The complexity of cost constrained vehicle scheduling problem

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ABSTRACT: This paper considered the cost constrained vehicle scheduling problem under the constraint that the total number of vehicles is known in advance. Each depot has a different time processing cost. The goal of this problem is to find a feasible minimum cost schedule for vehicles. A mathematical formulation of the problem is developed and the complexity of the problem when there are more than two depots is investigated. It is proved that in this case, the problem is NP-complete. Also, it is showed that there is not any constant ratio approximation algorithm for the problem, i.e., it is in the complexity class APX.

1. Introduction

Planning a transportation network is an important task for managing transportation systems [13]. This task consists of different steps. Step 1 is related to land use, transportation demand, etc. Having assessed a transportation network and its properties, such as multiple type vehicle size, operating cost and public transportation demand, the frequency setting for each line of different transportation modes is investigated in Step 2 [15]. Moreover, designing a timetable for servicing the related demand and frequencies is the aim of Step 3 [16]. Transportation scheduling provides a powerful approach for the problem of synchronizing different modes in Step 2 and Step 3. This synchronization is a way to integrate different modes efficiently. In Step 4, multiple vehicle types are assigned to trips and a schedule is created for each vehicle type. Crew assignment is carried out in Step 5. In this step, drivers are assigned to vehicles based on vehicle scheduling and the number of drivers and their work times are calculated as well.

This paper focues on vehicle scheduling problem, which is dealt with in Step 4 in which the designer seeks a set of schedules for vehicles so that each trip is performed exactly once with minimum cost. The problem is easy if there is only one depot for vehicles and efficient algorithms can be considered in this case, (see e.g. [10]).

When there are two or more depots for vehicles the problem is proved to be generally NP-hard ([3]). For this reason, several researchers have applied heuristic algorithms to solve the problem efficiently. The problem is well studied in the literature, see e.g. surveys of [27], [7]. Researchers focus on both heuristic and exact algorithms. See e.g. [4], [19], [29], [32], [33], [5], [12], [17] and [14] for exact algorithms and [3], [21], [20], [24] and [6] for heuristic solution. Also, studying the complexity of scheduling problem is one of the topics of interest in the literature. See e.g., [34] , [22], [26], [31] and [30].

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In this paper, the vehicle scheduling problem is studied when there are several depots and each depot has a different time processing cost. In addition, the total number of vehicles needed to perform all trips is known in advance. In Section 2, a mathematical programming model for the vehicle scheduling problem is developed. Section 3 provides a complexity analysis of the problem when there are more than two depots. In this section, it is proved that for this case, the problem is NP-complete. Also, it is shown that there is not any constant ratio approximation algorithm for the problem, i.e., it is in the complexity class APX. Section 4 ends the paper with a brief conclusion and future directions.

2. Vehicle schedule modeling

The objective of the problem is to create efficient schedules for vehicles so that they can perform all trips. A trip is the transportation of passengers from a specific station to a destination, which should be performed in a certain fixed time. Therefore, a trip $i (i \in \{1, 2, \ldots, N\})$ has a fixed start time s_i and a fixed finish time f_i. The start time and finish time of the trips are predetermined. To perform, a vehicle should be located at the start location exactly at the start time of that trip. Furthermore, the time needed to move from the destination location of trip i to the start location of trip j is $t_{i,j}$. This time can be considered as a set up time for performing trip j directly after trip i which is sequence-dependent. Fig. (1) shows an instance of vehicle scheduling problem with 7 trips.
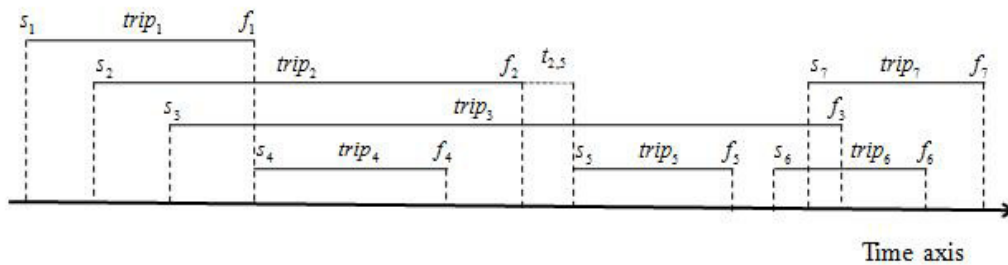

Figure 1: An instance of vehicle scheduling problem with 7 trips.

Assume that there are K depots with vehicles $b \in \{1, 2, \ldots, l_1, l_1 + 1, \ldots, l_2, \ldots, l_{k-1} + 1, \ldots, l_k, \ldots, l_{k-1} + 1, \ldots, l_k\}$ in which $\{l_{k-1} + 1, \ldots, l_k\}$ are the vehicles of depot k. Moreover, assume C_k is the time processing cost of vehicles in depot k and each vehicle has a time limit T. Due to the time limitation of each vehicle and time interval of each trip, some trips cannot be performed by the same vehicle. More precisely, if the start time of trip j is less than $f_i + t_{i,j}$, trip j cannot be performed directly after trip i by the same vehicle. In addition, if the difference between the finish time of trip j and the start time of trip i is less that T, trips i and j cannot be performed by the same vehicle due to the time limitation of the vehicle . For trip i , the set of trips which cannot be performed with i by the same vehicle, is denoted by Q_i and defined as follows:

$$Q_i = \{j|\ s_j < f_i + t_{i,j}\}. \tag{1}$$

The total number of vehicles needed for performing all trips is known in advance and is equal to B. The problem should select $B_1 < l_1$ vehicles of depot 1, $B_2 < l_2 - l_1$ vehicles of depot 2, $\cdots$, and $B_k < l_k - l_{k-1}$ vehicles of depot K so that $B = B_1 + B_2 + \cdots + B_k$ and all trips are performed.

For every depot k, a network $G_k = (V_k, E_k)$ is considered so that V_k contains one node for every trip and two nodes o_k and d_k for depot k i.e. $V_k = \{o_k, 1, 2, \ldots, N, d_k\}$ and

$$E_k = \{(o_k, j)|\ j = 1, \ldots, N\} \cup \{(j, d_k)|\ j = 1, \ldots, N\} \cup \{(i, j)|\ i = 1, \ldots, N, j = 1, \ldots, N, j \notin Q_i\}.$$

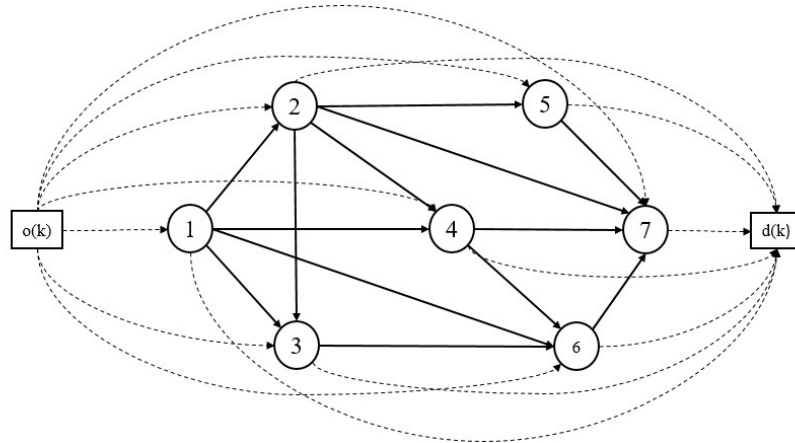See Fig. (2) for the network with 7 trips.

Figure 2: network for depot k.

For each arc $(i, j) \in A^k$, let $X_{i,j}^k$ be the flow of commodity k through arc (i, j). Also the variable B_k is defined as the number of vehicles from depot k. Now the model of vehicle scheduling problem can be stated as the following model.

$$\min z = \sum_{k=1}^{K} C_k \left(\sum_{(i,j) \in E_k} t_{i,j} x_{i,j}^k + \sum_{i=1}^{N} (f_i - s_i) \sum_{(i,j) \in E_k} x_{i,j}^k \right), \tag{2}$$

s.t.

$$\begin{cases} \sum_{k=1}^{K} \sum_{(i,j) \in E_k} x_{i,j}^k = 1 & \forall i = 1, \ldots, N, \\ \sum_{\{j | (o_k, j) \in E_k\}} x_{o_k j}^k = B_k & \forall k = 1, \ldots, K, \\ \sum_{\{j | (i,j) \in E_k\}} x_{i,j}^k - \sum_{\{j | (i,j) \in E_k\}} x_{j,i}^k = 0, & \forall i = 1, \ldots, N,, \\ B_1 + B_2 + \cdots + B_K = B, \\ z \leq \overline{C}, \\ x_{i,j}^k \in \{0, 1\}, & \forall (i, j) \in E_k. \end{cases} \tag{3}$$

Where $\bar{C}$ is the upper bound of the total cost and detemined by the manager. The objective function minimizes transportation cost which is stated as the total transportation time between trips and trip times performed by each depot multiplied by depot cost. The first constraint ensures that each trip is assigned to exactly one depot. The second constraint stipulates that the number of vehicles used from depot k should be equal to B_k. The third constraint deals with flow conservation constraint and the forth one ensures that the total number of vehicles is equal to B. The fifth constraint is the cost constraint on the total cost. Finally, the binary restriction is applied in the last constraint.

3. The complexity of the problem for more than two depots

In this section, we show that the cost constrained vehicle scheduling problem is NP-complete. To prove this, we first in the next theorem show that there is not any constant factor approximation algorithm for this problem when the number of depots is more than two.

Theorem 3.1. *For $r \geqslant 1$ (r is a constant), assuming $P \neq NP$, there is no polynomial time approximation algorithm with factor r for the cost constrained vehicle scheduling problem when the number of depots is at least 3.*

Proof. The proof is by contradiction. Suppose that for some $r \geqslant 1$, there is a polynomial time approximation algorithm A for the cost constraint vehicle scheduling problem with factor r, i.e. $C_A/C^* \leqslant r$ where C_A is the cost returned by the algorithm A and C^* is the optimal solution. We will show how to use the algorithm A to solve the Numerical 3-Dimensional Matching (N3DM) problem in polynomial time. Since N3DM problem is NP-complete [11] our theorem follows.

$\square$

First we recall the definition of N3DM problem [11].

Definition 3.2. *Given the integers t, d and $0 < a_i, b_i, c_i < d$ for $i = 1, 2, \ldots, t$, satisfying the following relation:*

$$\sum_{i=1}^{t}(a_i + b_i + c_i) = td \tag{4}$$

Are there permutations ρ and σ so that $a_i + b_{\rho(i)} + c_{\sigma(i)} = d, i = 1, 2, \ldots, t$?

Consider a particular instance of N3DM. We construct an instance of the cost constrained vehicle scheduling problem as follows. Define

$$U = 49dt^4r^2 \tag{5}$$

$$V = U - 7dt^2r = 49dt^4r^2 - 7dt^2r \tag{6}$$

$$W = U + 7dt^2r + 3d = 49dt^4r^2 + 7dt^2r + 3d \tag{7}$$

$$Z = W + U + d = 98dt^4r^2 + 7dt^2r + 4d \tag{8}$$

Moreover, let $K = 3, C_1 = (14dt^3 + 14dt^2 - 5dt/r + 5d/r)/Z, C_2 = 1/r$ and $C_3 = 7t^2$. It can be easily verified that $0 < C_1 < C_2 < C_3$. Assume $\overline{C} = 49dt^4, B_1 = t, B_2 = t^2 - t$ and $B_3 = t^2$. In this instance of the cost constrained vehicle scheduling problem, the total number of vehicles is $B = B_1 + B_2 + B_3 = 2t^2$. Next, we choose $t^2 + 2t$ distinct rational numbers E_i, F_j and $X_{i,j}$ for $i, j = 1, 2, \ldots, t$ in $(U, U + 3d)$ so that:

$$U < F_j < U + d < E_J < U + 2d < X_{i,j} < U + 3d \tag{9}$$

We define $\mid T \mid = 6t^2 + t$ as the number of trips. The start time and finish time pairs (s_i, f_i) of these trips are as follows:
$(0, E_i)$ for $i = 1, \ldots, t$,
$(t - 1)times(V, F_j)$ for $j = 1, \ldots, t$,
$(X_{i,j}, W + a_i + b_j)$ for $i, j = 1, \ldots, t$,
$(t - 1)times(U, E_i)$ for $i = 1, \ldots, t$,
$(X_{i,j}, U + 3d)$ for $i, j = 1, \ldots, t$,
$(E_i, X_{i,j})$ for $i, j = 1, \ldots, t$,
$(F_j, X_{i,j})$ for $i, j = 1, \ldots, t$,
$(W + d - c_k, Z)$ for $k = 1, \ldots, t$,
(U, F_j) for $j = 1, \ldots, t$.
Also we set $t_{i,j} = 0, \forall i, j \in T$. In this way, an instance of the cost constraint vehicle scheduling problem is constructed in polynomial time. In what follows we show that if the algorithm A is applied to solve this problem, then $C_A \leqslant r\overline{C}$ for all $r \geqslant 1$, if and only if the corresponding instance of N3DM problem has a solution. To show this we prove the following lemmas:

Theorem 3.3. *Trips $(0, E_i)$ for $i = 1, \ldots, t$ can only be assigned to depot 1.*

Proof. Suppose a trip $(0, E_i)$ is assigned to another depot. In this case the unit time processing cost for this trip is at least $1/r$, thus the processing time of this trip is $E_i > U + d > U = 49dt^4r = r\overline{C}$, which is a contradiction.

$\square$

Theorem 3.4. *Trips $(t - 1)times(V, F_j) for j = 1, \ldots, t$ can only be assigned to depot 2.*

Proof. Due to Lemma 3.3, trips $(t - 1)times(V, F_j) for j = 1, \ldots, t$ cannot be assigned to depot 1. The rest of the proof is similar to the proof of lemma 3.3.

$\square$

Theorem 3.5. *Trips $(t - 1)times(U, E_i)$ for $i = 1, \ldots, t$ and (U, F_j) for $i = 1, \ldots, t$ can only be assigned to depot 3.*

Proof. The proof is trivial from the result of Lemma 3.3 and Lemma 3.4.

$\square$

Theorem 3.6. *Trips $(W + d - c_k, Z)$ for $k = 1, \ldots, t$ can only be assigned to depot 1.*

Proof. The proof is similar to the proof of Lemma 3.3.

$\square$

Theorem 3.7. *Trips $(X_{i,j}, W + a_i + b_j)$ for $i, j = 1, \ldots, t$ can not be assigned to depot 3. Furthermore, there is no idle time for each vehicles from depot 1 and depot 2 during the time $[U + 3d, W]$.*

Proof. The first part of the proof is similar to the proof of Lemma 3.3. For the second part, it is obvious that all t^2 trips start before time $U + 3d$ and end after time W.

$\square$

Theorem 3.8. *Trips $(X_{i,j}, U + 3d) for i, j = 1, \ldots, t$ can only be assigned to depot 3.*

Proof. The proof is the immediate result of Lemma 3.7.

$\square$

Theorem 3.9. *There is no idle time for each of $2t^2$ vehicles during the time interval $[U, U + 3d)$.*

Proof. The proof is obvious from the definition of the trips.

$\square$

The rest of the proof for theorem 3.1: Now we show that there is no idle time for the vehicles of depot 1 during the time interval $[W, W + d]$ and that the instance of N3DM problem has a solution. From the above lemmas we see that trips $(0, E_i)$ for $i = 1, \ldots, t, (E_i, X_{i,j})$ for $i, j = 1, \ldots, t, (X_{i,j}, W + a_i + b_j)$ for $i, j = 1, \ldots, t$, and $(W + d - c_k, Z) for i, k = 1, \ldots, t$ in which each i, j and k occurs exactly once, should be assigned only to depot 1. Moreover, the trips $(t - 1) times (V, F_j)$ for $j = 1, \ldots, t$ and $(X_{i,j}, W + a_i + b_j)$ for $i, j = 1, \ldots, t$ in which each $j(j = 1, \ldots, t)$ occurs exactly $(t - 1)$ times must be assigned to depot 2. From the previous lemmas it follows that trips $(t - 1) times (U, E_i)$ for $i = 1, \ldots, t, (E_i, X_{i,j})$ for $i, j = 1, \ldots, t$, and $(X_{i,j}, U + 3d)$ for $i, j = 1, \ldots, t$, where each $i(i = 1, \ldots, t)$ occurs exactly $(t - 1)$ times, must be assigned to depot 3. Also the trips (U, F_j) and $(F_j, X_{i,j})$ for $j = 1, \ldots, t$ should be assigned to depot 3. Finally from the fact that $\sum_{i=1}^{t}(a_i + b_i + c_i) = td$ and each i, j and k occurs exactly once for trips $(X_{i,j}, W + a_i + b_j)$ and $(W + d - c_k, Z)$ assigned to depot 1, it can be cocluded that there is no idle time for each of depot 1 vehicles durig time interval $[W, W + d]$. Thus $W + a_i + b_j = W + d - c_k$. If we define $\rho(i) = j$ and $\sigma(i) = k$, then $a_i + b_{\rho(i)} + c_{\sigma(i)} = d$ for $i = 1, \ldots, t$ and the instance of N3DM problem has a solution.

Now if the instance of N3DM problem has a solution, the total processing cost of the trip assignment is as follows:

$$
\begin{aligned}
C_A &= tZ(14dt^3 + 14dt^2 - 5dt/r + 5d/r)/Z + \\
&\quad + (t^2 - t)(W + 2d_V)/r + 21dt^4 \\
&= 49dt^4 \\
&= \bar{C} \\
&\leq r\bar{C}.
\end{aligned}
\tag{10}
$$

It follows that algorithm A can solve N3DM problem in polynomial time which is a contradiction.

Corollary 3.10. *The cost constraint vehicle scheduling problem is NP-complete.*

Proof. Follow the proof of theorem 3.1 by setting $r = 1$.

$\square$

4. Conclusion

This paper investigated the cost constrained vehicle scheduling problem in transportation networks. It is considered a case in which there are several depots each of which has a different cost and the total number of vehicles is predetermined. A mathematical integer programming model for the problem is developed. Then, it is investigated the complexity of the problem for the case in which there are more than two depots. It is proved that when there are at least three depots, the problem is Np-complete. Moreover, there is not an approximation algorithm of constant ratio for this problem, i.e. it is in the complexity class APX. In the future work, the authors plan to investigate the vehicle scheduling problem in multimodal transportation networks.

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