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Pulse Delay Compensation for High Velocity Moving Vehicle accurate Localization in the Low Frequency Positioning System

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ABSTRACT: The purpose of this paper is to provide a positioning algorithm for high-velocity moving vehicles by a low frequency local positioning system, such as Loren-C navigation system. The performance of the Linear Digital Averaging (LDA) depends on similarity of the reception period of consecutive pulses (i.e. Pulse Code Interval (PCI)). The velocity of the receiver changes the period of pulse reception in each PCI and distorts the average pulse. The distortion of the average pulse depends on the number of pulses and the amount of pulse delays (i.e. the difference between pulse reception period and PCI). In this paper, pulse delay threshold and consequently the velocity of receiver threshold of the acceptable average pulse distortion is analyzed. It is shown that the determined threshold of the velocity of receiver is very low for a wide variety of applications. The proposed solution to increase the velocity threshold is to compensate the pulse delays using the last estimation of the location and the velocity vector of the receiver. The proposed algorithm can be applied to design receivers for high-velocity vehicles. The simulation results confirm the convergence of the proposed positioning algorithm and the feasibility of increasing the velocity threshold by means of pulse delay compensation before the LDA.

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1- Introduction

Local positioning systems, such as Loran-C, use a chain of at least three transmitters to localize the stationary or low speed Loran-C receivers. Research related to Loran-C system covers a wide variety of problems. These studies have focused on topics such as receiver-positioning algorithms based on Time Difference (TD) measurements [1-3], reducing system errors by synchronizing the transmitters [4], analysis, modeling, mitigation of atmospheric noise and interference effects [5-7], identification and modeling the system errors [8, 9] and determining the system error level by performing practical experiments [7]. A group of important research has focused on various sources of positioning error [10]. Error sources in a Loran-C-based navigation system can be grouped into four classes; errors that cause the receiver to detect a zero crossing of the received pulse other than the third one which cause an error by a factor of 10 microseconds, errors that cause the receiver to identify the third zero crossing with non-zero error which causes an error for less than 10 microseconds, errors that cause the TDs as time references not match to the distance differences [11, 12], and the errors in the conversion of time-derived coordinates to spatial coordinates which occur due to approximations applied in the mapping of TDs to geographical coordinates and Geometric Dilution of Precision (GDOP) [13]. These error sources can be categorized into two groups. Some error sources, including clock error and transmitter control parameters errors, cause delay in the pulse transmission and cannot be mitigated by the receiver. These error sources are decreased by the chain control station. Other error sources reduce the Signal-to-Interference-and-Noise Ratio (SINR). These errors, such as thermal noise, that have Gaussian distribution are compensated by the various methods. The Linear Digital Average (LDA) algorithm [14] is one of the well-known of these methods. The LDA algorithm is commonly used in Loran-C receivers as a pre-processor for increasing the SINR of the average pulse. The long distance between the transmitters and the receiver intensifies the Gaussian noise power, reduces the SNR of the received pulses [15], and consequently strengthens the necessity of LDA usage. This paper studies the effect of the velocity of the receiver on the cycle identification accuracy and analyses the LDA pre-processing behavior in this condition. The LDA increases the SNR of the received pulse by averaging the several consecutive Phase Code Intervals (PCIs) [14]. If the receiver is stationary and there is no time distortion in the transmitters and the receiver, the consecutive pulses will be received by a time period of the PCI. The minimum required number of PCIs to be averaged depends on the SNR of the received pulses and the SNR required for accurate third zero

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crossing detection. However, the receivers that installed on maneuverable and high-velocity moving vehicles obviously encountered more complicated situations. For example, it is obvious that the receiving period of the similar pulses change when the receiver has high speed. Variations in the period of reception of pulses affect the LDA performance. The pulse delay in LDA distorts the average pulse. To the best of the authors' knowledge, there is no valuable published research for LDA application in high-velocity moving receivers. Additionally, the relationship between average pulse distortion and velocity of the receiver has not been addressed in any reference. In addition, there is no algorithm specifically proposed for positioning high-velocity moving receivers in the literature. The errors that occurred for the high velocity receivers in the low frequency local radio navigation systems are studied in this paper. Furthermore, the methods for compensating these errors are proposed. It is shown that averaging would not be advantageous if the maximum delay of *N* similar pulses is greater than 10% of the carrier period of SLP (i.e. 10 microseconds). In addition to the parameters of the chain, the pulse delay depends on the receiver position at the instant of receiving the similar pulses and the velocity of the receiver. Therefore, the pulse delay threshold imposes a threshold on the velocity of the receiver. To overcome this limitation on Loran-C usage for high-velocity moving receivers, this paper proposes an algorithm which compensates the pulse delay by estimating the position and velocity vector of the receiver. In this algorithm, the pulses in the successive PCIs match up to each other with the maximum error of 10% of the carrier period to increase the SNR of the average pulse.

2- Loran-C Positioning Error Sources for a High-Velocity Moving Vehicle

Various error sources of the Loran-C system, such as atmospheric noise, do not depend on the velocity of the receiver [16-19]. However, the velocity causes static precipitations, Doppler Effect, LDA pre-processing error, and TD measurements error. The velocity of the vehicle equipped with Loren-C receiver may cause static precipitations and change the received pulse frequency spectrum due to the Doppler Effect. In addition to the velocity, static precipitation depends on the shape and size of the vehicle. Since the focus of this paper is on providing a positioning algorithm for a receiver mounted on a generic vehicle, the analysis of the effects of static precipitation is ignored. The Doppler Effect depends on the Loran chain parameters and the motion parameters of the receiver. More importantly, the variation in the frequency of receiving the similar pulses (pulse delay) affects the LDA performance, distorting the average pulse. Furthermore, the pulse delay caused by Emission Delay (ED) adds velocity dependent values to TD measurements. In the following, the impact of Doppler Effect on the cycle identification accuracy, the effects of ED on TDs, and LDA performance on a high-velocity moving receiver are investigated.

2- 1- Doppler Effect and Cycle Identification

If the carrier frequency of a signal is f_0 and the velocities of receiver and transmitter are v_r and v_s, respectively, the received signal frequency is calculated by the following equation [20]:

$$f = \left(\frac{c \pm v_r}{c \mp v_r}\right) f_0 \tag{1}$$

where c is the signal velocity in the emission medium. If the transmitter and the receiver are moving toward each other, the upper symbols (+ in the nominator and - in the denominator) are used, and if they are moving away from each other, the lower symbols are used. For example, if the Loran-C receiver moves toward the transmitter at the velocity of 1000 m/s, the carrier frequency increases to 100.334 Hz that causes about 100 picoseconds error in the third zero crossing measurement. Comparing this source of error with the time deviation of the clock of the Loran-C transmitters that is inherently about 7 nanoseconds [21], the error created by Doppler Effect can be ignored.

2- 2- The Effect of ED on TD Measurements

In standard Loran-C, the secondary transmitters wait until receiving the master pulses, stop as much as the Coding Delay (CD), and start transmitting the slave pulses [22]. The summation of the signal arrival time from the master station to the secondary station, (called the Base Time (BT)) and CD is called ED. A moving receiver changes its location during the time interval of the EDs. Therefore, the master and the slaves' pulses are received at different positions of the receiver. Fig. 1 shows the effect of ED on pulse arrival time. The secondary stations 1 and 2 transmit pulses ED_1 and ED_2 seconds later than the master station transmission, respectively. Without loss of generality, it is assumed that $ED_1 < ED_2$ in Fig. 1. Since the receiver moves at the velocity of v m/s, the receiver moves $\left(ED_i + \varepsilon_i\right) v$ meters at the time interval of $ED_i + \varepsilon_i$. ε_i is the difference between the travel time of the master signal and the i[th] secondary station signal to the receiver. Assuming that the receiver is stationary, the pulse travel time to the receiver from secondary station 1 and 2 will be T_1 and T_2, respectively. Since the receiver is moving, the pulse travel times T_1' , T_2' are different from T_1 and T_2. The pulse travel time delays are given as follows:

$$\begin{aligned} \delta T_1 &= T_1 - T_1' \\ \delta T_2 &= T_2 - T_2' \\ \delta T_M &= T_M - T_M' = 0 \end{aligned} \tag{2}$$

Based on these descriptions, the measurement error of the TDs in a moving receiver with respect to a stationary receiver

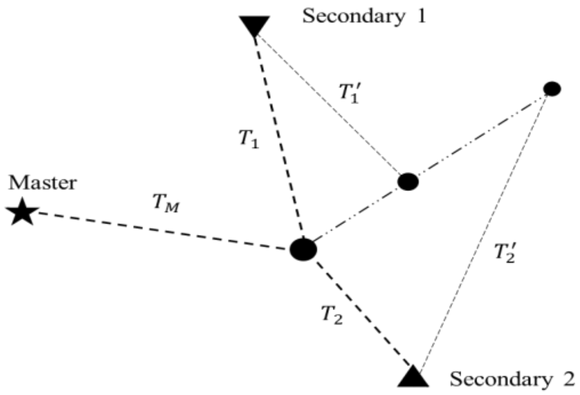


Fig. 1. Pulse travel time of a stationary and a moving receiver. The circles indicate the receiver position at different moments. The larger circle, the medium circle, and the smaller circle indicate the receiver position at the reception moment of the first pulse of the master station, the secondary station 1, and the secondary station 2, respectively.

is as follows:

$$\begin{aligned} \Delta T_{M,1} &= \delta T_M - \delta T_1 = -\delta T_1 \\ \Delta T_{M,2} &= \delta T_M - \delta T_2 = -\delta T_2 \end{aligned} \tag{3}$$

$\Delta T_{M,i}$ is the TD measurement error corresponding to the i[th] secondary station. These measurement errors cause position loci to intersect at locations other than the real position of the receiver. Based on the above equations and Fig. 1, the positioning error caused by the TD measurement error depends on the value of EDs, the velocity of the receiver vector, and the relative geometry of the stations and the receiver. As an example to show the order of this source of positioning error, the maximum of the position error for a receiver with one Mach velocity is 10 meters.

2- 3- LDA in a Moving Receiver

Often the Loran-C received pulses are drowned in noise, and the pulses are barely detectable. To solve this problem, the receivers divide the total received signal into PCI sized windows and perform LDA for cycle identification. Assuming that the noise is Wide-Sense Stationary (WSS), its density function is Gaussian and the similar pulses' period equals PCI, LDA reduces the noise power and the pulses of average PCI will have a higher SNR than the original pulses. The number of participated PCIs in LDA depends on the initial SNR, and the required SNR of the average pulse (the response time of the receivers) is changed according to the required accuracy. For example, the RAYNAV 780 the response time of the receivers vary between 30 and 300 seconds to determine the position depending on user request [23].

It should be noted that LDA will be useful when the reception period of the similar pulses equal PCI, which is true for stationary receiver and synchronous transmitters. Conversely, in a moving vehicle, the receiver's displacement changes the reception period of the similar pulses. In this case, averaging with PCI sized windows may not increase the SNR of pulses, and rather distort the average pulse. The distortion of the average pulse is proportional to the velocity of the receiver. For velocities higher than a threshold, the cycle identification algorithm is not applicable for the distorted average pulse. This will be discussed in detail further in this paper.

2- 4- Conclusion on Positioning Error Sources for a Moving Receiver

The Loran-C positioning starts with the execution of LDA as a pre-processing. Afterwards, the cycle identification algorithm detects the third zero crossing of the average pulse. Finally, the TDs are calculated and the positioning algorithm determines the position. As previously discussed, there are three vehicle independent error sources for moving receivers. Pulse delay distorts the average pulse, Doppler Effect shifts the third zero crossing and causes a negligible cycle identification error, and the EDs add an error on TDs.

The shape of the average pulse calculated from the received PCIs by a moving receiver may differ largely from the shape of SLP. The cycle identification algorithms are sensitive to the shape of the pulse under test. Therefore, the cycle identification error is sensitive to LDA and consequently, to the velocity of the receiver that in high-velocity receivers the cycle identification algorithm may be incapable of detecting the third zero crossing or may detect a wrong zero crossing.

Since the receiver is normally far away from the transmitters, in a moving receiver with a nearly constant velocity vector, the positioning error caused by EDs are approximately equivalent in each participated PCI in LDA and consequently in the average pulse. Therefore, the error caused by EDs is biased and can be eliminated appropriately by approximating the position of the receiver.

3- Usual Approach for Cycle Identification

The primary task of the Loran-C receiver is to accurately determine the transmitter pulse reception time used for TD determination. To avoid the effect of the sky wave interference, the third zero crossing which occurs 30 microseconds after the beginning of the pulse is defined as the Standard Zero Crossing (SZC) and pulse arrival time [24]. There are several methods for detecting SZC, including the correlation matching method [7], the half-cycle peak ratio detection method [25], and the peak-peak ratio detection method [14]. The accuracy of these methods depend on the SNR received pulse, ECD, and the distortion rate which shows the similarity of the received signal to SLP.

3- 1- Linear Digital Average

The Loran-C system receiver is usually far from transmitters and consequently the pulse received by the receiver is drowned in noise, which makes it difficult to detect the presence of a pulse [15]. The common approach to increase the SNR of the received pulses is to apply the LDA algorithm. LDA effectively compensates for WSS and zero mean Gaussian distributed noise [8, 9]. In LDA, the receiver divides the received signal to PCI sized windows after sampling and computes the average of corresponding samples of all similar pulses [26]. The averaging of several samples of similar pulses reduces noise power and increases SNR and cycle identification accuracy [14].

If the receiver is stationary and there is no time distortion in the transmitters and the receiver, similar pulses in PCIs will be received with an exact PCI period. Therefore, in order to average the similar pulses, it is sufficient to divide the received signal to PCI sized windows. In this case, if the PCI is the correct coefficient of the pulse sampling period, the similar samples would be averaged with each other. Two samples of similar pulses are called similar if they have the same time interval from the beginning of the pulses. The SNR of the i[th] sample of the received pulse and the average pulse are calculated by the following equations [27]:

$$\begin{aligned}
&\hat{s}_i^t = s_i + n_i^t \\
&SNR_i^t = P_{s_i} / P_{n_i} \\
&\hat{s}_i^{mean} = \frac{1}{N}\sum_{t=1}^{N}\hat{s}_i^t = s_i + \frac{1}{N}\sum_{t=1}^{N} n_i^t \\
&SNR_i^{mean} = NP_{s_i} / P_{n_i}
\end{aligned} \tag{4}$$

where $\hat{s}_i^t$ is the i[th] sample of the i[th] pulse that includes the standard pulse sample s_i and noise n_i^t. P_{s_i} is the power of the i[th] sample of the standard pulse, and P_{n_i} is the power of noise accumulated with the i[th] sample of the received pulse. After averaging the i[th] similar sample of the N received pulses, the noise power is reduced by the factor of $1/N$, and the SNR is increased by the factor of N.

3- 2- Standard Zero Crossing Identification

After LDA pre-processing and SNR increment, the SZC is determined. The accuracy of the SZC process depends on the algorithm. For example, the proposed algorithm in [14] performs the cycle identification with an accuracy higher than 150 nanoseconds for a pulse with the SNR higher than 15 dBs. The minimum number of PCIs required to be averaged depend on the SNR of the participated pulses in LDA and the required SNR of the average pulse to determine SZC with the specified accuracy.

4- Receiver Velocity and the Pulse Reception Time

Similar pulses are received with the PCI period in a stationary receiver. Movement of the receiver increases/decreases the periodicity of the similar pulses reception, since the displacement of the receiver changes the distance between the transmitter and the receiver. It creates a time delay in receiving the similar pulses of the next PCI. Therefore, the period of the arrival of the similar pulses will be larger or smaller than the PCI in the moving receivers. Fig. 2 shows the concept of pulse delay in a receiver moving away from the master station. For simplicity, each Loran pulse is shown by an impulse function.

4- 1- Velocity Vector and Pulse Delay

According to Fig. 3 and assuming that the surface is flat and the receiver is located initially at a position with the angle of α_i relative to the transmitter, and moves with the velocity of v to the direction with the angle of the β_i similar pulse of $(i+1)^{th}$, PCI will be received t_{p_i} seconds later than the similar pulse of the ith PCI:

$$t_{P_i} = PCI + \tau_{d_i} = PCI + d_{i+1} + d_i / c \tag{5}$$

Where τ_{d_i} is the pulse delay and c is the velocity of the signal in the medium. According to Fig. 3, the distance between the receiver and the transmitter at $(i+1)^{th}$ PCI is calculated by:

$$d_{i+1}^2 = d_i^2 + v_i^2 t_{P_i}^2 - 2d_i v_i t_{P_i}\cos(\beta_i - \alpha_i) \tag{6}$$

According to (5) and (6), and assuming $\gamma_i = \beta_i - \alpha_i$, the following is given:

$$d_{i+1} = \frac{v_i^2(cPCI - d_i) - cd_i v\cos\gamma_i}{c^2 - v_i^2} + \frac{c\sqrt{\begin{aligned}&d_i v_i\cos\gamma_i(d_i v_i\cos\gamma_i + 2cd_i - 2PCIc^2)\\&+PCIcv_i^2(PCIc - 2d_i) + c^2 d_i^2\end{aligned}}}{c^2 - v_i^2} \tag{7}$$

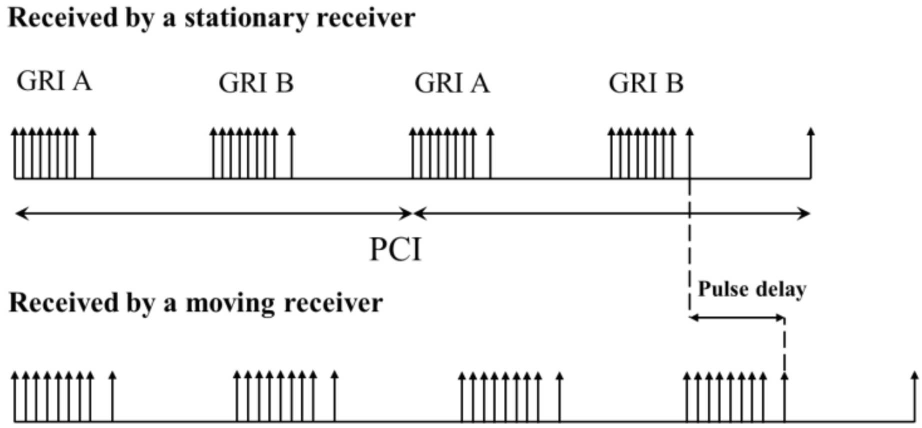


Fig. 2. Pulse delay in a moving receiver relative to a stationary receiver

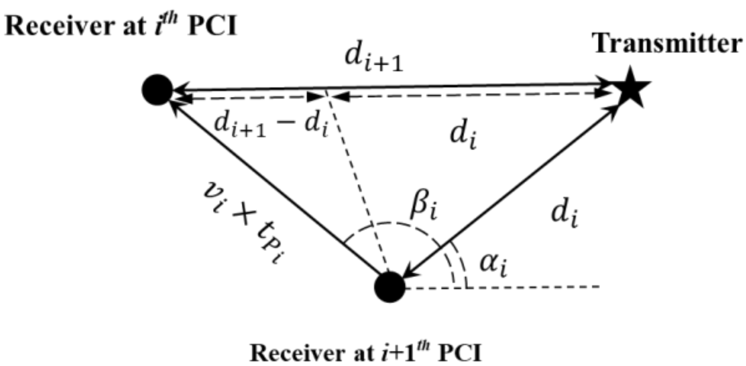


Fig. 3. Calculation of pulse delay in a flat surface

According to (7), the pulse delay is a function of PCI, velocity vector of receiver, v and β_i, signal propagation velocity (c), distance between the receiver and the transmitter at the pulse arrival time in the previous PCI (d_i), and the initial angle of the receiver relative to the transmitter (α_i). Briefly, pulse delay depends on the vehicle dynamics and the chain parameters. The parameters of the chain include PCI and c, and the vehicle dynamics include d_i, v_i and $\beta_i - \alpha_i$. α_i depends on the position of the transmitter $\boldsymbol{x}_i^{tr} = \begin{bmatrix} x_i^{tr} & y_i^{tr} \end{bmatrix}^T$ and the position of the receiver $\boldsymbol{x}_i^{rc} = \begin{bmatrix} x_i^{rc} & y_i^{rc} \end{bmatrix}^T$ at the ith PCI. The position of the transmitter is a parameter of the chain and is known for the receiver. Defining the distance based on the positions $d_i = \left\| \boldsymbol{x}_i^{rc} - \boldsymbol{x}_i^{tr} \right\|$ and $\gamma_i = \beta_i - \alpha_i$, the pulse delay would be a function of three unknown parameters, γ_i, x_i^r, and v_i.

Continuing the path to N consecutive PCIs, a sequence of τ_{d_i} is observed. The delay of receiving the first PCI is assumed to be zero. Therefore, the delay sequence of the successive PCIs is $\tau_d = \left[\tau_{d_0}, \tau_{d_1}, \ldots, \tau_{d_{N-1}} \right]$, where $\tau_{d_0} = 0$. According to (7), the distribution of τ_d is proportional to the parameters of the chain, $\gamma = \left[\gamma_0, \gamma_1, \ldots, \gamma_{N-1} \right]$, $\boldsymbol{X}^{rc} = \left[\boldsymbol{x}_0^r, \boldsymbol{x}_1^r, \ldots, \boldsymbol{x}_{N-1}^r \right]$, and $\boldsymbol{v} = \left[v_0, v_1, \ldots, v_{N-1} \right]$. As a result, distribution of τ_d in addition to the chain parameters depends on the velocity vector of receiver.

4- 2- LDA in High-Velocity Receivers

The Loran-C pulse waveform is an asymmetric exponential function represented in the following equation:

$$\boldsymbol{s}(t,\tau,\varphi) = \begin{cases} A\left(t-\tau\right)^2 e^{-\frac{2(t-\tau)}{65}} \sin\left(2\pi f_c t + \varphi\right) & t \geq \tau \\ 0 & t < \tau \end{cases} \tag{8}$$

where A is a normalization constant of the magnitude of the antenna current in amperes, t is the time in microseconds, τ is ECD in microseconds, f_c is the carrier frequency that equals 100 kHz, and φ is the code phase parameter which is zero or π radians. If the receiver receives N consecutive PCIs so that the $(i+1)^{th}$ pulse has a delay of τ_{d_i} seconds relative to the ith pulse, the delay sequence will be $\tau_d=\left[\tau_{d_0},\tau_{d_1},\ldots,\tau_{d_{N-1}}\right]$, so that $\tau_{d_0}=0$. Considering the initial PCI pulse arrival time as the reference time, the pulse delay of the subsequent PCIs is defined as $\boldsymbol{t}_d=\left[t_{d_0},t_{d_1},\ldots,t_{d_{N-1}}\right]$, where t_{d_0} and $t_{d_i}=\tau_{d_{i-1}}+\tau_{d_i};i=1,\ldots,N-1$.

Assuming $\tau=0$ and $\varphi=0$, the average pulse is as follows:

$$\begin{aligned}\boldsymbol{s}_N(t,\boldsymbol{t}_d)&=\frac{1}{N}\sum_{i=0}^{N-1}\boldsymbol{s}\left(t-t_{d_i}\right)=\\&\frac{1}{N}\sum_{i=0}^{N-1}A\left(t-t_{d_i}\right)^2e^{-\frac{2\left(t-t_{d_i}\right)}{65}}\sin\left(2\pi f_c\left(t-t_{d_i}\right)\right);\ t\geq 0\end{aligned}\tag{9}$$

4- 2- 1- Desirable and Undesirable Parts of the Average Pulse

The primary purpose of this paper is to determine the threshold of the velocity of the receiver according to the average pulse distortion. The velocity of the receiver causes the average pulse distortion and reduces the accuracy of the cycle identification. Therefore, by determining the threshold of the distortion and determining the relation between the amount of pulse distortion and the velocity of the receiver, the threshold of the velocity of the receiver can be obtained. The average pulse computed by (9) can be rewritten as follows:

$$\begin{aligned}\boldsymbol{s}_N(t,\boldsymbol{t}_d)&=\frac{AB_N(\boldsymbol{t}_d)}{N}t^2e^{-\frac{2t}{65}}\sin\left(2\pi f_ct-\psi_N(\boldsymbol{t}_d)\right)+\\&\frac{A}{N}\sum_{i=1}^{N-1}\left(t_{d_i}^2-2tt_{d_i}\right)e^{-\frac{2\left(t-t_{d_i}\right)}{65}}\sin\left(2\pi f_c\left(t-t_{d_i}\right)\right);\ t\geq 0\end{aligned}\tag{10}$$

Where $B_N(t_d)$ and $\psi(t_d)$ are given as follows:

$$\begin{aligned}B_N(\boldsymbol{t}_d)&=\sqrt{\left(\sum_{i=0}^{N-1}e^{-\frac{2t_{d_i}}{65}}\cos\left(2\pi f_ct_{d_i}\right)\right)^2+\left(\sum_{i=0}^{N-1}e^{-\frac{2t_{d_i}}{65}}\sin\left(2\pi f_ct_{d_i}\right)\right)^2}\\\psi_N(\boldsymbol{t}_d)&=\arctan\\&\left(\sum_{i=0}^{N-1}e^{-\frac{2t_{d_i}}{65}}\sin\left(2\pi f_ct_{d_i}\right)\middle/\sum_{i=0}^{N-1}e^{-\frac{2t_{d_i}}{65}}\cos\left(2\pi f_ct_{d_i}\right)\right)\end{aligned}\tag{11}$$

The desirable pulse $\boldsymbol{S}_{N_{ds}}(t,\boldsymbol{t}_d)$ is a pulse with an exact equivalent envelope to the standard pulse, multiplied by an attenuation coefficient $\lambda_N(\boldsymbol{t}_d)=B_N(\boldsymbol{t}_d)/N$ and a carrier with a phase of $\psi(t_d)$. If the carrier phase is rewritten as $\psi(t_d)=2\pi f_c\hat{t}$, $\boldsymbol{S}_{N_{ds}}(t,\boldsymbol{t}_d)$ is converted to:

$$\boldsymbol{s}_{N_{ds}}(t,\boldsymbol{t}_d)=A\lambda_N(\boldsymbol{t}_d)t^2e^{-\frac{2t}{65}}\sin\left(2\pi f_c\left(t-\hat{t}\right)\right)\tag{12}$$

By defining a new variable $t'=t-\hat{t}$, (13) is rewritten as follows:

$$\boldsymbol{s}_{N_{ds}}\left(t'+\hat{t},\boldsymbol{t}_d\right)=A\lambda_N(\boldsymbol{t}_d)\left(t'+\hat{t}\right)^2e^{-\frac{2\left(t'+\hat{t}\right)}{65}}\sin\left(2\pi f_ct'\right)\tag{13}$$

Equation (14), which is the shifted desirable pulse, is a standard pulse with $\tau=-\hat{t}$. Therefore, the desirable pulse differs from the SLP in the ECD, in addition to the amplitude.

According to (12), the envelope of the undesirable pulse is dissimilar to the SLP and is also multiplied by t_{d_i}. Furthermore, the phase of the undesirable pulse carrier is different from the desirable pulse carrier phase. Therefore, the phase and the envelope of the undesirable pulse are different from the ones of the desirable pulse. The undesirable pulse cannot be defined by SLP. Consequently, the undesirable pulse encloses the total distortion of the average pulse. To measure the extent of the distortion, a proportional constant is defined as follows:

$$\rho_N(\boldsymbol{t}_d)=s_{N_{ds}}^{max}(\boldsymbol{t}_d)\big/s_{N_{un}}^{max}(\boldsymbol{t}_d)\tag{14}$$

Where $S_{N_{ds}}^{max}(\boldsymbol{t}_d)$ and $S_{N_{un}}^{max}(\boldsymbol{t}_d)$ are the maximum value of the desirable and undesirable part of the average pulse, respectively.

4- 2- 2- Average Pulse SNR

Since the accumulated noise of the received pulse is zero mean Gaussian and WSS, the LDA reduces the noise power of the average pulse $\left(P_N^n\right)$ by the coefficient of $1/N$, compared to the noise power of N received pulses $\left(P_{in}^n\right)$, i.e.:

$$\frac{P_N^n}{P_{in}^n}=\frac{1}{N}\tag{15}$$

Assuming $\psi(t_d)\approx 0$ and $\rho_N(\boldsymbol{t}_d)\gg 1$, the average pulse power $\left(P_N^s\right)$ equals the received pulse power $\left(P_{in}^s\right)$ multiplied by $\lambda_N^2(\boldsymbol{t}_d)$:

$$\frac{P_N^s}{P_{in}^s}=\lambda_N^2(\boldsymbol{t}_d)\tag{16}$$

Accordingly, the SNR of the average pulse is given as follows:

$$SNR_N = \frac{P_N^s}{P_N^n} = \frac{P_{in}^s \lambda_N^2 \left(\boldsymbol{t}_d\right)}{P_{in}^n \left(1/N\right)} = SNR_{in} N \lambda_N^2 \left(\boldsymbol{t}_d\right) \qquad (17)$$

Where SNR_{in} is the SNR of the received pulses. Therefore, if $N \lambda_N^2 \left(\boldsymbol{t}_d\right) > 1$, LDA of the delayed received pulses increases the SNR.

4- 3- LDA in High-Velocity Receivers

The ideal result of the LDA of N noisy pulses with the same parameters is that the LDA does not distort the average pulse and reduces the SNR by the factor of N. According to (10) and (11), we have $\lambda_N \left(\boldsymbol{t}_d\right) = 1, \psi\left(\boldsymbol{t}_d\right) = 0, S_{N_{un}}\left(t, \boldsymbol{t}_d\right) = 0$ for the desirable pulse in the case of $t_{d_i} = 0; i = 0, \ldots, N-1$, and the undesirable pulse equals zero. However, if only for one i $t_{d_i} \neq 0$, then $\lambda_N \left(\boldsymbol{t}_d\right) \neq 1, \psi\left(\boldsymbol{t}_d\right) \neq 0, S_{N_{un}}\left(t, \boldsymbol{t}_d\right) \neq 0$. The condition of $t_{d_i} = 0; i = 0, \ldots, N-1$ may be true only for $v_i = 0; i = 0, \ldots, N-1$ (a stationary receiver). Generally, the delays of the received pulses cause $ECD \neq 0$ and $\lambda_N \left(\boldsymbol{t}_d\right) \neq 1$ in the desirable pulse and $\rho_N \left(\boldsymbol{t}_d\right) \neq 0$ because the undesirable pulse is nonzero. The error distribution of a cycle identification algorithm depends on ECD, SNR_N, and the average pulse distortion. Consequently, if $\rho_N \left(\boldsymbol{t}_d\right)$ is large enough to ignore the undesirable pulse, it is enough for SNR_N to be larger than a threshold and for ECD to be lower than a threshold to achieve a specific accuracy.

According to (7), N and $\boldsymbol{t}_d$ can be calculated for a particular velocity vector of the receiver and the given chain parameters. The values of ECD, $\lambda_N \left(\boldsymbol{t}_d\right), SNR_N / SNR_{in}$, and $\rho_N \left(\boldsymbol{t}_d\right)$ are also functions of N and $\boldsymbol{t}_d$. The value of ECD is obtained by computing $\psi\left(t_d\right)$ from (11): $ECD = \psi\left(t_d\right)/2\pi f_c$. The attenuation coefficient is obtained by calculating $B_N \left(\boldsymbol{t}_d\right)$ from (11): $\lambda_N \left(\boldsymbol{t}_d\right) = B_N \left(\boldsymbol{t}_d\right)/N$. The values of SNR_N / SNR_{in} and $\rho_N \left(\boldsymbol{t}_d\right)$ can also be obtained from (15) and (18), respectively. Therefore, having the velocity vector of the receiver and the parameters of the chain, the accuracy of the cycle identification algorithm and the average pulse time of arrival could be estimated. Consequently, the threshold of the velocity of the receiver can be calculated for the required cycle identification accuracy.

5- Pulse Delay Compensation

In a moving receiver, a limited number of delayed pulses can be averaged $\left(N_{min}\right)$. Generally, increasing the number of pulses participating in LDA increases the delay time, and not only may not increase the SNR, the average pulse also may be subjected to severe distortion. To increase the number of pulses participating in the LDA, it is proposed to compensate for the pulse delays according to the velocity vector of the receiver and initial position estimation.

5- 1- Velocity Vector of the Receiver and Initial Position Estimation

According to (7), the pulse delay could be calculated and compensated by the determination of the velocity of the receiver vector and the initial position. Having the true values of the velocity vector and the initial position help to calculate the exact values of the pulse delays and compensate for them before LDA to have a distortion free average pulse. Obviously, the exact values of the parameters are unknown to the receiver. Nevertheless, the receiver can estimate the parameters according to the previous measurements. Therefore, pulse delay compensation accuracy is related to the estimation accuracy of the velocity vector and the initial position of the receiver. To increase the accuracy of pulse delay estimation, each PCI can be used in several LDA groups. This is obtained by groping each PCI with $N-1$ consecutive PCIs. Accordingly, there would be a velocity vector and an initial position estimation for each PCI.

5- 2- The Proposed Compensation Algorithm

The pulse delay compensation is performed to reduce the distortion of the average pulse and the effect of ED on TDs. In this algorithm, the number of PCIs (*N*) required for averaging is determined by the SNR of the received pulses and the required accuracy of the cycle identification. Pulse delay compensation requires the initial position and velocity vector of the receiver. Since the parameters are unknown, the algorithm requires an initial estimation. According to the estimations, the pulse delay of each PCI could be compensated. If the SNR of the average pulse is not high enough to identify SZC, it is necessary to reduce the number of PCIs in each group, since the reason of the cycle identification failure is possibly that the maximum delay between the pulses is more than 10% of the SLP carrier signal's period. Obviously, if even a couple of PCIs cannot be averaged, it means that the estimations are not accurate enough and should be updated. The calculated position of the average pulse of each PCI group is the receiver position at the moment of receiving the first PCI, since in LDA, all subsequent pulses are matched to the first PCI. If the threshold of 10% of the SLP carrier signal's period is satisfied, the SZC of the average pulse in each PCI group has a maximum error of one microsecond relative to the receiver position in the first PCI. According to the position estimated in the current and previous PCI groups, the velocity vector of receiver is updated. The velocity vector is updated according to the distance between the current and the previous position divided by the elapsed time and the direction of movement. The flowchart of the proposed method is shown in Fig. 4.

6- Simulation Results

The average pulse of the delayed pulses is characterized by the parameters $\lambda_N \left(\boldsymbol{t}_d\right)$, and ECD of the desirable pulse and SNR_N and $\rho_N \left(\boldsymbol{t}_d\right)$ in this paper. The velocity of the receiver threshold can be calculated according to the desired values of *N*, $\lambda_N \left(\boldsymbol{t}_d\right)$, ECD, SNR_N, the direction of the movement, and the chain parameters. It is also possible to

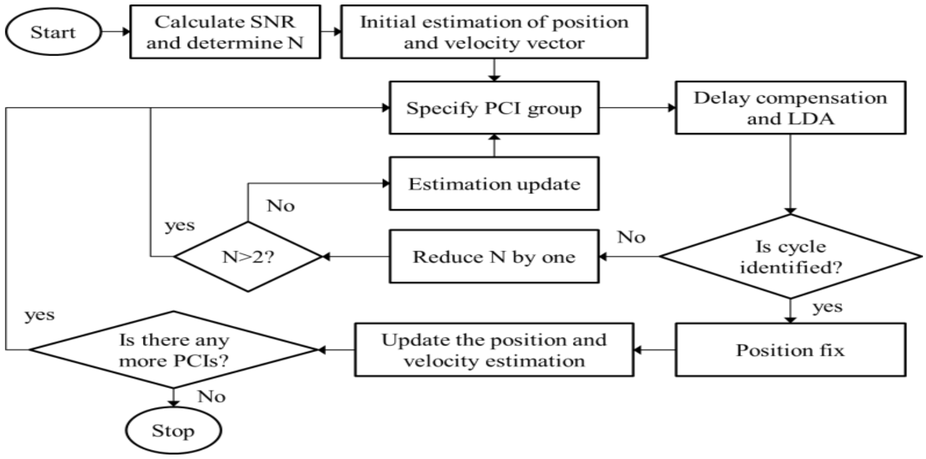


Fig. 4. The Flowchart of the proposed method for positioning a high-velocity moving receiver

calculate *N*, $\lambda_N(t_d)$, ECD, and SNR_N, and estimate the cycle identification accuracy by having the velocity of the receiver vector and the chain parameters. In this section, the values of the average pulse parameters are investigated for different scenarios.

6- 1- Effects of the Number and Delay of Pulses on Distortion Factors

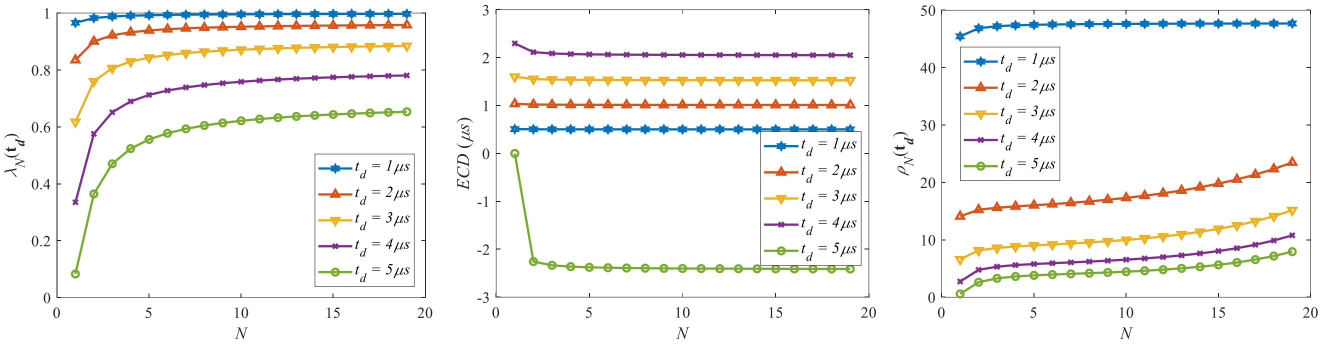
In this section, the values of $\lambda_N(t_d)$, ECD, and $\rho_N(t_d)$ are calculated for different number and delay of pulses involved in LDA. The results help to determine the threshold for number and delay of the pulses according to distortion factors of the average pulse. For simplicity, all the results are based on the assumption $t_{d_i} = \frac{it_d}{N-1}; i = 0,\ldots,N-1$. In Fig. 5, the values of $\lambda_N(t_d)$ are plotted relative to N and different values of t_d from 1 to 5µs.

According to Fig. 5.a, $\lambda_N(t_d)$ is increased by the growth of N and is near to one for $t_d \le 1\mu s$. According to (18) and assuming $\lambda_N(t_d) \approx 1$, the value of SNR_N is increased by the growth of N. Therefore, the required value of SNR_N is calculated with respect to the cycle identification accuracy, and the minimum value of N is obtained in accordance with (18) and SNR_{in}. The minimum value of $\lambda_N(t_d)$ in Fig. 5 belongs to $t_d = 5\mu s$ and $N = 2$, since the carrier period of the pulse is $10\mu s$, and summation of two pulses with 5µs delay results in summation of positive cycles of one pulse into negative cycles of the other pulse. The values of $\hat{t} = \psi(t_d)/2\pi f_c$, which represent the ECD, are shown in Fig. 5.b for different values of N and t_d. Fig. 6 shows that for $N > 3$, the ECD of the desirable pulse does not depend heavily on the number of involved pulses in LDA. For $t_d \le 1\mu s$, ECD is less than $0.6\mu s$. Fig. 5.c shows the ratio of the maximum amplitude of the desirable pulse to the maximum amplitude of the undesirable pulse, called the proportional constant. Accordingly, the proportional constant for $t_d < 5\mu s$ is increased by the increment of N, and is decreased by the increment of t_d. According to Fig. 5.a, the attenuation coefficient is increased by the increment of N for $t_d < 5\mu s$, and is higher than 0.6 for $N > 20$. $\lambda_N(t_d)$ approximately equals to one for $t_d = 1\mu s$ and $N > 5$. ECD approximately equals to 0.5µs for $t_d = 1\mu s$ and different number of pulses. Proportional constant $\rho_N(t_d)$ approximately equals to 47 for $t_d = 1\mu s$ and $N > 3$, i.e. the maximum amplitude of the desirable pulse is 47 times larger than the maximum amplitude of the undesirable pulse. Accordingly, Fig. 5.b and Fig. 5.c show that the average pulse is almost not distorted while $t_d = 1\mu s$ for $N > 3$. Consequently, the threshold of the maximum pulse delay in LDA for 3 or more PCIs is 1µs (i.e. 10% of the pulse's carrier period).

6- 1- 1- Determination of the Threshold of the Velocity of the Receiver Without Using the Compensation Algorithm

The receiver needs to average the received similar pulses to increase the SNR, and consequently increase the accuracy of the cycle identification. The motion of the receiver causes a delay in receiving pulses in successive PCIs. The delay in pulse reception is a function of the chain parameters, the initial position and the velocity vector of receiver. This paper presents the equations to determine the pulse reception time with a predefined accuracy. The accuracy of the cycle identification could also be determined by the velocity vector of the receiver. The threshold of the velocity of the receiver or the threshold of the accuracy of the cycle identification depends on the station that generates the maximum delay in the pulses. The maximum delay occurs when the receiver moves towards or in the opposite direction of the transmitter. Therefore, the maximum delay in a chain is related to the transmitter pulses, which the direction of the receiver's movement makes the minimum angle to the line connecting the receiver to the transmitter.

Suppose that a receiver moves at a velocity of one Mach (334 m/s) moving away from the transmitter that is 300 kilometers away in a chain $(PCI = 150ms)$. Based on the



(a) (b) (c)

Fig.5. a – The attenuation coefficient of the desirable pulse and b – The values of $\hat{t}=\psi(t_d)/(2\pi f_c)$ representing the ECD over N_s and t_d from 1 µs to 5 µs and c – The ratio of the maximum amplitude of the desirable pulse to the maximum amplitude of the undesirable pulse which is called proportional constant

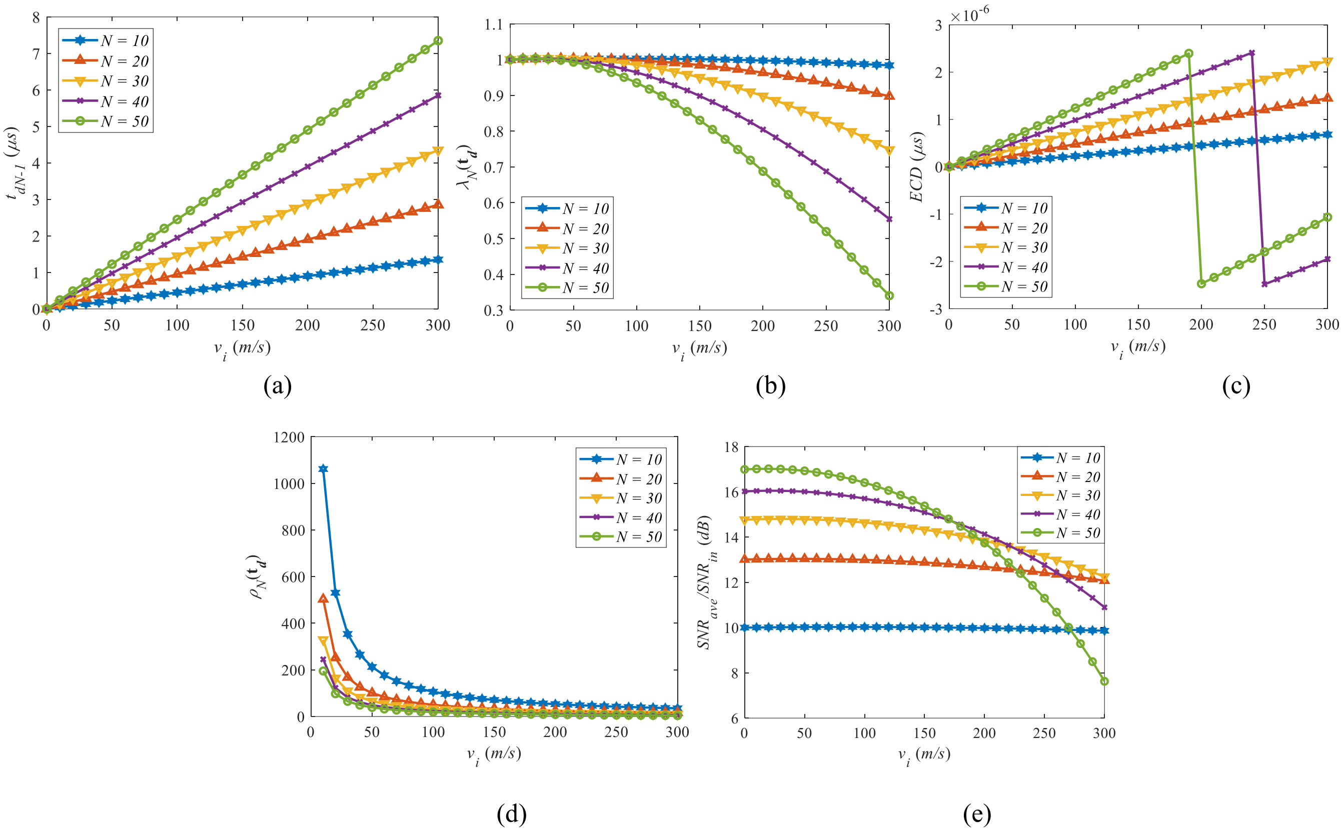


Fig. 6. a – The last pulse delay (t_{dN-1}) , b – The attenuation coefficient (λ_N (t_d)), c – The envelope to cycle difference (ECD), d – The proportional constant (ρ_N (t_d)) and e – The output to input signal to noise ratio SNR_{ave}/SNR_{in} for different velocities of the receiver moving away from the transmitter for different N_s

equations of this paper the SNR increment and the average pulse parameters can be calculated. The pulse delay $(\boldsymbol{t}_d)$ of 20 consecutive PCIs, the attenuation coefficient $\lambda_N(\boldsymbol{t}_d)$, the proportional constant $\rho_N(\boldsymbol{t}_d)$, ECD, and the average pulse signal to noise ratio SNR_N are given as follows:

$$\begin{aligned}
&\boldsymbol{t}_d = [0, 0.167, 0.334, \ldots, 3.173]_{1\times 20}\ \mu s \\
&\lambda_N(\boldsymbol{t}_d) = 0.8686, \qquad ECD = 1.617\mu s \\
&\rho_N(\boldsymbol{t}_d) = 14.22, \qquad SNR_N / SNR_{in} = 15.09
\end{aligned} \tag{18}$$

Therefore, for increasing SNR by the value of $10log15.09 = 11.76db$ at one Mach velocity, the average pulse ECD equals $1.617\mu s$ which is acceptable for some cycle identification algorithms. The average pulse parameters for different velocities from 0 to 300 m/s for different Ns are shown in Fig. 8.

According to Fig. 6(a) to (e), pulse delay and ECD are increased and the pulse attenuation coefficient of the desirable pulse and the proportional constant are decreased by the velocity and N increment. The trend of SNR_{ave} / SNR_{in} is different for each N, so that as N increases the decrement of SNR_N / SNR_{in} speeds up relative to the increment of the velocity. According to Fig. 6(c) to (e), for a desired value of ECD, $\rho_N(t_d)$, and SNR_N / SNR_{in} for the cycle identification algorithm, the velocity of receiver threshold can be calculated and the other parameters of the average pulse can be observed.

6- 1- 2- Moving Receiver Positioning Based on the Pulse Delay Compensation

Since the purpose of this simulation is to investigate the problem of pulse delay compensation, for simplicity in representations and calculations, an impulse function at third zero crossing point is considered instead of the Loran

Table 1. The estimates of the velocity vector and the position of the receiver and the position estimation error

PCI group	*Angle of velocity (degree)*	*Position estimation error (m)*	*Velocity (m/s)*
1	40	89.3	800
2	44.99	89.3	1001.3
3	43.04	6.68	907.2
4	45.76	35.36	1045.6
5	43.78	25.73	937.3
6	45.84	21.8	1052.1
7	44.06	28.62	948.3
8	45.77	16.72	1049.9
9	44.23	27.77	954.5
10	45.68	13.84	1046.1
11	44.36	26.2	959.1
12	45.59	11.64	1042.1
13	44.46	24.61	963.1
14	45.52	9.75	1038.4
15	44.56	23.14	966.6
16	45.45	8.08	1035.1
17	44.63	21.83	969.8
18	45.4	6.58	1032.1
19	44.7	20.66	972.2
20	45.35	5.25	1029.2

pulses. It is assumed that in a specific chain with GRI = 8990 and the secondary stations with $CD_1 = 11000\mu s$ and $CD_2 = 25000\mu s$, a receiver with the initial position of $26°N$ and $46°E$ at the velocity of $1000\ m/s$ is on the move to the northwest with 45° angle to the north. In this condition, the delay of received pulses from the first secondary transmitter between successive PCIs is about 0.2596 microseconds. According to the threshold of 10%, more than three PCIs cannot be averaged with no delay compensation. Therefore, if the LDA requires at least four PCIs, then the similar pulse delay created by receiver's motion should be compensated. In the case that the position and the velocity vector of the receiver are accurately known at any moment, the pulse delay can be compensated without error and infinite PCIs could be averaged to have a distortion free average pulse. Therefore, an accurate estimation of the position and velocity vector of the receiver compensates for the pulses delays accurately. Accordingly, an accurate pulse delay compensation results in an accurate position and velocity vector estimation in the next PCI group.

It is assumed in this simulation that the initial estimation of the velocity of the receiver vector is inaccurate. The position determination is done by the algorithm proposed in [28]. The initial estimate of the velocity of receiver is $800\ m/s$, which moves toward the northwest with 40° angle to the north. In this simulation, 100 PCIs are received and each five PCIs are averaged. It is also assumed that the initial estimate corresponds exactly to the actual initial position of the receiver. The estimated position error to the nearest point of the receiver path and the estimated velocity vector of the receiver are given in Table 1.

According to Table 1, position estimation errors in the consecutive PCI groups are decreasing, and velocity vector estimations are also getting closer to the real values which indicate the convergence of the proposed algorithm. According to the threshold of 10%, the receiver could move at a maximum velocity of $770\ m/s$ without pulse delay compensation. Applying the pulse delay compensation approach, the receiver could move at the velocity of $1000\ m/s$ with the maximum pulse delay of 3% of the carrier period. Therefore, the pulse delay compensation, in addition to increasing the velocity threshold, decreases the average pulse distortion and consequently increases the positioning accuracy. To compare, the velocity threshold increases to $3300\ m/s$ by applying the proposed delay compensation algorithm (i.e. about 4 times larger than the velocity threshold without compensation).

7- Conclusion

This paper has discussed the positioning error sources of a fast moving receiver and develops an approach for error compensation. For this purpose, the equations to calculate the threshold of the velocity of the receiver in a specified direction of movement in a specific chain were presented. The criteria for calculating the threshold of the velocity of the receiver were determined in accordance with the pulse distortion characteristics. The distortion characteristics were analyzed by defining the desirable and undesirable parts of the average pulse. This analysis showed that the distortion can be defined by known features, such as ECD, and the attenuation coefficient. Furthermore, the amount of delay of received pulses was defined as a function of the velocity vector of receiver. Therefore, an analytical framework was proposed to estimate the average pulse distortion relative to the velocity vector of the receiver. According to the simulations, 10% of the SLP's carrier period pulse delay (i.e. one microsecond) could be accepted as the rule of thumb for determining the threshold of the velocity of the receiver. The velocity of the receiver threshold obtained using mentioned 10% rule of thumb is very low for many applications. Simulations showed that for high-velocity vehicles, LDA would cause distortion to such an extent that the cycle identification algorithm could not obtain the desired accuracy. In order to increase the velocity threshold, a pulse delay compensation algorithm was proposed. In addition to providing the opportunity of increasing the velocity threshold, the proposed algorithm decreases the TD measurement error caused by EDs. The proposed algorithm compensates pulse delay by estimating the position and velocity vector of the receiver. Using this compensating approach, it is possible to reduce the average pulse distortion and increase the dynamic range of the velocity of the receiver using the estimated position and the velocity vector. Combining the proposed algorithm and the inertial navigation system using Kalman filter to increase the accuracy of positioning is one of important future research. Additionally, proposing a possible and low cost approach for practical validation of the mentioned results would be useful as one of the future works.

Nomenclature

ECD Envelop to Cycle Difference,
GRI Group Repetition Interval

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