} = \sum_{k=1}^{m} |V_i||V_k| G_{ik} + \sum_{\substack{k=1 \\ k\neq i}}^{m} |V_i||V_k| B_{ik} \delta_i - \sum_{\substack{k=1 \\ k\neq i}}^{m} |V_i||V_k| B_{ik} \delta_k + |V_i||K_i| \cos(\delta_i - k_i) \tag{26}$$

Equation (26) is a nonlinear relation in terms of the voltage angle of buses. There would be m-1 nonlinear equations like (26) because there are m-1 P-V buses in the network. These equations are solvable using the numerical methods. Relation (26) has four terms. Since the magnitude of the voltage of the P-V buses are known and considering the network admittance matrix, the first term of this relation is a known and constant value. There is a linear relationship between the second and third terms of this relation and the unknown variable of the equation (the angle of voltage of P-V buses). The forth term is the only nonlinear term of this relation which causes the problem to be nonlinear. Therefore, the numerical methods are employed to solve such equations. Given that only one term is nonlinear in (26) and considering the fact that the number of P-V buses in the distribution network is limited, numerical methods are converged to the desirable answer rapidly with a low number of iterations. In this paper, the Newton-Raphson method is used to solve the nonlinear equations. More details are provided in Appendix. After calculating the voltage of P-V buses, the voltages of other buses are computed linearly without needing any numerical methods using (27) which was obtained from (9).

$$V_B = Y_{BB}^{-1} I_B - Y_{BB}^{-1} Y_{BA} V_A \tag{27}$$

It is worth noting that, power system loads present different behavior to grid voltage variations. For example, active and reactive power consumption by fluorescent lamps are highly affected by the voltage magnitude, while personal computers are less sensitive to voltage variations. Other loads such as light bulbs, electric motors, etc. depend on the voltage applied to them. In general, loads connected to a node can be divided into two groups:

1- All loads connected to a node, behave as constant power loads:

In this case (which is far from reality), the introduced load flow cannot be implemented.

2- Only one load connected to the node, exhibits voltage-dependent behavior:

In this case, the introduced method is easily applicable.

4. Simulation Results

4-1- 14-, 33- and 70-buses Benchmark Distribution System

Simulations have been performed on the IEEE 14-, 33- and 70-buses benchmark distribution systems [25-27]. The details of the systems are presented in Table 1 and Figs. 4, 5 and 6 depicts the schematic of these networks. The simulations

Table 1. Information of the systems

Test Case No.	Node	Branch	S_b in (MVA)	V_b in (kV)	Load in (MW+jMVAR)
1	14	13	100	23	28.7+j17.3
2	33	32	1	12.66	3.715+j2.3
3	70	69	1	11	4.468+j3.059

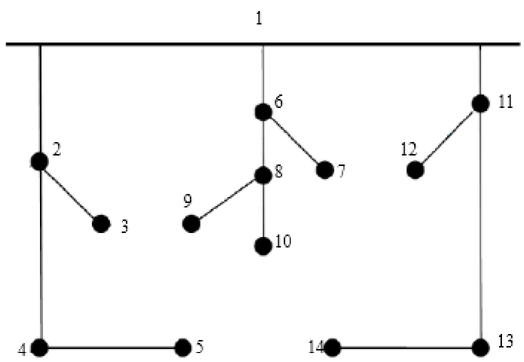


Fig. 4. 14 bus case study

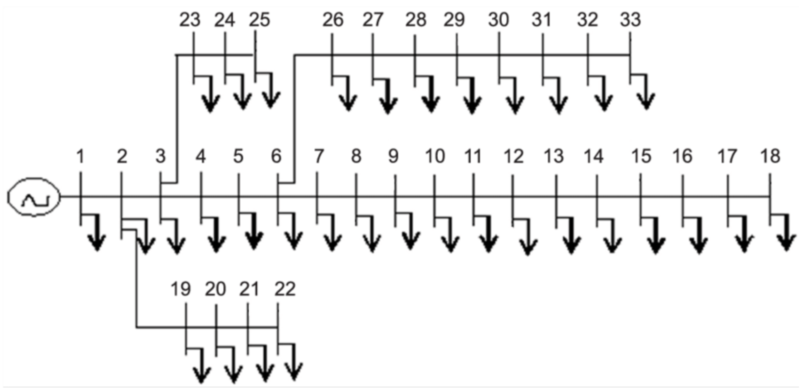


Fig. 5. 33 bus case study

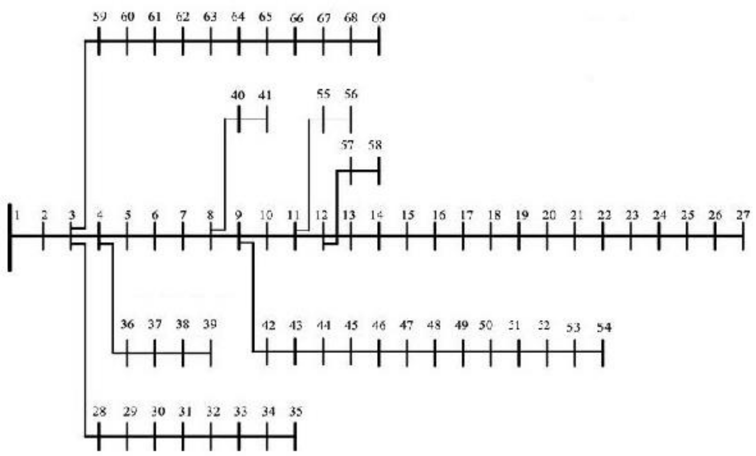


Fig. 6. 70 bus case study

accomplished in this paper has been carried out by MATLAB 2012b software.

The subscript b indicates the base value. For the sake of simplicity, it's assumed that all loads of the network are similar

Table 2. Information of the DG sources

Test Case No.	Number of DG	Location at Bus	Size (MW)
14-Nodes	2	2 and 5	1.5
33-Nodes	2	2 and 3	0.25
70-Nodes	3	3, 4 and 12	0.5

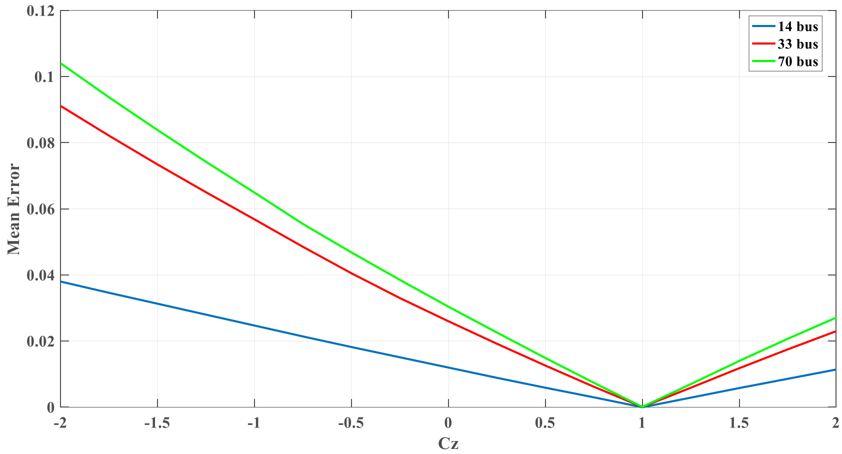


Fig. 7. Mean error between the proposed and Gauss-Seidel methods for each system

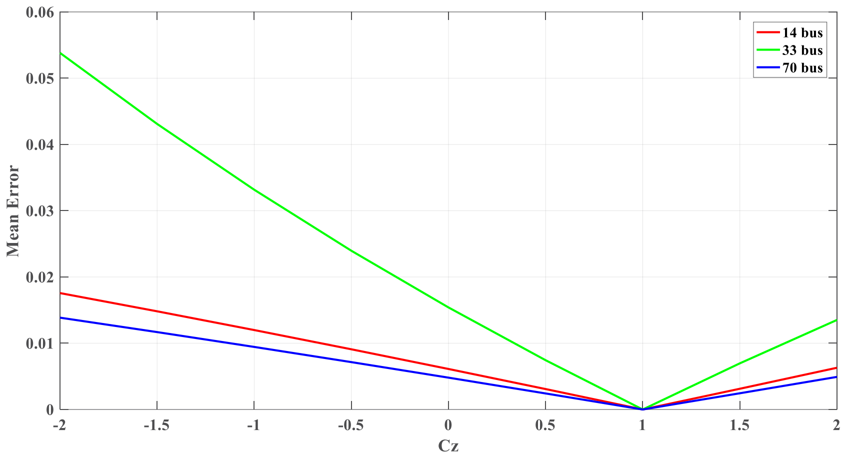


Fig. 8. Mean error between the proposed and Newton-Raphson methods for each system

from the viewpoint of voltage dependency and also, it's considered that the dependence of active and reactive power to voltage are identical [15]. In other words, for all loads in the system the following equations hold:

$$C_Z = C'_Z \quad \& \quad C_I = C'_I \tag{28}$$

Considering the equation above and (3), the parameter C_Z is the only independent variable of the PF problem. To verify the proposed method, a comparison has been carried out between the results obtained from the Gauss-Seidel and Newton-Raphson methods considering voltage-dependent load model with the introduced approach by means of (29) which calculates the mean error [15].

$$\eta = \frac{1}{N}\sum_{i=1}^{N}\frac{\left|\overline{V}_i' - \overline{V}_i\right|}{\left|\overline{V}_i'\right|} \tag{29}$$

Where:

V_i': The voltage of the buses obtained from the Gauss-Seidel or Newton-Raphson method considering voltage-dependent load model.

V_i : The voltage achieved using the introduced procedure.

The information related to the penetration of DG sources (DG sources are assumed to behave as P-V bus) are presented

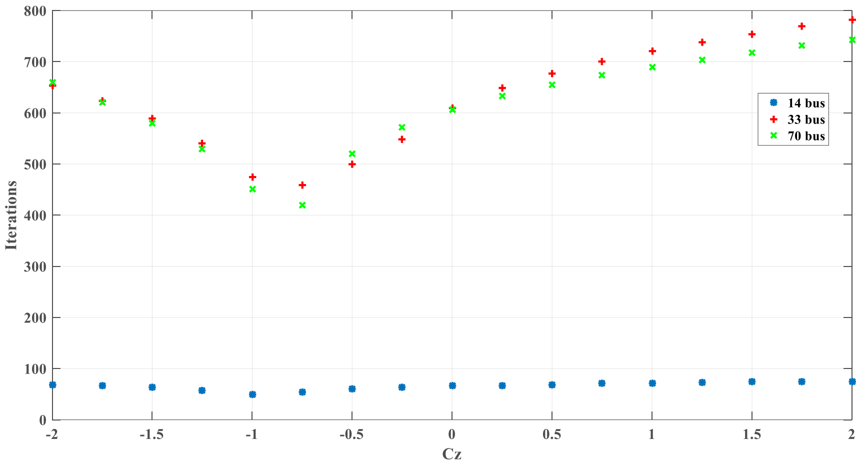


Fig. 9. Number of iterations for the Gauss-Seidel method

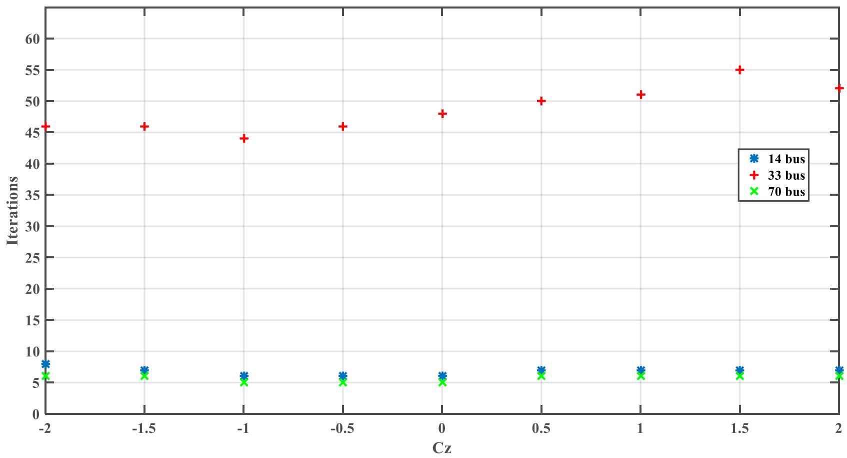


Fig. 10. Number of iterations for the Newton-Raphson method

in Table 2. This information includes number, location and size of the DG sources in the case study systems. It should be noted that the DGs of all systems have the same size.

Fig. 7 shows the relationship between the mean error versus the parameter C_Z for the proposed and Gauss-Seidel methods on the 14-, 33- and 70-bus distribution systems. Also, this comparison is presented for the proposed and Newton-Raphson methods in Fig. 8. In according to Fig. 7 and Fig. 8, it's clear that the mean error value is in the acceptable range and reaches its least value near $C_Z = 1$.

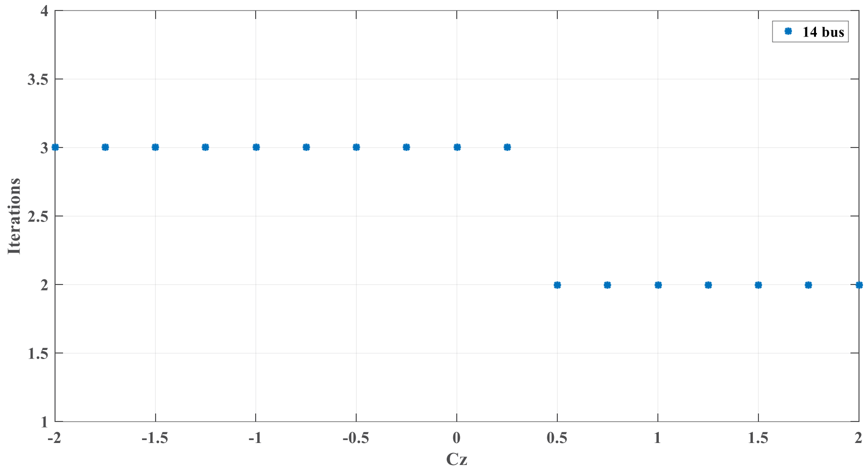
As can be seen in Figures 7 and 8, the amount of deviation of the proposed load flow method from the Gauss-Seidel and Newton-Raphson methods, is in an acceptable range. In other words, the results obtained from the new approach presented in this paper are valid and reliable. It is also observed that the mean error between the proposed method and the Newton-Raphson method is less than Gauss-Seidel method. The reason for this problem, may be that the proposed method also uses the numerical Newton-Raphson method to calculate the angles of PV buses.

It is noteworthy that the proposed method is faster than the Gauss-Seidel and Newton-Raphson methods and converges to the appropriate value with a very low number of iterations. Fig. 9, Fig. 10 and Fig. 11 illustrate the relationship between the number of required iterations versus the parameter C_Z for convergence of the proper response in the tolerance of 10^{-5} for the Gauss-Seidel, Newton-Raphson and the introduced methods, respectively. Considering these three Figures, it is obvious that the proposed method converges to the suitable response with a lower number of iterations comparing to the Gauss-Seidel and Newton-Raphson method.

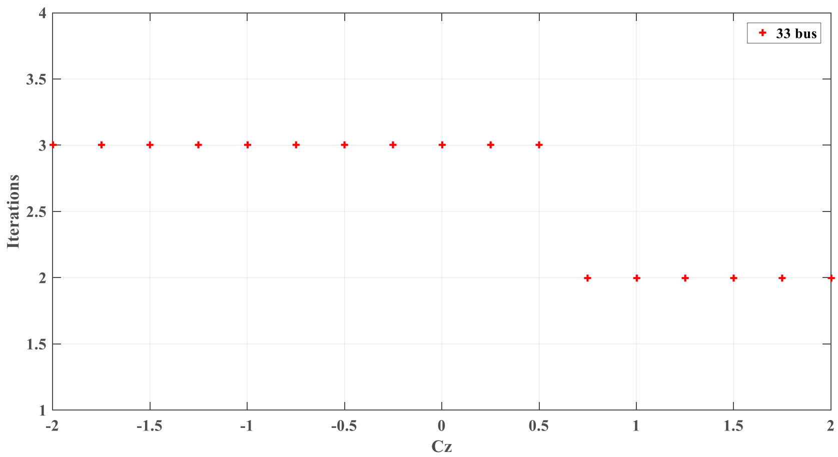
A comparison between these three methods during the processing time of the algorithm is done and the results of each system are provided in Table 3.

The angle values of the voltage of the controllable buses (P-V buses) are obtained for different values of the parameter in radian for both the Gauss-Seidel and Newton-Raphson methods with the introduced method in Tables 4, 5 and 6. These Tables are for the 14-, 33- and 70-bus systems, respectively.

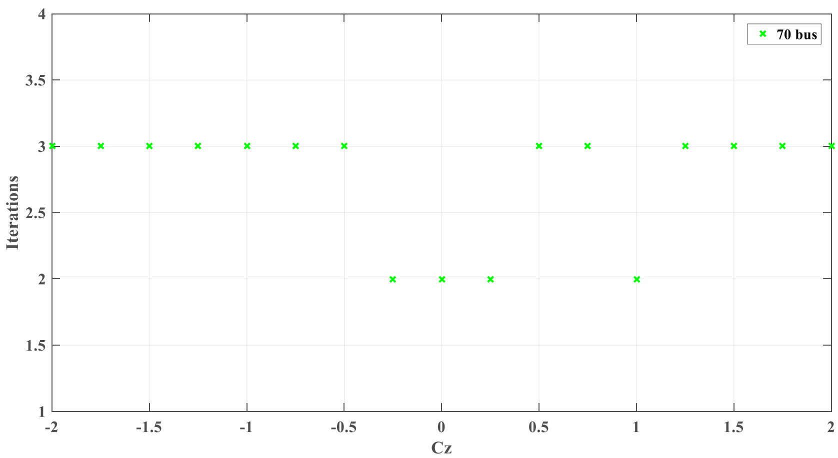
It is easy to see that the answer obtained with the proposed method, with an acceptable deviation, is similar



(a)



(b)



(c)

Fig. 11. Number of iterations for the proposed method:
(a) 14-bus system
(b) 33-bus system and
(c) 70-bus system

to the classic load flow methods such as Gauss-Seidel and Newton-Raphson.

4-2- Large Scale Networks

In order to demonstrate the efficiency of the proposed

Table 3. Average time of power flow process for the classic methods (Gauss-Seidel (G.S.) and Newton-Raphson (N.R.) methods) and the proposed method for 16 times in second

Test Case	The G.S. Method	The N.R. Method	The Proposed Method
14-bus	0.017	0.006	0.004
33-bus	0.3646	0.04	0.0021
70-bus	1.1376	0.06	0.01

Table 4. The voltage angle of the P-V buses in Radian for the Gauss-Seidel (G. S.), Newton-Raphson (N. R.) and the proposed methods (P. M.) for the 14-Bus System

Bus	$C_Z = 0.5$		$C_Z = 0.75$		$C_Z = 1$	
	G. S. & N. R.	P. M.	G. S. & N. R.	P. M.	G. S. & N. R.	P. M.
2	-0.012	-0.017	-0.012	-0.014	-0.012	-0.012
3	-0.021	-0.03	-0.02	-0.02	-0.02	-0.02

Table 5. The voltage angle of the P-V buses in Radian for the Gauss-Seidel (G. S.), Newton-Raphson (N. R.) and the proposed methods (P. M.) for the 33-Bus System

Bus	$C_Z = 0.75$		$C_Z = 1$		$C_Z = 1.25$	
	G. S. & N. R.	P. M.	G. S. & N. R.	P. M.	G. S. & N. R.	P. M.
2	-0.005	-0.006	-0.005	-0.005	-0.005	-0.005
3	-0.03	-0.03	-0.03	-0.03	-0.03	-0.03

Table 6. The voltage angle of the P-V buses in Radian for the Gauss-Seidel (G. S.), Newton-Raphson (N. R.) and the proposed methods (P. M.) for the 70-Bus System

Bus	$C_Z = -1.5$		$C_Z = -1.25$		$C_Z = -1$	
	G. S. & N. R.	P. M.	G. S. & N. R.	P. M.	G. S. & N. R.	P. M.
2	0.03	0.04	0.03	0.03	0.028	0.028
3	0.03	0.03	0.03	0.0245	0.03	0.02
4	0.03	0.03	0.03	0.0245	0.03	0.02

Table 7. The general data of the balanced 874-nodes distribution network

Test Case	Node	Branch	S_b in (MVA)	V_b in (kV)	Load in (MW+jMVAR)
874-node	874	873	100	23	12487+j7436

method, this method is applied to large scales networks, so that these networks are closer to reality. For this purpose, the balanced 874-node distribution network is used. The general data of this system can be found in Table 7 and the detail data in [29].

In this network, 15 DGs are placed in different buses which behave as a PV bus. DGs are similar and the active power of each one is 100 KW. Considering the loads of the network as the mentioned voltage-dependent loads and applying to the Gauss-Seidel and Newton-Raphson methods, it can be seen

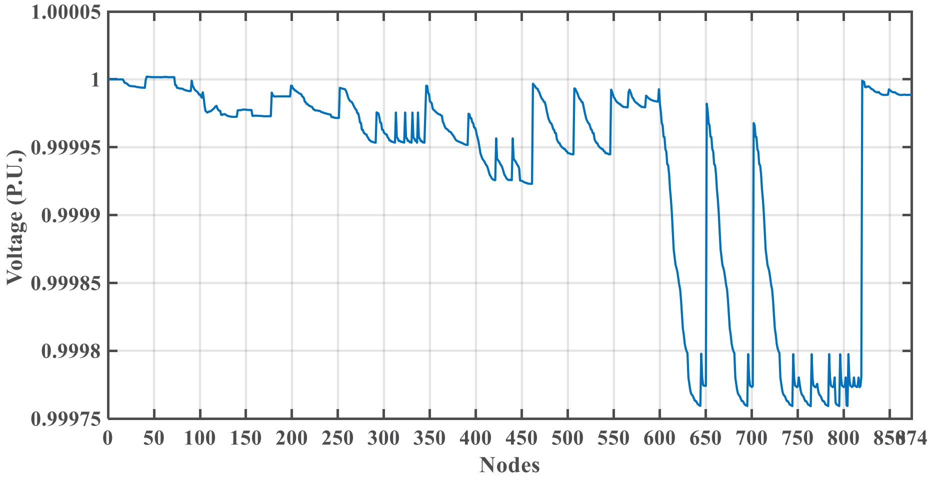


Fig. 12. The voltage diagram of the 874-node distribution network for $C_z = -1.5$.

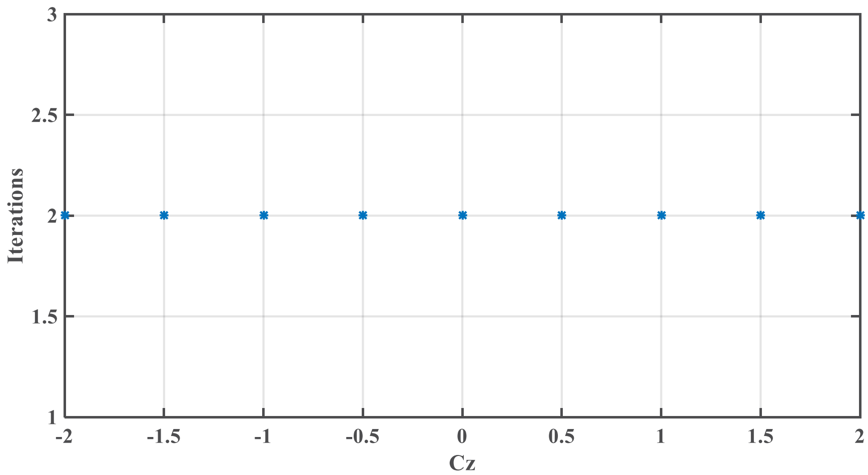


Fig. 13. Number of iterations versus the different values of C_z

Table 8. The simulation results for large scales networks

Test case	Branch	No. of DGs	Average Simulation Time (sec)	Average Iterations	Average Voltage of Nodes (P.U.)
3496-bus	3495	20	8.95	3	0.98
5244-bus	5244	30	31.23	3	0.97
6118-bus	6117	50	56.99	4	0.96

that after a long time and a lot of iterations, these methods do not converge toward an appropriate response. But the proposed method converges toward an acceptable response at a very high speed and a low iteration.

Fig. 12 shows the voltage diagram of the network nodes for $C_z = -1.5$. Also, Fig. 13 shows the number of iterations required to achieve the suitable response versus the different values of C_z.

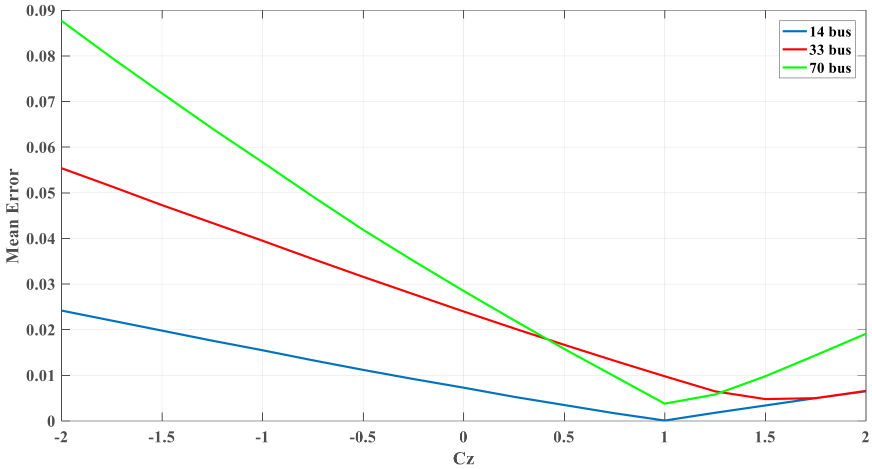


Fig. 14. Mean error for weakly meshed systems

It should be noted that the average time required for the convergence of the proposed method is 0.23 seconds.

By connecting the 874-node network to each other with different combinations, new networks with more nodes are created and the proposed method has been applied to them. The simulations results is shown in Table 8. In this Table, The average iteration is the average number of iterations for the different values of C_z .

4-3- Robustness against Meshed Distribution Systems

The proposed PF method is reliable and robust against meshed networks. Thus, this method is applied to meshed distribution systems. To demonstrate the efficiency of the approach on these networks, some lines are added to create a loop to the IEEE 14-, 33- and 70-bus systems. To do this, 3, 5 and 9 lines have been added to each system, respectively.

These networks consist of loops by the mentioned added lines. Fig. 14 shows the value of the mean error versus for the proposed method and the Gauss-Seidel method.

5. Conclusion

Considering the high penetration of DG sources in distribution networks, the need for a reliable PF method is rising nowadays. This method must be capable of being applied to networks which contain DG sources with different operational modes. In this paper, the proposed PF method supports both of the two performance modes of DG sources, namely P-Q bus and P-V bus modes.

The load model used in this paper is the voltage-dependent load model. This model provides a more accurate representation of the load behavior and also leads to linearized PF equations. Voltage of load buses (P-Q bus) are linearly calculated by the introduced approach and the numerical methods are only used for calculating the angles of P-V buses which results in a neglectable calculation burden. The Newton-Raphson numerical method is utilized to solve the nonlinear equations. In this method, the non-diagonal elements of the Jacobean matrix are constant and do not need to update at each iteration but the diagonal elements need to update at each iteration because they are not constant (refer to Appendix). Therefore, the numerical method has a very high speed and converges to the appropriate response in very few iterations. Generally, advantages of the proposed method are summarized in the following paragraphs:

- Making use of the voltage-dependent load model which represents a more accurate behavior of load.
- Being applicable to networks containing DGs with different performance modes.
- Providing PF equations that are close to linear equations which can be processed with very high speed.
- Being applicable to meshed and radial networks.

This method is used for three-phase balanced networks. For future works, the proposed method can be developed for three-phase unbalanced networks.

6. Nomenclature

P	Active Power
Q	Reactive Power
V	Voltage Magnitude of Terminal
G	Conductance of load
B	Susceptance of load
I_p	Real part of load current
I_q	Imaginary part of load current
N	Number of nodes of system
m	Number of DGs behave as P-V buses (regard to the main feeder)
$\vec{I}_{gi}$	Current generated by the DG connected to *ith* bus

$\vec{I}_{di}$	Modelled current source for the load connected to ith bus
I_i	Injected current to ith bus
P_{gi}	The produced active power by the DG connected to ith bus
Q_{gi}	The produced reactive power by the DG connected to ith bus
δ_i	Voltage angle of ith bus
C_Z, C_I, C_Z' and C_I'	The parameters of voltage-depended load model

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Appendix

This appendix addresses how the Newton-Raphson numerical method is applied for solving the nonlinear equation (26). Since power generation of the main feeder (slack bus) is unknown, this bus is ignored in the equations. Thus, the angle of this bus has been considered zero. Equation (26) is modified again as follows:

$$P_{gi} = \sum_{k=1}^{m}|V_i||V_k|G_{ik} + \sum_{\substack{k=1\\k\neq i}}^{m}|V_i||V_k|B_{ik}\delta_i^t - \sum_{\substack{k=1\\k\neq i}}^{m}|V_i||V_k|B_{ik}\delta_k^t + |V_i||K_i|\cos\left(\delta_i^t - k_i\right) \overset{\Delta}{=} f_i^t \tag{30}$$

In (30), the subscript i is $2,3,\ldots,m$.

The superscript t is tth iteration in the Newton-Raphson method. Parameter P_{gi} is constant and known and called as the power generation of the DG connected to ith bus. Parameter f_i^t should be calculated at every iteration.

$$P_{gi} - f_i^t = 0 \overset{\Delta}{=} \Delta P_i^t \quad \& \quad i = 2,3,\ldots,m \tag{31}$$

In other words, ΔP_i^t is the power mismatch in tth iteration. By applying the Newton-Raphson numerical method on equation (31) and forming the Jacobian matrix, relation (32) is obtained. The Newton-Raphson process continues as far as the difference in the power mismatch of two consecutive iterations for all P-V buses (except reference bus) is less than 10^{-5} . Equation (31) has been expanded at each iteration as follows:

$$\begin{bmatrix} \Delta P_2^t \\ \Delta P_3^t \\ \vdots \\ \Delta P_m^t \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{\partial f_2^{\ t}}{\partial \delta_2} & \frac{\partial f_2^{\ t}}{\partial \delta_3} & \cdots & \frac{\partial f_2^{\ t}}{\partial \delta_m} \\ \frac{\partial f_3^{\ t}}{\partial \delta_2} & \frac{\partial f_3^{\ t}}{\partial \delta_3} & \vdots & \frac{\partial f_3^{\ t}}{\partial \delta_m} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_m^{\ t}}{\partial \delta_2} & \frac{\partial f_m^{\ t}}{\partial \delta_3} & \cdots & \frac{\partial f_m^{\ t}}{\partial \delta_m} \end{bmatrix}}_{Jacobian\ Matrix} \begin{bmatrix} \Delta \delta_2^t \\ \Delta \delta_3^t \\ \vdots \\ \Delta \delta_m^t \end{bmatrix} \quad (32)$$

The elements of Jacobian matrix are calculated at each iteration as follows:

$$J_{ii} = \frac{\partial f_i^t}{\partial \delta_i} = \sum_{\substack{k=1 \\ k \neq i}}^{m} |V_i||V_k|G_{ik} - |V_i||K_i|\sin(\delta_i^t - k_i) \qquad \& \quad i = 2,3,\ldots,m \quad (33)$$

$$J_{iq} = \frac{\partial f_i^t}{\partial \delta_q} = -|V_i||V_q|B_{iq} \qquad i,q = 2,3,\ldots,m \quad \& \quad q \neq i \quad (34)$$

J_{ii} : The diagonal elements of the Jacobian matrix.

J_{iq} :The non-diagonal elements of the Jacobian matrix.

It is obvious that the main diagonal elements are only dependent to the parameter δ. The non-diagonal elements of the Jacobian matrix are constant and do not need to update at each iteration. The parameter is calculated at each iteration and then the new value of the parameter is computed using the equation below to be used in the next iteration.

$$\delta_i^{\ t+1} = \delta_i^{\ t} + \Delta\delta_i^{\ t} \qquad \& \quad i = 2,3,\ldots,m \quad (35)$$

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