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Path Following of an Underactuated Autonomous Underwater Vehicle Using Backstepping and Disturbance Observer-Based Sliding Mode Control

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ABSTRACT: In this paper, a new hierarchical robust nonlinear control scheme is designed for the horizontal plane path following control problem of an underactuated autonomous underwater vehicle in the presence of the model uncertainties and fast-time-varying external disturbances. First, the path following error model is established based on the virtual guidance method. Afterwards, the controller design starts at a kinematic level and evolves to a dynamic setting, building on the kinematic controller derived, using backstepping technique and a disturbance observer-based sliding mode control, respectively. A Lyapunov-based stability analysis proves that all the signals are ultimately bounded, and path following errors converge to an arbitrarily small neighborhood of the origin. Following achievements are highlighted in this paper: (I) in order to simplify the control design, the derivative of the virtual control is estimated by the disturbance observer which avoids explosion of complexity without common filtering techniques; (II) the proposed controller can be easily implemented with no information of the bounds on the parameter uncertainties and external disturbances in a continuously changing environment. Furthermore, computer simulations have shown that the overall closed-loop system achieves a good path following performance, which proves the feasibility and good robustness of the proposed control law.

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1- Introduction

Autonomous Underwater Vehicles (AUVs) have been invaluable tools for researching on marine environment. This class of underwater vehicles has proven its merit in a wide range of applications such as inspection, exploration, oceanography, biology, and so on [1]. The motion control of AUVs is challenging due to the nonlinear coupled terms, uncertain hydrodynamic parameters, and significant external disturbances. In addition to these, due to the consideration of weight, cost, and energy consumption, most AUVs are underactuated (i.e., they have fewer actuators than the number of degrees of freedom). Hence, one cannot control every state variable directly and the effects of disturbances on the uncontrollable variables are not easy to be compensated either [2]. Therefore, due to the low maneuverability character of underactuated AUVs, it is particularly important to investigate path following control. In the path following problem, the vehicle is regulated to follow a path in the absence of the temporal specifications. Typically, smoother convergence to a path can be achieved when path following strategies are used instead of trajectory tracking controllers, and the control inputs are less likely to reach saturation.

Backstepping Control (BSC) has provided a powerful tool to design controllers for underactuated AUVs by setting candidate Lyapunov functions, and later producing a stabilizing control law [3, 4]. However, there are some problems with the traditional BSC. One is the so-called "explosion of complexity", which results from tedious differential calculations of virtual controls, especially when the system order grows. In recent years, novel strategies introducing "command filters" have been used sometimes to deal with this problem [5, 6].

Another problem of the traditional BSC is that its robustness against the uncertainties requires further strength. As a common solution, Disturbance Observer- based (DO) controllers have been widely used for AUV control, where DO plays a key role. In references [7-9], different types of DOs were introduced to provide an adequate estimation for the dynamic uncertainties and external disturbances. In reference [10], a path following controller was designed for an underactuated AUV with the dynamic and velocity measurement uncertainties. The method consists of a DO-based kinematic controller and a linear-parameter-varying-based dynamic controller. In reference [11], an adaptive Extended State Observer (ESO) was proposed to estimate the unknown submarine velocity, parameter uncertainties and external disturbances for an AUV trajectory tracking problem. Guerrero et al. developed an adaptive DO based on the generalized super-twisting algorithm through ESO

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technique [12]. The developed DO was introduced into BSC and nonlinear proportional-derivative control laws. Although the DO-based control methods have better control performance, the design process is often more complicated, and/or the control scheme needs the adjustment of many controller gains, which can be time-consuming.

Compared to the conventional BSC, adaptive controllers are considered to be better for the systems exposed to uncertainties since they can enhance their performance with little or no information of the bounds on uncertainties. For instance, one can refer to [13], model-based output feedback control [14], and adaptive output feedback control based on the dynamic recurrent fuzzy neural network [15] for underactuated AUVs. In reference [16], a neural network and an adaptive compensator were used for the approximation of the unknown dynamics, and the compensation of the unknown effects like external disturbances and the reconstruction error of the neural network, respectively. However, these adaptive controllers impose intensive computational burden in the case of higher order systems and are effective only for constant or slowly-varying disturbances.

Unlike the above adaptive controllers, robust adaptive control approaches have shown the special characteristics in motion control of underactuated AUVs with uncertain dynamics and environmental disturbances. In reference [17], a robust adaptive controller was proposed by using Lyapunov direct method, BSC, and parameter projection techniques. A novel and adaptive dynamical SMC scheme was presented for the trajectory tracking control problem of an underactuated AUV in the presence of systematical uncertainty and environmental disturbances [18], where the robustness of the controller was enhanced by the combination of BSC and SMC. It used a virtual velocity variable to represent the attitude error in order to avoid the representation singularities and simplify the analytical expression of the control law. In reference [19], an adaptive robust path following controller was presented by integrating BSC and SMC, and fuzzy logic was used to deal with the problem of nonlinearity, uncertainties and external disturbances. Wang et al. proposed a robust adaptive controller based on the command filtered BSC for path following task, where a neuro-adaptive technique was employed to deal with the problem of parameter uncertainties and external disturbances [5].

The SMC-based control strategies suffer from the chattering phenomenon. As usual, this problem can be tackled by approximating discontinuous function by continuous terms [20], and by increasing the order of the sliding surfaces [21, 22]. As an alternative way, one can use intelligent control approaches like fuzzy logic and neural network control to estimate uncertainty items online to reduce system chattering [19, 23]. However, checking the stability of these intelligent controllers has been found to be very difficult [24]. In addition, the controllers based on neural networks or fuzzy systems greatly depend on the number of the neural network nodes or the number of the fuzzy rule bases, thereby resulting in more computational burden and online learning time. It should be mentioned that, in practice, a simpler controller with less computational burden is acceptable for its implementation [25].

Motivated by aforementioned considerations, in this paper a nonlinear robust control with a hierarchical structure is proposed for path following of an underactuated AUV on the horizontal plane exposed to the dynamic uncertainties and fast-time-varying external disturbances. First, we establish the path following error model based on the “virtual guidance” method, then we design the control law using BSC and SMC strategies with respect to the kinematic and dynamic models of the system. The disturbances of the ocean currents can vary considerably even on a small journey, making it difficult to obtain the bounds on uncertainties. In order to overcome this problem, a new DO is proposed in this paper that accurately estimates the whole effects of the uncertainties and includes the effects of them in the control inputs. Additionally, the chattering problem can be obviated with the presence of DO. Thus, the proposed controller can be designed without knowing the exact parameters of the dynamic model and the bounds on the uncertainties. Meanwhile, is relatively easy to be applied. The stability of the proposed controller is also discussed by Lyapunov stability criteria to demonstrate the ultimate boundedness of all the path following errors. Based on the stability analysis result, the characteristics of the closed-loop system and tuning guidelines of the control gains are addressed. The work presented here has several advantages over many techniques available in the literature such as [3, 5]. These advantages include easy derivation of the control law and low computational burden. To illustrate the effectiveness of the developed controller, simulation results for path following problem of an underactuated AUV on the horizontal plane are presented and discussed. The main contributions of this paper are as follows:

Unlike the conventional SMC, no knowledge of bounds on the parameter uncertainties and time-varying external disturbances is required.

Unlike the other approaches based on BSC, there is no need for analytical calculation or command filtering to obtain the derivative of virtual control.

The proposed controller has a strong robustness against the parameter uncertainties and time-varying external disturbances, and has characteristics such as simplicity and continuous control signals.

Stability of the overall system is proved.

The remainder of this paper is structured as follows. A brief introduction to the AUV dynamics is presented in section 2. Section 3 details the design of the proposed method. In section 4, the stability of the developed method is addressed by Lyapunov sense. Simulation results and a brief discussion of the proposed control system are presented in section 5 and section 6, respectively. Finally, the conclusions are given in section 7.

2- Problem Formulation

2- 1- Underactuated AUV Modelling

The motion equations of an underactuated AUV on the horizontal X-Y plane are presented in this section.

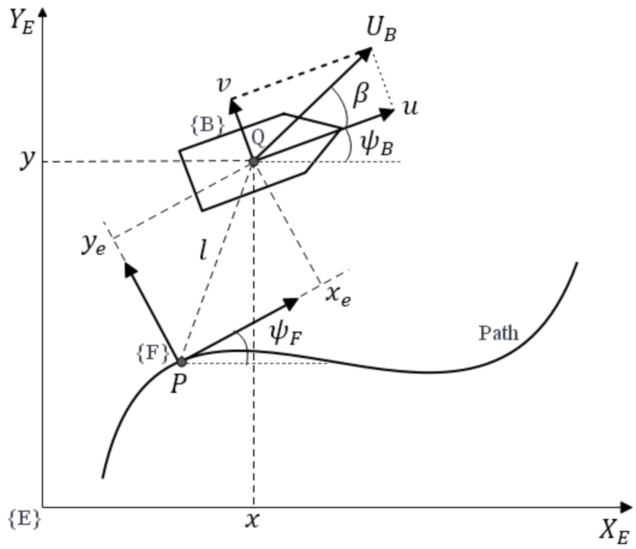


Fig. 1. Diagram of two-dimensional path following of the AUV.

The kinematic and dynamic equations for the AUV can be developed using an earth-fixed coordinate frame $\{E\}$ and a body-fixed coordinate frame $\{B\}$, as depicted in Fig. 1. By assuming that (I) is the Center of Mass (COM) of the vehicle is coincident with the origin of $\{B\}$, (II) the mass distribution is homogeneous, and (III) the hydrodynamic drag terms of order higher than two are negligible, the AUV model can be given as follows.

The AUV kinematic equations are [5]:

$$\begin{cases} \dot{x} = u\cos(\psi_B) - v\sin(\psi_B) \\ \dot{y} = u\sin(\psi_B) + v\cos(\psi_B) \\ \dot{\psi}_B = r \end{cases} \tag{1}$$

The AUV dynamic equations are [26]:

$$\begin{cases} \dot{u} = \frac{m_{22}}{m_{11}} vr - \frac{f_1}{m_{11}} u + \frac{\tau_u}{m_{11}} - \frac{D_u(t)}{m_{11}} \\ \dot{v} = -\frac{m_{11}}{m_{22}} ur - \frac{f_2}{m_{22}} v - \frac{D_v(t)}{m_{22}} \\ \dot{r} = \frac{m_{11} - m_{22}}{m_{33}} uv - \frac{f_3}{m_{33}} r + \frac{\tau_r}{m_{33}} - \frac{D_r(t)}{m_{33}} \end{cases} \tag{2}$$

where x and y denote coordinates of the AUV in $\{E\}$, and ψ_B is the yaw angle that parameterizes the rotation matrix from $\{B\}$ to $\{E\}$. u and v denote the surge (forward) and sway (lateral) velocities expressed in $\{B\}$, respectively. r denotes the angular velocity (yaw rate); the variables τ_u and τ_r represent the control force along the surge motion of the AUV, and the torque control that is applied in order to produce angular motion around the z_b axis of $\{B\}$, respectively; $D_i(t), i = u, v, r$ represent the external disturbances induced by ocean currents, waves and wind; The constants $f_i > 0, i = 1,2,3$ and $m_{ii}, i = 1,2,3$ represent the combined inertia and added mass terms. Note that since there is no actuator

for direct controlling the lateral motion, the AUV model is an underactuated dynamical system, and that u and r are the kinematic system inputs.

2- 2- AUV Path Following Error Dynamics

The path following error model in the horizontal plane is presented in this section [3] (See Fig. 1). In general, a path following controller should compute: (I) the distance between the COM of the vehicle Q, and the virtual guidance point P, on the path, and (II) the angle between the total velocity vector of the vehicle and the tangent to the path at P, making both close to zero. This intuitive explanation motivates the development of a kinematic model in terms of the Serret-Frenet (SF) coordinate frame $\{F\}$ that progresses along the path; $\{F\}$ plays the role of the body axis of the virtual guidance vehicle that should be followed by AUV. Using this set-up, the mentioned distance and angle form the coordinates of the path following error space where the control problem is formulated. Let $\psi_B^* = \psi_B + \beta$ be the angle of the total velocity vector, where $\beta = \text{atan}(v/u)$ is the drift (side-slip) angle, with the assumption that $|u| + |v| \neq 0$. Thus, the path following error coordinates can be defined as (x_e, y_e, ψ_e), where (x_e, y_e) are the coordinates of the vehicle in $\{F\}$, and $\psi_e = \psi_B^* - \psi_F$, where ψ_F is the angle of the tangent to the path at P.

In the path following problem, the path is parameterized by a scalar parameter s, which s may be defined as the arc length from a given point. In order to describe the path precisely, we define $c(s)$ as the path curvature at P. Afterwards, the velocity of P is $(d\boldsymbol{P}/dt)_F = (\dot{s},0,0)^T$ in $\{F\}$, and the angular velocity of $\{F\}$ can be expressed as $\dot{\boldsymbol{u}}_F = (0,0,c(s)\dot{s})^T$. It is also straightforward to compute the velocity of Q in $\{F\}$ as:

$$\boldsymbol{R}_E^F \left(\frac{d\boldsymbol{Q}}{dt}\right)_E = \left(\frac{d\boldsymbol{P}}{dt}\right)_F + \left(\frac{d\boldsymbol{l}}{dt}\right)_F + \left(\boldsymbol{\omega}_F \times \boldsymbol{l}\right). \quad (3)$$

where $\boldsymbol{l}$ denotes the vector from P to Q, $(d\boldsymbol{l}/dt)_F = (\dot{x}_e,\dot{y}_e,0)^T$, $(d\boldsymbol{Q}/dt)_E = (\dot{x},\dot{y},0)^T$, $(\dot{\boldsymbol{u}}_F \times \boldsymbol{l}) = (-c(s)\dot{s}y_e, c(s)\dot{s}x_e, 0)$, and $\boldsymbol{R}_E^F$ is the rotation matrix from $\{E\}$ to $\{F\}$:

$$\boldsymbol{R}_E^F = \begin{bmatrix} \cos(\psi_F) & \sin(\psi_F) & 0 \\ -\sin(\psi_F) & \cos(\psi_F) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Equation (3) can be rewritten as:

$$\boldsymbol{R}_E^F \begin{bmatrix} \dot{x} \\ \dot{y} \\ 0 \end{bmatrix} = \begin{bmatrix} \dot{s}\left(1-c(s)y_e\right)+\dot{x}_e \\ c(s)\dot{s}x_e + \dot{y}_e \\ 0 \end{bmatrix} \quad (4)$$

Let $U_B = \sqrt{u^2+v^2}$ be the total velocity. Later, we rewrite the kinematic Eq. (1) with respect to U_B as:

$$\begin{cases} \dot{x} = U_B \cos(\psi_B^*) \\ \dot{y} = U_B \sin(\psi_B^*) \\ \dot{\psi}_B^* = r + \dot{\beta} \end{cases} \quad (5)$$

Hence, the path following error dynamic model can be obtained using Eq. (4) and Eq. (5) as:

$$\begin{cases} \dot{x}_e = \dot{s}\left(c(s)y_e - 1\right) + U_B \cos(\psi_e) \\ \dot{y}_e = -c(s)\dot{s}x_e + U_B \sin(\psi_e) \\ \dot{\psi}_e = r + \dot{\beta} - c(s)\dot{s} \end{cases} \quad (6)$$

Remark 1: The term $\dot{\beta}$ in the third relation of Eq. (6) is the hiding acceleration ($\dot{u}$ and $\dot{v}$) terms which can be obtained by differentiating or direct measuring. The former may accentuate high-frequency noises and the latter increases the costs. Here, we obtain ψ_e in two steps to bypass these issues: first by integrating the term $r - c(s)\dot{s}$, then by adding β to the integrated term.

2- 3- Control Objective

In this paper, the control objective can be expressed as follows: Consider the vehicle's model with the kinematic and dynamic equations given by Eq. (1) and Eq. (2). Given a path (parameterized in terms of its length) to be followed and a desired profile for the forward velocity, $0 < u_d$. Derive a feedback control law for the thrusting force τ_u, the heading torque τ_r, and the rate of progression $\dot{s}$, of the virtual guidance point P, along the path so the path following error variables x_e, y_e, ψ_e, and $u - u_d$ converge to a neighborhood around the origin that can be made arbitrarily small in the presence of the dynamic uncertainties and fast-time-varying external disturbances,.

3- Path Following Controller Design

This section introduces a nonlinear robust double-closed-loop strategy to steer the dynamic model of the AUV described by Eq. (1) and Eq. (2) along a desired path. The first step in the proposed scheme with a specific hierarchical structure is related to the design of a virtual control input using BSC technique in the outer closed-loop that ensures the path following errors converge to zero. Afterwards, the true control inputs are built on the virtual control derived using the DO-based SMC approach in the inner closed-loop.

3- 1- Kinematic Control

First, by considering the path following error dynamics in Eq. (6), the kinematic controller is designed to compute the progression rate $\dot{s}$ of the virtual guidance point P, along the path and the yaw rate r, as the virtual control inputs. The kinematic control is synthesized using conventional BSC. In the kinematic design, it is conventional to assume that the actual surge velocity u is equal to the desired surge velocity u_d [3] in order to allow the system to be considered as autonomous.

3- 1- 1- Position Control

Consider the candidate Lyapunov function V_1 as:

$$V_1 = \frac{1}{2}\left(x_e^2 + y_e^2\right) \quad (7)$$

Differentiating V_1 with respect to time and using Eq. (6) yields:

$$\dot{V}_1 = x_e\left(U_B\cos(\psi_e) - \dot{s}\right) + y_e U_B \sin(\psi_e) \tag{8}$$

Consider the progression rate $\dot{s}$ of the virtual guidance point P, along the path and the desired approach angle ψ_c as the virtual control values:

$$\begin{aligned} &\dot{s} = U_B\cos(\psi_c) + K_x x_e \\ &\psi_c(y_e) = -\lambda_y \frac{e^{2K_y y_e}-1}{e^{2K_y y_e}+1},\ 0<\lambda_y<\frac{\pi}{2} \end{aligned} \tag{9}$$

where K_x and K_y are the positive constants that will be selected later. The desired approach angle, ψ_c, is useful in shaping the transient response during the path approach phase [3].

Let suppose that $\psi_e = \psi_c$, since $y_e\psi_c \le 0$ for all y_e, then substituting Eq. (9) in Eq. (8) yields:

$$\dot{V}_1 = -K_x x_e^2 - y_e U_B \sin\left(\lambda_y \frac{e^{2K_y y_e}-1}{e^{2K_y y_e}+1}\right) \le 0 \tag{10}$$

Therefore, the system can be asymptotically stabilized using Eq. (9) if $\lim_{t\to\infty}\psi_e = \psi_c$.

3- 1- 2- Attitude Control

By considering that ψ_c is not a true control, we have to introduce the angular error variable $\tilde{\psi} = \psi_e - \psi_c$ and try to stabilize it. Define the candidate Lyapunov function V_2 as:

$$V_2 = \frac{1}{2}\tilde{\psi}^2 \tag{11}$$

Differentiating V_2 along with Eq. (6) yields:

$$\dot{V}_2 = \tilde{\psi}\left(r + \dot{\beta} - c(s)\dot{s} - \dot{\psi}_c\right) \tag{12}$$

By considering the yaw rate r as the virtual control input, its desired value r_c can be given by:

$$r_c = -\dot{\beta} + c(s)\dot{s} + \dot{\psi}_c - K_\psi\tilde{\psi} \tag{13}$$

where K_ψ is a positive constant to be selected later. Later, substituting Eq. (13) in Eq. (12) yields:

$$\dot{V}_2 = -K_\psi\tilde{\psi}^2 + \tilde{\psi} r_e \tag{14}$$

where $r_e = r - r_c$. Therefore, it can be concluded that the system asymptotically follows the desired approach angle if $\lim_{t\to\infty} r = r_c$.

Remark 2: It should be noticed that Eq. (13) appears in a noncausal form. In fact, the term $\dot{\beta}$ contains acceleration terms, and through them a loop are formed that makes r dependent on itself. An approach is suggested by Lapierre and Soetanto where the dynamic model is used to yield an algebraic solution for r [3]. However, considering a dynamic model to estimate system accelerations can be problematic in the case of uncertain dynamics. Here, we obtain $\dot{\beta}$ by passing β through a high-pass filter to avoid this problem, effectively simplifying the expression of the kinematic control r_c.

3- 2- Dynamic Control

The feedback control laws in the first equation of Eq. (9) and Eq. (13) are only applied to the kinematic model of the vehicle. Here these control laws will be extended to deal with the AUV's dynamics. In the kinematic design, the total velocity of the AUV was left free, but implicitly dependent on the desired surge velocity u_d. In the dynamic design, the surge velocity u will be explicitly taken into account. Notice that the AUV's yaw rate r was supposed to be a true control input. This assumption is removed here by considering the dynamics of the AUV. In this section, the true control inputs of the AUV will be derived so that $u - u_d$ and $r - r_c$ get close to zero. Here SMC along with the DO is introduced for the dynamic control law. The key idea of this section is to estimate the whole uncertainties by DO, and to later use the estimated values in the true control inputs to negate the effect of the uncertainties.

3- 2- 1- Sliding Mode Control

In the sliding mode approach, suitable sliding surfaces of the desired dynamics are defined, and the control laws are derived that sliding conditions are always satisfied [19]. This makes the system insensitive to uncertainties and to behave according to the definition of the sliding surfaces. In this section, the sliding surfaces are defined and the control law is derived.

We define the sliding surfaces as:

$$S_i = e_i(t),\ i = u, r \tag{15}$$

where $e_u = u - u_d$, $e_r = r - r_c$. Differentiating Eq. (15) with respect to the time and using Eq. (2) yields:

$$\dot{S}_i = a_i + b_i \left(\tau_i - D_i\left(t\right)\right) - \alpha_i,\; i = u, r \tag{16}$$

where $\alpha_u = \dot{u}_d$, $\alpha_r = \dot{r}_c$, and $a_i, b_i, i = u, r$ are defined to shorten the equations as:

$$\begin{cases} a_u = \dfrac{m_{22}}{m_{11}} vr - \dfrac{f_1}{m_{11}} u,\; b_u = \dfrac{1}{m_{11}} \\ a_r = \dfrac{m_{11} - m_{22}}{m_{33}} uv - \dfrac{f_3}{m_{33}} r,\; b_r = \dfrac{1}{m_{33}} \end{cases} \tag{17}$$

We rewrite Eq. (16) with respect to the known and unknown components:

$$\dot{S}_i = \hat{a}_i + \hat{b}_i \tau_i + d_i,\; i = u, r \tag{18}$$

Where:

$$\begin{cases} a_i = \hat{a}_i + \Delta a_i \\ b_i = \hat{b}_i + \Delta b_i \end{cases},\; i = u, r \tag{19}$$

where $\hat{a}_i, \hat{b}_i, i = u, r$ are the nominal constant values, and $\Delta a_i, \Delta b_i, i = u, r$ are the additive uncertainties with unknown bounds in the dynamic equations. $d_i, i = u, r$ are terms due to the uncertainties and external disturbances (and also the acceleration commands) that will be named the lumped uncertainties henceforth:

$$d_i = \Delta a_i + \Delta b_i \left(\tau_i - D_i\left(t\right)\right) - \hat{b}_i D_i\left(t\right) - \alpha_i,\; i = u, r \tag{20}$$

Remark 3: Notice that the acceleration command $\dot{r}_c$ is incorporated in the lumped uncertainty term and so its effect will be included in the true control input by DO, and thereby, obviating the need for analytical operations or command filtering.

The true control inputs to be designed are composed of two components, τ_i^{eq} and τ_i^n. The former is used for compensating the known terms, and the latter is used for compensating the lumped uncertainty term in the dynamics of the sliding surfaces:

$$\tau_i = \tau_i^{eq} + \tau_i^n,\; i = u, r \tag{21}$$

With:

$$\tau_i^{eq} = -\frac{1}{\hat{b}_i}\left(\hat{a}_i + K_i S_i\right),\; i = u, r \tag{22}$$

$$\tau_i^n = -\frac{1}{\hat{b}_i} \hat{d}_i,\; i = u, r \tag{23}$$

where $\hat{d}_i, i = u, and\ r$ are the estimations of the lumped uncertainty terms by DO, and $K_i, i = u, r$ are positive constants to be selected later. The procedure to estimate the lumped uncertainties by DO is described in the next subsection. Substituting Eq. (21) and Eq. (22) in Eq. (18) yields:

$$\dot{S}_i = -K_i S_i + \hat{b}_i \tau_i^n + d_i,\; i = u, r \tag{24}$$

Equation (24) will be used in designing of DO.

Let $\tilde{d}_i = d_i - \hat{d}_i, i = u, r$ be estimation errors. Afterwards, substituting Eq. (24) in Eq. (23) yields the dynamics of the sliding surfaces excited by the estimation errors:

$$\dot{S}_i = -K_i S_i + \tilde{d}_i,\; i = u, r \tag{25}$$

Equation (25) will be used in the stability analysis.

If DO behaves so that the estimation errors tend to zero, the sliding surfaces will tend to zero, thereby the AUV will be placed on the target path and will progress with the desired surge velocity in spite of the parameter uncertainties and external disturbances. Next, we will design the DO for estimating the lumped uncertainties so that the estimation errors tend to zero.

3- 2- 2- Disturbance Observer

The DO designed here is a modified version developed by Chen et al. [27]. The estimations of the lumped uncertainties can be expressed as:

$$\hat{d}_i = \gamma_i\left(t\right) + p_i\left(S_i\right),\; i = u, r \tag{26}$$

where $p_i\left(S_i\right), i = u, r$ are linear or nonlinear scalar functions of the sliding surfaces. Now, the auxiliary variables, $\gamma_i\left(t\right), i = u, r$, have to be updated in such a way that the estimation errors, $\tilde{d}_i, i = u, r$, tend to zero. Differentiating Eq. (26) yields:

$$\dot{\hat{d}}_i = \dot{\gamma}_i(t) + \frac{\partial p_i}{\partial S_i}\dot{S}_i,\ i = u, r \quad (27)$$

where $\partial p_i / \partial S_i, i = u, r$ are called the DO gains. We substitute Eq. (24) in Eq. (27) to yield:

$$\dot{\hat{d}}_i = \dot{\gamma}_i(t) + \frac{\partial p_i}{\partial S_i}\left(-K_i S_i + \hat{b}_i \tau_i^n + d_i\right),\ i = u, r \quad (28)$$

As mentioned, the auxiliary variables have to be updated in such a way that the estimation error dynamics are stable. Therefore, the update law for $\gamma_i(t), i = u, r$ can be suggested as:

$$\dot{\gamma}_i(t) = -\frac{\partial p_i}{\partial S_i}\left(-K_i S_i + \hat{b}_i \tau_i^n + \hat{d}_i\right),\ i = u, r \quad (29)$$

Substituting Eq. (29) in Eq. (28) yields:

$$\dot{\hat{d}}_i(t) = \frac{\partial p_i}{\partial S_i}\tilde{d}_i,\ i = u, r \quad (30)$$

We subtract both sides of Eq. (30) from $\dot{d}_i, i = u, r$ to obtain the estimation error dynamics excited by the rates of the lumped uncertainties:

$$\dot{\tilde{d}}_i(t) = -\frac{\partial p_i}{\partial S_i}\tilde{d}_i + \dot{d}_i,\ i = u, r \quad (31)$$

Equation (31) recommends that for stability of the estimation errors, $\tilde{d}_i, i = u, r$, the choice of $p_i(S_i), i = u, r$ have to be so that $\partial p_i / \partial S_i, i = u, r$ are positive functions. In addition, from this equation, it can be found that for the estimation errors to be bounded, it is necessary to make the following assumption.

Assumption 1: The values of the lumped uncertainties can be arbitrarily large, but their rates are bounded:

$$\left|\dot{d}_i\right| < \mu_i,\ i = u, r \quad (32)$$

where $\mu_i, i = u, r$ are positive constants.

4- Stability Analysis

In this section, the stability conditions are discussed and the ultimate bounds on the relevant variables are obtained. Consider candidate Lyapunov function V_3 as:

$$V_3 = \frac{1}{2}\left(S_u^2 + \tilde{d}_u^2\right) + \frac{1}{2}\left(S_r^2 + \tilde{d}_r^2\right) + V_2 \quad (33)$$

Differentiating V_3 along with Eq. (14), Eq. (25) and Eq. (31) yields:

$$\begin{aligned}\dot{V}_3 = & -K_u S_u^2 + S_u \tilde{d}_u - \frac{\partial p_u}{\partial S_u}\tilde{d}_u^2 + \tilde{d}_u \dot{d}_u \\ & -K_r S_r^2 + S_r \tilde{d}_r - \frac{\partial p_r}{\partial S_r}\tilde{d}_r^2 + \tilde{d}_r \dot{d}_r - K_\psi \tilde{\psi}^2 + S_r \tilde{\psi}\end{aligned} \quad (34)$$

By using Young's inequality, $AB \le \frac{1}{2}\left(A^2 + B^2\right)$ and Eq. (32) we get:

$$\begin{aligned}\dot{V}_3 \le & -\left(K_u - \frac{1}{2}\right)S_u^2 - \left(\frac{\partial p_u}{\partial S_u} - \frac{1}{2}\right)\tilde{d}_u^2 + \left|\tilde{d}_u\right|\mu_u \\ & -\left(K_r - 1\right)S_r^2 - \left(\frac{\partial p_r}{\partial S_r} - \frac{1}{2}\right)\tilde{d}_r^2 - \left(K_\psi - \frac{1}{2}\right)\tilde{\psi}^2 + \left|\tilde{d}_r\right|\mu_r\end{aligned} \quad (35)$$

To make ensure that the whole system is stable, the control parameters and DO gains can always be selected as:

$$\begin{aligned}& K_u - \frac{1}{2} > 0,\ K_r - 1 > 0, \frac{\partial p_u}{\partial S_u} - \frac{1}{2} > 0, \\ & \frac{\partial p_r}{\partial S_r} - \frac{1}{2} > 0,\ K_\psi - \frac{1}{2} > 0\end{aligned} \quad (36)$$

From Eq. (35), it can be concluded that the dynamics of the estimation errors, $\tilde{d}_i, i = u, r$, the sliding surfaces, $S_i, i = u, r$, and the angular error variable $\tilde{\psi}$ are not asymptotically stable but can be ultimately bounded in the sense of Corless and Leitman [28], provided that the control parameters are adopted appropriately by satisfying Eq. (36). By avoiding the details of derivation, the ultimate bounds on the mentioned variables can be given as:

$$\left|\tilde{d}_i\right| \le \frac{\mu_i}{\partial p_i / \partial S_i},\ i = u, r \quad (37)$$

$$\left|S_i\right| \le \frac{\mu_i}{\left(\partial p_i / \partial S_i\right)K_i},\ i = u, r \quad (38)$$

Table 1. Simulation conditions for performance evaluation of the controllers.

Condition	Description	Time period (s)
(1)	no parametric uncertainties and external disturbances	$0 \le t < 60$
(2)	30% parametric uncertainties and constant external disturbances	$60 \le t < 120$
(3)	30% parametric uncertainties and time-varying external disturbances	$120 \le t < 200$

Table 2. The control parameters.

$K_x = 2.2$	$K_y = 1.1$	$K_\psi = 20$	$\partial p_u / \partial S_u = 300$
$\lambda_y = \pi / 3$	$K_u = 1$	$K_r = 1.2$	$\partial p_r / \partial S_r = 300$

$$|\tilde{\psi}| \le \frac{\mu_r}{(\partial p_r / \partial S_r) K_r K_\psi} \quad (39)$$

Additionally, it is straightforward to calculate the ultimate bounds on the path following errors as:

$$|\psi_e| \le \frac{2\mu_r}{(\partial p_r / \partial S_r) K_r K_\psi} \quad (40)$$

$$|y_e| \le \frac{\mu_r}{(\partial p_r / \partial S_r) \lambda_y K_r K_\psi K_y} \quad (41)$$

$$|x_e| \le \frac{\mu_r U_B |c(s)|}{(\partial p_r / \partial S_r) \lambda_y K_r K_\psi K_y K_x} \quad (42)$$

Thus, we conclude that the above variables are ultimately bounded and their ultimate bounds can be made arbitrarily small by appropriately selecting the control parameters.

5- Simulation Results

In this section, to evaluate the control performance of the proposed controller, numerical simulation of the closed loop system is carried out under three different conditions as given in Table (1), and a comparison of the controller is done with conventional BSC [3]. The model is represented in Eq. (1) and Eq. (2), and the model parameters can be found in [26]. The planar target path is parameterized as:

$$\begin{cases} x_d(s) = 50\sin(s/50) \\ y_d(s) = 50\cos(s/50) \end{cases}$$

where s is updated using Eq. (9). The desired surge velocity is $u_d = 1m/s$, $p_i(S_i) = (\partial p_i / \partial S_i) S_i, i = u, r$, and the controller design parameters are listed in Table (2). The initial position, attitude angle, and the initial velocity of AUV are given by:

$$(x(0), y(0), \psi(0)) = (25m, 25m, 0rad)$$

$$(u(0), v(0), r(0)) = (0.1m/s, 0m/s, 0rad/s)$$

We suppose there are inaccuracies of the order of 30% in all the AUV's hydrodynamic parameters. In addition, the effect of the constant external disturbances is selected as $D_u = 50N, D_v = 50N, D_r = 40Nm$, and the fast-time-varying external disturbances is considered as follows:

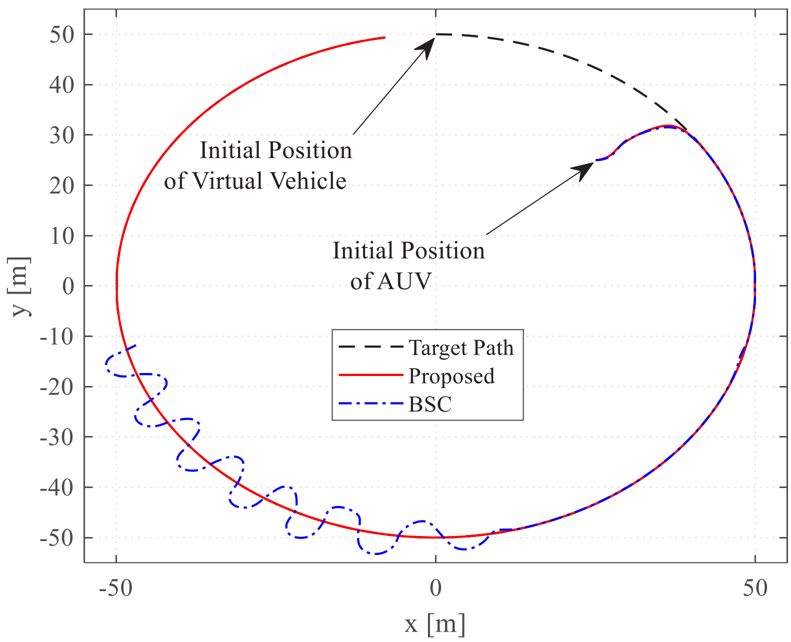


Fig. 2. Path following of the AUV in XY-plane.

$$\begin{cases} D_u(t) = 35sin(0.4\pi t)N \\ D_v(t) = 35sin(0.4\pi t)N \\ D_r(t) = 20sin(0.2\pi t)cos(0.1\pi t)Nm \end{cases}$$

The simulation results given by the conventional BSC and the proposed controller schemes are shown in Fig. 2 to Fig. 6. Fig. 2 and Fig. 3 show the path following performance under all the condition described in Table 1. It can be seen that the proposed controller succeeded in following the position and orientation of the virtual vehicle under all the conditions, although the conventional BSC can only exactly follow the path under the ideal condition (condition (1)). However, under the uncertain condition with constant external disturbances, conventional BSC produces a slightly steady state error in position and orientation following, whereas the performance gets degraded greatly in the presence of fast-time-varying external disturbances. The time history of the true control inputs during path following control can be observed in Fig. 5.

As can be seen from the simulation results, under the guidance of the proposed controller, the AUV can almost accurately complete the planar path following in the presence of the model perturbations and external disturbances. As shown in Fig. 3 and Fig. 4, the path following errors, x_e, y_e, ψ_e, and the surge velocity error, $u - u_d$ eventually converge to a small area around the origin under the proposed controller. This is equivalent to state that: (I) the AUV's COM approaches the position of the virtual target, and (II) the AUV moves with the desired forward velocity along the path. Moreover, the control law developed is not sensitive to the parameter uncertainties and external disturbances, which indicates that the proposed controller offers strong robustness to the uncertainties in hydrodynamic parameters and constant as well as fast-time-varying external disturbances.

6- Discussion

Fig. 6 shows that under uncertain condition with constant external disturbances (condition (2)) the estimation errors are null (after the transient phases). This can be inferred from Eq. (31) where the estimation error dynamics are excited by the rate of the lumped uncertainties. This means in the case of constant or slowly-varying disturbances (i.e. ($\dot{d}_i \approx 0$)), the DO is able to estimate the lumped uncertainty almost accurately and the path following errors asymptotically converge to zero. The disturbances induced by the waves and the wind are rarely constant, but the fast-varying sinusoidal type is a common model to include the effects of environmental disturbances in the motion control problems of the AUVs [7, 29].

7- Conclusions

In this paper, a nonlinear robust controller using BSC and a new DO-based SMC are proposed for the horizontal plane path following of an underactuated AUV. The path following

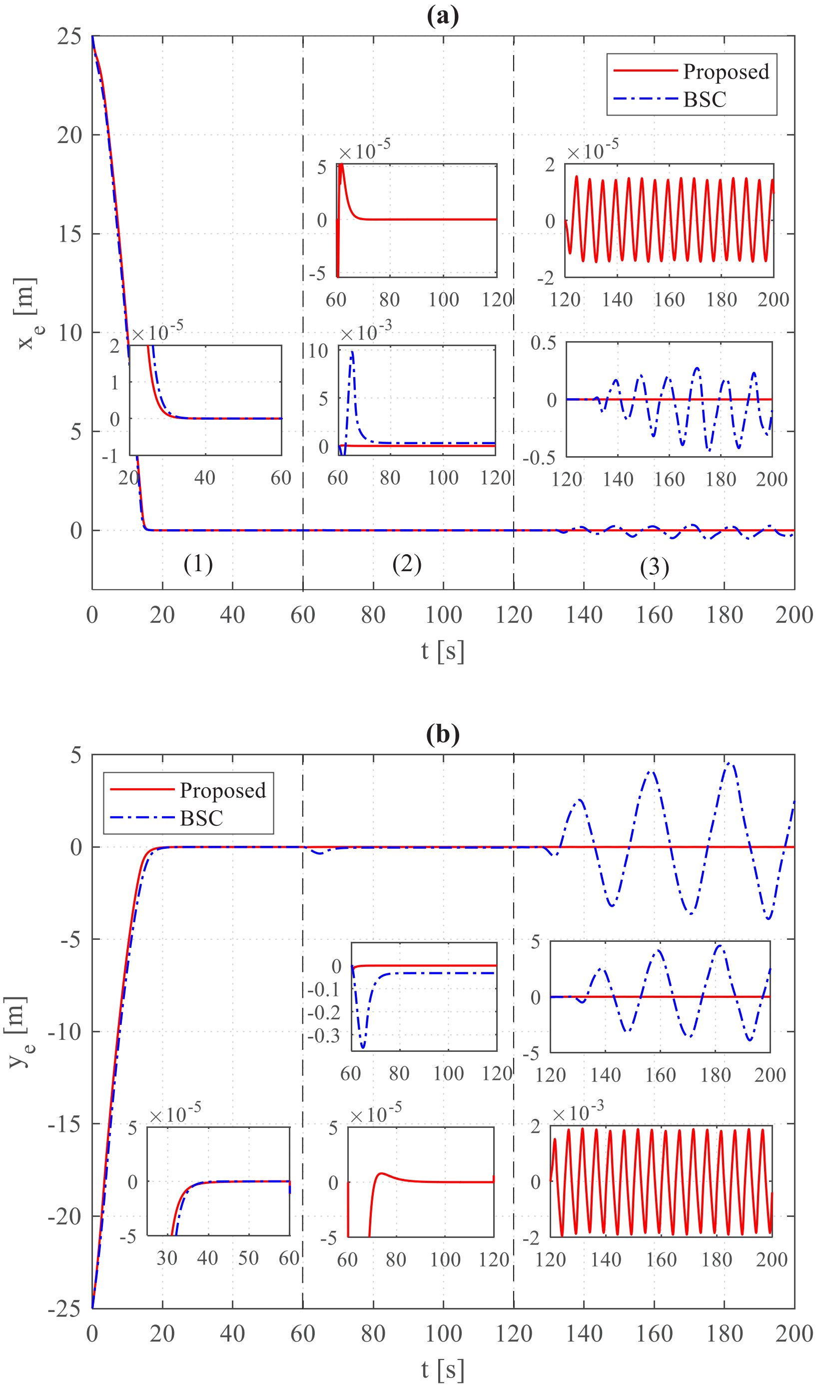


Fig. 3. Path following errors of the AUV: (a) x_e, (b) y_e and (c) ψ_e. (Continude)

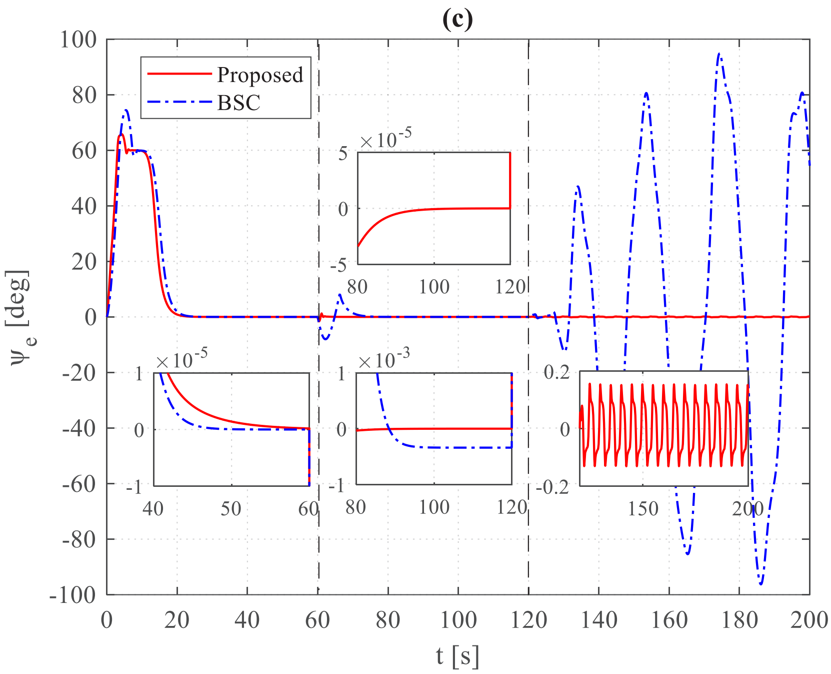


Fig. 3. Path following errors of the AUV: (a) x_e, (b) y_e and (c) ψ_e.

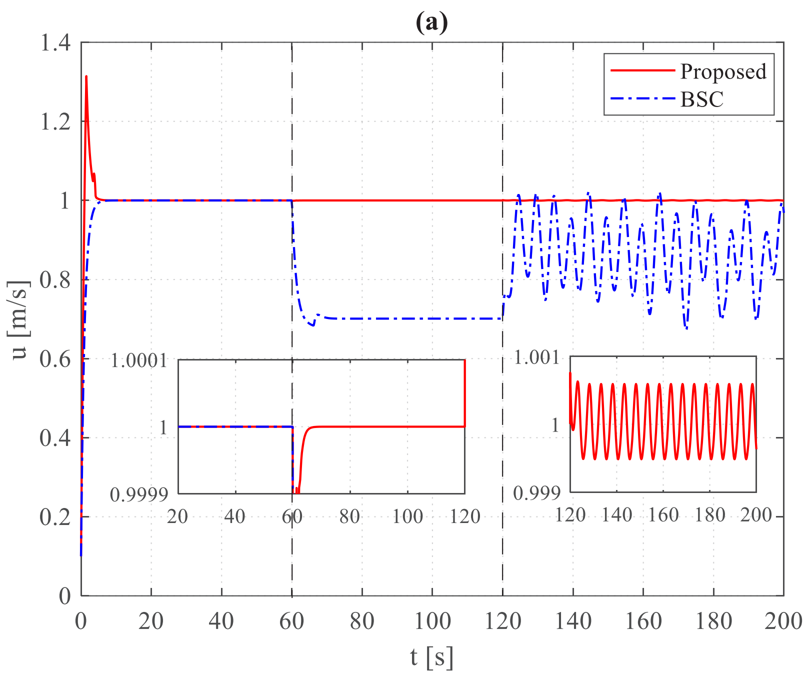
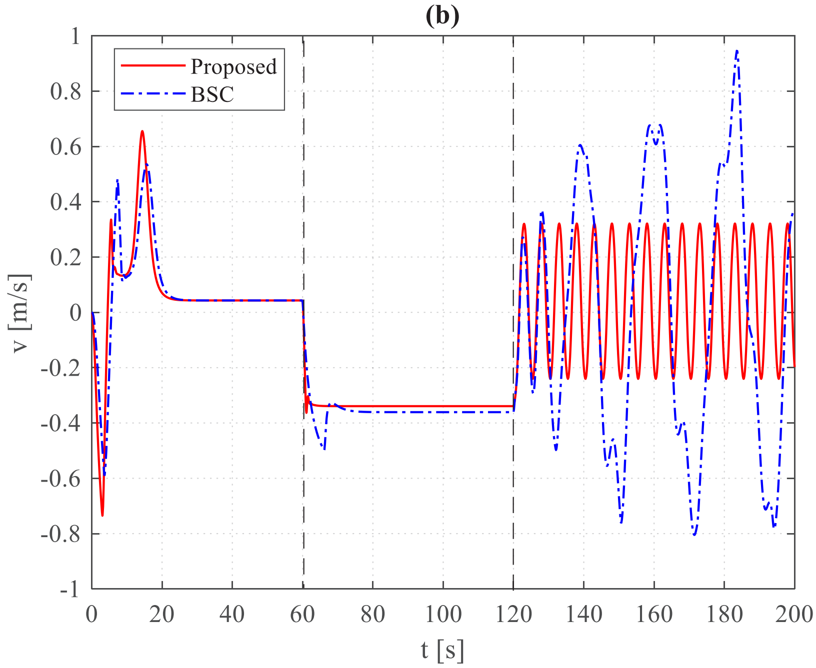


Fig. 4. Velocity response curves of the AUV: (a) surge velocity, (b) sway velocity and (c) yaw rate. (Continude)



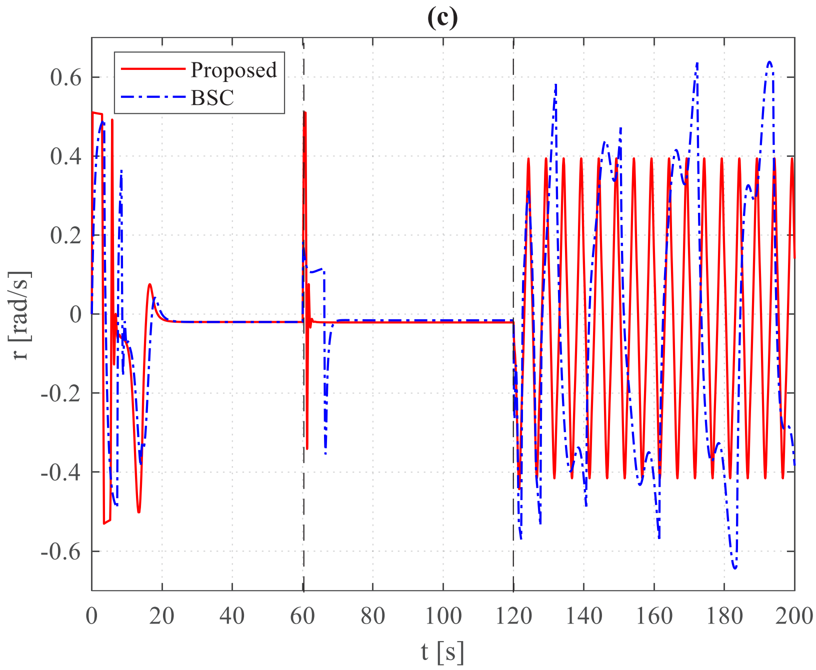


Fig. 4. Velocity response curves of the AUV: (a) surge velocity, (b) sway velocity and (c) yaw rate.

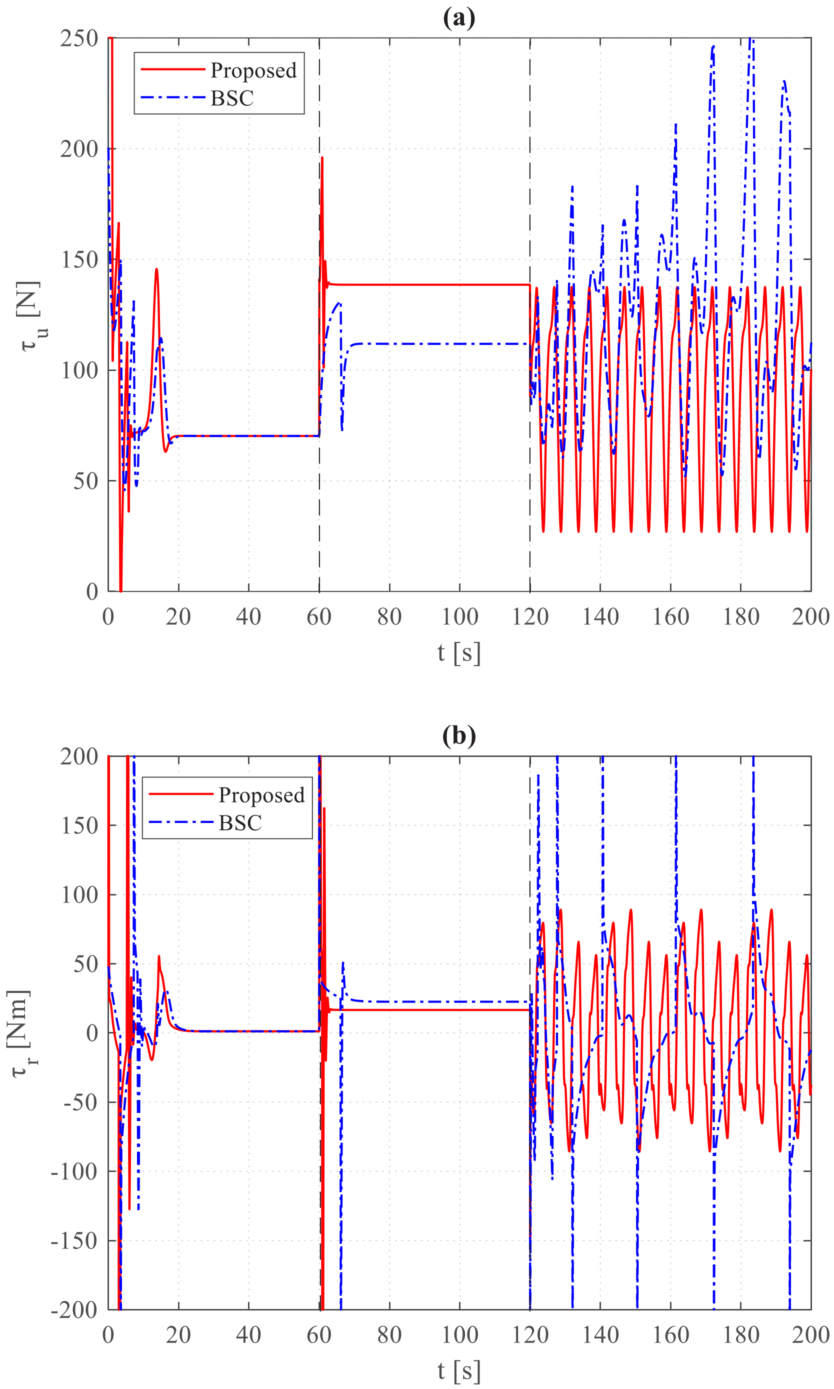


Fig. 5. Control inputs of the AUV: (a) force and (b) moment.

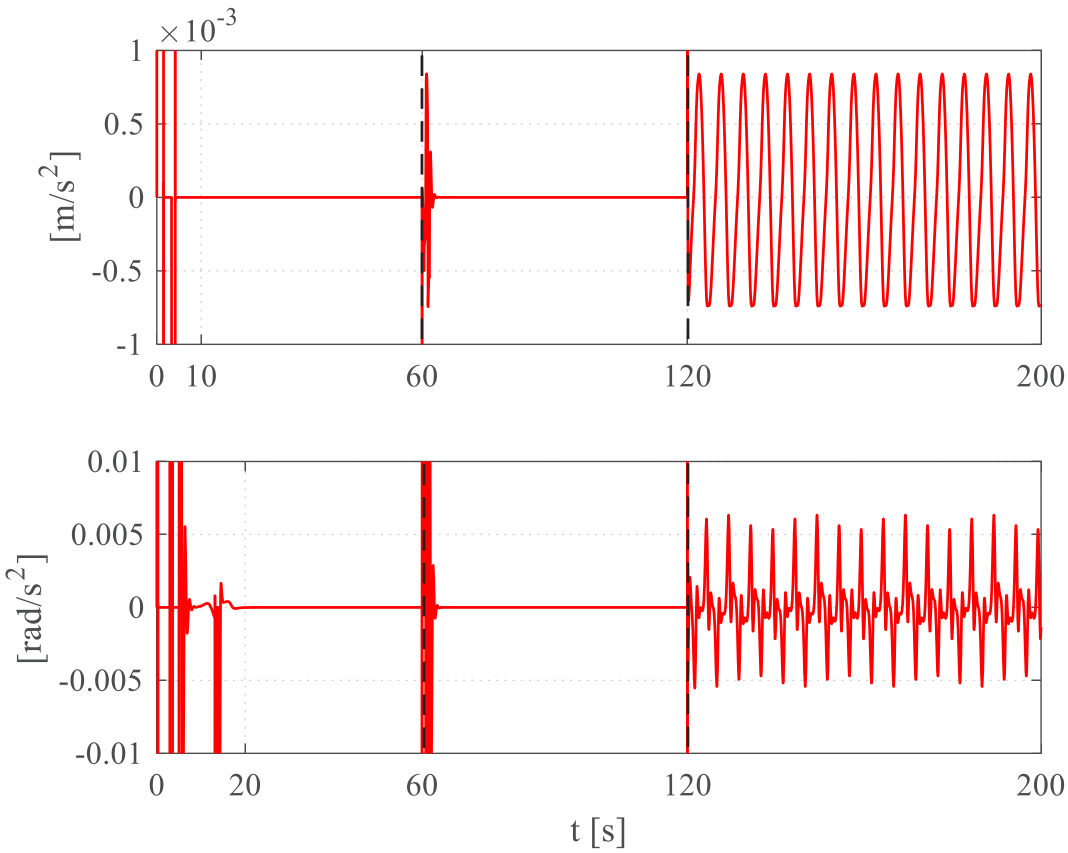


Fig. 6. Estimation errors.

error model of the AUV in SF coordinate frame is established based on virtual guidance method. The control scheme is made up of a double-looped structure: a kinematic controller and a dynamic controller. First, a kinematic controller is designed based on BSC to guarantee the position errors to go close to zero. Afterwards, a dynamic controller is developed using the DO-based SMC by considering the output of the kinematic controller as a reference velocity command. By using the designed kinematic and dynamic controllers, the stability of the whole closed-loop cascaded system is proved by Lyapunov stability criteria. The performance of the presented control scheme is validated by the computer simulations under different conditions and compared with conventional BSC. The proposed controller greatly reduces the complexities of the control law derivations in the approaches based on the traditional BSC and meanwhile ensures rigorous robustness of the scheme thanks to its simple and effective compensation using the DO.

Future research will address the extension of these results by designing a path following controller for three-dimensional maneuvers.

Declaration of conflicting interest

The authors declare that there is no conflict of interest.

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Nomenclature

AUV	Autonomous Underwater Vehicle
BSC	Backstepping Control
COM	Center of Mass
DO	Disturbance Observer
ESO	Extended State Observer
SMC	Sliding Mode Control
$\{B\}$	body-fixed frame
$\{E\}$	earth-fixed frame
$\{F\}$	Serret-Frenet frame
u,v	surge and sway velocities in $\{B\}$
U_B	AUV's net velocity
r	AUV's yaw rate
(x,y)	AUV's coordinates in $\{E\}$
ψ_B	AUV's yaw angle
$f_i(t), i=1,2,3$	unknown nonlinear dynamics of the AUV
$D_i(t), i=u,v,r$	external disturbances
τ_u,τ_r	control inputs
$m_{ii}, i=1,2,3$	combined inertia and added mass
Q	AUV's center of mass
P	virtual guidance point
(x_e,y_e)	AUV's coordinates in $\{F\}$
ψ_e	yaw angular error
ψ_F	desired angle
ψ_B^*	Angle of the total velocity vector
β	drift angle
s	arc length
$c(s)$	path curvature at P
$\dot{s}$	update rate of P along the path
$\boldsymbol{\omega}_F$	angular velocity of $\{F\}$
$\boldsymbol{l}$	vector from P to Q
$\boldsymbol{R}$	rotation matrix from $\{E\}$ to $\{F\}$
u_d	desired surge velocity

V_1, V_2, V_3	Lyapunov functions
ψ_c	desired approach angle
$K_x, K_y, K_\psi, K_u, K_r$	control gains
λ_y	positive design parameter
$\tilde{\psi}$	angular error variable
r_c	desired yaw rate
r_e	yaw rate error
t	time
$S_i, i = u, r$	sliding surfaces
$c_i, i = u, r$	positive constants
a_i, b_i	dynamic terms
$\alpha_i, i = u, r$	acceleration commands
$\hat{a}_i, \hat{b}_i, i = u, r$	nominal values of dynamic terms
$\Delta a_i, \Delta b_i, i = u, r$	additive uncertainties of dynamic terms
$d_i, i = u, r$	lumped uncertainties
$\hat{d}_i, i = u, r$	estimation of lumped uncertainties
$\tau_i^{eq}, \tau_i^n, i = u, r$	components of actual controls
$\tilde{d}_i, i = u, r$	estimation errors
$\gamma_i(t), i = u, r$	auxiliary variables
$p_i(S_i), i = u, r$	functions of the sliding surfaces
$\partial p_i / \partial S_i, i = u, r$	DO gains
$\mu_i, i = u, r$	bounds of the lumped uncertainties' rate

References

[1] Z. Peng, J. Wang, J. Wang, Constrained control of autonomous underwater vehicles based on command optimization and disturbance estimation, IEEE Transactions on Industrial Electronics, 66(5) (2019) 3627-3635.

[2] H. Khalil, Nonlinear systems, third ed. New Jersey: Prentice Hall, 2002.

[3] L. Lapierre, D. Soetanto, Nonlinear path-following control of an AUV, Ocean Engineering, 34(11-12) (2007) 1734-1744.

[4] X. Liang, X. Qu, Y. Hou, J. Zhang, Three-dimensional path following control of underactuated autonomous underwater vehicle based on damping backstepping, International Journal of Advanced Robotic Systems, 14(4) (2017) 1-9.

[5] J. Wang, C. Wang, Y. Wei, C. Zhang, Three-Dimensional Path Following of an Underactuated AUV Based on Neuro-Adaptive Command Filtered Backstepping Control, IEEE Access, 6 (2018) 74355-74365.

[6] Z. Zheng, Y. Huang, L. Xie, B. Zhu, Adaptive Trajectory Tracking Control of a Fully Actuated Surface Vessel With Asymmetrically Constrained Input and Output, IEEE Transactions on Control Systems Technology, 26(5) (2018) 1851-1859.

[7] S. Liu, Y. Liu, N. Wang, Nonlinear disturbance observer-based backstepping finite-time sliding mode tracking control of underwater vehicles with system uncertainties

and external disturbances, Nonlinear Dynamics, 88 (2017) 465–476.

[8] J. Miao, S. Wang, Z. Zhao, Y. Li, M. Tomovic, Spatial curvilinear path following control of underactuated AUV with multiple uncertainties, ISA Transactions, 67 (2017) 107-130.

[9] Z. Yan, Y. Liu, J. Zhou, D. Wu, Path Following Control of an AUV under the Current Using the SVR-ADRC, Journal of Applied Mathematics, 2014 (2014) 1-12.

[10] Y. Zhang, X. Wang, S. Wang, J. Miao, DO-LPV-based robust 3D path following control of underactuated autonomous underwater vehicle with multiple uncertainties, ISA Transactions, 101 (2020) 189-203.

[11] R. Cui, L. Chen, C. Yang, M. Chen, Extended State Observer-Based Integral Sliding Mode Control for an Underwater Robot With Unknown Disturbances and Uncertain Nonlinearities, IEEE Transactions on Industrial Electronics, 64(8) (2017) 6785-6795.

[12] J. Guerrero, J. Torres, V. Creuze, A. Chemori, Adaptive disturbance observer for trajectory tracking control of underwater vehicles, Ocean Engineering, 200 (2020).

[13] B. Subudhi, K. Mukherjee, S. Ghosh, A static output feedback control design for path following of autonomous underwater vehicle in vertical plane, Ocean Engineering, 63 (2013) 72–76.

[14] J. Refsnes, A. Sorensen, K. Pettersen, Model-based output feedback control of slender-body underactuated AUVs: theory and experiments, IEEE Transactions on Control Systems Technology, 16(5) (2008) 930-946.

[15] L. Zhang, X. Qi, Y. Pang, Adaptive output feedback control based on DRFNN for AUV, Ocean Engineering, 36(9-10) (2009).

[16] N. Kumar, M. Rani, An efficient hybrid approach for trajectory tracking control of autonomous underwater vehicles, Applied Ocean Research, 95 (2020) 102053.

[17] K. Do, J. Pan, Z. Jiang, Robust and adaptive path following for underactuated autonomous underwater vehicles, Ocean Engineering, 31(16) (2004) 1967-1997.

[18] J. Xu, M. Wang, L. Qiao, Dynamical sliding mode control for the trajectory tracking of underactuated unmanned underwater vehicles, Ocean Engineering, 105 (2015) 54–63.

[19] X. Liang, X. Qu, L. Wan, Q. Ma, Three-Dimensional Path Following of an Underactuated AUV Based on Fuzzy Backstepping Sliding Mode Control, International Journal of Fuzzy Systems, 20 (2018) 640-649.

[20] T. Elmokadem, M. Zribi, K. Youcef-Toumi, Trajectory tracking sliding mode control of underactuated AUVs, Nonlinear Dynamics, 84 (2015) 1079–1091.

[21] H. Joe, M. Kim, S. Yu, Second-order sliding-mode controller for autonomous underwater vehicle in the presence of unknown disturbances, Nonlinear Dynamics, 78 (2014) 183–196.

[22] G. Zhang, H. Huang, H. Qin, L. Wan, Y. Li, J. Cao, Y. Su, A novel adaptive second order sliding mode path following control for a portable AUV, Ocean Engineering, 151 (2018) 82–92.

[23] J. Zhou, X. Zhao, T. Chen, Z. Yan, Z. Yang, Trajectory Tracking Control of an Underactuated AUV Based on Backstepping Sliding Mode With State Prediction, IEEE Access, 7 (2019) 181983-181993.

[24] B. Patre, P. Londhe, R. Nagarale, Fuzzy Sliding Mode Control for Spatial Control of Large Nuclear Reactor, IEEE Transactions on Nuclear Science, 62(5) (2015) 2255-2265.

[25] Z. Yan, M. Wang, J. XU, Robust adaptive sliding mode control of underactuated autonomous underwater vehicles with uncertain dynamics, Ocean Engineering, 173 (2019) 802-809.

[26] K. Pettersen, O. Egeland, Time-varying exponential stabilization of the position and attitude of an underactuated autonomous underwater vehicle, IEEE Transactions on Automatic Control, 44(1) (1999) 112-115.

[27] W. Chen, Nonlinear Disturbance Observer-Enhanced Dynamic Inversion Control of Missiles, Journal of Guidance, Control and Dynamics, 26(1) (2003) 161-166.

[28] M. Corless, G. Leitman, Continuous state feedback guaranteeing uniform ultimate boundedness for uncertain dynamic systems, IEEE Transactions on Automatic Control, 26(5) (1981) 1139-1144.

[29] C. Yu, X. Xiang, L. Lapierre, Q. Zhang, Robust Magnetic Tracking of Subsea Cable by AUV in the Presence of Sensor Noise and Ocean Currents, IEEE Journal of Oceanic Engineering, 43(2) (2018) 311-322.

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