Combination of Adomian Decomposition Method with Fourier Transform for Solving the Squeezing Flow Influenced by a Magnetic Field

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ABSTRACT: In this paper the Fourier Transform Combined with Adomian Decomposition Method (FTADM) is applied for solving the squeezed unsteady flow between the parallel plates influenced by an inclined magnetic field. By moving these plates towards each other, the squeezing flow which is perpendicular to the plates is appeared. We assume that inclination varies from 0 to 90 degrees. The momentum and energy equations are solved using the Fourier Transform Combined with Adomian Decomposition Method transform. The effect of the squeezed number, the angle of the magnetic inclination, and the bottom plate suction/injection on the velocity and temperature are studied. The results show that by increasing the squeeze number, the intensity of the magnetic field and the magnetic inclined angle may increase the velocity and bottom plates in the longitudinal direction. However, the velocity near the midway of up and bottom plates may decrease. Moreover, our results show more accuracy and a smaller number of calculations when compared to the previous numerical simulations. This may be attributed to the fact that in FTADM, one is able to incorporate all boundary conditions into the solution. However, in the semi-analytical methods, the solution may be accurate in a limited portion of the solution domain, since only a part of the boundary conditions is imposed into the solution.

1- Introduction

The study of squeezing flow between two infinite parallel plates has been very popular for researchers since last decade. By moving these plates towards each other, the squeezing flow perpendicular to the plates appears. The squeezing flow has a vast majority of applications such as power transmission, polymer processing, hydraulic lifts, and cooling water. Many of these flows, such as the lubrication problem, may be considered as squeezing flow [1]. Due to these applications, the study of momentum and heat transfer in squeezing flow has attracted a lot of attention. The primary study of squeezed flow between two parallel plates was performed by Stefan [1], where he studied the lubrication mechanism. This study was a beginning for more extensive works to understand the behavior of the squeezed flow between two planar plates. Ran et al. [2] used the Homotopy Analysis Method (HAM) to investigate the squeezed flow between two planar plates slowly moving towards each other. Mustafa et al. [3] reported characteristics of the heat and mass transfer in a viscous fluid squeezed between two infinite parallel plates. Sheikholeslami et al. [4] utilized a neural network to estimate the heat transfer rate for Nanofluid of alumina through a channel. Khan et al. [5] used the Variation of Parameters Method (VPM) to study the behavior of copper nanoparticles between parallel plates. Dib et al. [6] employed the Duan-Rach method to obtain a recursive convergent solution for momentum and energy equation of an unsteady squeezing problem of a Nanofluid flow. Hayat et al. [7] studied heat and mass transfer for electromagnetic squeezed flow through a Riga plate considering thermal radiation and chemical reaction. Hayat et al. [8] also considered the squeezed flow for Nanotubes of carbon as a Darcy-Forchheimer porous media. Ahmed et al. [9] carried out a theoretical study on characteristics of natural and forced convection flow of Sutterby fluid in a squeezing flow. Investigation of momentum and heat transfer in squeezing flow influenced by a magnetic field has been very important for many researchers with a variety of applications such as MHD (Magneto-Hydro-Dynamic) generator, nuclear reactors, studies of plasma, geometrical extractions, and control of boundary layer [10-25]. Recently, many researchers studied the behavior of the squeezed flow influenced by a magnetic field using different methods. Siddique et al. [26] studied the 2-D flow of a viscous MHD fluid between parallel plates influenced by a magnetic field by using the Homotopy Perturbation Method. Domairry and Aziz [27] performed a study on MHD squeezed flow between parallel disks given the suction/injection of the fluid. Haq et al. [28] studied MHD squeezing flow of a fluid through a sensor plate. Investigation of Magneto Hydro Dynamic (MHD) squeezing flow of a nanofluid through a stretching porous media was performed by Hayat et al. [29]. The Adomian Decomposition Method (ADM) [30-37] is a well-known systematic method

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for the practical solution of linear, nonlinear, deterministic, and stochastic operator equations. These equations include Ordinary Differential Equations (ODEs), Partial Differential Equations (PDEs), integral equations, and integrodifferential equations. The ADM method is a powerful technique, which provides efficient algorithms for analytic approximate solutions for real-world applications in the applied sciences and engineering. A.M. Wazwaz studied the Bratu problem and Lane-Emden, using the Adomian Decomposition Method [38-39]. J.S. Duan solved nonlinear fractional ordinary differential equations [40]. Y.T. Yang used the Adomian Decomposition Method to solve the periodic base temperature in convective longitudinal fins [41]. Recently, Nourazar et al. [42] developed a new method by a modification to the Adomian Decomposition Method using the Fourier transform. In the present work, we use the method of Nourazar et al. [42] to solve the momentum and energy equations of squeezing flow influenced by a magnetic field. Effects of different parameters such as squeezing number, the intensity of the magnetic field, the bottom plate suction/injection parameter, and inclination angle on velocity and temperature profiles are studied. In using the semi-analytical methods ADM, the convergence of the solution is not violated when the number of terms of series solutions is increased. In other words, the convergence of the ADM is independent of the number of terms of series solutions, since the convergence of the ADM is verified. However, in the numerical methods, the accuracy of the results, mesh refinement, is limited by convergency problems.

2- The Adomian Decomposition Method (ADM)

The Adomian Decomposition Method (ADM) is proposed by Adomian [43-45]. Assume the following equation:

$$G(u) = g(\eta),$$

(1)

where G is an arbitrary operator that may be partitioned into the linear operator and a non-linear operator as:

$$G(u) = L(u) + N(u) = g(\eta).$$

(2)

The unknown function u of the linear operator, L, may be written as a series solution:

$$u = \sum_{n=0}^{\infty} u_n,$$

(3)

where $n \geq 0$ and u_n can be calculated by the recursive equations:

$$L(u) = L\left(\sum_{n=0}^{\infty} u_n\right),$$

(4)

and non-linear operator N can be written as:

$$N(u) = \sum_{n=0}^{\infty} A_n = \sum_{n=0}^{\infty} \frac{1}{n!} \frac{d^n}{d\lambda^n}\left[N\left(\sum_{n=0}^{\infty} \lambda^i u_i\right)\right]_{\lambda=0},$$

(5)

$N(u)$ may be rewritten as follows:

$$N(u) = N(u_0) + \left(\sum_{n=0}^{\infty} u_n\right) N'(u_0) +$$
$$\frac{1}{2!}\left(\sum_{n=0}^{\infty} u_n\right)^2 N''(u_0) + \frac{1}{3!}\left(\sum_{n=0}^{\infty} u_n\right)^3 N'''(u_0) + ...$$
$$= N(u_0) + (u - u_0) N'(u_0) +$$
$$\frac{1}{2!}(u - u_0)^2 N''(u_0) + \frac{1}{3!}(u - u_0)^3 N'''(u_0) + ...,$$

(6)

substituting Eq. (3) and (4) into Eq. (2):

$$L\left(\sum_{n=0}^{\infty} u_n\right) + \sum_{n=0}^{\infty} A_n = g(\eta),$$

(7)

where u_n can be obtained by solving the recursive equations obtained from Eq. (7). Assume the overall error associated with series solution as follows [46]:

$$R(\eta) = L\left(\sum_{n=0}^{\infty} u_n\right) + \sum_{n=0}^{\infty} A_n - g(\eta).$$

(8)

Assuming that the overall error in connection with the series solution, Eq. (7), defined by Eq. (8) is square integrable as $\text{Re}s = \int_\Omega R(\eta)^2 d\eta$. For a proper approximation, Res must tend to zero when the number of series terms approache infinity. In our calculations, the Duan and Rach [47] controlling parameter h for convergence is chosen to be equal to zero.

3- The Basic Idea of FTADM

Taking Fourier transform from Eq. (7) [48]:

$$L\left(\sum_{n=0}^{\infty} u_n\right) + \sum_{n=0}^{\infty} A_n = g(\omega),$$

(9)

expanding Eq. (9) as a recursive equation:

$$\sum_{n=1}^{\infty} L(u_i) + \sum_{n=0}^{\infty} A_i = 0, \tag{10}$$

the recursive equations, Eq. (10), could be written as:

$$L(u_k) + A_{k-1} = 0. \tag{11}$$

Where $u_0, u_1, u_2, u_3, ..., u_n$ are the components of series solution of $u = \sum_{n=0}^{\infty} u_n$ and Adomian polynomials are written as:

$$\sum_{k=0}^{\infty} A_k = \sum_{k=0}^{\infty} \frac{1}{k!} \frac{d^k}{d\lambda^k} \left[N\left(\sum_{k=0}^{\infty} \lambda^i u_i \right) \right]_{\lambda=0}, \tag{12}$$

4- Governing Equations

The problem to be considered is the unsteady squeezing flow of an incompressible electrically conducting fluid confined between two infinite parallel plates subjected to an inclined external magnetic field B (Fig.1). The x-axis is along the direction of the lower plate of the channel and the y-axis is normal. The time-variable magnetic field is $B = (B_m \cos\gamma, B_m \sin\gamma, 0)$, where B_m donates $B_0 = (1-\alpha t)^{-\frac{1}{2}}$, is applied at an inclination angle γ with respect to the x-axis. The induced magnetic field is assumed to be negligible for a small magnetic Reynolds number. The gap between $H(t) = l(1-\alpha t)^{\frac{1}{2}}$, changes with the time t, where l is the initial gap between the plates at the time $t = 0$ [49]. The case of $\alpha > 0$ corresponds to the squeezing motion of plates, as $\alpha > 0$ the plates move apart. Along the direction normal to the xy-plane, the velocity and temperature can be seen as unchanged. The Ohm's law gives the form of the current density vector J [50]:

$$J = \sigma(V \times B) = (0, 0, uB_m \sin\gamma - vB_m \cos\gamma) \tag{13}$$

in which σ is the electrical conductivity, and $V = (u, v, 0)$ is the velocity vector. Using eq.13, we obtain the Lorentz force:

$$\begin{aligned} J \times B = (&\sigma B_m^2 v \sin\gamma\cos\gamma - \sigma B_m^2 u \sin^2\gamma, \\ &\sigma B_m^2 u \sin\gamma\cos\gamma - \sigma B_m^2 v \sin\gamma\cos^2\gamma, 0) \end{aligned} \tag{14}$$

and the Joule heating:

$$\frac{1}{\sigma} J J = \sigma B_m^2 (u^2 \sin^2\gamma + v^2 \cos^2\gamma - 2uv \sin\gamma\cos\gamma) \tag{15}$$

The conservation equations for mass, momentum and energy in the presence of the suction or injection across the stretching lower plate, viscous dissipation, and Joule heating may be written as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{16}$$

$$\begin{aligned} \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \\ -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{\mu}{\rho}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) + \\ \frac{\sigma B_m^2}{\rho}\sin\gamma(v\cos\gamma - u\sin\gamma) \end{aligned} \tag{17}$$

$$\begin{aligned} \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = \\ -\frac{1}{\rho}\frac{\partial p}{\partial y} + \frac{\mu}{\rho}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) + \\ \frac{\sigma B_m^2}{\rho}\cos\gamma(u\sin\gamma - v\cos\gamma) \end{aligned} \tag{18}$$

$$\begin{aligned} \frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho C_p}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \\ \frac{\mu}{\rho C_p}\left[2\left(\frac{\partial u}{\partial x}\right)^2 + 2\left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2 \right] \\ + \frac{\sigma B_m^2}{\rho C_p}(u\sin\gamma - v\sin\gamma)^2 \end{aligned} \tag{19}$$

Here u and v are the velocity components of the fluid along the x and the y directions, respectively. T is the temperature. μ, ρ, C_p and k denote the effective dynamic viscosity, density, specific heat capacity at constant pressure, and the thermal conductivity of fluids, respectively.

The following boundary conditions are imposed at the lower and upper plates:

$$\begin{aligned} u = u_s = \frac{bx}{1-\alpha t}, v = v_s = -\frac{v_0}{\sqrt{1-\alpha t}}, \\ T = T_0 \quad at \quad y = 0 \end{aligned} \tag{20}$$

$$\begin{aligned} u = 0, v = v_H = \frac{dH}{dt} = -\frac{\alpha l}{2\sqrt{1-\alpha t}}, \\ T = T_H = T_0 + \frac{T_0}{1-\alpha t} \quad at \quad y = H(t) \end{aligned} \tag{21}$$

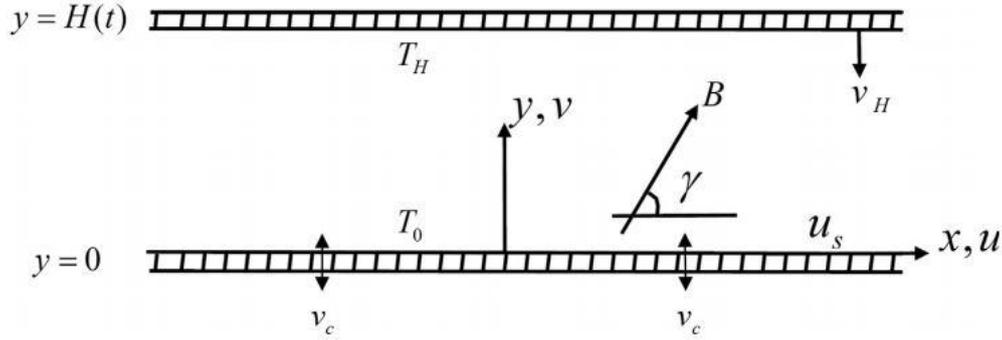Here u_s is the stretching velocity of the lower plate, v_c

Fig. 1. Geometry configuration of the problem

is the mass flux velocity through the lower plate, V_H is the velocity of the upper plates, T_0 denotes the temperature at the upper-plate surface, and T_H is the temperature at the upper-plate surface.

The mathematical analysis of the problem is simplified by introducing the following change of variables:

$$\eta = \frac{y}{H(t)}, \ v = v_H f(\eta),$$

$$u = \frac{-x}{H(t)} v_H f'(\eta), \ \theta(\eta) = \frac{T - T_0}{T_H - T_0} \tag{22}$$

By substituting (22) into Eqs. (16) – (19), the continuity equation is satisfied. Afterwards, Eqs. (17)– (19) become:

The governing dimensionless momentum and energy equations for the squeezed flow influenced by a magnetic field can be written as [51]:

$$f^{iv} - S(\eta f''' + 3f'' + ff'' - ff''') - M^2 \sin\gamma(\sin\gamma f'' + 2\delta\cos\gamma f') = 0, \tag{23}$$

Where S is the squeeze number, η is similarity coordinate, M is the magnetic parameter, γ is inclination angle with respect to the x axis, and δ is the electrical conductivity.

$$\theta'' + \Pr S(f\theta' - \eta\theta' - 2\theta) + \Pr Ec\left(f''^2 + 4\delta^2 f'^2 + M^2(f'^2\sin^2\gamma + f^2\delta^2\cos^2\gamma)\right) \tag{24}$$
$$+2\Pr Ecff'\delta\cos\gamma\sin\gamma = 0,$$

The boundary conditions are:

$$f(0) = S_b, \ f'(0) = R, \ f(1) = 1, \ f'(1) = 0 \tag{25}$$

$$\theta(0) = 0, \ \theta(1) = 1. \tag{26}$$

Where $\Pr$ is Prandtl number which is defined as the ratio of momentum boundary thickness to thermal boundary thickness, and the Eckert number, Ec, is defined as the ratio of mass transport by advective velocity (u^2, where u is the advective velocity), to the enthalpy difference of the plate and the fluid thickness ($C_p \Delta T$, where C_p is the thermal conductivity coefficient at constant pressure and T is the temperature).

5- Momentum Equation

By taking the third and fourth order integrals from Eq. (23), we may obtain $f''(0)$ and $f'''(0)$:

$$\int_0^1 \int_0^\eta \int_0^\eta \left(f^{iv} - S(\eta f''' + 3f'' + ff'' - ff''') - \right. \tag{27}$$

$$\left. M^2 \sin\gamma(\sin\gamma f'' + 2\delta\cos\gamma f')\right)d\eta d\eta d\eta = 0,$$

$$\int_0^1 \int_0^\eta \int_0^\eta \int_0^\eta \left(f^{iv} - S(\eta f''' + 3f'' + ff'' - ff''') - \right. \tag{28}$$

$$\left. M^2 \sin\gamma(\sin\gamma f'' + 2\delta\cos\gamma f')\right)d\eta d\eta d\eta d\eta = 0,$$

Taking Fourier transform of Eq. (23):

$$F\left(f^{iv} - S(\eta f''' + 3f'' + ff'' - ff''') - \right. \tag{29}$$

$$\left. M^2 \sin\gamma(\sin\gamma f'' + 2\delta\cos\gamma f')\right) = 0,$$

$$F(f'(\eta)) = -f(0) + i\omega F(f(\eta)), \tag{30}$$

$$F(f''(\eta)) = -f'(0) - i\omega f(0) - \omega^2 F(f(\eta)), \tag{31}$$

$$F\left(f'''(\eta)\right) = -f''(0) - i\,\omega f'(0) + \omega^2 f(0) - i\,\omega^3 F(f(\eta)), \tag{32}$$

$$F\left(f^{iv}(\eta)\right) = -f'''(0) - i\,\omega f''(0) + \omega^2 f'(0) + i\,\omega^3 f(0) + \omega^4 F(f(\eta)), \tag{33}$$

$$f(\eta) = \sum_{n=0}^{\infty} f_n(\eta), \quad F\left(\sum_{n=0}^{\infty} f_n(\eta)\right) = \sum_{n=0}^{\infty} f_n(\omega), \tag{34}$$

$$\eta f''(\eta) = \sum_{n=0}^{\infty} A_n(\eta), \quad F\left(\sum_{n=0}^{\infty} A_n(\eta)\right) = \sum_{n=0}^{\infty} A_n(\omega), \tag{35}$$

$$ff'' + ff''' = \sum_{n=0}^{\infty} B_n(\eta),$$
$$F\left(\sum_{n=0}^{\infty} B_n(\eta)\right) = \sum_{n=0}^{\infty} B_n(\omega), \tag{36}$$

substituting Eqs. (30)-(33), $f''(0)$ and $f'''(0)$ into Eq. (29) and making use of Eqs. (34)-(36), Eq. (29) may be rewritten as:

(For more details, see the appendix)

$$\sum_{n=0}^{\infty} \hat{f}_n = \hat{f}_0 + \left(-\frac{12}{\omega^4} + \frac{6i}{\omega^3}\right) \times$$
$$\left(-S\left(\int_0^1\int_0^\eta\int_0^\eta\int_0^\eta \sum_{n=0}^{\infty}\hat{A}_n\,d\eta d\eta d\eta d\eta + 3\int_0^1\int_0^\eta \sum_{n=0}^{\infty}\hat{f}_n\,d\eta d\eta\right)\right)$$
$$+\left(-\frac{12}{\omega^4} + \frac{6i}{\omega^3}\right)\left(\int_0^1\int_0^\eta\int_0^\eta\int_0^\eta \sum_{n=0}^{\infty}\hat{B}_n\,d\eta d\eta d\eta d\eta\right)+$$
$$\left(-\frac{12}{\omega^4} + \frac{6i}{\omega^3}\right)\left(-M^2 sin\gamma(sin\gamma\int_0^1\int_0^\eta \sum_{n=0}^{\infty}\hat{f}_n\,d\eta d\eta)\right)+$$
$$\left(-\frac{12}{\omega^4} + \frac{6i}{\omega^3}\right)\left(2\delta cos\gamma\int_0^1\int_0^\eta\int_0^\eta \sum_{n=0}^{\infty}\hat{f}_n\,d\eta d\eta d\eta\right)+$$
$$\left(\frac{6}{\omega^4} - \frac{2i}{\omega^3}\right)\left(-S\int_0^1\int_0^\eta\int_0^\eta \sum_{n=0}^{\infty}\hat{A}_n\,d\eta d\eta d\eta\right) \tag{37}$$
$$+\left(\frac{6}{\omega^4} - \frac{2i}{\omega^3}\right)\left(-S(3\int_0^1 \sum_{n=0}^{\infty}\hat{f}_n\,d\eta + \int_0^\eta\int_0^\eta \sum_{n=0}^{\infty}\hat{B}_n\,d\eta d\eta d\eta)\right)+$$
$$\left(\frac{6}{\omega^4} - \frac{2i}{\omega^3}\right)\left(-M^2 sin\gamma(sin\gamma\int_0^1 \sum_{n=0}^{\infty}\hat{f}_n\,d\eta)\right)+$$
$$\left(\frac{6}{\omega^4} - \frac{2i}{\omega^3}\right)\left(2\delta cos\gamma\int_0^1\int_0^\eta \sum_{n=0}^{\infty}\hat{f}_n\,d\eta d\eta\right)+$$
$$\frac{S}{\omega^4}\left(\sum_{n=0}^{\infty}\hat{A}_n - 3\omega^2\sum_{n=0}^{\infty}\hat{f}_n + \sum_{n=0}^{\infty}\hat{B}_n\right)$$
$$+\frac{M^2 \sin\gamma}{\omega^4}\left(-\omega^2 \sin\gamma\sum_{n=0}^{\infty}\hat{f}_n + 2i\,\omega\delta\cos\gamma\sum_{n=0}^{\infty}\hat{f}_n\right),$$

The recursive equations for the momentum equation may be written as:

$$\hat{f}_k = \left(-\frac{12}{\omega^4} + \frac{6i}{\omega^3}\right)\times$$
$$\left(-S\left(\int_0^1\int_0^\eta\int_0^\eta\int_0^\eta \hat{A}_{k-1}\,d\eta d\eta d\eta d\eta + 3\int_0^1\int_0^\eta \hat{f}_{k-1}\,d\eta d\eta\right)\right)+$$
$$\left(-\frac{12}{\omega^4} + \frac{6i}{\omega^3}\right)\left(\int_0^1\int_0^\eta\int_0^\eta\int_0^\eta \hat{B}_{k-1}\,d\eta d\eta d\eta d\eta\right)+$$
$$\left(-\frac{12}{\omega^4} + \frac{6i}{\omega^3}\right)\left(-M^2 sin\gamma(sin\gamma\int_0^1\int_0^\eta \hat{f}_{k-1}\,d\eta d\eta)\right)+$$
$$\left(-\frac{12}{\omega^4} + \frac{6i}{\omega^3}\right)\left(2\delta cos\gamma\int_0^1\int_0^\eta\int_0^\eta \hat{f}_{k-1}\,d\eta d\eta d\eta\right)+$$
$$\left(\frac{6}{\omega^4} - \frac{2i}{\omega^3}\right)\left(-S\int_0^1\int_0^\eta\int_0^\eta \hat{A}_{k-1}\,d\eta d\eta d\eta\right)\times \tag{38}$$
$$\left(\frac{6}{\omega^4} - \frac{2i}{\omega^3}\right)\left(-S(3\int_0^1\hat{f}_{k-1}\,d\eta + \int_0^\eta\int_0^\eta \sum_{n=0}^{\infty}\hat{B}_{k-1}\,d\eta d\eta d\eta)\right)+$$
$$\left(\frac{6}{\omega^4} - \frac{2i}{\omega^3}\right)\left(-M^2 sin\gamma(sin\gamma\int_0^1 \hat{f}_{k-1}\,d\eta)\right)$$
$$+\left(\frac{6}{\omega^4} - \frac{2i}{\omega^3}\right)\left(2\delta cos\gamma\int_0^1\int_0^\eta \hat{f}_{k-1}\,d\eta d\eta\right) + \frac{S}{\omega^4}\left(\hat{A}_{k-1} - 3\omega^2\hat{f}_{k-1} + \hat{B}_{k-1}\right)$$
$$+\frac{M^2 \sin\gamma}{\omega^4}\left(-\omega^2 \sin\gamma\hat{f}_{k-1} + 2i\,\omega\delta\cos\gamma\hat{f}_{k-1}\right)f_k = F^{-1}(\hat{f}_k),$$

where $\widehat{A}_0, \widehat{A}_1, \widehat{A}_2, ..., \widehat{A}_{k-1}, \widehat{B}_0, \widehat{B}_1, \widehat{B}_2, ..., \widehat{B}_{k-1}$ are the Fourier transforms of the Adomian Polynomials.

$$f_0(\eta) = (R + 2S_b - 2)\eta^3 + (3 - 3S_b - 2R)\eta^2 + R\eta +$$

$$f_1(\eta) = \left(\frac{SR^2}{70} + \frac{2SRS_b}{35} - \frac{2SR}{35} + \frac{2SS_b^2}{35} - \frac{4SS_b}{35} + \frac{2S}{35}\right)\eta^7$$
$$+\left(\begin{array}{c}\dfrac{\delta sin(2\gamma)M^2 R}{120} + \dfrac{\delta sin(2\gamma)M^2 S_b}{120} + \\ \dfrac{\delta sin(2\gamma)M^2}{60} - \dfrac{SR^2}{15} - \dfrac{7SRS_b}{30} + \dfrac{7SR}{30} - \dfrac{S}{5}\end{array}\right)\eta^6$$
$$+\left(\begin{array}{c}\dfrac{\sigma_2}{20} - \dfrac{SR}{5} - \dfrac{SS_b}{5} - \dfrac{S}{10} + \dfrac{M^2 R}{40} + \dfrac{M^2 S_b}{20} + \\ \dfrac{2SR^2}{15} + \dfrac{3SS_b}{10} - \dfrac{M^2}{20} + \dfrac{2SRS_b}{5}\end{array}\right)\eta^5$$
$$+\left(\begin{array}{c}\dfrac{3S}{4} - \dfrac{SR}{4} - \dfrac{SS_b}{4} - \dfrac{\sigma_2}{8} - \dfrac{M^2 R}{12} - \dfrac{M^2 S_b}{8} - \\ \dfrac{SR^2}{6} - \dfrac{SS_b^2}{2} + \dfrac{M^2}{8} - \dfrac{SRS_b}{2} + \dfrac{\sigma_4}{12} + \dfrac{\sigma_1}{8} + \dfrac{\sigma_3}{24}\end{array}\right)\eta^4 \tag{39}$$
$$+\left(\begin{array}{c}\dfrac{19SR}{42} - \dfrac{24S}{35} + \dfrac{SS_b}{14} + \dfrac{\sigma_2}{10} + \dfrac{11M^2 R}{120} + \\ \dfrac{M^2 S_b}{10} + \dfrac{9SR^2}{70} + \dfrac{43SS_b^2}{70} - \dfrac{M^2}{10} + \\ \dfrac{47SRS_b}{105} - \dfrac{11M^2 Rcos(2\gamma)}{120} - \dfrac{\sigma_1}{10} - \dfrac{\sigma_2}{12} - \dfrac{\sigma_3}{60} + \dfrac{\sigma_6}{12}\end{array}\right)\eta^3$$

$$+\left(\begin{array}{l}\dfrac{5S}{28}-\dfrac{5SR}{28}+\dfrac{13SS_b}{140}-\dfrac{\sigma_2}{40}-\dfrac{M^2R}{30}-\dfrac{M^2S_b}{40}-\dfrac{3SR^2}{70}\\ -\dfrac{19SS_b}{70}+\dfrac{M^2}{40}-\dfrac{6SRS_b}{35}+\dfrac{\sigma_4}{30}+\dfrac{\sigma_1}{40}+\dfrac{\sigma_5}{20}+\dfrac{\sigma_6}{20}\end{array}\right)\eta^2,$$

$$\sigma_1 = M^2 S_b cos(2\gamma)$$

$$\sigma_2 = M^2 cos(2\gamma)$$

$$\sigma_3 = M^2 R \delta sin(2\gamma) \tag{39}$$

$$\sigma_4 = M^2 R cos(2\gamma)$$

$$\sigma_5 = M^2 \delta sin(2\gamma)$$

$$\sigma_6 = M^2 S_b \delta sin(2\gamma).$$

6- Energy Equation

$$\begin{aligned}&\theta'' + \Pr S(f\theta' - \eta\theta' - 2\theta) +\\ &\Pr Ec\left(f''^2 + 4\delta^2 f'^2 + M^2(f'^2\sin^2\gamma +\right.\\ &\left. f^2\delta^2\cos^2\gamma + 2ff'\delta\cos\gamma\sin\gamma)\right) = 0,\end{aligned} \tag{24}$$

$$\theta(0) = 0, \quad \theta(1) = 1, \tag{25}$$

Taking double integral from Eq. (14):

$$\int_0^1\int_0^\eta\left(\begin{array}{l}\theta'' + \Pr S(f\theta' - \eta\theta' - 2\theta)\\ +\Pr Ec\left(\begin{array}{l}f''^2 + 4\delta^2 f'^2 +\\ M^2(f'^2\sin^2\gamma + f^2\delta^2\cos^2\gamma +\\ 2ff'\delta\cos\gamma\sin\gamma)\end{array}\right)\end{array}\right)d\eta d\eta = 0, \tag{40}$$

$$\begin{aligned}&\theta'(0) = 1 + \Pr S\int_0^1\int_0^\eta(f\theta' - \eta\theta' - 2\theta)d\eta d\eta +\\ &\Pr Ec\int_0^1\int_0^\eta(f''^2 + 4\delta^2 f'^2)d\eta d\eta\\ &+\Pr Ec\int_0^1\int_0^\eta M^2(f'^2\sin^2\gamma + f^2\delta^2\cos^2\gamma + 2ff'\delta\cos\gamma\sin\gamma)d\eta d\eta.\end{aligned} \tag{41}$$

Taking Fourier transform of Eq. (24):

$$F\left(\begin{array}{l}\theta'' + \Pr S(f\theta' - \eta\theta' - 2\theta)\\ +\Pr Ec\left(\begin{array}{l}f''^2 + 4\delta^2 f'^2 +\\ M^2(f'^2\sin^2\gamma + f^2\delta^2\cos^2\gamma)\end{array}\right)\\ +2\Pr Ecff'\delta\cos\gamma\sin\gamma\end{array}\right) = 0, \tag{42}$$

$$F(\theta'(\eta)) = -\theta(0) + i\,\omega F(\theta(\eta)), \tag{43}$$

$$F(\theta''(\eta)) = -\theta'(0) - i\,\omega\theta(0) - \omega^2 F(\theta(\eta)), \tag{44}$$

Substituting Eqs. (43) and (44) into Eq. (42):

$$F(\theta(\eta)) = \frac{1}{\omega^2}\left(\begin{array}{l}-\theta'(0) + \Pr S\left(F(f\theta' - \eta\theta' - 2\theta)\right) +\\ \Pr Ec\left(F(f''^2) + 4\delta^2 F(f'^2)\right) + \Pr EcM^2 \times\\ \left(F(f'^2)\sin^2\gamma + F(f^2)\delta^2\cos^2\gamma + 2F(ff')\delta\cos\gamma\sin\gamma\right)\end{array}\right), \tag{45}$$

Substituting Eq. (41) into Eq. (45):

$$\begin{aligned}&F(\theta(\eta)) = -\frac{1}{\omega^2}\\ &-\frac{\Pr}{\omega^2}\left(\begin{array}{l}S\int_0^1\int_0^\eta(f\theta' - \eta\theta' - 2\theta)d\eta d\eta\\ +Ec\int_0^1\int_0^\eta(f''^2 + 4\delta^2 f'^2 + M^2(f'^2\sin^2\gamma +\\ \qquad f^2\delta^2\cos^2\gamma + 2ff'\delta\cos\gamma\sin\gamma))\\ -S\left(F(f\theta' - \eta\theta' - 2\theta)\right)\\ -Ec\left(\begin{array}{l}F(f''^2) + 4\delta^2 F(f'^2) + M^2(F(f'^2)\sin^2\gamma +\\ F(f^2)\delta^2\cos^2\gamma + 2F(ff')\delta\cos\gamma\sin\gamma\end{array}\right)\end{array}\right.\ \ d\eta d\eta, \tag{46}\end{aligned}$$

$$\theta = \sum_{n=0}^\infty \theta_n(\eta), \quad F\left(\sum_{n=0}^\infty \theta_n(\eta)\right) = \sum_{n=0}^\infty \theta_n(\omega), \tag{47}$$

$$f\theta' - \eta\theta' - 2\theta = \sum_{n=0}^\infty C_n(\eta), \quad F\left(\sum_{n=0}^\infty C_n(\eta)\right) = \sum_{n=0}^\infty C_n(\omega), \tag{48}$$

$$f''^2 = \sum_{n=0}^\infty E_n(\eta), \quad F\left(\sum_{n=0}^\infty E_n(\eta)\right) = \sum_{n=0}^\infty E_n(\omega), \tag{49}$$

$$f'^2 = \sum_{n=0}^\infty G_n(\eta), \quad F\left(\sum_{n=0}^\infty G_n(\eta)\right) = \sum_{n=0}^\infty G_n(\omega), \tag{50}$$

$$f^2 = \sum_{n=0}^\infty H_n(\eta), \quad F\left(\sum_{n=0}^\infty H_n(\eta)\right) = \sum_{n=0}^\infty H_n(\omega), \tag{51}$$

$$ff' = \sum_{n=0}^{\infty} K_n(\eta), \quad F\left(\sum_{n=0}^{\infty} K_n(\eta)\right) = \sum_{n=0}^{\infty} K_n(\omega), \tag{52}$$

$$
\begin{aligned}
&f''^2 + 4\delta^2 f'^2 + \\
&\quad M^2 (f'^2 \sin^2 \gamma + f^2 \delta^2 \cos^2 \gamma + 2ff'\delta \cos \gamma \sin \gamma) = P(\eta) \\
&P(\eta) = \sum_{n=0}^{\infty} P_n(\eta),
\end{aligned}
\tag{53}
$$

$$
\begin{aligned}
&\sum_{n=0}^{\infty} E_n(\eta) + 4\delta^2 \sum_{n=0}^{\infty} G_n(\eta) + \\
&\quad M^2 \left(\sum_{n=0}^{\infty} G_n(\eta) \sin^2 \gamma + \sum_{n=0}^{\infty} H_n(\eta) \delta^2 \cos^2 \gamma \right. \\
&\quad + 2\sum_{n=0}^{\infty} K_n(\eta) \delta \cos \gamma \sin \gamma = \sum_{n=0}^{\infty} P_n(\eta),
\end{aligned}
\tag{54}
$$

$$
\begin{aligned}
&\sum_{n=0}^{\infty} E_n(\omega) + 4\delta^2 \sum_{n=0}^{\infty} G_n(\omega) + \\
&M^2 \left(\sum_{n=0}^{\infty} G_n(\omega) \sin^2 \gamma + \sum_{n=0}^{\infty} H_n(\omega) \delta^2 \cos^2 \gamma + \right. \\
&2\sum_{n=0}^{\infty} K_n(\omega) \delta \cos \gamma \sin \gamma = \sum_{n=0}^{\infty} P_n(\omega).
\end{aligned}
\tag{55}
$$

Substituting Eqs. (47)-(55) into Eq. (46):

$$
\begin{aligned}
\sum_{n=0}^{\infty} \theta_n(\omega) &= -\frac{1}{\omega^2} \\
&- \frac{\Pr}{\omega^2} \left(\begin{array}{c} S\left(\int_0^1 \int_0^{\eta} \sum_{n=0}^{\infty} C_n(\eta) d\eta d\eta - \sum_{n=0}^{\infty} C_n(\omega)\right) \\ Ec\left(\int_0^1 \int_0^{\eta} \sum_{n=0}^{\infty} P_n(\eta) d\eta d\eta - \sum_{n=0}^{\infty} P_n(\omega)\right) \end{array} \right),
\end{aligned}
\tag{56}
$$

The recursive equations for energy equation can be written as:

$$\theta_0(\omega) = -\frac{1}{\omega^2}, \theta_0 = F^{-1}(\theta_0),$$

$$
\begin{aligned}
\theta_1(\omega) &= -\frac{\Pr}{\omega^2}\left[S\left(\int_0^1 \int_0^{\eta} C_0(\eta) d\eta d\eta - C_0(\omega)\right)\right] + \\
&\left[Ec\left(\int_0^1 \int_0^{\eta} P_0(\eta) d\eta d\eta - P_0(\omega)\right)\right], \theta_1 = F^{-1}(\theta_1),
\end{aligned}
\tag{57}
$$

$$
\begin{aligned}
\theta_2(\omega) &= -\frac{\Pr}{\omega^2}\left[S\left(\int_0^1 \int_0^{\eta} C_1(\eta) d\eta d\eta - C_1(\omega)\right)\right] + \\
&\left[Ec\left(\int_0^1 \int_0^{\eta} P_1(\eta) d\eta d\eta - P_1(\omega)\right)\right], \theta_2 = F^{-1}(\theta_2),
\end{aligned}
$$

$$
\begin{aligned}
\theta_k(\omega) &= -\frac{\Pr}{\omega^2}\left[S\left(\int_0^1 \int_0^{\eta} C_{k-1}(\eta) d\eta d\eta - C_{k-1}(\omega)\right)\right] + \\
&\left[Ec\left(\int_0^1 \int_0^{\eta} P_{k-1}(\eta) d\eta d\eta - P_{k-1}(\omega)\right)\right] \\
\theta_k &= F^{-1}(\theta_k).
\end{aligned}
$$

Where $\widehat{C_0}, \widehat{C_1}, \widehat{C_2}, ..., \widehat{C_{k-1}}, \widehat{P_0}, \widehat{P_1}, \widehat{P_2}, ..., \widehat{P_{k-1}}$ are the Fourier transforms of the Adomian Polynomials, respectively. The overall error for momentum and energy equations can be written as:

$$
\begin{aligned}
R\_1 &= f^{iv} - S(\eta f''' + 3f'' + ff'' - ff''') - \\
&\quad M^2 \sin \gamma (\sin \gamma f'' + 2\delta \cos \gamma f'),
\end{aligned}
\tag{58}
$$

$$
\begin{aligned}
R\_2 &= \theta'' + \Pr S(f\theta' - \eta\theta' - 2\theta) + \Pr Ec \times \\
&\left(f''^2 + 4\delta^2 f'^2 + M^2(f'^2 \sin^2 \gamma + f^2 \delta^2 \cos^2 \gamma)\right) \\
&+ 2\Pr Ec ff'\delta \cos \gamma \sin \gamma,
\end{aligned}
\tag{59}
$$

where R_1 and R_2 are the overall errors for momentum and energy equations, respectively.

7- Results and Discussion

In the present work, we use the method of Nourazar et al. [42] to solve the momentum and energy equations of the squeezing flow influenced by a magnetic field. Effects of different parameters such as squeezing number, intensity of magnetic field, the bottom plate suction/injection parameter, and the inclination angle on velocity and temperature profiles are studied. Figs.2 and 3 show the variations of the overall error for momentum and energy equations versus the number of recursive terms in the FTADM recursive equations (58) and (81), respectively. As can be illustrated from Figs. 2 and 3, once the number of recursive terms is increased, the error associated with results decreases monotonically.

Figs. 4 and 5 show the effect of the intensity of the magnetic field (M) on the velocity $f'(\eta)$ and temperature in the longitudinal direction. Fig. 4 shows that increasing the intensity of the magnetic field results in increasing the velocity profile in the longitudinal direction near the up and bottom plates. However, the velocity near in the middle of up and bottom plates decreases slightly. This may be attributed to the intensity of the magnetic field. Fig. 5 shows that the temperature of the fluid rises as the magnetic intensity field (M) increases, and this increase is monotonic for small values of M. However, a maximum value of temperature occurs in central region between up and bottom plates for large values of M.

Figs. 6 and 7 show the effect of the magnetic inclined angle γ on the velocity $f'(\eta)$, and temperature $\theta(\eta)$ in the longitudinal direction. We may see the same behavior for velocity and temperature profile as is seen in Figs. 4 and 5 due to the intensity of the magnetic field (M).
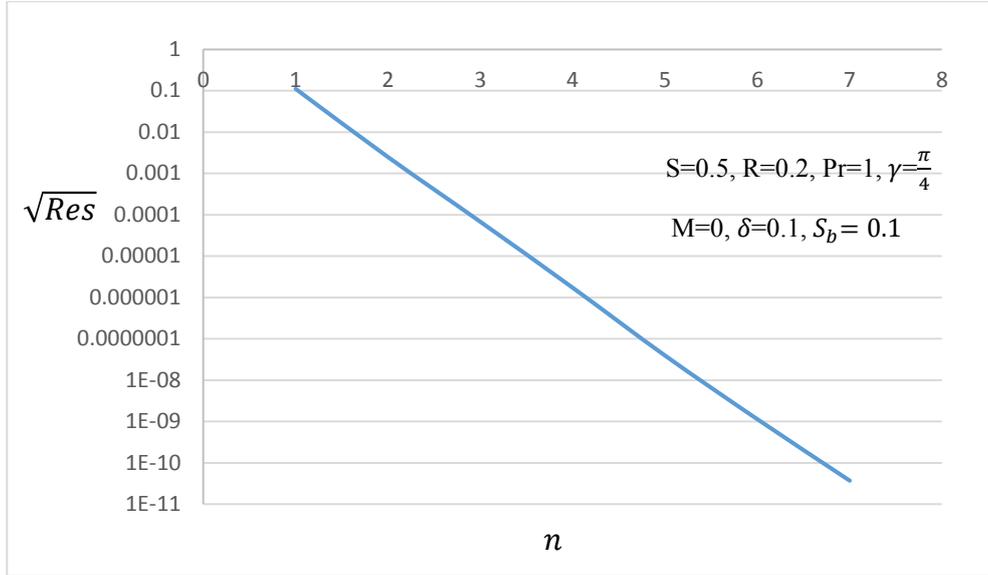
Fig. 2. variations of overall error of momentum equation versus the number of terms in recursive equation, n=1,2,3,4,5,6,7.
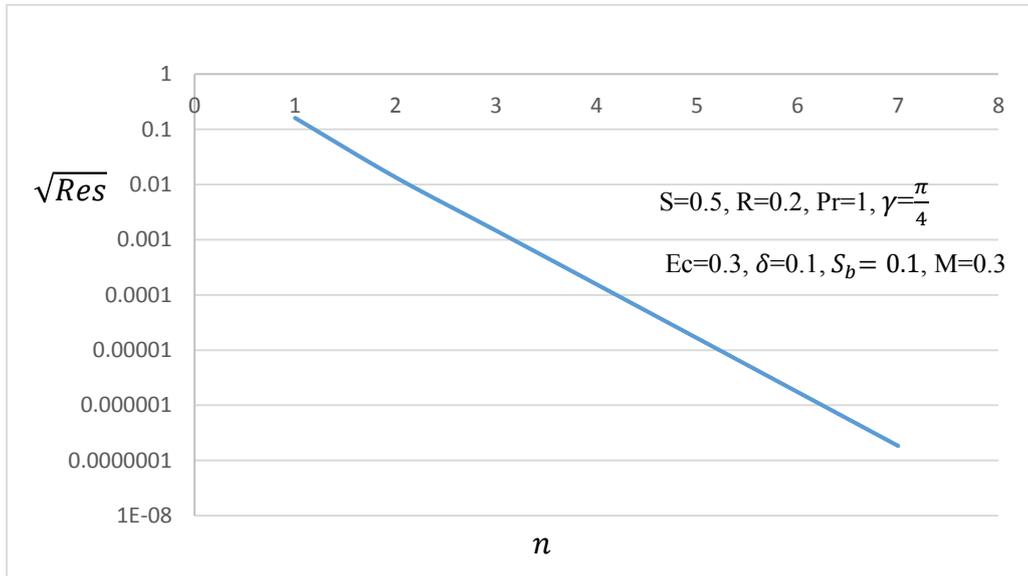

Fig. 3. variations of overall error of energy equation versus the number of terms in recursive equation, n=1,2,3,4,5,6,7.
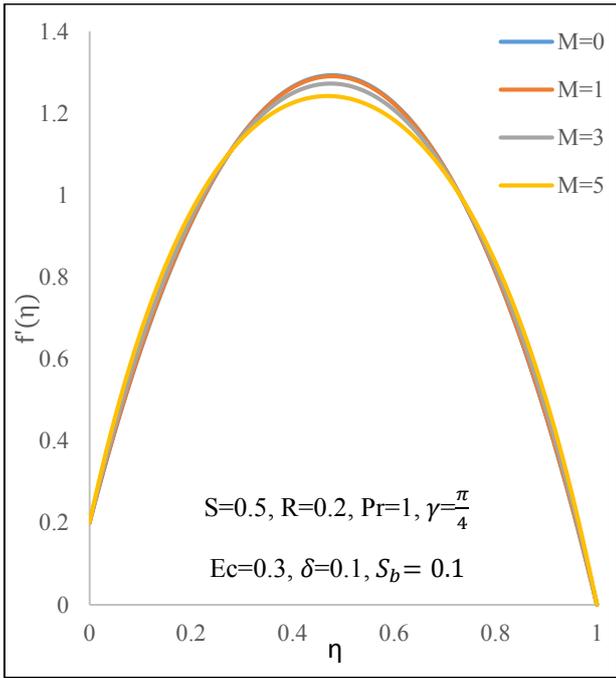
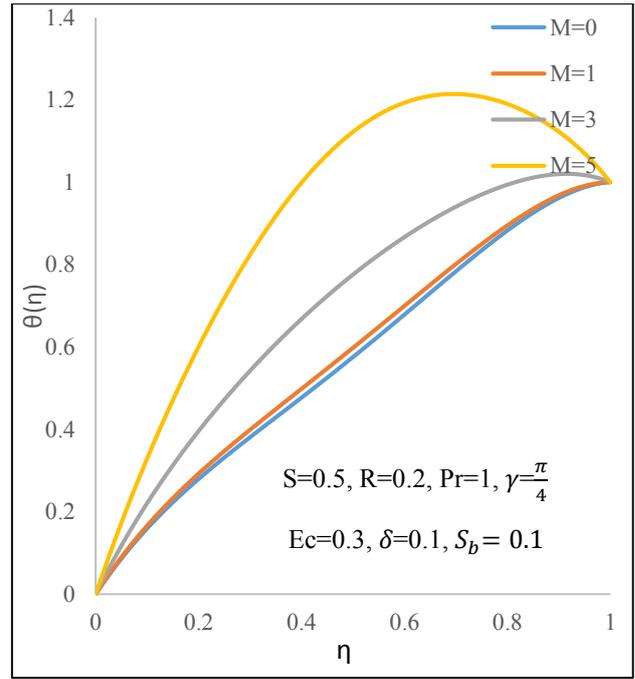**Fig. 4. shows the effect of the intensity of magnetic field M on the velocity profile in the longitudinal direction**

Fig. 5. shows the effect of the intensity of magnetic field M on the temperature profile in the longitudinal direction
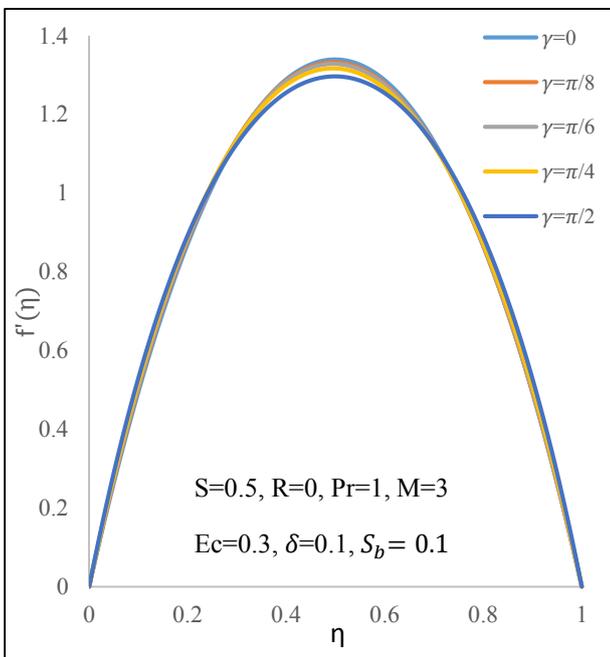
Fig. 6. shows the effect of the inclined angle γ on velocity profile in the longitudinal direction
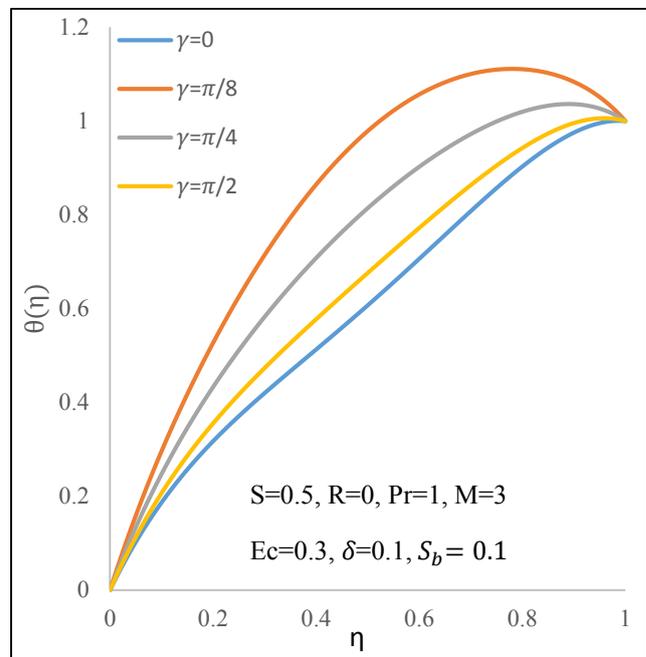
Fig. 7. shows the effect of the magnetic inclination γ on the temperature profile in the longitudinal direction
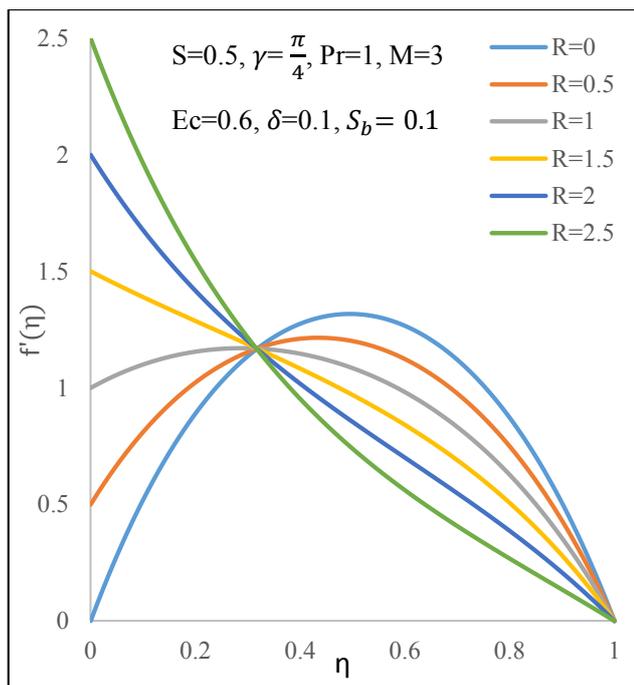
Fig. 8. shows the effect of the lower-plate stretching parameter R on velocity profile in the longitudinal direction
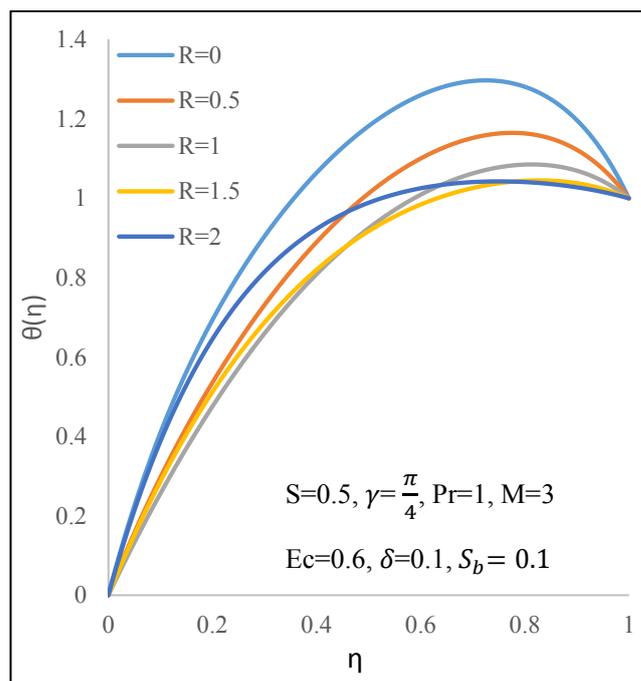
Fig. 9. shows the effect of the lower-plate stretching parameter R on the temperature profile in the longitudinal direction

Figs. 8 and 9 show the influence of bottom-plate stretching parameter (R) on the velocity $f'(\eta)$ and temperature $\theta(\eta)$ in the longitudinal direction. Fig. 8 reveals that increasing R results in increasing the velocity near the bottom plate, where the velocity near the upper plate decreases. By increasing the stretching parameter (R), maximum velocity occurring in central region between up and bottom plates occurs on surface of the bottom plate. We conclude from Fig. 9 that by increasing R, the temperature decreases initially but increases subsequently.

Figs. 10 and 11 show the effect of squeeze number S on the velocity $f'(\eta)$ and temperature $\theta(\eta)$ in the longitudinal direction. Fig. 10 shows that increasing the squeeze number results in decreasing the velocity of the fluid in the longitudinal direction near the up and bottom plates. However, there is an opposite behavior for velocity in central region between up and bottom plates. Fig. 11 reveals that by increasing the value of S, the temperature decreases.

Figs. 12 and 13 show the effect of bottom-plate suction/injection parameter S_b on the velocity $f'(\eta)$ and temperature $\theta(\eta)$ in the longitudinal direction. Fig. 12 shows that increasing the S_b induces the decrease in maximum velocity in the longitudinal direction. The temperature profile shows a decrease while the S_b increases.

8- Conclusion

In the present study, we propose a new method using the Fourier Transform Combined with the Adomian Decomposition Method (FTADM) for solving the squeezed unsteady flow between the parallel plates influenced by an inclined magnetic field. The influence of intensity of the magnetic field, magnetic inclined angle, bottom-plate stretching parameter, squeeze number, and bottom-plate suction/injection parameter on the velocity and temperature in the longitudinal direction is studied. The results show that by increasing the squeeze number, the intensity of magnetic field and the magnetic inclined angle velocity near up and bottom plates in the longitudinal direction may enhance the velocity near up and bottom plates. However, the velocity near the central region between up and bottom plates decreases. By enhancing the intensity of the magnetic field and the magnetic inclined angle, the temperature rises. Although, by increasing the squeeze number the temperature decreases. As an intensive conclusion, using the FTADM leads to more accurate results with a smaller number of calculations in comparison with the previous simulations. This may be due to the fact that using the FTADM enables us to incorporate all boundary conditions. However, in the previous semi-analytical methods only a part of boundary conditions may have been imposed into the solution.
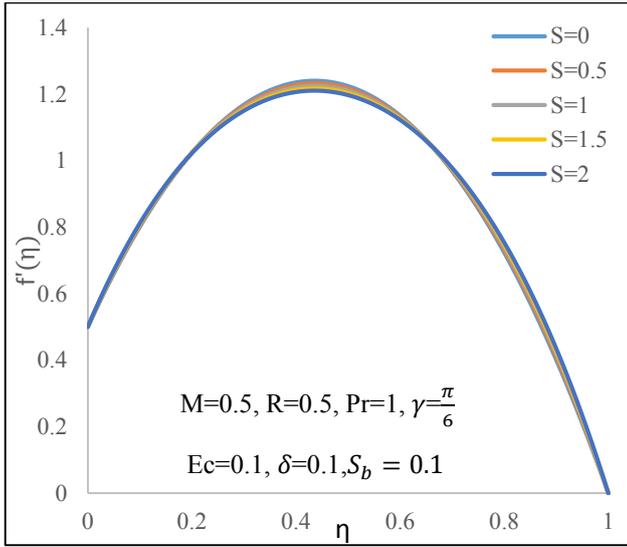
Fig. 10. shows the effect of the squeeze number S on the velocity profile in the longitudinal direction
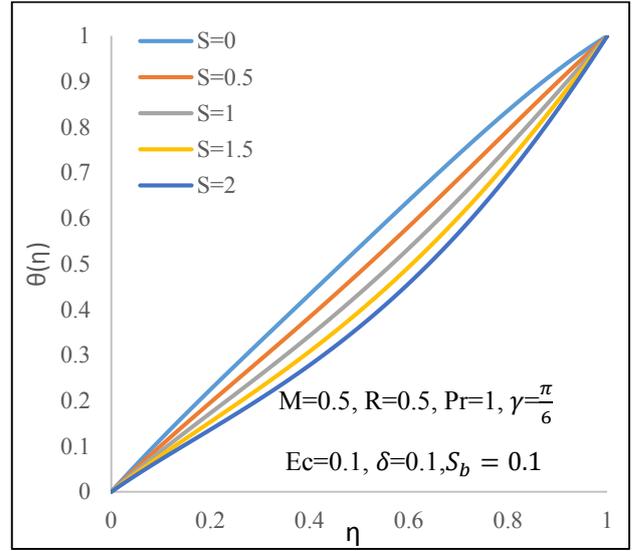
Fig. 11. shows the effect of the squeeze number S on the temperature profile in the longitudinal direction
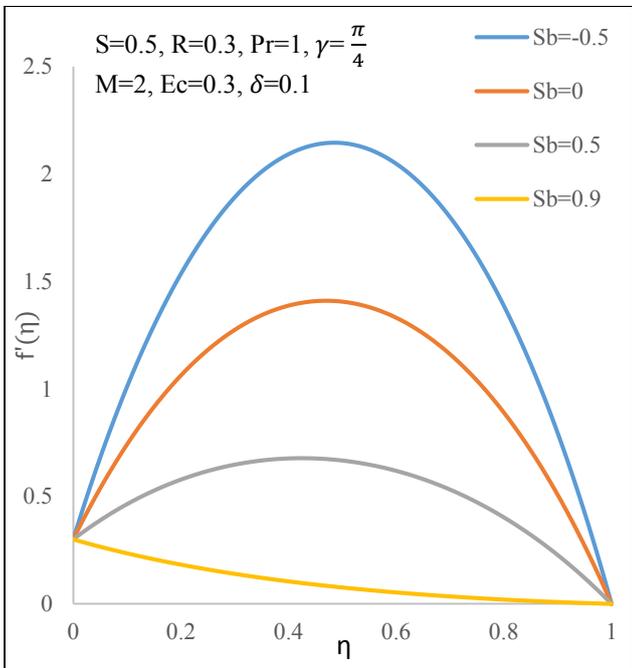
Fig. 12. shows the effect of the lower-plate suction/injection parameter S_b on the velocity profile in the longitudinal direction
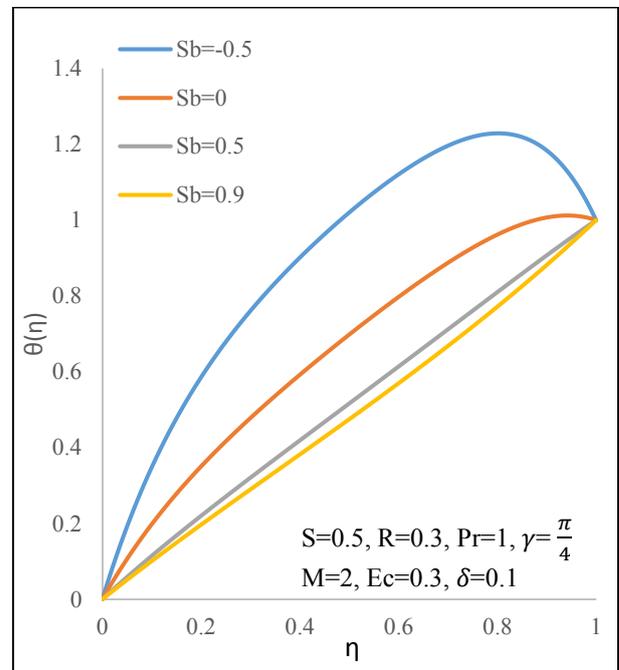
Fig. 13. shows the effect of the lower-plate suction/injection parameter S_b on the on the temperature profile in the longitudinal direction

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Appendix [1]

Momentum equation

Taking the fourth order integral from Eq. (23):

$$\int_0^1\int_0^\eta\int_0^\eta\int_0^\eta \left(f^{iv} - S(\eta f''' + 3f'' + ff'' - ff''') - M^2\sin\gamma(\sin\gamma f'' + 2\delta\cos\gamma f')\right)d\eta\, d\eta\, d\eta\, d\eta = 0, \qquad (60)$$

$$1 - S_b - R - \frac{1}{2}f''(0) - \frac{1}{6}f'''(0) - S\left(I_1 + 3(I_2 - \frac{1}{2}S_b - \frac{1}{6}R) + I_3\right)$$

$$-M^2\sin\gamma\left(\sin\gamma(I_2 - \frac{1}{2}S_b - \frac{1}{6}R) + 2\delta\cos\gamma(I_4 - \frac{1}{6}S_b)\right) = 0, \qquad (61)$$

$$I_1 = \int_0^1\int_0^\eta\int_0^\eta\int_0^\eta \eta f'''(\eta)d\eta\, d\eta\, d\eta\, d\eta, \qquad (62)$$

$$I_2 = \int_0^1\int_0^\eta f(\eta)d\eta\, d\eta, \qquad (63)$$

$$I_3 = \int_0^1\int_0^\eta\int_0^\eta\int_0^\eta (ff'' - ff''')d\eta\, d\eta\, d\eta\, d\eta, \qquad (64)$$

$$I_4 = \int_0^1\int_0^\eta\int_0^\eta f(\eta)d\eta\, d\eta\, d\eta, \qquad (65)$$

taking triple integral from Eq. (23):

$$\int_0^1\int_0^\eta\int_0^\eta \left(f^{iv} - S(\eta f''' + 3f'' + ff'' - ff''') - M^2\sin\gamma(\sin\gamma f'' + 2\delta\cos\gamma f')\right)d\eta\, d\eta\, d\eta = 0, \qquad (66)$$

$$-R - f''(0) - \frac{1}{2}f'''(0) - S\left(I_{11} + 3(I_{22} - S_b - \frac{1}{2}R) + I_{33}\right)$$

$$-M^2\sin\gamma\left(\sin\gamma(I_{22} - S_b - \frac{1}{2}R) + 2\delta\cos\gamma(I_{44} - \frac{1}{2}S_b)\right) = 0, \qquad (67)$$

$$I_{11} = \int_0^1\int_0^\eta\int_0^\eta \eta f'''(\eta)d\eta\, d\eta\, d\eta, \qquad (68)$$

$$I_{22} = \int_0^1 f(\eta)d\eta, \qquad (69)$$

$$I_{33} = \int_0^1\int_0^\eta\int_0^\eta (ff'' - ff''')d\eta\, d\eta\, d\eta, \qquad (70)$$

$$I_{44} = \int_0^1\int_0^\eta f(\eta)d\eta\, d\eta, \qquad (71)$$

Considering C_1, C_2, C_{11}, C_{22} as:

$$C_1 = 1 - S_b - R, \qquad (72)$$

$$C_2 = -S\left(I_1 + 3(I_2 - \frac{1}{2}S_b - \frac{1}{6}R) + I_3\right) - M^2 sin\gamma[sin\gamma(I_2 - \frac{1}{2}S_b - \frac{1}{6}R) + 2\delta cos\gamma(I_4 - \frac{1}{6}S_b)], \quad (73)$$

$$C_{11} = -R, \quad (74)$$

$$C_{22} = -S\left(I_{11} + 3(I_{22} - S_b - \frac{1}{2}R) + I_{33}\right) - M^2 sin\gamma[sin\gamma(I_{22} - S_b - \frac{1}{2}R) + 2\delta cos\gamma(I_{44} - \frac{1}{2}S_b)], \quad (75)$$

substituting Eqs. (72)-(75) into Eqs. (61) and (67):

$$-\frac{1}{2}f''(0) - \frac{1}{6}f'''(0) + C_1 + C_2 = 0, \quad (76)$$

$$-f''(0) - \frac{1}{2}f'''(0) + C_{11} + C_{22} = 0, \quad (77)$$

$$f''(0) = -2(C_{11} + C_{22}) + 6(C_1 + C_2), \quad (78)$$

$$f'''(0) = 6(C_{11} + C_{22}) - 12(C_1 + C_2). \quad (79)$$

Taking Fourier transform of Eq. (23):

$$F\left(f^{iv} - S(\eta f''' + 3f'' + ff'' - ff''') - M^2 \sin\gamma(\sin\gamma f'' + 2\delta\cos\gamma f')\right) = 0, \quad (80)$$

$$F(f'(\eta)) = -f(0) + i\omega F(f(\eta)), \quad (81)$$

$$F(f''(\eta)) = -f'(0) - i\omega f(0) - \omega^2 F(f(\eta)), \quad (82)$$

$$F\left(f'''(\eta)\right) = -f''(0) - i\omega f'(0) + \omega^2 f(0) - i\omega^3 F(f(\eta)), \quad (83)$$

$$F\left(f^{iv}(\eta)\right) = -f'''(0) - i\omega f''(0) + \omega^2 f'(0) + i\omega^3 f(0) + \omega^4 F(f(\eta)), \quad (84)$$

substituting Eqs. (81)-(82) into Eq. (80):

$$f = \frac{1}{\omega^4}f'''(0) + \frac{i}{\omega^3}f''(0) - \frac{R}{\omega^2} - \frac{i}{\omega}S_b + \frac{S}{\omega^4}\left(\eta f''' + 3(-R - i\omega S_b - \omega^2 f + (ff'' + ff'''))\right) \\ + \frac{M^2 \sin\gamma}{\omega^4}\left(\sin\gamma(-R - i\omega S_b - \omega^2 f) + 2\cos\gamma(-S_b + i\omega f)\right), \quad (85)$$

substituting Eqs. (78) and (79) in Eq. (86):

$$f = \frac{1}{\omega^4}\left(6(C_{11} + C_{22}) - 12(C_1 + C_2)\right) + \frac{i}{\omega^3}\left(-2(C_{11} + C_{22}) + 6(C_1 + C_2)\right) + \frac{S}{\omega^4}\left(\eta f''' + 3(-R - i\omega S_b)\right) \\ \frac{S}{\omega^4}\left(-\omega^2 f\right) + \frac{S}{\omega^4}(ff'' + ff''') + \frac{M^2 \sin\gamma}{\omega^4}\left(\sin\gamma(-R - i\omega S_b - \omega^2 f) + 2\cos\gamma(-S_b + i\omega f)\right), \quad (86)$$

$$f = (-\frac{12}{\omega^4} + \frac{6i}{\omega^3})\left(3S(\frac{1}{2}S_b + \frac{1}{6}R) + M^2\sin\gamma\left(\sin\gamma(\frac{1}{2}S_b + \frac{1}{6}R) + 2\delta\cos\gamma(\frac{1}{6}S_b)\right)\right)$$

$$+ (\frac{6}{\omega^4} - \frac{2i}{\omega^3})\left(3S(S_b + \frac{1}{2}R) + M^2\sin\gamma\left(\sin\gamma(S_b + \frac{1}{2}R) + 2\delta\cos\gamma S_b\right)\right) - R - \frac{3S}{\omega^4}(R + i\omega)$$

$$- \frac{M^2\sin\gamma}{\omega^4}(\sin\gamma(R + i\omega S_b) + 2\delta\cos\gamma S_b) - \frac{i}{\omega}S_b - \frac{R}{\omega^2} + (-\frac{12}{\omega^4} + \frac{6i}{\omega^3})(-S(I_1 + 3I_2 + I_3))$$

$$+ (-\frac{12}{\omega^4} + \frac{6i}{\omega^3})(-M^2\sin\gamma(\sin\gamma I_2 + 2\delta\cos\gamma I_4)) + (\frac{6}{\omega^4} - \frac{2i}{\omega^3})(-S(I_{11} + 3I_{22} + I_{33}))$$

$$+ (\frac{6}{\omega^4} - \frac{2i}{\omega^3})(-M^2\sin\gamma(\sin\gamma I_{22} + 2\delta\cos\gamma I_{44})),$$

$$+ \frac{S}{\omega^4}\left(\eta f''' - 3\omega^2 f + (ff'' + ff''')\right) + \frac{M^2\sin\gamma}{\omega^4}\left(-\omega^2\sin\gamma f + 2i\omega\delta\cos\gamma f\right), \tag{87}$$

$$f(\eta) = \sum_{n=0}^{\infty} f_n(\eta), \quad F(\sum_{n=0}^{\infty} f_n(\eta)) = \sum_{n=0}^{\infty} f_n(\omega), \tag{88}$$

$$\eta f''(\eta) = \sum_{n=0}^{\infty} A_n(\eta), \quad F(\sum_{n=0}^{\infty} A_n(\eta)) = \sum_{n=0}^{\infty} A_n(\omega), \tag{89}$$

$$ff'' + ff''' = \sum_{n=0}^{\infty} B_n(\eta), \quad F(\sum_{n=0}^{\infty} B_n(\eta)) = \sum_{n=0}^{\infty} B_n(\omega), \tag{90}$$

Making use of Eqs. (88)-(90), Eq. (87) may be rewritten as:

$$\sum_{n=0}^{\infty} \hat{f}_n = \hat{f}_0 + (-\frac{12}{\omega^4} + \frac{6i}{\omega^3})\left(-S\left(\int_0^1\int_0^\eta\int_0^\eta\int_0^\eta \sum_{n=0}^{\infty} \hat{A}_n d\eta d\eta d\eta d\eta + 3\int_0^1\int_0^\eta \sum_{n=0}^{\infty} \hat{f}_n d\eta d\eta\right)\right)$$

$$+ (-\frac{12}{\omega^4} + \frac{6i}{\omega^3})\left(\int_0^1\int_0^\eta\int_0^\eta\int_0^\eta \sum_{n=0}^{\infty} \hat{B}_n d\eta d\eta d\eta d\eta\right) + (-\frac{12}{\omega^4} + \frac{6i}{\omega^3})\left(-M^2\sin\gamma(\sin\gamma\int_0^1\int_0^\eta \sum_{n=0}^{\infty} \hat{f}_n d\eta d\eta)\right)$$

$$+ (-\frac{12}{\omega^4} + \frac{6i}{\omega^3})\left(2\delta\cos\gamma\int_0^1\int_0^\eta \sum_{n=0}^{\infty} \hat{f}_n d\eta d\eta\right) + (\frac{6}{\omega^4} - \frac{2i}{\omega^3})\left(-S\int_0^1\int_0^\eta\int_0^\eta \sum_{n=0}^{\infty} \hat{A}_n d\eta d\eta d\eta\right)$$

$$+ (\frac{6}{\omega^4} - \frac{2i}{\omega^3})\left(-S(3\int_0^1 \sum_{n=0}^{\infty} \hat{f}_n d\eta + \int_0^1\int_0^\eta\int_0^\eta \sum_{n=0}^{\infty} \hat{B}_n d\eta d\eta d\eta)\right) + (\frac{6}{\omega^4} - \frac{2i}{\omega^3})\left(-M^2\sin\gamma(\sin\gamma\int_0^1 \sum_{n=0}^{\infty} \hat{f}_n d\eta)\right) \tag{91}$$

$$+ (\frac{6}{\omega^4} - \frac{2i}{\omega^3})\left(2\delta\cos\gamma\int_0^1\int_0^\eta \sum_{n=0}^{\infty} \hat{f}_n d\eta d\eta\right) + \frac{S}{\omega^4}\left(\sum_{n=0}^{\infty} \hat{A}_n - 3\omega^2\sum_{n=0}^{\infty} \hat{f}_n + \sum_{n=0}^{\infty} \hat{B}_n\right)$$

$$+ \frac{M^2\sin\gamma}{\omega^4}\left(-\omega^2\sin\gamma\sum_{n=0}^{\infty} \hat{f}_n + 2i\omega\delta\cos\gamma\sum_{n=0}^{\infty} \hat{f}_n\right),$$

where f_0 can be written as:

$$f_0 = (-\frac{12}{\omega^4} + \frac{6i}{\omega^3})\left(3S(\frac{1}{2}S_b + \frac{1}{6}R) + M^2 \sin\gamma\left(\sin\gamma(\frac{1}{2}S_b + \frac{1}{6}R) + 2\delta\cos\gamma(\frac{1}{6}S_b)\right)\right)$$

$$(\frac{6}{\omega^4} - \frac{2i}{\omega^3})\left(3S(S_b + \frac{1}{2}R) + M^2\sin\gamma\left(\sin\gamma(S_b + \frac{1}{2}R) + \delta\cos\gamma(S_b)\right)\right) - R - \frac{3S}{\omega^4}(R + i\omega) \qquad (92)$$

$$-\frac{M^2\sin\gamma}{\omega^4}\left(\sin\gamma(R + i\omega S_b) + 2\delta\cos\gamma S_b\right) - \frac{i}{\omega}S_b - \frac{R}{\omega^2},$$

The recursive equations for momentum equation may be written as:

$$f_0 = F^{-1}(f_0), \qquad (93)$$

$$\hat{f_1} = (-\frac{12}{\omega^4} + \frac{6i}{\omega^3})\left(-S\left(\int_0^1\int_0^\eta\int_0^\eta\int_0^\eta \hat{A}_0 d\eta d\eta d\eta d\eta + 3\int_0^1\int_0^\eta \hat{f}_0 d\eta d\eta\right)\right)$$

$$+(-\frac{12}{\omega^4} + \frac{6i}{\omega^3})\left(\int_0^1\int_0^\eta\int_0^\eta\int_0^\eta \hat{B}_0 d\eta d\eta d\eta d\eta\right) + (-\frac{12}{\omega^4} + \frac{6i}{\omega^3})\left(-M^2\sin\gamma(\sin\gamma\int_0^1\int_0^\eta \hat{f}_0 d\eta d\eta)\right)$$

$$+(-\frac{12}{\omega^4} + \frac{6i}{\omega^3})\left(2\delta\cos\gamma\int_0^1\int_0^\eta\int_0^\eta \hat{f}_0 d\eta d\eta d\eta\right) + (\frac{6}{\omega^4} - \frac{2i}{\omega^3})\left(-S\int_0^1\int_0^\eta\int_0^\eta \hat{A}_0 d\eta d\eta d\eta\right)$$

$$(\frac{6}{\omega^4} - \frac{2i}{\omega^3})\left(-S(3\int_0^1 \hat{f}_0 d\eta + \int_0^1\int_0^\eta\int_0^\eta \sum_{n=0}^{\infty}\hat{B}_0 d\eta d\eta d\eta)\right) + (\frac{6}{\omega^4} - \frac{2i}{\omega^3})\left(-M^2\sin\gamma(\sin\gamma\int_0^1 \hat{f}_0 d\eta)\right) \quad (94)$$

$$+(\frac{6}{\omega^4} - \frac{2i}{\omega^3})\left(2\delta\cos\gamma\int_0^1\int_0^\eta \hat{f}_0 d\eta d\eta\right) + \frac{S}{\omega^4}\left(\hat{A}_0 - 3\omega^2\hat{f}_0 + \hat{B}_0\right)$$

$$+\frac{M^2\sin\gamma}{\omega^4}\left(-\omega^2\sin\gamma\hat{f}_0 + 2i\omega\delta\cos\gamma\hat{f}_0\right)$$

$$f_1 = F^{-1}(\hat{f_1}),$$

$$\hat{f_2} = (-\frac{12}{\omega^4} + \frac{6i}{\omega^3})\left(-S\left(\int_0^1\int_0^\eta\int_0^\eta\int_0^\eta \hat{A}_1 d\eta d\eta d\eta d\eta + 3\int_0^1\int_0^\eta \hat{f}_1 d\eta d\eta\right)\right)$$

$$+(-\frac{12}{\omega^4} + \frac{6i}{\omega^3})\left(\int_0^1\int_0^\eta\int_0^\eta\int_0^\eta \hat{B}_1 d\eta d\eta d\eta d\eta\right) + (-\frac{12}{\omega^4} + \frac{6i}{\omega^3})\left(-M^2\sin\gamma(\sin\gamma\int_0^1\int_0^\eta \hat{f}_1 d\eta d\eta)\right)$$

$$+(-\frac{12}{\omega^4} + \frac{6i}{\omega^3})\left(2\delta\cos\gamma\int_0^1\int_0^\eta\int_0^\eta \hat{f}_1 d\eta d\eta d\eta\right) + (\frac{6}{\omega^4} - \frac{2i}{\omega^3})\left(-S\int_0^1\int_0^\eta\int_0^\eta \hat{A}_1 d\eta d\eta d\eta\right)$$

$$(\frac{6}{\omega^4} - \frac{2i}{\omega^3})\left(-S(3\int_0^1 \hat{f}_1 d\eta + \int_0^1\int_0^\eta\int_0^\eta \sum_{n=0}^{\infty}\hat{B}_1 d\eta d\eta d\eta)\right) + (\frac{6}{\omega^4} - \frac{2i}{\omega^3})\left(-M^2\sin\gamma(\sin\gamma\int_0^1 \hat{f}_1 d\eta)\right) \quad (95)$$

$$+(\frac{6}{\omega^4} - \frac{2i}{\omega^3})\left(2\delta\cos\gamma\int_0^1\int_0^\eta \hat{f}_1 d\eta d\eta\right) + \frac{S}{\omega^4}\left(\hat{A}_1 - 3\omega^2\hat{f}_1 + \hat{B}_1\right)$$

$$+\frac{M^2\sin\gamma}{\omega^4}\left(-\omega^2\sin\gamma\hat{f}_1 + 2i\omega\delta\cos\gamma\hat{f}_1\right)$$

$$f_2 = F^{-1}(\hat{f_2}),$$

$$\hat{f}_k = (-\frac{12}{\omega^4} + \frac{6i}{\omega^3})\left(-S\left(\int_0^1\int_0^\eta\int_0^\eta\int_0^\eta \hat{A}_{k-1}d\eta d\eta d\eta d\eta + 3\int_0^1\int_0^\eta \hat{f}_{k-1}d\eta d\eta\right)\right)$$

$$+(-\frac{12}{\omega^4}+\frac{6i}{\omega^3})\left(\int_0^1\int_0^\eta\int_0^\eta\int_0^\eta \hat{B}_{k-1}d\eta d\eta d\eta d\eta\right)+(-\frac{12}{\omega^4}+\frac{6i}{\omega^3})\left(-M^2\sin\gamma(\sin\gamma\int_0^1\int_0^\eta \hat{f}_{k-1}d\eta d\eta)\right)$$

$$+(-\frac{12}{\omega^4}+\frac{6i}{\omega^3})\left(2\delta\cos\gamma\int_0^1\int_0^\eta\int_0^\eta \hat{f}_{k-1}d\eta d\eta d\eta\right)+(\frac{6}{\omega^4}-\frac{2i}{\omega^3})\left(-S\int_0^1\int_0^\eta\int_0^\eta \hat{A}_{k-1}d\eta d\eta d\eta\right)$$

$$(\frac{6}{\omega^4}-\frac{2i}{\omega^3})\left(-S(3\int_0^1 \hat{f}_{k-1}d\eta + \int_0^1\int_0^\eta\int_0^\eta\sum_{n=0}^\infty \hat{B}_{k-1}d\eta d\eta d\eta)\right)+(\frac{6}{\omega^4}-\frac{2i}{\omega^3})\left(-M^2\sin\gamma(\sin\gamma\int_0^1 \hat{f}_{k-1}d\eta)\right)\ (96)$$

$$+(\frac{6}{\omega^4}-\frac{2i}{\omega^3})\left(2\delta\cos\gamma\int_0^1\int_0^\eta \hat{f}_{k-1}d\eta d\eta\right)+\frac{S}{\omega^4}\left(\hat{A}_{k-1}-3\omega^2\hat{f}_{k-1}+\hat{B}_{k-1}\right)$$

$$+\frac{M^2\sin\gamma}{\omega^4}\left(-\omega^2\sin\gamma\hat{f}_{k-1}+2i\omega\delta\cos\gamma\hat{f}_{k-1}\right)$$

$$f_k = F^{-1}(\hat{f}_k),$$

where $\hat{A}_0, \hat{A}_1, \hat{A}_2,...,\hat{A}_{k-1}, \hat{B}_0, \hat{B}_1, \hat{B}_2,....,\hat{B}_{k-1}$ are the Fourier transforms of the Adomian Polynomials.

$$f_0(\eta) = (R + 2S_b - 2)\eta^3 + (3 - 3S_b - 2R)\eta^2 + R\eta + S_b,$$

$$f_1(\eta) = \left(\frac{SR^2}{70} + \frac{2SRS_b}{35} - \frac{2SR}{35} + \frac{2SS_b^2}{35} - \frac{4SS_b}{35} + \frac{2S}{35}\right)\eta^7$$

$$+\left(\frac{\delta\sin(2\gamma)M^2R}{120} + \frac{\delta\sin(2\gamma)M^2S_b}{120} + \frac{\delta\sin(2\gamma)M^2}{60} - \frac{SR^2}{15} - \frac{7SRS_b}{30} + \frac{7SR}{30} - \frac{S}{5}\right)\eta^6$$

$$+\left(\frac{\sigma_2}{20} - \frac{SR}{5} - \frac{SS_b}{5} - \frac{S}{10} + \frac{M^2R}{40} + \frac{M^2S_b}{20} + \frac{2SR^2}{15} + \frac{3SS_b}{10} - \frac{M^2}{20} + \frac{2SRS_b}{5}\right)\eta^5$$

$$+\left(\frac{3S}{4} - \frac{SR}{4} - \frac{SS_b}{4} - \frac{\sigma_2}{8} - \frac{M^2R}{12} - \frac{M^2S_b}{8} - \frac{SR^2}{6} - \frac{SS_b^2}{2} + \frac{M^2}{8} - \frac{SRS_b}{2} + \frac{\sigma_4}{12} + \frac{\sigma_1}{8} + \frac{\sigma_3}{24}\right)\eta^4$$

$$+\left(\begin{array}{l}\frac{19SR}{42} - \frac{24S}{35} + \frac{SS_b}{14} + \frac{\sigma_2}{10} + \frac{11M^2R}{120} + \frac{M^2S_b}{10} + \frac{9SR^2}{70} + \frac{43SS_b^2}{70} \\ -\frac{M^2}{10} + \frac{47SRS_b}{105} - \frac{11M^2R\cos(2\gamma)}{120} - \frac{\sigma_1}{10} - \frac{\sigma_2}{12} - \frac{\sigma_3}{60} + \frac{\sigma_6}{12}\end{array}\right)\eta^3$$

$$+\left(\begin{array}{l}\frac{5S}{28} - \frac{5SR}{28} + \frac{13SS_b}{140} - \frac{\sigma_2}{40} - \frac{M^2R}{30} - \frac{M^2S_b}{40} - \frac{3SR^2}{70} \\ -\frac{19SS_b}{70} + \frac{M^2}{40} - \frac{6SRS_b}{35} + \frac{\sigma_4}{30} + \frac{\sigma_1}{40} + \frac{\sigma_5}{20} + \frac{\sigma_6}{20}\end{array}\right)\eta^2,$$

$$\sigma_1 = M^2 S_b cos(2\gamma)$$

$$\sigma_2 = M^2 cos(2\gamma)$$

$$\sigma_3 = M^2 R \delta sin(2\gamma)$$

$$\sigma_4 = M^2 R cos(2\gamma)$$

$$\sigma_5 = M^2 \delta sin(2\gamma)$$

$$\sigma_6 = M^2 S_b \delta sin(2\gamma). \tag{97}$$

$$\sigma_1 = M^2 S_b cos(2\gamma)$$