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Delay-dependent Robust Control for Uncertain Linear Systems with Distributed and Multiple Delays

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ABSTRACT: This paper considers robust control of uncertain linear neutral systems with multiple state and state derivative delays. With equivalent descriptor representation, the stabilization problem is extended to more general class of neutral-type uncertain linear systems with discrete and distributed delays. The parametric uncertainties are time varying and unknown but norm bounded. Two delay-dependent/independent approaches are proposed to design robust controllers for a class of uncertain linear neutral systems with parametric uncertainty, discrete and distributed multiple delays. Using a presented descriptor model and an appropriate Lyapunov functional, sufficient conditions for closed loop stability are given in terms of linear matrix inequalities (LMIs). Solving the LMI problems, a robust memoryless state feedback is designed for all admissible uncertainties. The results depend on the size and varying rate of the delays. Two examples are provided to show the effectiveness of the proposed strategy.

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1- Introduction

Many engineering applications are involved with time–delay systems such as networks, mechanical, energy, process control, computer – based control systems and stochastic systems [1-5]. Time-delay in dynamical systems is often a source of instability and poor performance, which presents in many applications such as stochastic perturbations [5], H∞ control theory [6-7], optimal control of time – delay systems [8], and output-feedback stabilization approaches [9]. Recently, neutral systems with multiple delays in both state and/or inputs and the derivatives of states have attracted attention in practical applications such as distributed networks, population ecology and control process [10-16].

Based on the size of a delay, stabilization problem can be classified into two categories: delay-independent and delay-dependent, with either single or multiple delays. Most of the robust control results based on the Riccati or Lyapunov approach are independent of size of the delay (the time delay might be arbitrarily large), and they are thereby conservative. In neutral systems, delay dependent methods are generally less conservative, especially when the size of the delay is small [7,10-12]. Based on the Lyapunov – Krasovskii stability theory, using linear matrix inequality (LMI) techniques and descriptor system representation, stability conditions are proposed [10-18].

Descriptor systems present a general mathematical framework for the modelling, simulation and control of the complex dynamical systems. In this paper the descriptor system representation transforms the original system into a distributed system to reduce conservatism [19]. In this direction, a part of the proposed strategies to reduce conservatism either do not consider uncertainty [8-11,17-21] or are not extendable to the class of neutral-type systems with distributed or multiple delays [7,12,21-25]. To resolve these shortcomings, [26] proposed strategies to robust stabilize neutral systems with multiple distributed delays and non-parametric uncertainties. On the other hand, [13-26] derived stability conditions for neutral systems with multiple delays. However, the stability conditions are not extended to systems with distributed delays. In this paper the stabilization problem is extended to more general class of neutral-type uncertain linear systems with multiple delays. Two delay-dependent/independent approaches are proposed to design robust controllers for a class of uncertain linear neutral systems with parametric uncertainty, discrete and distributed multiple delays. The closed loop stability is guaranteed by solving a set of appropriately derived LMIs.

The rest of this paper is organized as follows: In Section 2, a new stability sufficient condition for uncertain neutral linear systems with discrete multiple state delay is presented based on the descriptor system representation. The robust stability of the delay system and delay-dependent is formulated in

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appropriately defined LMIs. In Section 3, by introducing a new Lyapunov functional, the stability condition is extended to structured uncertain neutral systems with discrete and distributed multiple delay. Based on the stability condition, designing delay dependent/independent state feedback control is formulated in terms of LMI. Several examples are presented to show the effectiveness of the proposed solutions. Finally, concluding results are given in section 4.

2- Problem Statement

Consider the uncertain time-delay systems described by the following state equations:

$$\dot{x}(t)-\sum_{i=1}^{m}\tilde{D}_i\dot{x}(t-h_i)= \sum_{i=0}^{m}\tilde{A}_i x(t-h_i) \quad ,x(t)=\varphi(t)t\epsilon[0\ h] \tag{1}$$

Where :

$$\begin{aligned}&\tilde{D}_i=\bar{D}_i+\Delta\overline{D_i}\ ,\\ &\tilde{A}_i=A_i+\Delta A_i\\ &\tilde{A}_0=A_0+\Delta A_0,\\ &(A_0=A+BK,\quad \Delta A_0=\Delta A+\Delta BK)\end{aligned} \tag{2}$$

and $x(t)\in R^n$, $h_0=0,0<h_i\le h,i=1,\ldots,m$, A_i and D_i are constant n×n matrices, and φ is a continuously differentiable initial function.

In this paper, the admissible uncertainties are described by:

$$\begin{aligned}&\Delta A_i=D_iF_i(x,t)E_i\\ &\Delta A_0=D_0F_0(x,t)E_0\\ &[\Delta A\quad \Delta B]=D_0F_0(x,t)[E_a\quad E_b],\\ &E_0=E_a+E_bK\\ &\Delta\overline{D_i}=D_iF_i(x,t)\bar{E}_i\end{aligned} \tag{3}$$

where D_0, E_a, E_b, D_i $\bar{E}_i$, $\bar{E}_0$ and E_i are known constant real matrices of appropriate dimensions, and $F_i(x,t)$ and are unknown real-value time varying matrices with Lebesgue measurable elements satisfying the following bounds:

$$\begin{aligned}&F_i^T(x,t)F_i(x,t)\le I\\ &F_0^T(x,t)F_0(x,t)\le I,\quad \forall t\end{aligned} \tag{4}$$

Assumption A1:

$$\sum_{i=1}^{m}\left|\tilde{D}_i\right|<1 \tag{5}$$

Where is any matrix norm.

Under assumption A1, both stability conditions associated with continuous and continuously differentiable initial functions are equivalent [10]. The following lemmas are essential in deriving stability conditions in the rest of the paper.

Lemma I[28,29] : For any $z,y\in R^n$ and any positive definite matrix $X\in R^{n\times n}$ $-2z^Ty\le z^TX^{-1}z+y^TXy$

Lemma 2 [28,29]: Let A, D, E and F be real matrices of appropriate dimensions with $\|F\|\le I$. Then we have:

For any $\varepsilon>0$, $DFE+E^TF^TD^T\le\varepsilon^{-1}DD^T+\varepsilon E^TE$

For any matrix P> 0 and scalar

$\hat{a}>0$ satisfying $\varepsilon I-EPE^T>0$,

$(A+DFE)P(A+DFE)^T\le APA^T+APE^T(\varepsilon I-EPE^T)^{-1}EPA^T+\varepsilon DD^T$

For any matrix P > 0 and scalar >0 , satisfying $P-\varepsilon DD^T>0$

,$(A+DFE)^TP^{-1}(A+DFE)\le A^T(P-\varepsilon DD^T)^{-1}A+\varepsilon^{-1}E^TE$

2- 1- Stability of Delay-dependent Uncertain Linear Systems

The system given in (1) can be represented in an equivalent descriptor representation as follows:

$$\dot{x}(t)=y(t),\quad y(t)= \sum_{i=1}^{m}\tilde{D}_iy(t-h_i)+\sum_{i=0}^{m}\tilde{A}_ix(t-h_i) \tag{6}$$

$$\begin{aligned}&\dot{x}(t)=y(t),\\ &0=-y(t)+\sum_{i=1}^{m}\tilde{D}_iy(t-h_i)\\ &+\left(\sum_{i=0}^{m}\tilde{A}_i\right)x(t)-\sum_{i=0}^{m}\tilde{A}_i\int_{t-hi}^{t}y(s)ds\end{aligned} \tag{7}$$

Consider the following Lyapunov–Krasovskii functional:

$$V(t)=\begin{bmatrix} x^T(t) & y^T(t)\end{bmatrix} \times EP\begin{bmatrix} x(t) \\ y(t)\end{bmatrix}+V_1+V_2 \quad (8)$$

Where:

$$E=\begin{bmatrix} I & 0 \\ 0 & 0\end{bmatrix},\quad P=\begin{bmatrix} P_1 & 0 \\ P_2 & P_3\end{bmatrix},\quad P_1=P_1^T>0 \quad (9)$$

$$V_1=\sum_{i=1}^{m}\int Q_{t-h_i}^{t}\, y^T(s)_i\, y(s)ds,\ Q_i>0, \quad (10)$$

$$V_2=\sum_{i=1}^{m}\int_{-h_i}^{0}\int_{t+\theta}^{t} y^T(s)R_i y(s)dsd\theta \quad R_i>0, \quad (11)$$

The first terms of equation (8), V_1 and V_2 are associated with the descriptor system, delay independent stability with respect to the discrete delays, and delay-dependent stability with respect to the distributed delays.

In this Section, we present a sufficient condition for stability of system (1) by a memoryless feedback based on the Lyapunov functional method. The condition is in terms of solvability of a linear matrix inequality as stated in the following theorem.

Theorem 1. The system given in (1) is stable for all under A1, if there exist $X=\begin{bmatrix} X_1 & 0 \\ X_2 & X_3\end{bmatrix}$, $X_1=X_1^T>0, X_2, X_3$, $\bar{Q}_i=\bar{Q}_i^T>0$, $\bar{R}_i=\bar{R}_i^T>0$, $\xi_i>0,\ i=1,\dots,m$ such that:

$$W=\begin{bmatrix} \bar{\bar{\psi}} & \bar{\theta}_1 & 0 & \bar{\theta}_5 & X^T\begin{bmatrix} 0 \\ vec\{I\}\end{bmatrix} & X^T\begin{bmatrix} 0 \\ vec\{h_i I\}\end{bmatrix} \\ * & \bar{\theta}_2 & \bar{\theta}_3 & 0 & 0 & 0 \\ * & * & \bar{\theta}_4 & 0 & 0 & 0 \\ * & * & * & \bar{\theta}_6 & 0 & 0 \\ * & * & * & * & -diag(\bar{Q}_i) & 0 \\ * & * & * & * & * & -diag(h_i\bar{R}_i)\end{bmatrix}<0 \quad (12)$$

Where

$$\overset{\blacklozenge}{\psi}=\begin{bmatrix} 0 & 0 \\ AX_1+BY & 0\end{bmatrix}+\begin{bmatrix} 0 & (AX_1)^T+(BY)^T \\ 0 & 0\end{bmatrix}$$

$$+\begin{bmatrix} 0 & I \\ \sum_{i=1}^{m}A_i & -I\end{bmatrix}X+X^T\begin{bmatrix} 0 & \sum_{i=1}^{m}A_i^T \\ I & -I\end{bmatrix}$$

$$+\begin{bmatrix} 0 & 0 \\ 0 & 2\sum_{i=1}^{m}\xi_i^{-1}D_iD_i^T\end{bmatrix}+X^T\begin{bmatrix} \sum_{i=0}^{m}\xi_i E_i^T E_i & 0 \\ 0 & 0\end{bmatrix}X$$

$$+\begin{bmatrix} 0 & 0 \\ 0 & \xi_0^{-1}D_0D_0^T\end{bmatrix}+\begin{bmatrix} 0 & 0 \\ 0 & \sum_{i=1}^{m}h_i\xi_i D_iD_i^T\end{bmatrix}+\sum_{i=1}^{m}h_i\begin{bmatrix} 0 \\ A_i\end{bmatrix}\bar{R}_i\begin{bmatrix} 0 & A_i^T\end{bmatrix},$$

$$\bar{\theta}_1=vec\left\{\begin{bmatrix} 0 \\ \bar{D}_i\end{bmatrix}(\bar{Q}_i)\right\}^T,\ \bar{\theta}_2=-diag(\bar{Q}_i),\bar{\theta}_3=vec\{\bar{E}_i^T\}^T,$$

$$\bar{\theta}_4=-diag(\xi_i^{-1}I),$$

$$\bar{\theta}_5=vec\left\{h_i\begin{bmatrix} 0 \\ A_i\end{bmatrix}\bar{R}_i\begin{bmatrix} 0 & E_i^T\end{bmatrix}\right\}^T,$$

$$\bar{\theta}_6=-diag\left(h_i\left(\xi_i I-\begin{bmatrix} 0 \\ E_i\end{bmatrix}\bar{R}_i\begin{bmatrix} 0 & E_i^T\end{bmatrix}\right)\right),\ i=1,\dots,m.$$

And

$$\left(\xi_i I-\begin{bmatrix} 0 \\ E_i\end{bmatrix}\bar{R}_i\begin{bmatrix} 0 & E_i^T\end{bmatrix}\right)>0.$$

Therefore, the state-feedback gain is given by .

Proof: By presenting system (1) in the equivalent descriptor form we have:

$$\frac{d}{dt}\begin{bmatrix} x^T(t) & y^T(t)\end{bmatrix}EP\begin{bmatrix} x(t) \\ y(t)\end{bmatrix}=2x^T(t)P_1\dot{x}(t)=$$
$$2\begin{bmatrix} x^T(t) & y^T(t)\end{bmatrix}P^T\begin{bmatrix} \dot{x}(t) \\ 0\end{bmatrix}=$$
$$2\begin{bmatrix} x^T(t) & y^T(t)\end{bmatrix}P^T. \quad (13)$$
$$\begin{bmatrix} y(t) \\ -y(t)+\sum_{i=1}^{m}\tilde{D}_i y(t-h_i)+\left(\sum_{i=0}^{m}\tilde{A}_i\right)x(t)-\sum_{i=0}^{m}\tilde{A}_i\int_{t-hi}^{t}y(s)ds\end{bmatrix}$$

Taking derivative of equation (8) with respect to t and applying (13) we obtain:

$$\frac{dV_1}{dt}=\sum_{i=1}^{m}y^T(t)Q_i y(t),-\sum_{i=1}^{m}y^T(t-h_i)Q_i y(t-h_i)$$

also

$$\frac{dV_2}{dt}=\sum_{i=1}^{m}\int_{-hi}^{0}y^T(t)R_i y(t)d\theta,$$
$$-\sum_{i=1}^{m}\int_{-hi}^{0}y^T(t+\theta)R_i y(t+\theta)d\theta= \quad (14)$$
$$\sum_{i=1}^{m}h_i y^T(t)R_i y(t)-\sum_{i=1}^{m}\int_{t-hi}^{t}y^T(s)R_i y(s)ds$$

Therefore we will have:

$$
\begin{aligned}
&\frac{dV}{dt}=2\left[x^T(t) \quad y^T(t)\right]P^T\begin{bmatrix}0 & I\\ 0 & -I\end{bmatrix}\begin{bmatrix}x(t)\\ y(t)\end{bmatrix}+\\
&2\left[x^T(t) \quad y^T(t)\right]P^T\begin{bmatrix}0\\ \sum_{i=1}^{m}\tilde{D}_i y(t-h_i)\end{bmatrix}+\\
&2\left[x^T(t) \quad y^T(t)\right]P^T\begin{bmatrix}0 & 0\\ \sum_{i=0}^{m}\tilde{A}_i & 0\end{bmatrix}\begin{bmatrix}x(t)\\ y(t)\end{bmatrix}-\\
&2\left[x^T(t) \quad y^T(t)\right]P^T\begin{bmatrix}0\\ \sum_{i=1}^{m}\tilde{A}_i\int_{t-hi}^{t}y(s)ds\end{bmatrix}+\\
&\sum_{i=1}^{m}y^T(t)\left[\mathrm{Q_i}+h_iR_i\right]y(t)-\\
&\sum_{i=1}^{m}y^T(t-h_i)\mathrm{Q_i}y(t-h_i)\\
&-\sum_{i=1}^{m}\int_{t-hi}^{t}y^T(s)R_iy(s)ds\\
&=\zeta\begin{bmatrix}\tilde{\psi} & P^T\begin{bmatrix}0\\ \tilde{D}_1\end{bmatrix} & P^T\begin{bmatrix}0\\ \tilde{D}_2\end{bmatrix} & \cdots & P^T\begin{bmatrix}0\\ \tilde{D}_m\end{bmatrix}\\ [0 \quad \tilde{D}_1^T]P & -\mathrm{Q}_1 & 0 & 0 & 0\\ [0 \quad \tilde{D}_2^T]P & 0 & -\mathrm{Q}_2 & 0 & \vdots\\ \vdots & \vdots & 0 & \ddots & 0\\ [0 \quad \tilde{D}_m^T]P & 0 & 0 & 0 & -\mathrm{Q}_\mathrm{m}\end{bmatrix}\zeta^T\\
&+\sum_{i=1}^{m}\tilde{\eta}_i-\sum_{i=1}^{m}\int_{t-hi}^{t}y^T(s)R_iy(s)ds.
\end{aligned}
\tag{15}
$$

Where:

$$
\begin{aligned}
&\zeta=\left[x^T(t) \quad y^T(t) \quad y^T(t-h_1) \quad \ldots \quad y^T(t-h_m)\right]\\
&\tilde{\psi}=P^T\begin{bmatrix}0 & I\\ \sum_{i=0}^{m}\tilde{A}_i & -I\end{bmatrix}+\\
&\begin{bmatrix}0 & \sum_{i=0}^{m}\tilde{A}_i^T\\ I & -I\end{bmatrix}P+\begin{bmatrix}0 & 0\\ 0 & \sum_{i=1}^{m}\left[\mathrm{Q_i}+h_iR_i\right]\end{bmatrix}
\end{aligned}
\tag{16}
$$

$$
\tilde{\eta}_i=-2\int_{t-hi}^{t}\left[x^T(t) \quad y^T(t)\right]P^T\begin{bmatrix}0\\ \tilde{A}_i\end{bmatrix}y(s)ds \tag{17}
$$

Using Lemmas 1 and 2:

$$
\begin{aligned}
&\tilde{\eta}_i\le h_i\left[x^T(t) \quad y^T(t)\right]P^T\begin{bmatrix}0\\ \tilde{A}_i\end{bmatrix}R_i^{-1}\left[0 \quad \tilde{A}_i^T\right]\\
&\times P\begin{bmatrix}x(t)\\ y(t)\end{bmatrix}+\int_{t-hi}^{t}y^T(s)R_iy(s)ds
\end{aligned}
\tag{18}
$$

Afterwards, by using Lemma 2 we will have:

$$
\begin{aligned}
&(18)\le\left[x^T(t) \quad y^T(t)\right]h_iP^T\times\\
&\left(\begin{bmatrix}0\\ A_i\end{bmatrix}R_i^{-1}\left[0 \quad A_i^T\right]+\begin{bmatrix}0\\ A_i\end{bmatrix}R_i^{-1}\left[0 \quad E_i^T\right]\left(\xi_iI-\begin{bmatrix}0\\ E_i\end{bmatrix}R_i^{-1}\left[0 \quad E_i^T\right]\right)^{-1}\begin{bmatrix}0\\ E_i\end{bmatrix}R_i^{-1}\left[0 \quad A_i^T\right]\right)\\
&\times P\begin{bmatrix}x(t)\\ y(t)\end{bmatrix}+h_i\left[x^T(t) \quad y^T(t)\right]P^T\xi_i\begin{bmatrix}0 & 0\\ 0 & D_iD_i^T\end{bmatrix}P\begin{bmatrix}x(t)\\ y(t)\end{bmatrix}\\
&+\int_{t-hi}^{t}y^T(s)R_iy(s)ds
\end{aligned}
\tag{19}
$$

Where:

$$
\left(\xi_iI-\begin{bmatrix}0\\ E_i\end{bmatrix}R_i^{-1}\left[0 \quad E_i^T\right]\right)>0 \quad ,\xi_i>0,\ i=1,\ldots,m\ .
$$

Further, for $\tilde{\psi}$ we have:

$$
\begin{aligned}
&\tilde{\psi}=P^T\begin{bmatrix}0 & I\\ \sum_{i=0}^{m}\tilde{A}_i & -I\end{bmatrix}+\begin{bmatrix}0 & \sum_{i=0}^{m}\tilde{A}_i^T\\ I & -I\end{bmatrix}P\\
&+\begin{bmatrix}0 & 0\\ 0 & \sum_{i=1}^{m}\left[\mathrm{Q_i}+h_iR_i\right]\end{bmatrix}=P^T\begin{bmatrix}0 & I\\ \sum_{i=0}^{m}A_i & -I\end{bmatrix}\\
&+\begin{bmatrix}0 & \sum_{i=0}^{m}A_i^T\\ I & -I\end{bmatrix}P+\begin{bmatrix}0 & 0\\ 0 & \sum_{i=1}^{m}\left[\mathrm{Q_i}+h_iR_i\right]\end{bmatrix}\\
&+P^T\begin{bmatrix}0 & 0\\ \sum_{i=1}^{m}\Delta A_i & 0\end{bmatrix}+\begin{bmatrix}0 & \sum_{i=1}^{m}\Delta A_i^T\\ 0 & 0\end{bmatrix}P+\\
&P^T\begin{bmatrix}0 & 0\\ \Delta A_0 & 0\end{bmatrix}+\begin{bmatrix}0 & \Delta A_0^T\\ 0 & 0\end{bmatrix}P
\end{aligned}
\tag{20}
$$

In $\tilde{\psi}$ we have an uncertain term as:

$$P^T\begin{bmatrix}0&0\\ \Delta A_0&0\end{bmatrix}+\begin{bmatrix}0&\Delta A_0^T\\0&0\end{bmatrix}=$$

$$P^T\left\{\begin{bmatrix}0&0\\0&D_0\end{bmatrix}\begin{bmatrix}F_0&0\\0&F_0\end{bmatrix}\begin{bmatrix}0&0\\E_0&0\end{bmatrix}\right\}+$$

$$\left\{\begin{bmatrix}0&0\\0&D_0\end{bmatrix}\begin{bmatrix}F_0&0\\0&F_0\end{bmatrix}\begin{bmatrix}0&0\\E_0&0\end{bmatrix}\right\}^T P$$

$$\le \xi_0^{-1}P^T\begin{bmatrix}0&0\\0&D_0\end{bmatrix}\begin{bmatrix}0&0\\0&D_0\end{bmatrix}^T P+$$

$$\xi_0\begin{bmatrix}0&0\\E_0&0\end{bmatrix}^T\begin{bmatrix}0&0\\E_0&0\end{bmatrix}$$

$$=P^T\begin{bmatrix}0&0\\0&\xi_0^{-1}D_0D_0^T\end{bmatrix}P+\begin{bmatrix}\xi_0E_0^TE_0&0\\0&0\end{bmatrix}$$

So

$$\tilde{\psi}\le\psi=P^T\begin{bmatrix}0&I\\ \sum_{i=0}^{m} A_i&-I\end{bmatrix}+\begin{bmatrix}0&\sum_{i=0}^{m} A_i^T\\ I&-I\end{bmatrix}P+\begin{bmatrix}0&0\\0&\sum_{i=1}^{m}\left[Q_i+h_iR_i\right]\end{bmatrix}+P^T\begin{bmatrix}0&0\\0&\sum_{i=1}^{m}\xi_i^{-1}D_iD_i^T\end{bmatrix}P+\begin{bmatrix}\sum_{i=1}^{m}\xi_iE_i^TE_i&0\\0&0\end{bmatrix}+P^T\begin{bmatrix}0&0\\0&\xi_0^{-1}D_0D_0^T\end{bmatrix}P+\begin{bmatrix}\xi_0E_0^TE_0&0\\0&0\end{bmatrix} \tag{21}$$

Consequently from (15-20), we will have:

$$\frac{dV}{dt}\le\zeta\begin{bmatrix}\psi & P^T\begin{bmatrix}0\\ \tilde D_1\end{bmatrix} & \cdots & \dots & P^T\begin{bmatrix}0\\ \tilde D_m\end{bmatrix}\\ [0\ \ \tilde D_1^T]P & -Q_1 & 0 & \cdots & 0\\ \vdots & 0 & -Q_2 & 0 & \vdots\\ \vdots&\vdots&0&\ddots&0\\ [0\ \ \tilde D_m^T]P&0&0&0&-Q_m\end{bmatrix}\zeta^T+$$

The first part of (21) can be written as:

$$\sum_{i=1}^{m}[x^T(t)\ \ y^T(t)]h_iP^T\left(\begin{bmatrix}0\\A_i\end{bmatrix}R_i^{-1}[0\ \ A_i^T]+\begin{bmatrix}0\\A_i\end{bmatrix}R_i^{-1}[0\ \ E_i^T]\left(\xi_iI-\begin{bmatrix}0\\E_i\end{bmatrix}R_i^{-1}[0\ \ E_i^T]\right)^{-1}\begin{bmatrix}0\\E_i\end{bmatrix}R_i^{-1}[0\ \ A_i^T]\right)P\begin{bmatrix}x(t)\\y(t)\end{bmatrix}+\sum_{i=1}^{m} h_i[x^T(t)\ \ y^T(t)]P^T\xi_i\begin{bmatrix}0&0\\0&D_iD_i^T\end{bmatrix}P\begin{bmatrix}x(t)\\y(t)\end{bmatrix} \tag{22}$$

$$\zeta\begin{bmatrix}\psi & P^T\begin{bmatrix}0\\ \tilde D_1\end{bmatrix} & P^T\begin{bmatrix}0\\ \tilde D_2\end{bmatrix} & \dots & P^T\begin{bmatrix}0\\ \tilde D_m\end{bmatrix}\\ [0\ \ \tilde D_1^T]P & -Q_1 & 0&\dots&0\\ [0\ \ \tilde D_2^T]P&0&-Q_2&0&\vdots\\ \vdots&\vdots&0&\ddots&0\\ [0\ \ \tilde D_m^T]P&0&0&0&-Q_m\end{bmatrix}\zeta^T$$

$$=\zeta\left\{\begin{bmatrix}\psi & P^T\begin{bmatrix}0\\ \bar D_1\end{bmatrix} & P^T\begin{bmatrix}0\\ \bar D_2\end{bmatrix} & \dots & P^T\begin{bmatrix}0\\ \bar D_m\end{bmatrix}\\ [0\ \ \bar D_1^T]P & -Q_1 & 0 & \dots & 0\\ . & 0 & -Q_2 & 0 & \vdots \\ \vdots&\vdots&0&\ddots&0\\ [0\ \ \bar D_m^T]P & 0&0&0&-Q_m\end{bmatrix}+\begin{bmatrix}0 & P^T\begin{bmatrix}0\\ \Delta\bar D_1\end{bmatrix} & P^T\begin{bmatrix}0\\ \Delta\bar D_2\end{bmatrix}&\dots&P^T\begin{bmatrix}0\\ \Delta\bar D_m\end{bmatrix}\\ [0\ \ \Delta\bar D_1^T]P&0&0&\dots&0\\ .&0&0&0&0\\ \vdots&\vdots&\vdots&\ddots&\vdots\\ [0\ \ \Delta\bar D_m^T]P&0&0&0&0\end{bmatrix}\right\}\zeta^T$$

$$=\zeta\left\{\Gamma+\begin{bmatrix}0 & P^T\begin{bmatrix}0\\ \Delta\bar D_1\end{bmatrix} & P^T\begin{bmatrix}0\\ \Delta\bar D_2\end{bmatrix}&\dots&P^T\begin{bmatrix}0\\ \Delta\bar D_m\end{bmatrix}\\ [0\ \ \Delta\bar D_1^T]P&0&0&\dots&0\\ .&0&0&0&0\\ \vdots&\vdots&\vdots&\ddots&\vdots\\ [0\ \ \Delta\bar D_m^T]P&0&0&0&0\end{bmatrix}\right\}\zeta^T$$

According to the (2-4) we have:

$$\begin{bmatrix}0&P^T\begin{bmatrix}0\\ \Delta\bar D_1\end{bmatrix}&\dots&P^T\begin{bmatrix}0\\ \Delta\bar D_m\end{bmatrix}\\ [0\ \ \Delta\bar D_1^T]P&0&\cdots&0\\ \vdots&\vdots&\ddots&\vdots\\ [0\ \ \Delta\bar D_m^T]P&0&\dots&0\end{bmatrix}$$

$$=\begin{bmatrix}P^T&0&\dots&0\\0&I&\cdots&0\\ \vdots&\vdots&\ddots&\vdots\\0&0&\dots&I\end{bmatrix}\begin{bmatrix}0&0&\begin{bmatrix}0\\ \Delta\bar D_1\end{bmatrix}&\cdots&\begin{bmatrix}0\\ \Delta\bar D_m\end{bmatrix}\\0&0&0&\cdots&0\\ \vdots&\vdots&\vdots&\ddots&\vdots\\0&0&0&\dots&0\end{bmatrix}+$$

$$\begin{bmatrix}0&0&\dots&0\\ [0\ \ \Delta\bar D_1^T]&0&\cdots&0\\ \vdots&\vdots&\ddots&\vdots\\ [0\ \ \Delta\bar D_m^T]&0&\dots&0\end{bmatrix}\begin{bmatrix}P&0&\dots&0\\0&I&\cdots&0\\ \vdots&\vdots&\ddots&\vdots\\0&0&\dots&I\end{bmatrix}$$

Where:

$$\begin{bmatrix}0&0&\begin{bmatrix}0\\ \Delta\bar D_1\end{bmatrix}&\cdots&\begin{bmatrix}0\\ \Delta\bar D_m\end{bmatrix}\\0&0&0&\cdots&0\\ \vdots&\vdots&\vdots&\ddots&\vdots\\0&0&0&\dots&0\end{bmatrix}\begin{bmatrix}0&0&\begin{bmatrix}0\\ D_1\end{bmatrix}&\cdots&\begin{bmatrix}0\\ D_m\end{bmatrix}\\0&0&0&.&0\\ \vdots&&\vdots&\ddots&\vdots\\0&0&0&\dots&0\end{bmatrix} \tag{23}$$

$$\begin{bmatrix}0&0&0&\dots&0\\0&0&0&\dots&0\\0&0&F_1&.&0\\ \vdots&\vdots&.&\ddots&\vdots\\0&0&0&.&F_m\end{bmatrix}\begin{bmatrix}0&0&0&\dots&0\\0&0&0&\dots&0\\0&0&\bar E_1&.&0\\ \vdots&\vdots&.&\ddots&\vdots\\0&0&0&.&\bar E_m\end{bmatrix}$$

Using Lemma 2 we have:

$$\begin{bmatrix}0&P^T\begin{bmatrix}0\\ \Delta\bar D_1\end{bmatrix}&\dots&P^T\begin{bmatrix}0\\ \Delta\bar D_m\end{bmatrix}\\ [0\ \ \Delta\bar D_1^T]P&0&\cdots&0\\ \vdots&\vdots&\ddots&\vdots\\ [0\ \ \Delta\bar D_m^T]P&0&\dots&0\end{bmatrix}$$

$$\leq \begin{bmatrix} P^T\begin{bmatrix} 0 & 0 \\ 0 & \sum_{i=1}^{m}\xi_i^{-1}D_iD_i^T \end{bmatrix}P & 0 & \dots & 0 \\ 0\quad 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0\quad 0 & 0 & \dots & 0 \end{bmatrix} +$$

$$\begin{bmatrix} 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & \xi_1\overline{E}_1^T\overline{E}_1 & 0 & \vdots \\ \vdots & \vdots & 0 & \ddots & 0 \\ 0 & 0 & \dots & 0 & \xi_m\overline{E}_m^T\overline{E}_m \end{bmatrix}$$

So

$$(22) \leq \zeta\{\Gamma\}\zeta^T +$$

$$\zeta\left\{\begin{bmatrix} P^T\begin{bmatrix} 0 & 0 \\ 0 & \sum_{i=1}^{m}\xi_i^{-1}D_iD_i^T \end{bmatrix}P & 0 & \dots & 0 \\ 0\quad 0 & \xi_1\overline{E}_1^T\overline{E}_1 & . & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0\quad 0 & 0 & \dots & \xi_m\overline{E}_m^T\overline{E}_m \end{bmatrix}\right\}\zeta^T =$$

$$\zeta\{\Gamma + \overline{\Gamma}\}\zeta^T$$

Consequently, we will have:

$$\frac{dV}{dt} \leq \zeta \begin{bmatrix} \psi & P^T\begin{bmatrix} 0 \\ \overline{D}_1 \end{bmatrix} & P^T\begin{bmatrix} 0 \\ \overline{D}_3 \end{bmatrix} & \dots & P^T\begin{bmatrix} 0 \\ \overline{D}_m \end{bmatrix} \\ \begin{bmatrix} 0 & \overline{D}_1^T \end{bmatrix}P & -Q_1 & 0 & \dots & 0 \\ \begin{bmatrix} 0 & \overline{D}_2^T \end{bmatrix}P & 0 & -Q_2 & 0 & \vdots \\ \vdots & \vdots & 0 & \ddots & 0 \\ \begin{bmatrix} 0 & \overline{D}_m^T \end{bmatrix}P & 0 & 0 & 0 & -Q_m \end{bmatrix} \zeta^T + \zeta\{\overline{\Gamma}\}\zeta^T + \cdot \tag{24}$$

$$\sum_{i=1}^{m}\begin{bmatrix} x^T(t) & y^T(t) \end{bmatrix}h_iP^T$$

$$\left(\begin{bmatrix} 0 \\ A_i \end{bmatrix}R_i^{-1}\begin{bmatrix} 0 & A_i^T \end{bmatrix} + \begin{bmatrix} 0 \\ A_i \end{bmatrix}R_i^{-1}\begin{bmatrix} 0 & E_i^T \end{bmatrix}\left(\xi_i I - \begin{bmatrix} 0 \\ E_i \end{bmatrix}R_i^{-1}\begin{bmatrix} 0 & E_i^T \end{bmatrix}\right)^{-1}\right.$$

$$\left.\times\begin{bmatrix} 0 \\ E_i \end{bmatrix}R_i^{-1}\begin{bmatrix} 0 & A_i^T \end{bmatrix}\right)P\begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$$

$$+\sum_{i=1}^{m}h_i\begin{bmatrix} x^T(t) & y^T(t) \end{bmatrix}P^T\xi_i\begin{bmatrix} 0 & 0 \\ 0 & D_iD_i^T \end{bmatrix}P\begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$$

Thus, by Schur complement, $\frac{dV}{dt} \leq 0$ if the following LMI holds:

$$W = \begin{bmatrix} \overline{\psi} & \theta_1 & 0 & \theta_3 & \begin{bmatrix} 0 \\ vec\{I\} \end{bmatrix} & \begin{bmatrix} 0 \\ vec\{I\} \end{bmatrix} \\ * & \theta_2 & vec\{\overline{E}_i^T\} & 0 & 0 & 0 \\ * & * & -diag(\xi_i^{-1}I) & 0 & 0 & 0 \\ * & * & * & \theta_4 & 0 & 0 \\ * & * & * & * & -diag(Q_i^{-1}) & 0 \\ * & * & * & * & * & -diag(h_i^{-1}R_i^{-1}) \end{bmatrix} < 0,$$

And

$$\left(\xi_i I - \begin{bmatrix} 0 \\ E_i \end{bmatrix}R_i^{-1}\begin{bmatrix} 0 & E_i^T \end{bmatrix}\right) > 0.$$

Where:

$$\overline{\psi} = P^T\begin{bmatrix} 0 & I \\ \sum_{i=0}^{m}A_i & -I \end{bmatrix} + \begin{bmatrix} 0 & \sum_{i=0}^{m}A_i^T \\ I & -I \end{bmatrix}P +$$

$$P^T\begin{bmatrix} 0 & 0 \\ 0 & 2\sum_{i=1}^{m}\xi_i^{-1}D_iD_i^T \end{bmatrix}P + \begin{bmatrix} \sum_{i=1}^{m}\xi_iE_i^TE_i & 0 \\ 0 & 0 \end{bmatrix} + \tag{25}$$

$$P^T\begin{bmatrix} 0 & 0 \\ 0 & \xi_0^{-1}D_0D_0^T \end{bmatrix}P + \begin{bmatrix} \xi_0E_0^TE_0 & 0 \\ 0 & 0 \end{bmatrix} +$$

$$P^T\begin{bmatrix} 0 & 0 \\ 0 & \sum_{i=1}^{m}h_i\xi_iD_iD_i^T \end{bmatrix}P + \sum_{i=1}^{m}h_iP^T\begin{bmatrix} 0 \\ A_i \end{bmatrix}R_i^{-1}\begin{bmatrix} 0 & A_i^T \end{bmatrix}P$$

$$\theta_1 = vec\left\{P^T\begin{bmatrix} 0 \\ \overline{D}_i \end{bmatrix}\right\},$$

$$\theta_2 = -diag(Q_i),\ \theta_3 = vec\left\{h_iP^T\begin{bmatrix} 0 \\ A_i \end{bmatrix}R_i^{-1}\begin{bmatrix} 0 & E_i^T \end{bmatrix}\right\}^T,$$

$$\theta_4 = -diag\left(h_i\left(\xi_i I - \begin{bmatrix} 0 \\ E_i \end{bmatrix}R_i^{-1}\begin{bmatrix} 0 & E_i^T \end{bmatrix}\right)\right),\ i = 1, \dots, m.$$

By pre and post multiplying (25) by $\Delta_1 = diag\{I, diag(Q_i^{-1}), I, I, I, I\}$, we have the following inequality:

$$W = \begin{bmatrix} \overline{\psi} & vec\left\{P^T\begin{bmatrix} 0 \\ \overline{D}_i \end{bmatrix}(Q_i^{-1})\right\}^T & 0 & \theta_3 & \begin{bmatrix} 0 \\ vec\{I\} \end{bmatrix} & \begin{bmatrix} 0 \\ vec\{I\} \end{bmatrix} \\ * & -diag(Q_i^{-1}) & vec\{\overline{E}_i^T\}^T & 0 & 0 & 0 \\ * & * & -diag(\xi_i^{-1}I) & 0 & 0 & 0 \\ * & * & * & \theta_4 & 0 & 0 \\ * & * & * & * & -diag(Q_i^{-1}) & 0 \\ * & * & * & * & * & -diag(h_i^{-1}R_i^{-1}) \end{bmatrix} < 0 \tag{26}$$

Additionally, pre and post multiplying (26) by $\Delta_2 = diag\{X, I, I, I, I, I\}$ and Δ_2^T, where $X = P^{-1}$. Taking $Y = X_1 K$, $\bar{Q}_i = Q_i^{-1}$, $\bar{R}_i = R_i^{-1}$, and applying the Schur complement, we obtain inequality (12), and the proof is complete.

Functional V in equation (8) is degenerated, and has a negative derivative which results in asymptotic stability of (1) for continuous and based on assumption A1, continuously differentiable functions [27].

3- Delay-dependent/delay-independent Stability for Uncertain System with Discrete Distributed Delays

$$\dot{x}(t) - \sum_{i=1}^{k} \tilde{D}_i \dot{x}(t-g_i) = \sum_{i=0}^{m} \tilde{A}_i x(t-h_i)$$
$$+\sum_{i=0}^{m} \tilde{A}_{i0} \int_{t-\tau i}^{t} x(s)ds + \sum_{i=0}^{k} H_i x(t-g_i), \quad (27)$$

Where:

$$h_i, g_i \geq 0, \quad \tilde{A}_{i0} = A_{i0} + \Delta A_{i0},$$
$$\Delta A_{i0} = D_i F_i(x,t) E_{i0}, \tilde{A}_0 = A_0 + \Delta A_0,$$

In this section we extend the previous results to uncertain systems with discrete and distributed delays:

and D_i and E_{i0} are known constant real matrices of appropriate dimensions.

3- 1- Delay-dependent Stability

To find delay dependent stability criteria, we used the descriptor representation of the system as follows:

$$\dot{x}(t) = y(t),$$
$$y(t) = \sum_{i=1}^{k} \tilde{D}_i y(t-g_i) + \left(\sum_{i=0}^{m} \tilde{A}_i\right) x(t).$$
$$-\sum_{i=0}^{m} \tilde{A}_i \sum_{i=0}^{m} \tilde{A}_{i0} \int_{t-\tau i}^{t} x(s)ds \quad (28)$$
$$+\sum_{i=0}^{k} H_i x(t-g_i),$$

By defining the Lyapunov-Krasovskii functional candidate as:

$$V(t) = \begin{bmatrix} x^T(t) & y^T(t) \end{bmatrix} EP \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$$
$$+V_1 + V_2 + V_3 + V_4 \quad (29)$$

Where:

$$E = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}, \quad P = \begin{bmatrix} P_1 & 0 \\ P_2 & P_3 \end{bmatrix}, \quad P_1 = P_1^T > 0.$$

And:

$$V_1 = \sum_{i=1}^{k} \int_{t-g_i}^{t} Qy^T(s)_i y^T(s)ds, \; Q_i > 0, \quad (30)$$

$$V_2 = \sum_{i=1}^{m} \int_{-h_i}^{0} \int_{t+\theta}^{t} y^T(s) R_i y^T(s) ds d\theta \;\; R_i > 0, \quad (31)$$

$$V_1 = \sum_{i=1}^{k} \int_{t-g_i}^{t} y^T(s) Q_i y^T(s)ds, \; Q_i > 0, \quad (32)$$

$$V_3 = \sum_{i=1}^{k} \int_{t-g_i}^{t} x^T(s) U_i x^T(s)ds, \qquad U_i > 0, \quad (33)$$

$$V_4 = \sum_{i=1}^{m} \int_{-\tau_i}^{0} \int_{t+\theta}^{t} x^T(s) R_{i0} x^T(s) ds d\theta, \quad (34)$$

We obtain the following result:

Theorem 2. Under assumption A1, the given system in (27) is stable for all $g_i > 0, i = 1, \ldots, k$ if there Then,

$$\text{exist } X = \begin{bmatrix} X_1 & 0 \\ X_2 & X_3 \end{bmatrix},$$
$$X_1 = X_1^T > 0, X_2, X_3, \bar{Q}_i = \bar{Q}_i^T, \bar{U}_i = \bar{U}_i^T, i = 1, \ldots, k$$
$$\text{and } \bar{R}_j = \bar{R}_j^T > 0, \bar{R}_{j0} = \bar{R}_{j0}^T > 0, \quad (35)$$
$$j = 1, \ldots, m \text{ and } \xi_i > 0, \; i = 1, \ldots, m.$$

that satisfy the following LMI:

$$W=\begin{bmatrix}\bar{\bar{\psi}}_1 & \bar\theta_1 & 0 & \bar\theta_2 & \bar\theta_3 & \bar\theta_4 & X^T vec\{I\} & X^T vec\{h_i I\} & X^T vec\{I\} & X^T vec\{\tau_i I\}\\ * & \bar\theta_5 & \bar\theta_6 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ * & * & \bar\theta_7 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ * & * & * & \bar\theta_8 & 0 & 0 & 0 & 0 & 0 & 0\\ * & * & * & * & \bar\theta_9 & 0 & 0 & 0 & 0 & 0\\ * & * & * & * & * & \bar\theta_{10} & 0 & 0 & 0 & 0\\ * & * & * & * & * & * & \bar\theta_{11} & 0 & 0 & 0\\ * & * & * & * & * & * & * & \bar\theta_{12} & 0 & 0\\ * & * & * & * & * & * & * & * & \bar\theta_{13} & 0\\ * & * & * & * & * & * & * & * & * & \bar\theta_{14}\end{bmatrix}<0,$$

$$\bar{\bar{\psi}}_1=\begin{bmatrix}0 & 0\\ AX_1+BY & 0\end{bmatrix}+\begin{bmatrix}0 & (AX_1)^T+(BY)^T\\ 0 & 0\end{bmatrix}+$$

$$\begin{bmatrix}0 & I\\ \sum_{i=1}^{m}A_i & -I\end{bmatrix}X+X^T\begin{bmatrix}0 & \sum_{i=1}^{m}A_i^T\\ I & -I\end{bmatrix}+$$

$$\begin{bmatrix}0 & 0\\ 0 & 2\sum_{i=1}^{m}\xi_i^{-1}D_iD_i^T\end{bmatrix}+\begin{bmatrix}0 & 0\\ 0 & \xi_0^{-1}D_0D_0^T\end{bmatrix}+$$

$$X^T\begin{bmatrix}\sum_{i=0}^{m}\xi_iE_i^TE_i & 0\\ 0 & 0\end{bmatrix}X+\sum_{i=1}^{m}h_i\begin{bmatrix}0\\ A_i\end{bmatrix}\bar R_i[0 \quad A_i^T]+$$

$$\begin{bmatrix}0 & 0\\ 0 & 2\sum_{i=1}^{m}h_i\xi_iD_{i0}D_{i0}^T\end{bmatrix}+\sum_{i=1}^{m}h_i\begin{bmatrix}0\\ A_{i0}\end{bmatrix}\bar R_{i0}[0 \quad A_{i0}^T],$$

$$\bar\theta_1=vec\left\{\begin{bmatrix}0\\ \bar D_i\end{bmatrix}(\bar Q_i)\right\},\ \bar\theta_2=vec\left\{\begin{bmatrix}0\\ H_1\end{bmatrix}(\bar U_i)\right\},$$

$$\bar\theta_5=-diag(\bar Q_i),\ \bar\theta_8=-diag(\bar U_i),$$

$$\bar\theta_6=vec\{\bar E_i^T\},\ \bar\theta_7=-diag(\xi_i^{-1}I),\ i=1,\dots,k.$$

$$\bar\theta_3=vec\left\{h_i\begin{bmatrix}0\\ A_i\end{bmatrix}\bar R_i[0 \quad E_i^T]\right\}^T,$$

$$\bar\theta_9=-diag\left(h_i\left(\xi_iI-\begin{bmatrix}0\\ E_i\end{bmatrix}\bar R_i[0 \quad E_i^T]\right)\right),\ i=1,\dots,m.$$

$$\bar\theta_4=vec\left\{h_i\begin{bmatrix}0\\ A_{i0}\end{bmatrix}\bar R_{i0}[0 \quad E_{i0}^T]\right\}^T,\ \bar\theta_{10}=$$

$$-diag\left(h_i\left(\xi_iI-\begin{bmatrix}0\\ E_{i0}\end{bmatrix}\bar R_{i0}[0 \quad E_{i0}^T]\right)\right).$$

and

$\bar\theta_{11}=-diag(\bar Q_i)$, $i=1,\dots,k$. $\bar\theta_{12}=-diag(h_i\bar R_i)$, $i=1,\dots,m$

$\bar\theta_{13}=-diag(\bar U_i)$, i=1,…,k. $\bar\theta_{14}=-diag(\tau_i\bar R_{i0})$, $i=1,\dots,m$.

and

$$\left(\xi_iI-\begin{bmatrix}0\\ E_i\end{bmatrix}\bar R_i[0 \quad E_i^T]\right)>0,\ \left(\xi_iI-\begin{bmatrix}0\\ E_{i0}\end{bmatrix}\bar R_{i0}[0 \quad E_{i0}^T]\right)>0.$$

the state-feedback gain is then given by $K=X_1^{-1}Y$ where $(A_0=A+BK)$.

Proof: Similar to the proof of previous theorem we will have:

$$\frac{dV}{dt}=\zeta\begin{bmatrix}\tilde\psi_1 & P^T\begin{bmatrix}0\\ \tilde D_1\end{bmatrix} & \dots & P^T\begin{bmatrix}0\\ \tilde D_k\end{bmatrix} & P^T\begin{bmatrix}0\\ H_1\end{bmatrix} & \dots & P^T\begin{bmatrix}0\\ H_k\end{bmatrix}\\ [0 \quad \tilde D_1^T]P & -Q_1 & \dots & 0 & 0 & \dots & 0\\ \vdots & 0 & \ddots & 0 & 0 & \dots & .\\ [0 \quad \tilde D_k^T]P & 0 & \dots & -Q_k & 0 & \dots & 0\\ [0 \quad H_1^T]P & 0 & \dots & 0 & -U_1 & 0 & 0\\ \vdots & \vdots & \dots & \vdots & 0 & \ddots & 0\\ [0 \quad H_k^T]P & 0 & \dots & 0 & 0 & \dots & -U_k\end{bmatrix}\zeta^T \quad (36)$$

$$+\sum_{i=1}^{m}\tilde\eta_i+\sum_{i=1}^{m}\tilde\eta_{i0}\,.$$

$$-\sum_{i=1}^{m}\int_{t-hi}^{t}y^T(s)R_iy(s)\,ds-\sum_{i=1}^{m}\int_{t-\tau i}^{t}x^T(s)R_{i0}x(s)\,ds.$$

Where:

$$\zeta=[x^T(t) \quad y^T(t) \quad y^T(t-g_1)$$

$$\dots \quad y^T(t-g_k) \quad x^T(t-g_1) \quad \dots \quad x^T(t-g_k)],$$

$$\tilde\psi_1=P^T\begin{bmatrix}0 & I\\ \sum_{i=0}^{m}\tilde A_i & -I\end{bmatrix}+\begin{bmatrix}0 & \sum_{i=0}^{m}\tilde A_i^T\\ I & -I\end{bmatrix}P+$$

$$\begin{bmatrix}\sum_{i=1}^{k}U_i+\sum_{i=1}^{m}\tau_i R_{i0} & 0\\ 0 & \sum_{i=1}^{k}Q_i+\sum_{i=1}^{m}h_iR_i\end{bmatrix},$$

$$\tilde{\psi}_1 = P^T \begin{bmatrix} 0 & I \\ \sum_{i=0}^{m} \tilde{A}_i & -I \end{bmatrix} + \begin{bmatrix} 0 & \sum_{i=0}^{m} \tilde{A}_i^T \\ I & -I \end{bmatrix} P + \begin{bmatrix} \sum_{i=1}^{k} U_i + \sum_{i=1}^{m} \tau_i R_{i0} & 0 \\ 0 & \sum_{i=1}^{k} Q_i + \sum_{i=1}^{m} h_i R_i \end{bmatrix}, \tag{37}$$

$$\tilde{\eta}_i = -2 \int_{t-hi}^{t} [x^T(t) \quad y^T(t)]\, P^T \begin{bmatrix} 0 \\ \tilde{A}_i \end{bmatrix} y(s)ds, \tag{38}$$

$$\tilde{\eta}_{i0} = -2 \int_{t-hi}^{t} [x^T(t) \quad y^T(t)]\, P^T \begin{bmatrix} 0 \\ \tilde{A}_{i0} \end{bmatrix} x(s)ds. \tag{39}$$

By using Lemma 1 we will have:

$$\tilde{\eta}_{i0} \le h_i \left[x^T(t) \quad y^T(t) \right] P^T \begin{bmatrix} 0 \\ \tilde{A}_{i0} \end{bmatrix} \times R_{i0}^{-1} \left[0 \quad \tilde{A}_{i0}^T \right] P \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} + \int_{t-hi}^{t} x^T(s) R_{i0} x(s) ds \tag{40}$$

Then by using Lemma 2 we will have:

$$(40) \le \left[x^T(t) \quad y^T(t) \right] h_i P^T \left(\begin{bmatrix} 0 \\ A_{i0} \end{bmatrix} R_{i0}^{-1} \left[0 \quad A_{i0}^T \right] + \begin{bmatrix} 0 \\ A_{i0} \end{bmatrix} R_{i0}^{-1} \left[0 \quad E_{i0}^T \right] \left(\xi_i I - \begin{bmatrix} 0 \\ E_{i0} \end{bmatrix} R_{i0}^{-1} \left[0 \quad E_{i0}^T \right] \right)^{-1} \begin{bmatrix} 0 \\ E_{i0} \end{bmatrix} R_{i0}^{-1} \left[0 \quad A_{i0}^T \right] \right) \times P \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} + h_i \left[x^T(t) \quad y^T(t) \right] P^T \xi_i \times \begin{bmatrix} 0 & 0 \\ 0 & D_i D_i^T \end{bmatrix} P \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} + \int_{t-hi}^{t} x^T(s) R_{i0} x(s) ds \tag{41}$$

Where

$$\left(\xi_i I - \begin{bmatrix} 0 \\ E_{i0} \end{bmatrix} R_{i0}^{-1} \left[0 \quad E_{i0}^T \right] \right) > 0$$

and $\xi_i > 0,\ i = 1, \ldots, m$.

Further, using Lemma 1, 2 , for $\tilde{\psi}_1$ we have:

$$\tilde{\psi}_1 = P^T \begin{bmatrix} 0 & I \\ \sum_{i=0}^{m} \tilde{A}_i & -I \end{bmatrix} + \begin{bmatrix} 0 & \sum_{i=0}^{m} \tilde{A}_i^T \\ I & -I \end{bmatrix} P + \begin{bmatrix} \sum_{i=1}^{k} U_i + \sum_{i=1}^{m} \tau_i R_{i0} & 0 \\ 0 & \sum_{i=1}^{k} Q_i + \sum_{i=1}^{m} h_i R_i \end{bmatrix}$$
$$\le \psi_1 = P^T \begin{bmatrix} 0 & I \\ \sum_{i=0}^{m} A_i & -I \end{bmatrix} + \begin{bmatrix} 0 & \sum_{i=0}^{m} A_i^T \\ I & -I \end{bmatrix} P + \begin{bmatrix} \sum_{i=1}^{k} U_i + \sum_{i=1}^{m} \tau_i R_{i0} & 0 \\ 0 & \sum_{i=1}^{k} Q_i + \sum_{i=1}^{m} h_i R_i \end{bmatrix} + P^T \begin{bmatrix} 0 & 0 \\ 0 & \sum_{i=0}^{m} \xi_i^{-1} D_i D_i^T \end{bmatrix} P + \begin{bmatrix} \sum_{i=0}^{m} \xi_i E_i^T E_i & 0 \\ 0 & 0 \end{bmatrix} \tag{42}$$

Consequently from (19, 40,41), for the time derivative of V(x, t), (36), will have:

$$\frac{dV}{dt} \le \zeta \begin{bmatrix} \psi_1 & P^T \begin{bmatrix} 0 \\ \tilde{D}_1 \end{bmatrix} & \cdots & P^T \begin{bmatrix} 0 \\ \tilde{D}_k \end{bmatrix} & P^T \begin{bmatrix} 0 \\ H_1 \end{bmatrix} & \cdots & P^T \begin{bmatrix} 0 \\ H_k \end{bmatrix} \\ [0 \quad \tilde{D}_1^T]P & -Q_1 & 0 & 0 & 0 & \cdots & 0 \\ \vdots & 0 & \ddots & 0 & 0 & \cdots & 0 \\ [0 \quad \tilde{D}_k^T]P & 0 & 0 & -Q_k & 0 & \cdots & 0 \\ [0 \quad H_1^T]P & 0 & \cdots & 0 & -U_1 & 0 & 0 \\ \vdots & \vdots & \vdots & 0 & 0 & \ddots & 0 \\ [0 \quad H_k^T]P & 0 & 0 & 0 & 0 & 0 & -U_k \end{bmatrix} \zeta^T + \tag{43}$$
$$\sum_{i=1}^{m} [x^T(t) \quad y^T(t)] h_i P^T \left(\begin{bmatrix} 0 \\ A_i \end{bmatrix} R_i^{-1} [0 \quad A_i^T] + \begin{bmatrix} 0 \\ A_i \end{bmatrix} R_i^{-1} [0 \quad E_i^T] \left(\xi_i I - \begin{bmatrix} 0 \\ E_i \end{bmatrix} R_i^{-1} [0 \quad E_i^T] \right)^{-1} \begin{bmatrix} 0 \\ E_i \end{bmatrix} R_i^{-1} [0 \quad A_i^T] \right) P \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} +$$
$$\sum_{i=1}^{m} h_i [x^T(t) \quad y^T(t)] P^T \xi_i \begin{bmatrix} 0 & 0 \\ 0 & D_i D_i^T \end{bmatrix} P \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} +$$
$$\sum_{i=1}^{m} [x^T(t) \quad y^T(t)] h_i P^T \left(\begin{bmatrix} 0 \\ A_{i0} \end{bmatrix} R_{i0}^{-1} [0 \quad A_{i0}^T] + \begin{bmatrix} 0 \\ A_{i0} \end{bmatrix} R_{i0}^{-1} [0 \quad E_{i0}^T] \left(\xi_i I - \begin{bmatrix} 0 \\ E_{i0} \end{bmatrix} R_{i0}^{-1} [0 \quad E_{i0}^T] \right)^{-1} \begin{bmatrix} 0 \\ E_{i0} \end{bmatrix} R_{i0}^{-1} [0 \quad A_{i0}^T] \right) P \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} +$$
$$\sum_{i=1}^{m} h_i [x^T(t) \quad y^T(t)] P^T \xi_i \begin{bmatrix} 0 & 0 \\ 0 & D_i D_i^T \end{bmatrix} P \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$$

Further for the first part of eq.43 we will have:

$$\begin{bmatrix} \psi_1 & P^T\begin{bmatrix}0\\ \tilde{D}_1\end{bmatrix} & \dots & P^T\begin{bmatrix}0\\ \tilde{D}_k\end{bmatrix} & P^T\begin{bmatrix}0\\ H_1\end{bmatrix} & \dots & P^T\begin{bmatrix}0\\ H_k\end{bmatrix} \\ [0\quad \tilde{D}_1^T]P & -Q_1 & \dots & 0 & 0 & 0 & 0 \\ \vdots & 0 & \ddots & \vdots & \vdots & \dots & \vdots \\ [0\quad \tilde{D}_k^T]P & 0 & 0 & -Q_k & 0 & 0 & 0 \\ [0\quad H_1^T]P & 0 & 0 & 0 & -U_1 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & 0 & \ddots & 0 \\ [0\quad H_k^T]P & 0 & 0 & 0 & 0 & 0 & -U_k \end{bmatrix} \tag{44}$$

$$= \begin{bmatrix} \psi_1 & P^T\begin{bmatrix}0\\ \bar{D}_1\end{bmatrix} & \dots & P^T\begin{bmatrix}0\\ \bar{D}_k\end{bmatrix} & P^T\begin{bmatrix}0\\ H_1\end{bmatrix} & \dots & P^T\begin{bmatrix}0\\ H_k\end{bmatrix} \\ [0\quad \bar{D}_1^T]P & -Q_1 & 0 & 0 & 0 & 0 & 0 \\ \vdots & 0 & \ddots & 0 & \vdots & \dots & \vdots \\ [0\quad \bar{D}_k^T]P & \vdots & 0 & -Q_k & 0 & 0 & 0 \\ [0\quad H_1^T]P & 0 & 0 & 0 & -U_1 & 0 & 0 \\ \vdots & \vdots & \dots & \vdots & 0 & \ddots & 0 \\ [0\quad H_k^T]P & 0 & \dots & 0 & 0 & 0 & -U_k \end{bmatrix} +$$

$$\begin{bmatrix} 0 & P^T\begin{bmatrix}0\\ \Delta\bar{D}_1\end{bmatrix} & \dots & P^T\begin{bmatrix}0\\ \Delta\bar{D}_k\end{bmatrix} & 0 & \dots & 0 \\ [0\quad \Delta\bar{D}_1^T]P & 0 & \dots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \dots & \vdots & \vdots & \vdots & \vdots \\ [0\quad \Delta\bar{D}_k^T]P & 0 & \dots & 0 & 0 & 0 & 0 \\ 0 & 0 & \dots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \dots & \vdots & \vdots & \vdots & 0 \\ 0 & 0 & \dots & 0 & 0 & \dots & 0 \end{bmatrix}$$

By using Lemma 2 we will have:

$$\begin{bmatrix} 0 & P^T\begin{bmatrix}0\\ \Delta\bar{D}_1\end{bmatrix} & \dots & P^T\begin{bmatrix}0\\ \Delta\bar{D}_k\end{bmatrix} & 0 & \dots & 0 \\ [0\quad \Delta\bar{D}_1^T]P & 0 & \dots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \dots & \vdots & \vdots & \vdots & \vdots \\ [0\quad \Delta\bar{D}_k^T]P & 0 & \dots & 0 & 0 & 0 & 0 \\ 0 & 0 & \dots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \dots & \vdots & \vdots & \vdots & 0 \\ 0 & 0 & \dots & 0 & 0 & \dots & 0 \end{bmatrix} \le$$

$$\begin{bmatrix} P^T\begin{bmatrix}0 & 0\\ 0 & \sum_{i=1}^m \xi_i^{-1} D_i D_i^T\end{bmatrix}P & 0 & 0 & \dots & 0 & \dots & 0 \\ 0 & 2\xi_1\bar{E}_1\bar{E}_1^T & 0 & \dots & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & & \vdots \\ 0 & 0 & \dots & 2\xi_k\bar{E}_k\bar{E}_k^T & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 & 0 & \dots & 0 \end{bmatrix} = \bar{\bar{\Gamma}},$$

Also, according to the results of eq. (21-24 and 43-44), for (43) have:

$$\frac{dV}{dt} \le \zeta \begin{bmatrix} \psi_1 & P^T\begin{bmatrix}0\\ \bar{D}_1\end{bmatrix} & \dots & P^T\begin{bmatrix}0\\ \bar{D}_k\end{bmatrix} & P^T\begin{bmatrix}0\\ H_1\end{bmatrix} & \dots & P^T\begin{bmatrix}0\\ H_k\end{bmatrix} \\ [0\quad \bar{D}_1^T]P & -Q_1 & \dots & 0 & 0 & \dots & 0 \\ \vdots & 0 & \dots & 0 & \vdots & \dots & \vdots \\ [0\quad \bar{D}_k^T]P & 0 & \dots & -Q_k & 0 & \cdots & 0 \\ [0\quad H_1^T]P & 0 & \dots & 0 & -U_1 & 0 & 0 \\ \vdots & \vdots & \dots & \vdots & \vdots & \ddots & 0 \\ [0\quad H_k^T]P & 0 & \dots & 0 & 0 & \dots & -U_k \end{bmatrix} \zeta^T + \zeta\bar{\bar{\Gamma}}\zeta^T + \tag{45}$$

$$\sum_{i=1}^{m} [x^T(t) \quad y^T(t)] h_i P^T \left(\begin{bmatrix}0\\ A_i\end{bmatrix} R_i^{-1} [0 \quad A_i^T] + \right.$$

$$- \begin{bmatrix}0\\ A_i\end{bmatrix} R_i^{-1} [0 \quad E_i^T] \left(\xi_i I - \begin{bmatrix}0\\ E_i\end{bmatrix} R_i^{-1} [0 \quad E_i^T] \right)^{-1} \begin{bmatrix}0\\ E_i\end{bmatrix} R_i^{-1} [0 \quad A_i^T]$$

$$+ \begin{bmatrix}0\\ A_{i0}\end{bmatrix} R_{i0}^{-1} [0 \quad A_{i0}^T]$$

$$+ \begin{bmatrix}0\\ A_{i0}\end{bmatrix} R_{i0}^{-1} [0 \quad E_{i0}^T] \left(\xi_i I - \begin{bmatrix}0\\ E_{i0}\end{bmatrix} R_{i0}^{-1} [0 \quad E_{i0}^T] \right)^{-1}$$

$$\left. \times \begin{bmatrix}0\\ E_{i0}\end{bmatrix} R_{i0}^{-1} [0 \quad A_{i0}^T] \right) P \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$$

$$y^T(t)] P^T \xi_i \begin{bmatrix} 0 & 0 \\ 0 & 2D_i D_i^T \end{bmatrix} P \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}.$$

Where:

$$\zeta = \left[x^T(t) \quad y^T(t) \quad y^T(t-h_1) \quad \dots \quad y^T(t-h_m) \right].$$

Thus by Schur complements, $\frac{dV}{dt} \le 0$ if the following LMI holds:

$$W = \begin{bmatrix} \bar{\psi}_1 & \theta_1 & 0 & \theta_2 & \theta_3 & \theta_4 \\ * & \theta_9 & \theta_6 & 0 & 0 & 0 \\ * & * & \theta_7 & 0 & 0 & 0 \\ * & * & * & \theta_{10} & 0 & 0 \\ * & * & * & * & \theta_{11} & 0 \\ * & * & * & * & * & \theta_{12} \end{bmatrix} < 0, \tag{46}$$

$$\bar{\psi}_1 = P^T \begin{bmatrix} 0 & I \\ \sum_{i=0}^m A_i & -I \end{bmatrix} + \begin{bmatrix} 0 & \sum_{i=0}^m A_i^T \\ I & -I \end{bmatrix} P +$$

$$\begin{bmatrix} \sum_{i=1}^{k} U_i + \sum_{i=1}^{m} \tau_i R_{i0} & 0 \\ 0 & \sum_{i=1}^{k} Q_i + \sum_{i=1}^{m} h_i R_i \end{bmatrix} +$$

$$P^T \begin{bmatrix} 0 & 0 \\ 0 & 2\sum_{i=1}^{m} \xi_i^{-1} D_i D_i^T \end{bmatrix} P +$$

$$P^T \begin{bmatrix} 0 & 0 \\ 0 & \xi_0^{-1} D_0 D_0^T \end{bmatrix} P + \begin{bmatrix} \sum_{i=0}^{m} \xi_i E_i^T E_i & 0 \\ 0 & 0 \end{bmatrix} +$$

$$\sum_{i=1}^{m} h_i P^T \begin{bmatrix} 0 \\ A_i \end{bmatrix} R_i^{-1} [0 \quad A_i^T] P +$$

$$P^T \begin{bmatrix} 0 & 0 \\ 0 & 2\sum_{i=1}^{m} h_i \xi_i D_{i0} D_{i0}^T \end{bmatrix} P +$$

$$\sum_{i=1}^{m} h_i P^T \begin{bmatrix} 0 \\ A_{i0} \end{bmatrix} R_{i0}^{-1} [0 \quad A_{i0}^T] P,$$

$$\theta_1 = vec\left\{P^T \begin{bmatrix} 0 \\ D_i \end{bmatrix}\right\}^T, i = 1, \dots, k.$$

$$\theta_2 = vec\left\{P^T \begin{bmatrix} 0 \\ H_1 \end{bmatrix}\right\}^T, i = 1, \dots, k.$$

$$\theta_9 = -diag(Q_i), \theta_{10} = -diag(U_i),$$

$$\theta_6 = vec\{\bar{E}_i^T\}, \theta_7 = -diag(\xi_i^{-1} I), i = 1, \dots, k.$$

And:

$$\theta_3 = vec\left\{h_i P^T \begin{bmatrix} 0 \\ A_i \end{bmatrix} R_i^{-1} [0 \quad E_i^T]\right\}^T,$$

$$\theta_{11} = -diag\left(h_i \left(\xi_i I - \begin{bmatrix} 0 \\ E_i \end{bmatrix} R_i^{-1} [0 \quad E_i^T]\right)\right),$$

$$\theta_4 = vec\left\{h_i P^T \begin{bmatrix} 0 \\ A_{i0} \end{bmatrix} R_{i0}^{-1} [0 \quad E_{i0}^T]\right\}^T,$$

$$\theta_{12} = -diag\left(h_i \left(\xi_i I - \begin{bmatrix} 0 \\ E_{i0} \end{bmatrix} R_{i0}^{-1} [0 \quad E_{i0}^T]\right)\right)$$

For $i = 1, \dots, m$.

From the above LMI we have following inequality:

$$W = \begin{bmatrix} \bar{\bar{\psi}}_1 & \theta_1 & 0 & \theta_2 & \theta_3 & \theta_4 & vec\{I\} & vec\{h_i I\} & vec\{I\} & vec\{I\} \\ * & \theta_9 & \theta_6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & * & \theta_7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \theta_{10} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & 0 & 0 & \theta_{11} & 0 & 0 & 0 & 0 & 0 \\ * & 0 & 0 & 0 & 0 & \theta_{12} & 0 & 0 & 0 & 0 \\ * & 0 & 0 & 0 & 0 & 0 & \theta_{14} & 0 & 0 & 0 \\ * & 0 & 0 & 0 & 0 & 0 & 0 & \theta_{15} & 0 & 0 \\ * & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \theta_{16} & 0 \\ * & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \theta_{17} \end{bmatrix} < 0, \quad (47)$$

Where:

$$\bar{\bar{\psi}}_1 = P^T \begin{bmatrix} 0 & I \\ \sum_{i=0}^{m} A_i & -I \end{bmatrix} + \begin{bmatrix} 0 & \sum_{i=0}^{m} A_i^T \\ I & -I \end{bmatrix} P +$$

$$P^T \begin{bmatrix} 0 & 0 \\ 0 & 2\sum_{i=1}^{m} \xi_i^{-1} D_i D_i^T \end{bmatrix} P +$$

$$P^T \begin{bmatrix} 0 & 0 \\ 0 & \xi_0^{-1} D_0 D_0^T \end{bmatrix} P + \begin{bmatrix} \sum_{i=0}^{m} \xi_i E_i^T E_i & 0 \\ 0 & 0 \end{bmatrix} +$$

$$\sum_{i=1}^{m} h_i P^T \begin{bmatrix} 0 \\ A_i \end{bmatrix} R_i^{-1} [0 \quad A_i^T] P +$$

$$P^T \begin{bmatrix} 0 & 0 \\ 0 & 2\sum_{i=1}^{m} h_i \xi_i D_{i0} D_{i0}^T \end{bmatrix} P +$$

$$\sum_{i=1}^{m} h_i P^T \begin{bmatrix} 0 \\ A_{i0} \end{bmatrix} R_{i0}^{-1} [0 \quad A_{i0}^T] P,$$

$\theta_{14} = -diag(Q_i^{-1})$,i=1,…,k

.$\theta_{15} = -diag(h_i R_i^{-1})$,i=1,…,m.

$\theta_{16} = -diag(U_i^{-1})$, *i=1,…,k.*

$\theta_{17} = -diag(\tau_i^{-1} R_{i0}^{-1})$, *i=1,…,m.*

By pre and post multiplying (47), by $\Delta_2 = diag\left\{X, diag\left(Q_i^{-1}\right), I, diag(U_i^{-1}), I, I, I, I, I, I\right\}$ and Δ_2^T ,

where $X = P^{-1}$, also denote $X_1 K$ by Y and $\bar{Q}_i = Q_i^{-1}$, $\bar{R}_i = R_i^{-1}$, $\bar{U}_i = U_i^{-1}$, $\bar{R}_{i0} = R_{i0}^{-1}$, and applying the Schur formula, we will obtain the inequality shown in (35).

This implies asymptotic stability of (27) for continuous functions and under assumption A1, for continuously differentiable functions [27].

3- 2- Delay-independent Stability

Consider the system:

$$\dot{x}(t)-\sum_{i=1}^{k}\tilde{D}_i\dot{x}(t-g_i)=\tilde{A}_0x(t)+\sum_{i=0}^{k}H_ix(t-g_i), \quad (48)$$

Delay-independent stability conditions can be derived by applying Lyapunov–Krasovskii functional of (28), where V2 =V4 =0. Theorem 2 implies the following delay-independent stability criterion:

Corollary. Under A1, (45) is stable for all

$g_i > 0, i = 1,\ldots,k$ if there exist $X=\begin{bmatrix} X_1 & 0 \\ X_2 & X_3 \end{bmatrix}$, $X_1 = X_1^T > 0, X_2, X_3, \bar{Q}_i = \bar{Q}_i^T, \bar{U}_i = \bar{U}_i^T, \xi_i > 0, i = 1,\ldots,k$ that satisfy the following LMI:

$$W=\begin{bmatrix} \bar{\bar{\psi}}_1 & \bar{\theta}_1 & 0 & \bar{\theta}_2 & X^T vec\{I\} & X^T vec\{I\} \\ * & \bar{\theta}_5 & \bar{\theta}_6 & 0 & 0 & 0 \\ * & * & \bar{\theta}_7 & 0 & 0 & 0 \\ * & * & * & \bar{\theta}_8 & 0 & 0 \\ * & * & * & * & \bar{\theta}_{11} & 0 \\ * & * & * & * & * & \bar{\theta}_{13} \end{bmatrix} < 0,$$

$$\bar{\bar{\psi}}_1=\begin{bmatrix} 0 & 0 \\ AX_1+BY & 0 \end{bmatrix}+\begin{bmatrix} 0 & (AX_1)^T+(BY)^T \\ 0 & 0 \end{bmatrix}+\begin{bmatrix} 0 & I \\ \sum_{i=1}^{m} A_i & -I \end{bmatrix}X+X^T\begin{bmatrix} 0 & \sum_{i=1}^{m} A_i^T \\ I & -I \end{bmatrix}+\begin{bmatrix} 0 & 0 \\ 0 & 2\sum_{i=1}^{m}\xi_i^{-1}D_iD_i^T \end{bmatrix}+\begin{bmatrix} 0 & 0 \\ 0 & \xi_0^{-1}D_0D_0^T \end{bmatrix}+X^T\begin{bmatrix} \sum_{i=0}^{m}\xi_iE_i^TE_i & 0 \\ 0 & 0 \end{bmatrix}X, \bar{\theta}_1 = vec\left\{\begin{bmatrix} 0 \\ \bar{D}_i \end{bmatrix}(\bar{Q}_i)\right\}, \quad (49)$$

$\bar{\theta}_2 = vec\left\{\begin{bmatrix} 0 \\ H_1 \end{bmatrix}(\bar{U}_i)\right\}, \bar{\theta}_5 = -diag(\bar{Q}_i),$

$\bar{\theta}_8 = -diag(\bar{U}_i), \bar{\theta}_6 = vec\{\bar{E}_i^T\},$

$\bar{\theta}_7 = -diag(\xi_i^{-1} I), i = 1,\ldots,k.$ $\bar{\theta}_{11} = -diag(\bar{Q}_i),$

i=1,…,k. $\bar{\theta}_{13} = -diag(\bar{U}_i)$, i=1,…,k.

4- Numerical Simulations

In the current section, to evaluate the efficacy of the proposed method, a numerical example is brought as follows.

Example 1: Consider the following time-delay uncertain system with state delay7]]:

$$\dot{x}(t)=\tilde{A}_0x(t)+\tilde{A}_1x(t-h_1)+\tilde{D}_1\dot{x}(t-h_1), \tilde{A}_0 = A+BK+\Delta A_0$$

With

$$A=\begin{bmatrix} -0.9 & 0.2 \\ 0.1 & -0.9 \end{bmatrix}, B=\begin{bmatrix} 1 \\ 1 \end{bmatrix}, A_1=\begin{bmatrix} -1.1 & -0.2 \\ -0.1 & -1.1 \end{bmatrix}, \bar{D}_1=\begin{bmatrix} -0.2 & 0 \\ 0.2 & -0.1 \end{bmatrix}$$

where unknown matrices ΔA_0, ΔA_1 and $\Delta\bar{D}_1$ are satisfying $\Delta A_0 < \delta, \Delta A_1 < \delta, \Delta\bar{D}_1 < \delta$.

Using the stability condition proposed in theorem1, for maximum uncertainty bound δ =0.28, the maximum value of h_1 to guarantee asymptotically stability is h_{max} =2.1 with controller gain $K=\begin{bmatrix} -0.8974 & -1.1158 \end{bmatrix}$

Fig. 1 shows the uncertain system states convergence with proposed control strategy.

In addition, Table I shows effect of the uncertainty bound δ on maximum admissible delay before losing closed loop stability. this example again shows that the stability criterion in this paper gives a much less conservative result than these in [7].

Example 2: An uncertain system with a time-delay is assumed as the following:

$$\dot{x}(t)=(A+\Delta A)x(t)+A_1x(t-g(t))+\bar{D}_1\dot{x}(t-g(t))+Bu(t)$$
$$g(t)=0.2|\sin(t)|$$

Where:

$$A=\begin{bmatrix} -1.5 & -0.1 & 1 \\ 0 & -1.3 & 0.5 \\ 1 & 0 & -1 \end{bmatrix}, A_1=\begin{bmatrix} -1 & 0.1 & 0 \\ 0 & 0.2 & 0 \\ 0 & -1 & 0.2 \end{bmatrix}, B=\begin{bmatrix} -1 \\ 1.5 \\ 0.3 \end{bmatrix},$$

$$\bar{D}_1=\begin{bmatrix} -0.2 & -0.5 & 0 \\ 0 & 0.4 & 0 \\ 1 & 0 & 0.6 \end{bmatrix}, \text{ And } \Delta A=\begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0 & 0.1 \\ .2 & 0.01 & 0.01 \end{bmatrix}.$$

The total simulation time was set to 40sec, and the sampling time to 0.01sec. In our scenario, a delay is injected at time 12s with function $g(t)$ to all states. also disturbance signal d(t)=0.8 is injected at time 20s for 2 sec. Applying theorem 1 to this system, and using LMI (12), we have the controller gain $K=\begin{bmatrix} -0.5293 & 0.5615 & 0.1537 \end{bmatrix}$.

Fig. 1 shows system states trajectory and the results obtained by our proposed robust controller. Numerical results illustrate that the presented controlling algorithm performs perfectly. The obtained results in Fig 2 imply that the closed-loop system is stable and could tolerate the time variant delay and disturbance.

Example 3: Control of Mach Number in a Wind Tunnel

In steady-state operating, the dynamic response of the Mach Number perturbations δM to small perturbations in the guide vane angle actuator δθa in a driving fan is described by the following equations [30]:

$$\frac{1}{\alpha}\delta\dot{M}(t)+\delta M(t)=k\,\delta\theta(t-\tau(t))$$

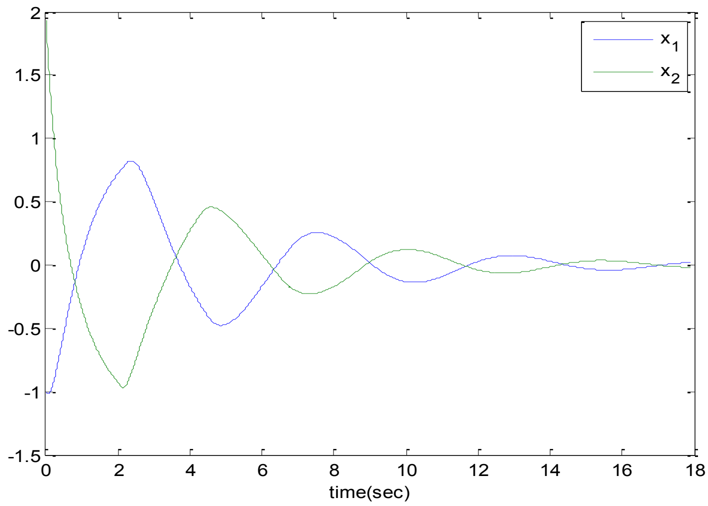

Fig. 1. states convergence for system with discrete delay

Table 1. The effect of the uncertainty bound δ on maximum admissible delay before losing closed loop stability

δ	**0.13**	0.28	**0.38**	**0.7**	**0.83**	**1.0**
$h_1(max)$	3.1	2.1	2.3	1.8	0.9	0.85

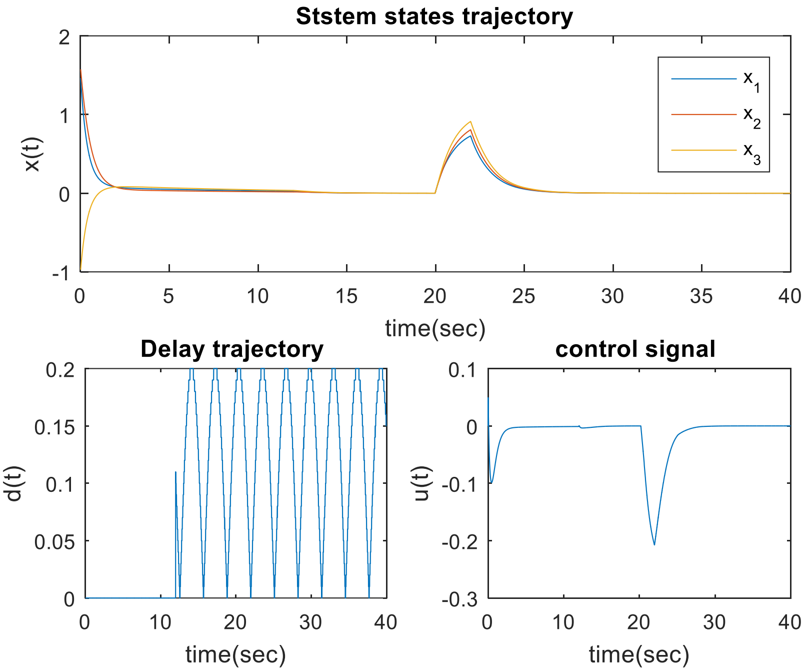

Fig. 2. system states convergence under proposed robust controller, and delay strategy g(t).

$$\ddot{\delta\theta}(t)+2\zeta\omega\delta\dot{\theta}(t)+\omega^2\delta\theta(t)=\omega^2\delta\theta_a(t)$$

where $\delta\theta$ is the guide vane angle, α , k, ξ, ω are parameters depending on the operating point which are presumed constant when the perturbation δM , $\delta\theta$, $\delta\theta_a$ are small and the delay $\tau(t)$ represents the time of the transport between the fan and the test section. The above equation in state space form yields $\dot{x}(t)=\tilde{A}_0x(t)+\tilde{A}_1x(t-\tau(t))+Bu(t)$,
Where: $\tilde{A}_0=A+\Delta A_0+BK$

$$x=\begin{bmatrix}\delta M\\ \delta\theta\\ \delta\dot{\theta}\end{bmatrix}, A=\begin{bmatrix}-\alpha & 0 & 0\\ 0 & 0 & 1\\ 0 & -\omega^2 & -2\zeta\omega\end{bmatrix}, B=\begin{bmatrix}0\\ 0\\ \omega^2\end{bmatrix}, A_1=\begin{bmatrix}0 & k\alpha & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$$

The control u(t) represents $\delta\theta_a$. Additionally for system parameters and uncertainty, we assume:

$$\frac{1}{\alpha}=1.964s\ ,k=-.0117deg^{-1},\zeta=0.8 and\ \omega=6rad/s$$

$$\tau(t)=\left|\sin(0.63t)\right|\ \ \Delta A_0=D_0F_0(x,t)E_0\ ,\ \Delta A_1=D_1F_1(x,t)E_1,$$

Where:

$$D_0=\begin{bmatrix}-1 & 0 & 0\\ 0 & 0.2 & 0\\ 0 & 0 & 0.4\end{bmatrix}, E_0=E_1=\begin{bmatrix}-.2 & 0 & 0\\ 0 & 0.1 & 0\\ 0 & 0 & 1\end{bmatrix}, D_1=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 0\\ 0 & 0 & -0.2\end{bmatrix},$$

The total simulation time was set to 10sec, and the sampling time to 0.01sec. Figure 3 shows the uncertain system sates under the designed controller converge to steady state.5-

5- Conclusion

This paper considers designing robust controllers for linear retarded and neutral type uncertain systems with discrete and distributed delays. Lyapunov–Krasovskii function and equivalent descriptor form of the original system have been introduced to guarantee closed loop stability.

The stabilization problem is extended to more general class of neutral-type uncertain linear systems with multiple delays. Two delay-dependent/independent approaches are proposed to design robust controllers for a class of uncertain linear neutral systems with parametric uncertainty, discrete and distributed multiple delays. The sufficient conditions, Delay-dependent/delay-independent are introduced in terms of LMIs. Afterwards, the strategy is extended to consider H∞ control of linear uncertain systems with delay. Numerical examples show the effectiveness of theoretical results.

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