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Analysis of Competition between Maritime and Multimodal Land-Sea Shipping in an International Freight Transportation Market

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ABSTRACT: Transportation, an integral element in international economic dynamics, plays a key role in both goods and service supply chains. This paper presents a game theoretic approach to the competition between maritime and multimodal (land-sea) freight forwarding companies in an international freight transportation market. Two scenarios of railway systems are considered: flexible and inflexible systems, whose difference lies in their capability of an immediate reduction in delivery time through swift infrastructural and operational improvements. The competition is modeled for each system and the effects of different policies are assessed through parametric and numerical analyses, yielding insightful results. It is found that with the improvement of the railway infrastructure and schedule, the equilibrium prices rise and fall, respectively, for the multimodal and maritime freight forwarding agents. Also, the analysis of changes in waterway tolls and rail access charges shows that if the government's purpose boosting customer satisfaction by reducing waterway tolls or rail access charges, which results in reduced equilibrium delivery price and time, they prefer the inflexible scenario over the flexible one. Therefore, counterintuitively, the flexibility of railway systems poses a serious challenge to governments when endeavoring to favor customers. The findings of the research provide a firm ground whereon transportation managers, policy makers, and service providers can make decisions more confidently in competitive freight transportation markets.

1-Introduction

Transportation, an integral element in international economic dynamics, plays a key role in both goods and service supply chains. Improving the efficiency of transportation systems is a crucial step to be taken toward an effective and highly competitive supply chain. With this end in view, shipping companies are inclined to adopt integrated transportation understanding its marked potential for reducing supply chain costs [1-3].

Maritime transportation is conceived of as the backbone of global trade given the key role it plays in the international movement of cargo [4]. With the globalization of economies, there has been an ever-increasing demand for intercontinental container shipping, economically benefitting countries that control international ports or waterways [5, 6]. This growth has been, by a large measure, owing to the progressive reduction in maritime shipping costs (compared to other modes of transportation), the gradual improvements in its speed of delivering large merchandise, and the security of deliveries.

Maritime transportation now moves approximately 80% of the international freight volume [7], while contributing less than 3% to the total global anthropogenic carbon emissions [8]. The presence of more than one alternative for shipping goods between origins and destinations often results in direct competition for attracting demand in a freight transportation market.

It is well-known fact that a manufacturing/distribution company is unable to survive the fierce competition in the freight transportation market unless it offers shipping services with competitive prices and delivery time [9]. Railway transportation offers an ideal potential for developing a complementary mode to maritime transportation [10]. This multimodality brings about a more competitive alternative to maritime transportation alone in terms of delivery time and flexibility [11].

Freight forwarding agents compete on the global stage to maximize their profits by attracting freight demands. These agents might utilize different modes of transportation to carry their share of freight through different international corridors, e.g. maritime, rail, road, air, or a combination of these modes which we refer to as multimodality [12].

In addition to the freight forwarding agents, countries that are situated along international freight corridors also benefit economically from the freight transported through these corridors [13]. Consequently, they are willing to provide incentives such as subsidies and discounts to increase the customers' utility and attract more of this traffic. Thus, customer satisfaction is a major factor to be taken into account by local governments as well as competing agents. Jiang, Li, Hua and Ru [14] studied governmental strategies for allotting subsidies to different modes in a multimodal transportation system. They analyzed the efficiency of these subsi-

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dies in the performance of multimodal transportation service providers under a Stackelberg game. Their findings suggest that logistics companies, producers, and freight forwarding companies find the highest profit, respectively, in transportation infrastructure subsidies, tax incentives, and international cooperation policies.

Various studies have been conducted on multimodal freight transportation. SteadieSeifi, Dellaert, Nuijten, Van Woensel, and Raoufi [15] presented a structured review of the literature on multimodal freight transportation that classifies problems into three levels: Strategic, tactical, and operational. With a time-cost-distance approach, Regmi and Hanaoka [16] assessed the performance of intermodal transportation corridors in North-East and Central Asia and identified the issues and challenges of their development and operation. Carrying out demand elasticity analysis, Beuthe, Jourquin, Geerts, and à Ndjang'Ha [17] analyzed the freight transportation market in the multimodal transportation network of Belgium regarding roadway, railway, and maritime modes. Vural, Roso, Halldórsson, Ståhle, and Yaruta [18] investigated the potential of different digital tools in mitigating barriers to increasing the utilization of intermodal transport. Jiang, Qiao, and Lu [19] analyzed the impacts of the New International Land-Sea Trade Corridor (a trade and multimodal transport corridor jointly built by the western Chinese provinces and the ASE-AN countries) on the freight transportation in the countries involved. Using a multinomial logit model, they modeled the choice behavior of freight owners and calculated the market share of the corridor. Tamannaei, Zarei, and Rasti-Barzoki [20] analyzed competition between road and intermodal roadrail transportation systems with consideration of government intervention. They found that improvements in the energy efficiency of the transportation systems may lead to adverse social and environmental impacts, which can effectively be eliminated by the government intervention.

Several factors are decisive in shipping companies' service quality and customers' harbor selection; some scholars have scrutinized the ones that are shown to be important to the customers. Slack [21] delved into the criteria that freight shippers consider when selecting harbors, having focused on the container traffic between the North American Mid-West and Western Europe. The results indicate that price, distance from the harbor, heavy traffic, multimodal relations, and several deliveries are relative of higher priority than other factors. Wang, Meng, and Zhang [22] developed three-game theory models for analyzing the competition between two maritime freight transportation firms for attracting demand in new markets. The results show that the players profit better in a Stackelberg equilibrium. Jiang, Fan, Luo and Xu [23] established a Stackelberg game theoretical model for sequential port competition in multimodal network design. It was found that reliability and large-scale freight shipment are the key factors for choosing multimodal transport. Tamannaei, Zarei, and Aminzadegan [24] addressed the pricing problem in a freight transportation market wherein a multimodal (railroad) service provider competes with a mono-modal (road) company. It is found that higher rates of tax on fuel consumption do not necessarily contribute toward a more sustainable transportation market. Mommens, van Lier and Macharis [25] used an agent-based model to quantify multimodal choice possibilities for five commonly used cargo types. The results suggested that the potential for a modal shift varies greatly between the considered cargo types, given that the sensitivity of the modal split to changes in shipping costs varies according to the cargo type used.

To the best of our knowledge, no study has comprehensively addressed the effects of different policies on the competition between maritime and multimodal freight transportation in an international market. In this paper, we endeavor to bridge this gap having adopted a game-theoretic approach.

The remainder of this article is organized as follows. The problem of concern is described in Section 2; regarding two railway flexibility scenarios, the competitive market of freight transportation is modeled in Section 3; the equilibrium solutions are revealed in Section 4; the effects of different technical and economic policies are assessed in Section 5, and the article is ultimately concluded in Section 6.

2- Problem description

A given tonnage of freight (Q) needs to be transported between two given points. There are two distinct freight forwarding agents available for the freight owners to choose from: maritime and multimodal freight forwarding agents. In the maritime transportation system, the freight is loaded onto a ship at the origin, and after traversing the sea, it is delivered to the customer at the destination. In a multimodal system, railway infrastructure is also utilized in addition to maritime transportation; i.e., the freight is initially loaded onto a ship at the origin and then, having traversed the sea up to an intermodal transshipment terminal (a seaport into which a railway station is integrated), it is transshipped from the ship to freight trains. Not to mention this terminal which incorporates a railway station from which the destination is accessible on the railway network-is regarded by the multimodal freight forwarding agent as the most efficient (to ship this specific freight through) among other alternatives alike. The freight of concern is then carried through the railway infrastructure to the destination where it is delivered to the customer. The freight owner's choice is a function of different parameters including delivery time and cost [26-28]. The aforementioned agents compete to maximize their profits by attracting a larger share of freight demand for the given origin-destination pair. Fig. 1. depicts the two competing modes considered as agents in this game.

There are two scenarios considered in this paper depending on the railway infrastructure conditions. In Scenario 1, the flexible system scenario, the railway system is capable of swift improvement and, as a result, immediate reduction of delivery time is possible (as in railway systems that are of high-quality infrastructure and scheduling). Conversely in Scenario 2, the inflexible system scenario, the railway system is incapable of swift improvement and immediate reduction of delivery time is infeasible (as in railway systems that are

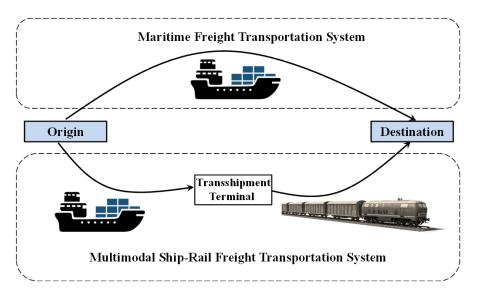


Fig. 1. The two competing modes considered as the agents in this game.

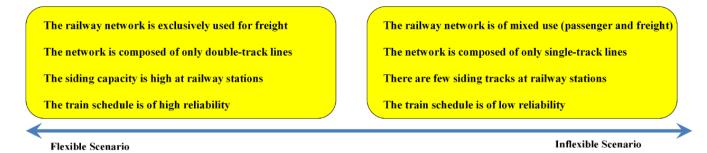


Fig. 2. Differentiation between Scenarios 1 and 2.

of low-quality infrastructure and scheduling).Fig. 2 demonstrates the differences between these two scenarios in further detail.

3- Model formulation

To formulate the competition of concern, the following nomenclature is first introduced.

The following assumptions are considered:

A1. All parameters are non-negative.

A2. Delivery time and price, as well as the demand function of both modes, are non-negative $(t_i > 0, p_i > 0, q_i > 0 \forall i \in \{M, H\})$.

A3. Ships and trains consume different types of fuel.

A4. To ensure the convexity of profit functions, the expression $\frac{\beta_t^2}{2}$ constantly holds.

$$4\beta_{\mu}$$

A5. The rate of fuel consumption per tonkilometer is lower in ships than in trains $(\theta_M < \theta_H)$. A6. For both price and delivery time, the sensitivity

A6. For both price and delivery time, the sensitivity of demand is higher than the cross-sensitivity of demand $(\beta_p > \gamma_p, \beta_t > \gamma_t)$; i.e., the demand for one mode is more sensitive to the changes in its delivery price and time than to those of its competitor's [29-32].

The total demand for freight shipping (Q) is assumed a fixed value and equals the sum of demand for both modes:

$$Q = q_h + q_m \tag{1}$$

The demand of each mode is considered a linear function of its delivery price and time over and above its competitor's delivery price and time, as assumed in previously published studies [33-36]. Consequently, the demand function of multimodal shipping (q_H) is calculated as follows:

$$q_H = \alpha_H - \beta_p P_H + \gamma_p P_M - \beta_t t_H + \gamma_t t_M \tag{2}$$

Obviously, the demand function of maritime shipping (q_M) then equals the subtraction of multimodal shipping demand from the total demand $(q_M = Q - q_H)$.

Table 1. List of notations.

	Indexes and parameters			
i	Index notation of the shipping mode (i=M for maritime shipping and i=H for multimodal shipping)			
Q	Total demand for freight shipping between the given origin and destination (tonne)			
$ heta_{\!\scriptscriptstyle M}$	Fuel consumption rate of ships (lit/tonne.km)			
$ heta_{\scriptscriptstyle H}$	Fuel consumption rate of trains (lit/tonne.km)			
$\alpha_{_H}$	Base market share of multimodal shipping (tonne)			
eta_p	Sensitivity of demand for multimodal shipping concerning its price			
eta_t	Sensitivity of demand for multimodal shipping concerning its delivery time			
γ_p	Cross-sensitivity of demand for multimodal shipping concerning the price of maritime shipping			
γ_t	Cross-sensitivity of demand for multimodal shipping concerning the delivery time in maritime shipping			
t_M	Delivery time of maritime shipping (hr)			
D_i	The shipping distance in mode i (km)			
d	The ratio of distance that the multimodal shipping traverses by ships $(0 \le d \le 1)$			
$\lambda_{_{H}}$	Coefficient of investment cost for reducing multimodal delivery time by one unit			
$f_{\scriptscriptstyle H}$	Train fuel cost (\$/litre)			
f_M	Cost of ship fuel (\$/liter)			
A_{H}	Railway access charge (\$/ton.km)			
A_M \overline{c}_i \overline{t}_H	Total waterway toll (\$/ton)			
\overline{c}_i	The operational cost of delivering a unit of freight by mode i (\$/tonne)			
\overline{t}_{H}	Delivery time of multimodal shipping in the status quo (hr)			
	Decision variables			
p_i^*	Equilibrium delivery price of delivering a unit of freight by mode i (\$/tonne)			
t_{H}^{*}	Equilibrium delivery time of multimodal shipping (hr)			
	Dependent variables			
\mathcal{C}_M	The total cost of delivering a unit of freight in maritime shipping			
\mathcal{C}_{H}	The total cost of delivering a unit of freight in multimodal shipping			
	Demand and profit functions			
q_i	Equilibrium demand function of mode i (tonne)			
$\pi_{_i}$	Equilibrium profit function of mode i			
	Abbreviations			
Sc_1	Scenario 1: Flexible railway system			
Sc_2	Scenario 2: Inflexible railway system			

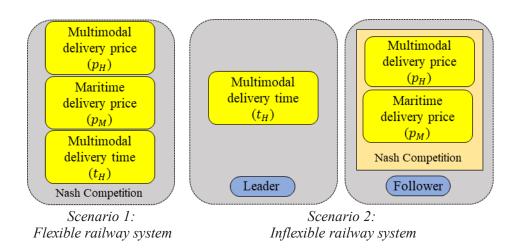


Fig. 3. Nash and Stackelberg game structures solved for Scenario 1 and Scenario 2, respectively.

In this paper, monetary costs of multimodal shipping other than rail fuel cost, rail access charge, and waterway tolls—are referred to as operational costs and encompass monetary expenses incurred by wage, maintenance, transshipment, loading at origin, and unloading at destination. The total cost of multimodal shipping, therefore, includes fuel, rail access charges, and operational costs. The total cost of maritime shipping includes fuel, international waterway tolls, and operational costs (i.e., wage, maintenance, loading at origin, and unloading at destination). Fuel cost for either mode is a function of the fuel consumption rate of the respective mode, shipping distance, and fuel cost.

The profit functions of each mode are as follows:

$$C_{H} = dD_{H}\theta_{M}f_{M} + (1-d)D_{H}\theta_{H}f_{H} +$$

$$\overline{A_{H}(1-d)D_{H} + c_{H}}$$
(3)

$$C_M = D_M \theta_M f_M + A_M + \overline{c_M} \tag{4}$$

$$\pi_H = (p_H - c_H)^* q_H - \lambda_H (\overline{t_H} - t_H)^2$$
(5)

$$\pi_M = (p_M - c_M)^* q_M \tag{6}$$

The expression $\lambda_H (\bar{t}_H - t_H)^2$ in the profit function of multimodal shipping denotes the investment cost required to reduce The delivery time (as similarly assumed by Madani and Rasti-Barzoki [37]). Based on Eqs. (2), (3), and (4), profit

functions can be rewritten as follows:

$$\pi_{M} = (p_{M} - c_{M})^{*} q_{M}$$

$$\pi_{H} = (\alpha_{H} - p_{H}\beta_{p} - t_{H}\beta_{t} + p_{M}\gamma_{p} + t_{M}\gamma_{t})$$

$$(-\overline{c}_{H} + p_{H} + D_{H}((d-1)(A_{H} + f_{H}\theta_{H}) - df_{M}\theta_{M})) - (t_{H} - \overline{t_{H}})^{2}\lambda_{H}$$
(7)

$$\pi_{M} = (Q - \alpha_{H} + p_{H}\beta_{p} + t_{H}\beta_{t} - p_{M}\gamma_{p} - t_{M}\gamma_{t})$$

$$(-(A_{M} + c_{M}) + p_{M} - D_{M}f_{M}\theta_{M})$$
(8)

Delivery time in maritime transportation is highly affected by marine weather conditions and thus measures taken toward improving it beyond our means; however, it is perfectly feasible in multimodal shipping to reduce this time by improving its railway part through plans to enhance transshipment infrastructure, scheduling, and prioritization. Accordingly, a fixed value is considered for maritime delivery time (t_M) , while multimodal delivery time (t_H) is considered a decision variable along with the multimodal (p_H) and maritime (p_M) delivery price.

4- Equilibrium solutions

In Scenario 1, the values of decision variables are acquired simultaneously in Nash equilibrium, whereas in Scenario 2, the competition follows a Stackelberg (leader-follower) structure wherein the multimodal delivery time (as the leader variable) is ascertained beforehand and then the equilibrium delivery prices of both modes (the followers) are attained in a Nash equilibrium (see Fig. 3).

4- 1- Scenario 1: Flexible railway system (Nash equilibrium) The Nash equilibrium in Scenario 1 is modeled as follows:

$$\begin{cases} \max_{t_H, p_H} \pi_H(p_H, p_M, t_H) \\ \max_{p_M} \pi_M(p_H, p_M, t_H) \end{cases}$$
(9)

Lemma 1: Profit functions π_H and π_M are concave (as proven in the Appendix).).

Theorem 1: The Nash equilibrium solutions for decision variables (p_H^*, p_M^*, t_H^*) as well as the demand and profit functions (q_H, q_M, π_H, π_M) in Scenario 1 are as follows.

$$(\beta_{t}^{2}(\bar{c}_{H} + D_{H}(-(d-1)(A_{H} + f_{H}\theta_{H}) + df_{M}\theta_{M})) - 2\lambda_{H}(Q + \alpha_{H} - \bar{t}_{H}\beta_{t} + (A_{M} + \bar{c}_{M})\gamma_{p} + t_{M}\gamma_{t} + 2\beta_{p}(\bar{c}_{H} - (-1 + d)D_{H}(A_{H} + f_{H}\theta_{H})) + (10)$$

$$p_{H}^{sc,1} = \frac{f_{M}\theta_{M}(2dD_{H}\beta_{p} + D_{M}\gamma_{p})))}{\beta_{t}^{2} - 6\beta_{p}\lambda_{H}}$$

$$\beta_{t}^{2}(Q + \gamma_{p}(c_{M} + D_{M}f_{M}\theta_{M})) - 2\beta_{p}\lambda_{H}(2Q - \alpha_{H} + \bar{t}_{H}\beta_{t} + 2(A_{M} + \bar{c}_{M})\gamma_{p} - t_{M}\gamma_{t} + \beta_{p}(\bar{c}_{H} - D_{H}(d - 1)(A_{H} + f_{H}\theta_{H})) + \beta_{M}^{sc,1} = \frac{f_{M}\theta_{M}(dD_{H}\beta_{p} + 2D_{M}\gamma_{p}))}{\alpha_{H}(\theta_{H}^{2} - \theta_{H}^{2} - \theta_{H}^{2}\lambda_{H})}$$
(11)

$$\beta_{t}(Q + \alpha_{H} + (A_{M} + c_{M})\gamma_{p} + t_{M}\gamma_{t} + \beta_{p}(-\overline{c}_{H} + (-1 + d)D_{H}(A_{H} + f_{H}\theta_{H}) + t_{H}^{sc,1} = \frac{f_{M}\theta_{M}(-dD_{H}\beta_{p} + D_{M}\gamma_{p})) - 6\overline{t}_{H}\beta_{p}\lambda_{H}}{\beta_{t}^{2} - 6\beta_{p}\lambda_{H}}$$
(12)

$$2\beta_{p}\lambda_{H}(Q + \alpha_{H} - \overline{t}_{H}\beta_{l} + (A_{M} + \overline{c}_{M})\gamma_{p} + t_{M}\gamma_{l} + \beta_{p}(-\overline{c}_{H} + (-1+d)D_{H}(A_{H} + f_{H}\theta_{H}) + q_{H}^{sc,1} = \frac{f_{M}\theta_{M}(-dD_{H}\beta_{p} + D_{M}\gamma_{p}))}{-\beta_{l}^{2} + 6\beta_{p}\lambda_{H}}$$
(13)

$$(Q \beta_t^2 + 2\beta_p \lambda_H - 2Q + \alpha_H - t_H \beta_t + (A_M + \overline{c}_M) \gamma_p + t_M \gamma_t + \beta_p (-\overline{c}_H + (-1+d)D_H)$$

$$q_M^{Sc,1} = \frac{(A_H + f_H \theta_H) + f_M \theta_M (-dD_H \beta_p + D_M \gamma_p)}{\beta_t^2 - 6\beta_p \lambda_H}$$
(14)

$$\begin{aligned}
\lambda_{H} (-\beta_{t}^{2} + 4\beta_{p}\lambda_{H}) \times (Q + \alpha_{H} - t_{H}\beta_{t} + (A_{M} + \overline{c}_{M})\gamma_{p} + t_{M}\gamma_{t} + \beta_{p}(-\overline{c}_{H} + (-1 + d)) \\
\pi_{H}^{Sc,1} &= \frac{D_{H}(A_{H} + f_{H}\theta_{H})) + f_{M}\theta_{M}(-dD_{H}\beta_{p} + D_{M}\gamma_{p}))^{2}}{(\beta_{t}^{2} - 6\beta_{p}\lambda_{H})^{2}}
\end{aligned}$$
(15)

$$(Q \beta_{t}^{2} + 2\beta_{p}\lambda_{H}(-2Q + \alpha_{H} - \bar{t}_{H}\beta_{t} + (A_{M} + \bar{c}_{M})\gamma_{p} + t_{M}\gamma_{t} + \beta_{p}(-\bar{c}_{H} + (A_{M} + \bar{c}_{M})\gamma_{p} + t_{M}\gamma_{t} + \beta_{p}(-\bar{c}_{H} + (A_{H} + f_{H}\theta_{H})) + f_{M}\theta_{M}(-dD_{H}\beta_{p} + D_{M}\gamma_{p})))^{2} (16)$$

$$\pi_{M}^{Sc,1} = \frac{(-1+d)D_{H}(A_{H} + f_{H}\theta_{H})) + f_{M}\theta_{M}(-dD_{H}\beta_{p} + D_{M}\gamma_{p}))}{\gamma_{p}(\beta_{t}^{2} - 6\beta_{p}\lambda_{H})^{2}}$$

For both modes to survive the competition and remain active in the market, equilibrium prices must be greater than the incurred shipping costs $(p_H^{Sc,1} > c_H, p_M^{Sc,1} > c_M)$. In addition, the values for multimodal delivery time $(t_H^{Sc,1})$ and demand functions $(q_H^{Sc,1}, q_M^{Sc,1})$ must be positive. To ensure this and other relational assumptions, the following relations must be established:

$$-Q - \alpha_{H} + \bar{t}_{H} \beta_{t} - t_{M} \gamma_{t} + \beta_{p} (\bar{c}_{H} - (d - 1)D_{H} (A_{H} + f_{H} \theta_{H})) + (17)$$

$$\bar{c}_{M} > \frac{f_{M} \theta_{M} (dD_{H} \beta_{p} - D_{M} \gamma_{p})}{\gamma_{p}} - A_{M}$$

$$-Q \beta_{i}^{z} + 2\beta_{p} \lambda_{H} \times (2Q - \alpha_{H} + t_{H} \beta_{i} - t_{M} \gamma_{i} + \beta_{p} (\overline{c}_{H} - D_{H} (d - 1)(A_{H} + f_{H} \theta_{H})) + \overline{c}_{M} < \frac{f_{M} \theta_{M} (dD_{H} \beta_{p} - D_{M} \gamma_{p}))}{2\beta_{p} \gamma_{p} \lambda_{H}} - A_{M}$$

$$(18)$$

4-2-Scenario 2: Inflexible railway system (Stackelberg equilibrium)

The equilibrium delivery time of multimodal shipping (t_{H}^{*}) is initially determined as the leader variable in Scenario 2 by solving the following:

$$\max_{t_H} \pi_H(t_H, p_H^*, p_M^*)$$
(19)

Subsequently, the equilibrium delivery prices (p_{H}^{*}, p_{M}^{*}) are determined by simultaneously solving the following functions:

$$\begin{cases} \max_{p_H} \pi_H(p_H) \\ \max_{p_M} \pi_M(p_M) \end{cases}$$
(20)

Theorem 2: The Stackelberg equilibrium solutions for decision variables (p_H^*, p_M^*, t_H^*) as well as equilibrium demand and profit functions (q_H, q_M, π_H, π_M) in Scenario 2 are as follows:

$$\beta_{t}^{2}(\bar{c}_{H} + D_{H}((-(d-1)(A_{H} + f_{H}\theta_{H}) + df_{M}\theta_{M})) - 3\lambda_{H}(Q + \alpha_{H} - \bar{t}_{H}\beta_{t} + (A_{M} + \bar{c}_{M})\gamma_{p} + t_{M}\gamma_{t} + 2\beta_{p}(\bar{c}_{H} - (-1 + d)D_{H}(A_{H} + f_{H}\theta_{H})) + (21)$$

$$p_{H}^{Sc,2} = \frac{f_{M}\theta_{M}(2dD_{H}\beta_{p} + D_{M}\gamma_{p}))}{\beta_{t}^{2} - 9\beta_{p}\lambda_{H}}$$

$$\beta_t^2 (Q + \gamma_p ((A_M + \overline{c}_M) + D_M f_M \theta_M) - 3\beta_p \lambda_H (2Q - \alpha_H + \overline{t}_H \beta_t + 2(A_M + \overline{c}_M) \gamma_p - t_M \gamma_t + \beta_p (\overline{c}_H - (-1 + d) D_H (A_H + f_H \theta_H)) + (22)$$

$$p_M^{Sc,2} = \frac{f_M \theta_M (dD_H \beta_p + 2D_M \gamma_p))}{\gamma_* (\beta_t^2 - 9\beta_* \lambda_H)}$$

$$\beta_{t}(Q + \alpha_{H} + (A_{M} + c_{M})\gamma_{p} + t_{M}\gamma_{t} + \beta_{p}(-\overline{c}_{H} + D_{H}(d - 1)(A_{H} + f_{H}\theta_{H})) + t_{H}^{Sc,2} = \frac{f_{M}\theta_{M}(-dD_{H}\beta_{p} + D_{M}\gamma_{p})) - 9\overline{t}_{H}\beta_{p}\lambda_{H}}{\beta_{t}^{2} - 9\beta_{p}\lambda_{H}}$$

$$(23)$$

$$3\beta_{p}\lambda_{H} \times (Q + \alpha_{H} - \bar{t}_{H}\beta_{t} + (A_{M} + \bar{c}_{M})\gamma_{p} + t_{M}\gamma_{t} + \beta_{p}(-\bar{c}_{H} + D_{H}(d - 1)(A_{H} + f_{H}\theta_{H})) + q_{H}^{Sc,2} = \frac{f_{M}\theta_{M}(-dD_{H}\beta_{p} + D_{M}\gamma_{p}))}{-\beta_{t}^{2} + 9\beta_{p}\lambda_{H}}$$
(24)

$$Q \beta_{t}^{2} + 3\beta_{p}\lambda_{H} \times (-2Q + \alpha_{H} - t_{H}\beta_{t} + (A_{M} + \overline{c}_{M})\gamma_{p} + t_{M}\gamma_{t} + \beta_{p}(-\overline{c}_{H} + D_{H}(d-1))$$

$$q_{M}^{Sc,2} = \frac{(A_{H} + f_{H}\theta_{H})(+f_{M}\theta_{M}) - (-dD_{H}\beta_{p} + D_{M}\gamma_{p}))}{\beta_{t}^{2} - 9\beta_{p}\lambda_{H}}$$
(25)

$$(Q \beta_t^2 + 3\beta_p \lambda_H \times (-2Q + \alpha_H - \bar{t}_H \beta_t + (A_M + \bar{c}_M)\gamma_p + t_M \gamma_t + \beta_p (-\bar{c}_H + D_H (d - 1)))$$

$$\pi_M^{Sc,2} = \frac{(A_H + f_H \theta_H) + f_M \theta_M (-dD_H \beta_p + D_M \gamma_p)}{\gamma_p (\beta_t^2 - 9\beta_p \lambda_H)^2}$$
(26)

$$\lambda_{H} (Q + \alpha_{H} - \overline{t}_{H} \beta_{t} + (A_{M} + \overline{c}_{M}) \gamma_{p} + t_{M} \gamma_{t} + \beta_{p} (-\overline{c}_{H} + D_{H} (d - 1)(A_{H} + f_{H} \theta_{H})) + (27)$$
$$\pi_{H}^{Sc,2} = \frac{f_{M} \theta_{M} (-dD_{H} \beta_{p} + D_{M} \gamma_{p}))^{2}}{-\beta_{t}^{2} + 9\beta_{p} \lambda_{H}}$$

To ensure that both modes survive the competition and remain active in the market, and to ensure the satisfaction of other relations assumed, the following relations must be established:

$$\lambda_{H} (Q + \alpha_{H} - \bar{t}_{H} \beta_{t} + (A_{M} + \bar{c}_{M}) \gamma_{p} + t_{M} \gamma_{t} + \beta_{p} (-\bar{c}_{H} + D_{H} (d - 1)(A_{H} + f_{H} \theta_{H})) + \pi_{H}^{Sc,2} = \frac{f_{M} \theta_{M} (-dD_{H} \beta_{p} + D_{M} \gamma_{p}))^{2}}{-\beta_{t}^{2} + 9\beta_{p} \lambda_{H}}$$
(28)

$$-Q - \alpha_{H} + t_{H} \beta_{t} - t_{M} \gamma_{t} + \beta_{p} (\overline{c}_{H} - D_{H} (d - 1)(A_{H} + f_{H} \theta_{H})) + \overline{c}_{N} > \frac{f_{M} \theta_{M} (dD_{H} \beta_{p} - D_{M} \gamma_{p})}{\gamma_{p}} - A_{M}$$

$$(29)$$

5- Policy analyses

In this section, we analyze the effects of multiple policies on delivery time and price, together with the demand and profit of maritime and multimodal freight forwarding agents. Five types of policies as below can be adopted by either the freight forwarding agents or the governments of countries situated along the respective international freight corridors.

Policy 1: Altering railway system flexibility

Policy 2: Altering international waterway tolls

Policy 3: Altering rail access charge

Policy 4: Improving railway systems (infrastructurally and operationally)

Policy 5: Employing ships with low-consumption technologies

Two types of policy analysis are conducted in this section - parametric and numerical analysis. The numerical analysis is carried out for the case of the north-south international rail corridor. This corridor has two main branches: the east and the west branches. The west one includes four different wings (see Fig. 4) [38, 39]. These wings enter Iran from the northern and eastern borders and then connect to India from Bandar Abbas port through the sea. In the present article, we have selected one of the wings of the western branch as a case study, which passes through Turkey and Iran (see Fig. 4.2); hence, the numerical analysis is conducted on this wing. It should be highlighted that the mentioned wing of the north-south international rail corridor is not currently active due to the railway infrastructure disjunction, lack of some links, and the capacity problems of the railway network in Iran and Turkey. By completing these links, the corridor will be capable to be widely exploited, which makes it a main competing corridor for the maritime shipping corridor shown in Fig. 5. Hence, the competition between the two corridors, i.e., the maritimeonly corridor and the multimodal maritime-railway corridor (the western branch of the North-South international rail corridor) is addressed. The parameters are set based on the realworld values reported in Table 2. The multimodal shipping agent delivers the freight at the Iran-Turkey border while the maritime shipping agent delivers it at the Port of Mersin. The freight owner is assumed to be situated at equal distances

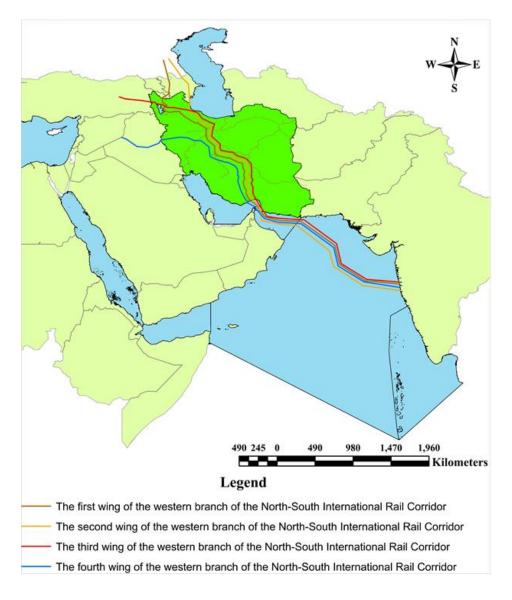


Fig. 4. The wings of the western branch of the North-South international rail corridor.

from these points, where they do not mind picking up their freight at either the Iran-Turkey border or the Port of Mersin.

In this case, the multimodal freight forwarding agent pays the Iranian government a fee—known as a rail access charge—for using its railway infrastructure. On the contrary, the maritime freight forwarding agent is charged two waterway tolls, once by the Djiboutian government for passing the Bab-el-Mandeb strait and once by the Egyptian government at The Suez Canal.

The observed values of the decision variables in the real world are also provided in Table 3 whereby it is feasible to estimate other parameters—whose values are not recorded in the real world—using the minimum error method. The details of this method have been provided in the Appendix.

Corollary: The previously unknown parameters are then

estimated and reported in Table 4.

. The error value of estimating these parameters equals 0.07 (7%) which is insignificant and therefore acceptable.

Policy 1: Altering railway system flexibility

Theorem 3: Given the aforementioned assumptions, the following relations are established between decision variables, demand functions, and profit functions in Scenarios 1 and 2:

$$\begin{cases} p_{H}^{Sc,1} > p_{H}^{Sc,2} \\ p_{M}^{Sc,1} < p_{M}^{Sc,2} \\ t_{H}^{Sc,1} < t_{H}^{Sc,2} \end{cases} \begin{cases} q_{H}^{Sc,1} > q_{H}^{Sc,2} \\ q_{M}^{Sc,1} < q_{M}^{Sc,2} \end{cases} \begin{cases} \pi_{H}^{Sc,1} < \pi_{H}^{Sc,2} \\ \pi_{M}^{Sc,1} < \pi_{M}^{Sc,2} \end{cases} \end{cases}$$

Parameter	Value	Reference
Q	40 million tons	www.trademap.org
$ heta_{\scriptscriptstyle H}$	0.005 liter/ton.km	English and Hackston [40]
$D_{_{H}}$	3700 km	www.searates.com
$D_{_M}$	7300 km	www.searates.com
d	0.55	Based on the case
- \mathcal{C}_M	40 \$/ton	www.searates.com
$\overline{\mathcal{C}}_{H}$	50 \$/ton	www.searates.com
t_M	480 hr	www.searates.com
A_{M}	4 \$/ton	www.wilhelmsen.com
$f_{\scriptscriptstyle H}$	0.4 \$/littre	ww.globalpetrolprices.com
$f_{\scriptscriptstyle M}$	0.5 \$/littre	www.bunkerworld.com
$ heta_{\!\scriptscriptstyle M}$	0.0037 liter/ton.km	English and Hackston [40]
$A_{\!H}$	0.01 \$/ton.km	www.RAI.com
$lpha_{_H}$	20 million tons	Assumed
\overline{t}_{H}	432 hr	Assumed

Table 2. Reported values of the parameters for the real-world case of India Turkey corridor.

 Table 3. The observed real-world values of the decision variables (source: www.searates.com).

Decision variable	Observed real-world values	
$p_{\scriptscriptstyle M}$	175 \$/ton	
$p_{\scriptscriptstyle H}$	180 \$/ton	
t_H	384 hr	

Table 4. Estimated values for unknown parameters us	-
ing minimum error method	

Parameter	Estimated values
$oldsymbol{eta}_p$	312077
$oldsymbol{eta}_t$	302298
γ_t	282850
${\gamma}_p$	71612
$\lambda_{_{H}}$	95661

There are two alternatives to be examined when considering this policy: 1. Improving railway system flexibility; 2. The do-nothing alternative (remaining in the inflexible condition).

Alternative 1: Improving the railway system in terms of flexibility brings about a reduction in the equilibrium delivery time of multimodal shipping which shifts the demand toward this mode. Accordingly, the equilibrium prices of maritime and multimodal shipping decrease and increase, respectively. This improvement, counterintuitively, cuts the equilibrium profit of both competing modes and not just maritime shipping. The underlying logic of this is that such an improvement entails heavy investment costs which reduce the equilibrium profit of the multimodal shipping agent concerning its pre-improvement, inflexible state. Therefore, it can be inferred that the upgrade from an inflexible railway system to a flexible one (a transition from Stackelberg Equilibrium to Nash Equilibrium) is unfavorable to both competing freight forwarding agents. Alternative 2: Doing nothing toward improving railway system flexibility is favorable to both competing freight forwarding agents as it prevents loss of profit. However, customers are less satisfied by the maritime shipping service due to its higher price compared to when alternative 1 is adopted; yet they prefer maritime shipping over multimodal shipping given that they find lower utility in the latter as a result of its increased delivery time.

To conclude, it is more profitable for both freight forwarding agents to compete in the Stackelberg game (Scenario 2) than in the Nash game (Scenario 1), whereas it is comparatively unprofitable and dissatisfactory for a significant portion of customers.

Policy 2: Altering international waterway tolls

Theorem 4: Given the assumptions made, the following can be uncovered:

Variable/ Function	Flexible System Scenario	Inflexible System Scenario	The difference between the two derivatives
$p_{\scriptscriptstyle H}$	$\frac{\partial p_{H}}{\partial A_{M}} > 0$	$\frac{\partial p_{H}}{\partial A_{M}} > 0$	$\left. \frac{\partial p_H}{\partial A_M} \right _{Flexible} - \frac{\partial p_H}{\partial A_M} \right _{Inflexible} > 0$
p_{M}	$\frac{\partial p_{_M}}{\partial A_{_M}} > 0$	$\frac{\partial p_{_M}}{\partial A_{_M}} > 0$	$\frac{\partial p_{M}}{\partial A_{M}}\bigg _{Flexible} - \frac{\partial p_{H}}{\partial A_{M}}\bigg _{Inflexible} > 0$
t _H	$\frac{\partial t_{H}}{\partial A_{M}} > 0$	$\frac{\partial t_{_H}}{\partial A_{_M}} < 0$	$\left. \frac{\partial t_H}{\partial A_M} \right _{Flexible} - \frac{\partial t_H}{\partial A_M} \right _{Inflexible} < 0$
$q_{\scriptscriptstyle H}$	$\frac{\partial q_{H}}{\partial A_{M}} > 0$	$\frac{\partial q_{H}}{\partial A_{M}} > 0$	
$q_{\scriptscriptstyle M}$	$\frac{\partial q_{\scriptscriptstyle M}}{\partial A_{\scriptscriptstyle M}} < 0$	$\frac{\partial q_{M}}{\partial A_{M}} < 0$	
$\pi_{_{H}}$	$\frac{\partial \pi_{H}}{\partial A_{M}} > 0$	$\frac{\partial \pi_{H}}{\partial A_{M}} > 0$	
$\pi_{_M}$	$\frac{\partial \pi_{_M}}{\partial A_{_M}} < 0$	$\frac{\partial \pi_{_M}}{\partial A_{_M}} < 0$	

With a rise in waterway tolls (A_M) , the costs of maritime shipping increase which results in an escalation of its equilibrium delivery price. This leaves the multimodal freight forwarder with the initiative in pricing, enabling it to boost its equilibrium profit. It is also worth noting that the delivery time of the multimodal freight forwarder increases with A_M . This policy decreases the demand and profit of the maritime freight forwarding agent while increasing those of the multimodal freight forwarding agent. Figs. 5 to 8 present the variations of the equilibrium solutions with A_M .

It is evident from this theorem that the policy of reducing

the international waterway tolls has synonymous effects on both flexible and inflexible scenarios. However, by decreasing the international waterway tolls by one unit, the delivery price of maritime shipping and multimodal shipping decreases more in the flexible scenario (compared to the inflexible scenario). Whereas the delivery time of multimodal shipping decreases more in the inflexible scenario (compared to the flexible scenario).

Policy 3: Altering rail access charge

Theorem 5: Given the assumptions made, the following can be given:

Variable/ Function	Flexible System Scenario	Inflexible System Scenario	The difference between the two derivatives
p_{H}	$\frac{\partial p_{_H}}{\partial A_{_H}} > 0$	$\frac{\partial p_{_H}}{\partial A_{_H}} > 0$	$\frac{\partial p_{H}}{\partial A_{H}}\bigg _{Flexible} - \frac{\partial p_{H}}{\partial A_{H}}\bigg _{Inflexible} < 0$
$p_{\scriptscriptstyle M}$	$\frac{\partial p_{_{M}}}{\partial A_{_{H}}} > 0$	$\frac{\partial p_{M}}{\partial A_{H}} > 0$	$\frac{\partial p_{_{M}}}{\partial A_{_{H}}}\bigg _{_{Flexible}} - \frac{\partial p_{_{M}}}{\partial A_{_{H}}}\bigg _{_{Inflexible}} > 0$
t _H	$\frac{\partial t_H}{\partial A_H} > 0$	$\frac{\partial t_{H}}{\partial A_{H}} > 0$	$\frac{\partial t_{H}}{\partial A_{H}}\bigg _{Flexible} - \frac{\partial t_{H}}{\partial A_{H}}\bigg _{Inflexible} > 0$
$q_{\scriptscriptstyle H}$	$\frac{\partial q_{_H}}{\partial A_{_H}} < 0$	$\frac{\partial q_{H}}{\partial A_{H}} < 0$	
$q_{\scriptscriptstyle M}$	$\frac{\partial q_{\scriptscriptstyle M}}{\partial A_{\scriptscriptstyle H}} > 0$	$\frac{\partial q_{M}}{\partial A_{H}} > 0$	
$\pi_{\scriptscriptstyle H}$	$\frac{\partial \pi_{_H}}{\partial A_{_H}} < 0$	$\frac{\partial \pi_{_H}}{\partial A_{_H}} < 0$	
$\pi_{_M}$	$\frac{\partial \pi_{_M}}{\partial A_{_H}} > 0$	$\frac{\partial \pi_{\scriptscriptstyle M}}{\partial A_{\scriptscriptstyle H}} > 0$	

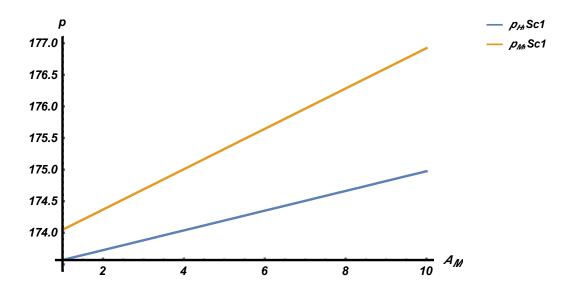


Fig. 6. Changes in the equilibrium values of delivery price with the waterway tolls in Scenario 1.

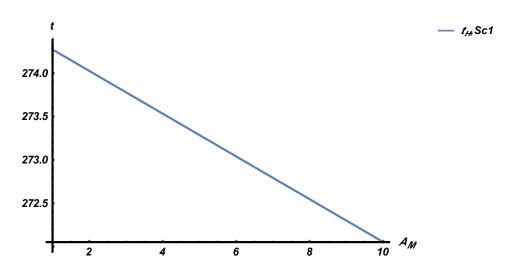


Fig. 7. Changes in the equilibrium values of delivery time with the waterway tolls in Scenario 1.

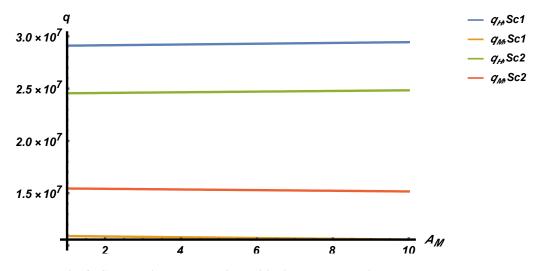


Fig. 8. Changes in each agent's equilibrium demand with the waterway tolls.

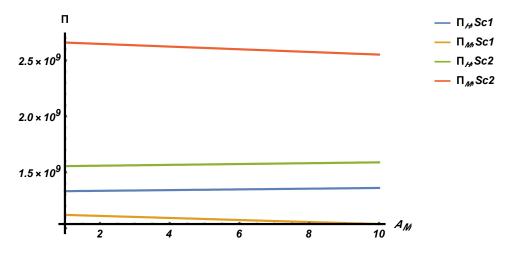


Fig. 9. Changes in each agent's equilibrium profit with the waterway tolls

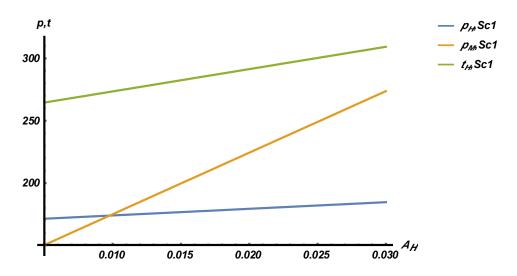


Fig. 10. Changes in the equilibrium values of the delivery price and time with rail access charge in Scenario 1.

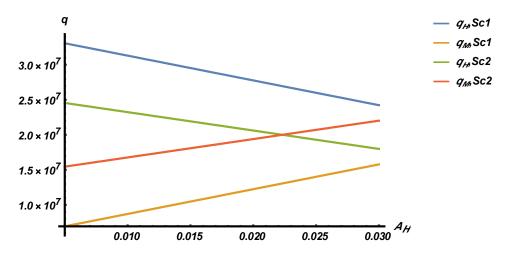


Fig. 11. Changes in each agent's equilibrium demand with a rail access charge.

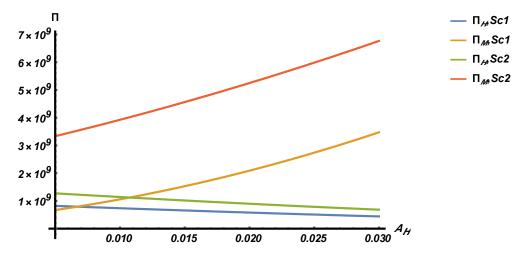


Fig. 12. Changes in each agent's equilibrium profit with a rail access charge.

The equilibrium delivery time and price of multimodal shipping increases with rail access charge (A_H) while its demand and utility decrease. On the contrary, not only does the equilibrium price of maritime shipping increase with this charge but its demand and profit also grow. Figs. 10 to 12 demonstrate the changes in the equilibrium values of decision variables, demand, and profit of each agent with a rail access charge.

It is evident from this theorem that the policy of increasing rail access charges has synonymous effects on both flexible and inflexible scenarios. However, by reducing the rail access charge by one unit, the delivery price of maritime shipping and the delivery time of multimodal shipping decreases more in the flexible scenario (compared to the inflexible scenario). Whereas the delivery price of multimodal shipping decreases more in the inflexible scenario (compared to the flexible scenario). This means that if governments purpose boosting customer satisfaction by reducing rail access charges, which results in reduced equilibrium delivery price and time, they prefer the inflexible scenario over the flexible one. Therefore, counterintuitively, the flexibility of shipping firms poses a serious challenge to governments when endeavoring to favor customers.

Policy 4: Improving railway systems (infrastructural and operational)

As mentioned in Section 0, the expression $\lambda_H (\bar{t}_H - t_H)^2$ in the profit function of multimodal shipping (Eq. (7) denotes the investment cost required to reduce the current delivery time by t_H . Since the multimodal shipping agent is assumed responsible for this investment, it will prove costineffective if it doesn't bring about a surge in demand great enough to compensate for these costs. Possible measures that can be taken to reduce multimodal delivery time comprise infrastructural and operational improvements. Infrastructural improvements include constructing new railways and upgrading existing ones as well as upgrading the railway fleet and transshipment machinery. On the other hand, operational improvements include enhancing scheduling, administrative efficiency, customs formalities, transshipment processes, and fleet movement. Infrastructural and operational improvements respectively decrease the values of λ_H and \bar{t}_H . With *S* defined as the amount of effort put into these improvements, its relation with λ_H and \bar{t}_H can be established as follows:

$$\lambda_{H} = \lambda_{H}^{initial} - K_{\lambda}, S \tag{30}$$

$$\bar{t}_H = \bar{t}_H^{initial} - K_t, S \tag{31}$$

Where $\lambda_{H}^{initial}$ and $\overline{t}_{H}^{initial}$ are the values of λ_{H} and \overline{t}_{H} before investment, and K_{λ} and K_{t} are the sensitivities of λ_{H} and t_{H} to the size of the investment.

Theorem 6: Given the assumptions made, the following can be given:

Variable/ Function	Flexible System Scenario	Inflexible System Scenario	The difference between the two derivatives
р _н	$\frac{\partial p_H}{\partial S} > 0$	$\frac{\partial p_H}{\partial S} > 0$	$\left. \frac{\partial p_H}{\partial S} \right _{Flexible} - \frac{\partial p_H}{\partial S} \right _{Inflexible} > 0$
p_M	$\frac{\partial p_{M}}{\partial S} < 0$	$\frac{\partial p_{M}}{\partial S} < 0$	$\frac{\partial p_{M}}{\partial S}\bigg _{Flexible} - \frac{\partial p_{M}}{\partial S}\bigg _{Inflexible} > 0$
t _H	$\frac{\partial t_H}{\partial S} < 0$	$\frac{\partial t_H}{\partial S} < 0$	$\left. \frac{\partial t_H}{\partial S} \right _{Flexible} - \frac{\partial t_H}{\partial S} \right _{Inflexible} > 0$
$q_{\scriptscriptstyle H}$	$\frac{\partial q_H}{\partial S} > 0$	$\frac{\partial q_{H}}{\partial S} > 0$	
$q_{\scriptscriptstyle M}$	$\frac{\partial q_M}{\partial S} < 0$	$\frac{\partial q_M}{\partial S} < 0$	
$\pi_{\scriptscriptstyle H}$	$\frac{\partial \pi_{H}}{\partial S} > 0$	$\frac{\partial \pi_{H}}{\partial S} > 0$	
$\pi_{_M}$	$\frac{\partial \pi_{_M}}{\partial S} < 0$	$\frac{\partial \pi_{_M}}{\partial S} < 0$	

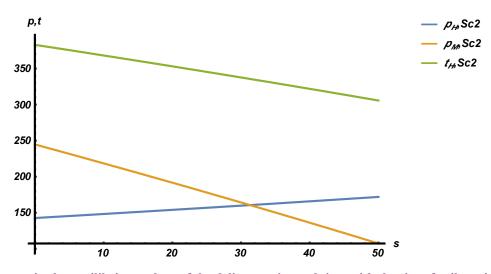


Fig. 13. Changes in the equilibrium values of the delivery price and time with the size of railway improvement investment in Scenario 2.

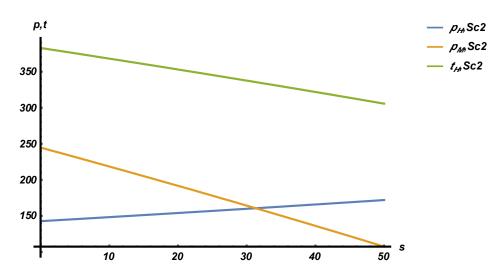


Fig. 14. Changes in each agent's equilibrium demand with the size of railway improvement investment.

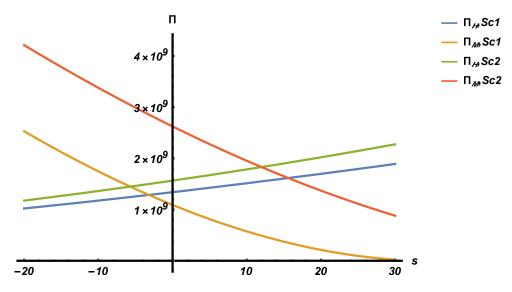


Fig. 15. Changes in each agent's equilibrium profit with the size of railway improvement investment.

With infrastructural and operational improvements, multimodal shipping demonstrates a better capability to reduce its delivery time more effectively. This reduction in delivery time brings about an enhanced level of service, endowing multimodal shipping with the initiative to set a higher price for its service. As the customers would find better utility in multimodal shipping, the maritime shipping agent has no choice but to lower their price to cushion their loss of market share.

As the railway system improves (in both infrastructure and operational conditions), the profit of multimodal shipping increases and, conversely, the profit of maritime shipping decreases. Therefore, it is expected that as the size of the investment increases, the profit of these two agents meets at a certain point where the agents equally benefit from the market. Figs. 13 to 15 illustrate how the equilibrium values of decision variables, demand, and profit of each agent change with the amount of effort put into railway system improvements.

Policy 5: Employing ships with low-consumption technologies

Modern technologies can be employed in ships to reduce the fuel consumed in maritime shipping and, consequently, its costs. Thus, the fuel consumption rate of ships (θ_M) can be a determining factor in the competitiveness of the agents against each other.

Theorem 7: Given the assumptions made, the following can be uncovered:

Variable/ Function	Flexible System Scenario	Inflexible System Scenario	The difference between the two derivatives
$p_{\scriptscriptstyle H}$	$\frac{\partial p_{_H}}{\partial \theta_{_M}} > 0$	$\frac{\partial p_{_H}}{\partial \theta_{_M}} > 0$	$\left. \frac{\partial p_H}{\partial \theta_M} \right _{Flexible} - \frac{\partial p_H}{\partial \theta_M} \right _{Inflexible} < 0$
$p_{\scriptscriptstyle M}$	$\frac{\partial p_{_M}}{\partial \theta_{_M}} > 0$	$\frac{\partial p_{_M}}{\partial \theta_{_M}} > 0$	$\left. \frac{\partial p_{M}}{\partial \theta_{M}} \right _{Flexible} - \frac{\partial p_{M}}{\partial \theta_{M}} \right _{Inflexible} > 0$
t_{H}	$\frac{\partial t_H}{\partial \theta_M} < 0IF(\frac{D_H \beta_p}{D_M} < \gamma_p < \beta_p)$	$\frac{\partial t_{H}}{\partial \theta_{M}} < 0IF(\frac{D_{H}\beta_{p}}{D_{M}} < \gamma_{p} < \beta_{p})$	$\frac{\partial t_{H}}{\partial \theta_{M}} \bigg _{Flexible} - \frac{\partial t_{H}}{\partial \theta_{M}} \bigg _{Inflexible} > 0$
11	$\frac{\partial t_{H}}{\partial \theta_{M}} > 0IF(0 < \gamma_{p} < \frac{D_{H}\beta_{p}}{D_{M}})$	$\frac{\partial t_H}{\partial \theta_M} > 0IF(0 < \gamma_p < \frac{D_H \beta_p}{D_M})$	$\partial heta_M _{Flexible} \partial heta_M _{Inflexible}$
a	$\frac{\partial q_{H}}{\partial \theta_{M}} > 0IF(\frac{D_{H}\beta_{p}}{D_{M}} < \gamma_{p} < \beta_{p})$	M M	
$q_{\scriptscriptstyle H}$	$\frac{\partial q_H}{\partial \theta_M} < 0 IF(0 < \gamma_p < \frac{D_H \beta_p}{D_M})$		
$q_{\scriptscriptstyle M}$	$\frac{\partial q_{M}}{\partial \theta_{M}} < 0IF(\frac{D_{H}\beta_{p}}{D_{M}} < \gamma_{p} < \beta_{p})$	$\frac{\partial q_{M}}{\partial \theta_{M}} < 0IF(\frac{D_{H}\beta_{p}}{D_{M}} < \gamma_{p} < \beta_{p})$	
\mathbf{q}_M	$\frac{\partial q_M}{\partial \theta_M} > 0IF(0 < \gamma_p < \frac{D_H \beta_p}{D_M})$	$\frac{\partial q_{M}}{\partial \theta_{M}} > 0IF(0 < \gamma_{p} < \frac{D_{H}\beta_{p}}{D_{M}})$	
$\pi_{\scriptscriptstyle H}$	$\frac{\partial \pi_{H}}{\partial \theta_{M}} > 0IF(\frac{D_{H}\beta_{p}}{D_{M}} < \gamma_{p} < \beta_{p})$	M M	
\mathcal{H}_{H}	$\frac{\partial \pi_{H}}{\partial \theta_{M}} < 0 IF(0 < \gamma_{p} < \frac{D_{H}\beta_{p}}{D_{M}})$	$\frac{\partial \pi_H}{\partial \theta_M} < 0 IF(0 < \gamma_p < \frac{D_H \beta_p}{D_M})$	
$\pi_{_M}$	$\frac{\partial \pi_{M}}{\partial \theta_{M}} < 0IF(\frac{D_{H}\beta_{p}}{D_{M}} < \gamma_{p} < \beta_{p})$	$\frac{\partial \pi_{M}}{\partial \theta_{M}} < 0IF(\frac{D_{H}\beta_{p}}{D_{M}} < \gamma_{p} < \beta_{p})$	
~ _M	$\frac{\partial \pi_{M}}{\partial \theta_{M}} > 0IF(0 < \gamma_{p} < \frac{D_{H}\beta_{p}}{D_{M}})$	$\frac{\partial \pi_{M}}{\partial \theta_{M}} > 0 IF(0 < \gamma_{p} < \frac{D_{H}\beta_{p}}{D_{M}})$	

It can be inferred from this theorem that the lower the fuel consumption rate of ships (θ_M), the lower both—and not only maritime—freight forwarding agents set their prices. Ship fuel is consumed by both agents and a reduction in its consumption rate reduces both agents' costs.

When θ_M is altered, the equilibrium solutions depend on the multimodal agent's cross-sensitivity of demand to the maritime agent's price (γ_p) , which indicates how customers alternate between the competing agents when the prices change. In other words, this parameter can be regarded as an index of the intensity of the competition between the freight forwarding agents. If the competition is relatively intense (i.e., $\frac{D_M \beta_p}{D_p} < \gamma_p < \beta_p$), a reduction in θ_M increases the equilibrium demand and profit of the maritime agent while decreasing those of the multimodal agent. Aware of the customers' high sensitivity to price, the multimodal freight forwarding agent cuts its delivery price to minimize its loss of demand. Also, given the constant value of γ_t , the delivery time of the multimodal agent increases. Conversely, if the competition is of low intensity (i.e., $0 < \gamma_p < \frac{D_H \beta_p}{D}$), reducing θ_M decreases the equilibrium demand and profit of the maritime agent while increasing those of the multimodal agent. In other words, in cases where the competition between the freight forwarding agents is not so intense, the policy of employing ships with low-consumption technologies proves ineffective in attracting demand to the maritime shipping agent.

Figs. 16 to 18 display the change in the equilibrium values of decision variables, demand, and profit of the competing agents with the fuel consumption rate of ships.

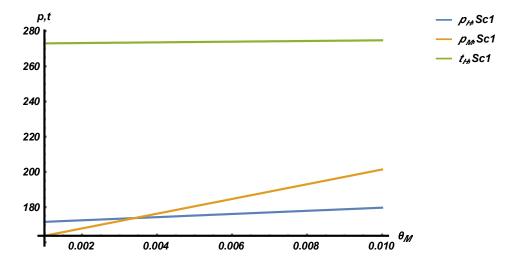


Fig. 16. Changes in the equilibrium values of the delivery price and time with the fuel consumption rate of ships in Scenario 1.

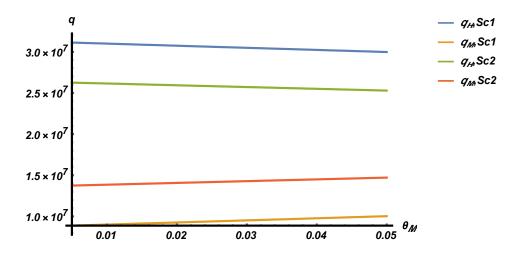


Fig. 17. Changes in each agent's equilibrium demand with the fuel consumption rate of ships.

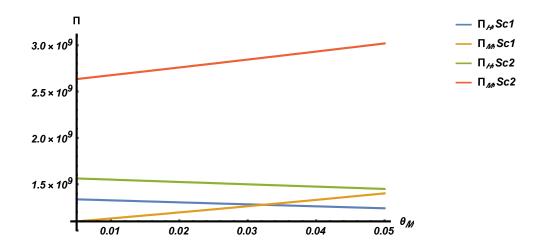


Fig. 18. Changes in each agent's equilibrium profit with the fuel consumption rate of ships.

It is evident from this theorem that the policy of employing ships with low-consumption technologies in maritime shipping has synonymous effects on both flexible and inflexible scenarios. However, by reducing ship fuel cost by one unit, the delivery price of maritime shipping and the delivery time of multimodal shipping decreases more in the flexible scenario (compared to the inflexible scenario). Whereas the delivery price of multimodal shipping decreases more in the inflexible scenario (compared to the flexible scenario).

6- Conclusion

Through this study, we investigated the competition between maritime and multimodal (land-sea) shipping agencies in the freight transportation market with a game-theoretic approach. Depending on the railway infrastructure condition, two scenarios of railway systems are considered: Flexible and inflexible systems, whose difference lies in their capability of an immediate reduction in delivery time through swift infrastructural and operational improvements. The shipping price of each freight forwarding agent and the delivery time of the multimodal agent is considered as the decision variables. The structure of the game in the flexible system scenario is based on Nash equilibrium wherein agents set the values of their decision variables simultaneously. On the contrary, the inflexible system scenario holds a Stackelberg (leader-follower) structure in which the multimodal shipping delivery time is ascertained beforehand and then the equilibrium delivery prices of both modes are attained in a Nash equilibrium.

The competition in the freight transportation market is modeled for each scenario and the effects of different technical and economic policies are analyzed by studying the equilibrium values of decision variables as well as the resultant equilibrium values of demand and profit functions. According to the results, as the flexibility state of the railway system upgrades from inflexible (Scenario 2) to flexible (Scenario 1), the demand gravitates toward multimodal shipping. But this improvement, counterintuitively, cuts the profit of both competing agents, and not just the maritime one.

The analysis of changes in waterway tolls and rail access charges shows that if governments purpose boosting customer satisfaction by reducing waterway tolls or rail access charges, which results in reduced equilibrium delivery price and time, they prefer the inflexible scenario over the flexible one. Therefore, counterintuitively, the flexibility of railway systems poses a serious challenge to governments when endeavoring to favor customers.

The fuel consumption rate of ships is shown to be a determining factor in the competitiveness of the freight forwarding agents against each other. When this parameter alters, the equilibrium solutions depend heavily on the intensity of the competition (i.e., the multimodal agent's cross-sensitivity of demand to the maritime agent's price).

In summary, Table 5. presents an overview of how railway system flexibility is either favorable or unfavorable in terms of different criteria from different perspectives.

Future work is encouraged on incorporating additional variables such as reliability, fleet size, and freight type in terms of perishability. Another avenue for future work is considering the effects of governmental interventions as well as political and security considerations on multi-agent freight shipping competition.

References

- [1] S. Chandra, M. Christiansen, K. Fagerholt, Analysing the modal shift from road-based to coastal shippingbased distribution–a case study of outbound automotive logistics in India, Maritime Policy & Management, 47(2) (2020) 273-286.
- [2] K. Tavassoli, M.J.J.o.C.P. Tamannaei, Hub network design for integrated Bike-and-Ride services: A competitive approach to reducing automobile dependence, 248 (2020) 119247.
- [3] S. Aminzadegan, M. Tamannaei, M.J.A.S.C. Fazeli, An integrated production and transportation scheduling problem with order acceptance and resource allocation decisions, 112 (2021) 107770.
- [4] K. Zavitsas, T. Zis, M.G. Bell, The impact of flexible environmental policy on maritime supply chain resilience, Transport Policy, 72 (2018) 116-128.
- [5] H. Hu, B. Du, M. Bernardo, Leasing or repositioning empty containers? Determining the time-varying guide leasing prices for decision making, Maritime Policy & Management, (2020) 1-17.
- [6] C. Wan, X. Yan, D. Zhang, Z. Yang, Analysis of risk factors influencing the safety of maritime container supply chains, International Journal of Shipping and Transport Logistics, 11(6) (2019) 476-507.
- [7] T. Wang, M. Li, H. Hu, Berth allocation and quay craneyard truck assignment considering carbon emissions in port area, International Journal of Shipping and Transport Logistics, 11(2-3) (2019) 216-242.
- [8] T.P. Zis, H.N. Psaraftis, F. Tillig, J.W. Ringsberg, Decarbonizing maritime transport: A Ro-Pax case study, Research in Transportation Business & Management, 37 (2020) 100565.
- [9] K. Shams, H. Asgari, X. Jin, Valuation of travel time reliability in freight transportation: A review and metaanalysis of stated preference studies, Transportation Research Part A: Policy and Practice, 102 (2017) 228-243.
- [10] J.-P. Rodrigue, T. Notteboom, Dry ports in European and North American intermodal rail systems: Two of a kind? Research in Transportation Business & Management, 5 (2012) 4-15.
- [11] V. Reis, J.F. Meier, G. Pace, R. Palacin, Rail and multimodal transport, Research in Transportation Economics, 41(1) (2013) 17-30.
- [12] C. Tawfik, S. Limbourg, Pricing problems in intermodal freight transport: Research overview and prospects, Sustainability, 10(9) (2018) 3341.
- [13] J.M. Pepe, Pepe, Valussi, Beyond Energy, Springer, 2018.
- [14] C. Jiang, H. Li, G. Hua, Y. Ru, Multimodal Transportation Government Subsidies Strategy Based Stackelberg Game and DEA, in: ICTE 2015, 2015, pp. 274-279.
- [15] M. SteadieSeifi, N.P. Dellaert, W. Nuijten, T. Van Woensel, R. Raoufi, Multimodal freight transportation

planning: A literature review, European journal of operational research, 233(1) (2014) 1-15.

- [16] M.B. Regmi, S. Hanaoka, Assessment of intermodal transport corridors: Cases from North-East and Central Asia, Research in Transportation Business & Management, 5 (2012) 27-37.
- [17] M. Beuthe, B. Jourquin, J.-F. Geerts, C.K. à Ndjang'Ha, Freight transportation demand elasticities: a geographic multimodal transportation network analysis, Transportation Research Part E: Logistics and Transportation Review, 37(4) (2001) 253-266.
- [18] C.A. Vural, V. Roso, Á. Halldórsson, G. Ståhle, M. Yaruta, Can digitalization mitigate barriers to intermodal transport? An exploratory study, Research in Transportation Business & Management, 37 (2020) 100525.
- [19] Y. Jiang, G. Qiao, J. Lu, Impacts of the new international land-sea trade corridor on the freight transport structure in China, central Asia, the ASEAN countries and the EU, Research in Transportation Business & Management, 35 (2020) 100419.
- [20] M. Tamannaei, H. Zarei, M.J.T.R.P.B.M. Rasti-Barzoki, A game theoretic approach to sustainable freight transportation: Competition between road and intermodal road-rail systems with government intervention, 153 (2021) 272-295.
- [21] B. Slack, Containerization, inter-port competition, and port selection, Maritime policy and management, 12(4) (1985) 293-303.
- [22] H. Wang, Q. Meng, X. Zhang, Game-theoretical models for competition analysis in a new emerging liner container shipping market, Transportation Research Part B: Methodological, 70 (2014) 201-227.
- [23] X. Jiang, H. Fan, M. Luo, Z. Xu, Strategic port competition in multimodal network development considering shippers' choice, Transport Policy, (2020).
- [24] M. Tamannaei, H. Zarei, S. Aminzadegan, A Game-Theoretic Approach to the Freight Transportation Pricing Problem in the Presence of Intermodal Service Providers in a Competitive Market, Networks and Spatial Economics, (2021).
- [25] K. Mommens, T. van Lier, C. Macharis, Multimodal choice possibilities for different cargo types: Application to Belgium, Research in Transportation Business & Management, 37 (2020) 100528.
- [26] A. Kanafani, Transportation demand analysis, (1983).
- [27] E. Kim, G.J. Hewings, H. Amir, Economic evaluation of transportation projects: An application of Financial Computable General Equilibrium model, Research in Transportation Economics, 61 (2017) 44-55.
- [28] G. Pace, S. Ricci, Multimodal, Intermodal and Terminals, in Sustainable Rail Transport, Springer, 2018, pp. 193-206.
- [29] S. Karray, S.P. Sigué, A game-theoretic model for co-promotions: Choosing a complementary versus an

independent product ally, Omega, 54 (2015) 84-100.

- [30] B. Dan, G. Xu, C. Liu, Pricing policies in a dual-channel supply chain with retail services, International Journal of Production Economics, 139(1) (2012) 312-320.
- [31]. Wei, J. Zhao, Y. Li, Pricing decisions for complementary products with firms' different market powers, European Journal of Operational Research, 224(3) (2013) 507-519.
- [32] H. Jafari, S.R. Hejazi, M. Rasti-Barzoki, Pricing decisions in dual-channel supply chain including monopolistic manufacturer and duopolistic retailers: a game-theoretic approach, Journal of Industry, Competition and Trade, 16(3) (2016) 323-343.
- [33] X. Chen, X. Wang, Free or bundled: channel selection decisions under different power structures, Omega, 53 (2015) 11-20.
- [34] E.J. Anderson, Y. Bao, Price competition with integrated and decentralized supply chains, European Journal of Operational research, 200(1) (2010) 227-234.
- [35] K. Pan, K.K. Lai, S.C. Leung, D. Xiao, Revenue-

sharing versus wholesale price mechanisms under different channel power structures, European Journal of Operational Research, 203(2) (2010) 532-538.

- [36] J. Chaab, M. Rasti-Barzoki, Cooperative advertising and pricing in a manufacturer-retailer supply chain with a general demand function; A game-theoretic approach, Computers & Industrial Engineering, 99 (2016) 112-123.
- [37] S.R. Madani, M. Rasti-Barzoki, Sustainable supply chain management with pricing, greening and governmental tariffs determining strategies: A gametheoretic approach, Computers & Industrial Engineering, 105 (2017) 287-298.
- [38] CAREC, A Railway Strategy for CAREC, 2017-2030, 2020.
- [39] CAREC, Railway Sector Assessment for Peoples Republic of China, 2021.
- [40] G. English, D.C. Hackston, Environmental and Social Impacts of Marine Transport in the Great Lakes-St. Lawrence Seaway Region, 2013.

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Appendix

Proof of Lemma 1. With $\partial^2_{PM^2} \pi_M = -2\gamma_p$ holding a negative value, it is obvious that π_M is concave in P_M . Also, $-2\beta_m = -\beta_n$

the Hessian matrix obtained for the profit of multimodal shipping (π_H) equals $\begin{pmatrix} -2\beta_p & -\beta_t \\ -\beta_t & -2\lambda_H \end{pmatrix}$ that, assuming

$$\lambda_H > \frac{\beta_t^2}{4\beta_p}$$
, is negative-definite; therefore π_H is concave in p_H .

Proof of Theorem 1. According to Lemma 1, the best responses of agents for P_M , p_H , and t_H have to be obtained by the first-order optimality condition of the agents' profit functions. Thus, the equilibrium solutions of these variables, i.e., Eqs. (6) to (8), are calculated through simultaneously solving the equations $\partial_{PH}\pi_H = 0$, $\partial_{PM}\pi_M = 0$ and $\partial_{tH}\pi_H = 0$. By replacing these values in Eqs. (1), (2), (3), and (4) the equilibrium solutions for demand and profit functions are obtained and presented in Eqs. (9) to (12).

Proof of Theorem 2. With $\partial_{PH^2}^2 \pi_H = -2\beta_p$ and $\partial_{PM^2}^2 \pi_M = -2\gamma_p$ holding negative values, it is obvious that equilibrium prices p_H and P_M , which are obtained by the first-order derivative of the agents' profit functions, are the best responses of the agents (followers) to each other's pricing. Thus, the equilibrium solutions of these variables are calculated by simultaneously solving the equations ($\partial_{PH}\pi_H = 0, \partial_{PM}\pi_M = 0$):

$$P_{H}^{*} = \frac{Q + \alpha_{H} - t_{H}\beta_{t} + \bar{c}_{M}\gamma_{p} + t_{M}\gamma_{t} + 2\beta_{p}(\bar{c}_{H} - D_{H}(d-1)(A_{H} + f_{H}\theta_{H})) + f_{M}\theta_{M}(2dD_{H}\beta_{p} + D_{M}\gamma_{p}))}{3\beta_{p}}$$

$$P_{M}^{*} = \frac{2Q - \alpha_{H} + t_{H}\beta_{t} + 2\bar{c}_{M}\gamma_{p} - t_{M}\gamma_{t} + \beta_{p}(\bar{c}_{H} - D_{H}(d-1)(A_{H} + f_{H}\theta_{H})) + f_{M}\theta_{M}(dD_{H}\beta_{p} + 2D_{M}\gamma_{p}))}{3\gamma}$$

By replacing these values in Eq. (6) the profit function of multimodal shipping is obtained:

$$\pi_{H} = \frac{(Q + \alpha_{H} - t_{H}\beta_{t} + c_{M}\gamma_{p} + t_{M}\gamma_{t} + \beta_{p}(D_{H}(d - 1)(A_{H} + f_{H}\theta_{H}) - c_{H}) + f_{M}\theta_{M}(D_{M}\gamma_{p} - dD_{H}\beta_{p}))^{2}}{9\beta_{p}} - \lambda_{H}(t_{H} - t_{H})^{2}$$

Regarding the assumption $\lambda_H > \frac{\beta_t^2}{4\beta_p}$, it is obvious that $\partial_{t_{H^2}}^2 \pi_H = \frac{2\beta_t^2}{9\beta_p} - 2\lambda_H$ is negative. Thus the best response of

multimodal shipping for t_H has to be obtained by the first-order derivative:

$$t_{H}^{*} = t_{H}^{Sc,2} = \frac{\beta_{t}(Q + \alpha_{H} + c_{M}\gamma_{p} + t_{M}\gamma_{t} + \beta_{p}(D_{H}(d-1)(A_{H} + f_{H}\theta_{H}) - c_{H}) + f_{M}\theta_{M}(D_{M}\gamma_{p} - dD_{H}\beta_{p})) - 9t_{H}\beta_{p}\lambda_{H}}{\beta_{t}^{2} - 9\beta_{p}\lambda_{H}}$$

The proof is completed by replacing t_H^* given above in p_H^* and p_M^* as well as Eqs. (1), (2), (3), and (4).

Corollary: To ensure estimation accuracy, the real-world sensitivity of the equilibrium maritime delivery price (P_M) to maritime delivery time (t_M) is estimated and equals -2.8; i.e., reducing the maritime delivery time by each hour incurs an

additional cost of 2.8 \$/tonne. This sensitivity is equivalent to $\frac{\partial p_M}{\partial t_M} = \frac{2\beta_p \gamma_t \lambda_H}{\gamma_p (\beta_t^2 - 6\beta_p \lambda_H)}$ in the mathematical model and

to ensure that the model conforms to the actual real-world value that is not so precise, $\frac{2\beta_p \gamma_t \lambda_H}{\gamma_p (\beta_t^2 - 6\beta_p \lambda_H)}$ is assumed to

be between -2.7 and -3.

Other relations between unknown parameters are derived from assumptions made in this paper, Eqs. (6), (7), and (8), and previous publications [29-32]. The consequent minimum error model is as follows.

$$Min_ERROR = \frac{\frac{|P_M - 175|}{175} + \frac{|P_H - 180|}{180} + \frac{|t_H - 384|}{384}}{3}$$

$$Subject to:$$
(28)

$$P_{H}^{\text{sc.l}} = \frac{(\beta_{t}^{2}(\bar{c}_{H} + D_{H}(-(d-1)(A_{H} + f_{H}\theta_{H}) + df_{M}\theta_{M})) - 2\lambda_{H}(Q + \alpha_{H} - \bar{t}_{H}\beta_{t} + \bar{c}_{M}\gamma_{p} + t_{M}\gamma_{t} + 2\beta_{p}(\bar{c}_{H} - (-1+d)D_{H}(A_{H} + f_{H}\theta_{H})) + f_{M}\theta_{M}(2dD_{H}\beta_{p} + D_{M}\gamma_{p})))}{\beta_{t}^{2} - 6\beta_{p}\lambda_{H}}}$$

$$P_{M}^{\text{sc.l}} = \frac{\beta_{t}^{2}(Q + \gamma_{p}(\bar{c}_{M} + D_{M}f_{M}\theta_{M})) - 2\beta_{p}\lambda_{H}(2Q - \alpha_{H} + \bar{t}_{H}\beta_{t} + 2\bar{c}_{M}\gamma_{p} - t_{M}\gamma_{t} + \beta_{p}(\bar{c}_{H} - D_{H}(d-1)(A_{H} + f_{H}\theta_{H})) + f_{M}\theta_{M}(dD_{H}\beta_{p} + 2D_{M}\gamma_{p}))}{\gamma_{p}(\beta_{t}^{2} - 6\beta_{p}\lambda_{H})}$$

$$t_{H}^{\text{sc.l}} = \frac{\beta_{t}(Q + \alpha_{H} + \bar{c}_{M}\gamma_{p} + t_{M}\gamma_{t} + \beta_{p}(-\bar{c}_{H} + (-1+d)D_{H}(A_{H} + f_{H}\theta_{H})) + f_{M}\theta_{M}(-dD_{H}\beta_{p} + D_{M}\gamma_{p})) - 6\bar{t}_{H}\beta_{p}\lambda_{H}}{\beta_{t}^{2} - 6\beta_{p}\lambda_{H}}$$

$$-3 < \frac{2\beta_{p}\gamma_{t}\lambda_{H}}{\gamma_{p}(\beta_{t}^{2} - 6\beta_{p}\lambda_{H})} \leq -2.7$$

$$\lambda_{H} > \frac{\beta_{t}^{2}}{4\beta_{p}}$$

$$0 < \gamma_{p} < \beta_{p}$$

$$\lambda_{H} > 0$$

$$0 < \gamma_{t} < \beta_{t}$$
Proof of Theorem 3. The equilibrium price of multimodal shipping in Scenario 1 equals:

$$P_{N}^{\text{sc.l}} = \frac{(\beta_{t}^{2}(C_{H} + D_{H}(-(d-1)(A_{H} + f_{H}\theta_{H})) + df_{M}\theta_{M})) - 2\lambda_{H}(Q + \alpha_{H} - \bar{t}_{H}\beta_{t} + \bar{c}_{M}\gamma_{p} + t_{M}\gamma_{t} + 2\beta_{p}(\bar{c}_{H} - (-1+d)D_{H}(A_{H} + f_{H}\theta_{H})) + f_{M}\theta_{M}(2dD_{H}\beta_{p} + D_{M}\gamma_{p}))}{\rho_{t}(\beta_{t}^{2} - 6\beta_{p}\lambda_{H})}$$

$$F_{H}^{c} = \frac{\beta_{t}^{2} - 6\beta_{p}\lambda_{H}}{\beta_{t}^{2} - 6\beta_{p}\lambda_{H}}$$
The equilibrium price of multimodal shipping in Scenario 2 equals:

$$P_{H}^{Sc,2} = \frac{\beta_{t}^{2}(\bar{c}_{H} + D_{H}(-(d-1)(A_{H} + f_{H}\theta_{H}) + df_{M}\theta_{M})) - 3\lambda_{H}(Q + \alpha_{H} - \bar{t}_{H}\beta_{t} + \bar{c}_{M}\gamma_{p} + t_{M}\gamma_{t} + 2\beta_{p}(\bar{c}_{H} - (-1+d)D_{H}(A_{H} + f_{H}\theta_{H})) + f_{M}\theta_{M}(2dD_{H}\beta_{p} + D_{M}\gamma_{p}))}{\beta_{t}^{2} - 9\beta_{n}\lambda_{H}}$$

The following is thus attained:

 $A = \beta_t^2 (\bar{c}_H + D_H (-(d-1)(A_H + f_H \theta_H) + df_M \theta_M))$ $B = \lambda_H (Q + \alpha_H - \bar{t}_H \beta_t + \bar{c}_M \gamma_p + t_M \gamma_t + 2\beta_p (\bar{c}_H - (-1+d)D_H (A_H + f_H \theta_H)) + f_M \theta_M (2dD_H \beta_p + D_M \gamma_p))$ Considering the feasibility conditions, i.e., Eqs. (13), (14), (24), and (25), the following relations must be established: (A - 3B) > (A - 2B) > 0

Also, regarding the assumption $\lambda_H > \frac{\beta_t^2}{4\beta_p}$, the following relations can be given:

$$(\beta_t^2 - 9\beta_p\lambda_H) < (\beta_t^2 - 6\beta_p\lambda_H) < 0 \rightarrow \frac{1}{(\beta_t^2 - 6\beta_p\lambda_H)} < \frac{1}{(\beta_t^2 - 6\beta_p\lambda_H)} < 0$$

Consequently, the following holds:

$$\frac{A-2B}{(\beta_t^2-6\beta_p\lambda_H)} > \frac{A-3B}{(\beta_t^2-9\beta_p\lambda_H)} \to P_H^{Sc,1} > P_H^{Sc,2}$$

Other relations between variables, demand, and profit functions can be similarly proven for both Scenarios. **Proof of Theorem 4.** The following is established for Scenario 2:

$$\frac{\partial p_M}{\partial A_M} = \frac{\beta_t^2 - 6\beta_p \lambda_H}{\beta_t^2 - 9\beta_p \lambda_H}$$

Given the assumptions $\lambda_H > \frac{\beta_t^2}{4\beta_p}$ and regarding the negative values of expressions ($\beta_t^2 - 6\beta_p \lambda_H$) and ($\beta_t^2 - 9\beta_p \lambda_H$)

), the inequality $\frac{\partial_{PM}}{\partial_{AM}} > 0$ holds.

Other relations as well as Theorems 5, 6, and 7 can be similarly proven for both Scenarios.