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Original Article

On m-th root metrics of isotropic projective Ricci curvature

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ABSTRACT: The Ricci curvature is introduced by spray on M^n. Sprays are deformed to projective sprays with a volume form dV on M^n. The projective Ricci curvature is defined as the expression of Ricci curvature with sprays. With this paper, we use the new notion that is called weakly isotropic projective Ricci curvature. We have introduced the idea of weakly isotropic projective Ricci curvature in [3]. Then we study and characterize m-th root metrics of weakly isotropic projective Ricci curvature. We obtain that every m-th root metric of weakly isotropic projective Ricci curvature is projectively Ricci-flat.

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1. Introduction

Finsler metrics are induced by sprays. Z. Shen has showed that the spray G can be deformed to a projective spray in [14] as follows:

$$\hat{\mathbf{G}} := \mathbf{G} + \frac{2\mathbf{S}}{n+1}\mathbf{Y},$$

where $\mathbf{S} = \mathbf{S}_{(\mathbf{G},\ dV)}$ is the S-curvature of $(\mathbf{G},\ dV)$ and on TM^n, $\mathbf{Y} := y^i \frac{\partial}{\partial y^i}$ is considered as the vertical field. The spray $\hat{\mathbf{G}}$ is a projective spray with respect to a fixed volume form dV. Thus the curvature of $\hat{\mathbf{G}}$ is the projective invariant of a spray $\mathbf{G}$ with respect to a fixed volume form dV. The Ricci curvature defined by $\hat{\mathbf{G}}$ is called projective Ricci curvature of $(\mathbf{G},\ dV)$:

$$\mathbf{PRic}_{(\mathbf{G},dV)} := \mathbf{Ric}_{\hat{G}},$$

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that can be expressed as follows ([15]):

$$\mathbf{PRic}_{(\mathbf{G},dV)} = \mathbf{Ric} + (n-1)\Big\{\frac{\mathbf{S}_{|0}}{n+1} + \Big[\frac{\mathbf{S}}{n+1}\Big]^2\Big\}, \tag{1}$$

where $\mathbf{Ric} = \mathbf{Ric}_{\mathbf{G}}$ is the Ricci curvature of the spray $\mathbf{G}$, and $\mathbf{S}_{|0}$ is the covariant derivative of $\mathbf{S}$ along the geodesics of $\mathbf{G}$. The projective Ricci curvature can be acceptable as a special weighted Ricci curvature. A spray $\mathbf{G}$ on M^n that is defined by a volume form dV on M^n is called *projectively Ricci-flat*, namely:

$$\mathbf{PRic}_{(\mathbf{G},dV)} = 0.$$

One can easily see that if $\mathbf{G}$ is Ricci-flat and $\mathbf{S} = dh$ for some scalar function $h = h(x)$, then $\mathbf{G}$ is projectively Ricci-flat. A Finsler metric F on M^n is said to be projectively Ricci-flat if the induced spray $\mathbf{G} = \mathbf{G}_F$ is projectively Ricci-flat. It is remarkable that every weak Einstein Finsler metric $\mathbf{Ric} = (n-1)[\frac{3\theta}{F} + c]F^2$ with vanishing S-curvature satisfy $\mathbf{PRic}_{(\mathbf{G}_F,dV)} = (n-1)[\frac{3\theta}{F} + c]F^2$. For some research on Finsler metrics of projectively Ricci-flat, one can see [1, 7, 6, 8, 15].

Definition 1.1. *Let F be a Finsler metric on M^n and $\mathbf{G} = \mathbf{G}_F$ be the induced spray of F.*

- *F is of isotropic projective Ricci curvature: if $\boldsymbol{PRic}_{(\mathbf{G}_F,dV)} = (n-1)c(x)F^2$;*
- *F is of constant projective Ricci curvature: if $\boldsymbol{PRic}_{(\mathbf{G}_F,dV)} = (n-1)cF^2$, where c is a real constant;*
- *F is called projectively Ricci-flat: if $\boldsymbol{PRic}_{(\mathbf{G}_F,dV)} = 0$.*

Example 1.1. *Every Einstein Finsler metric $\boldsymbol{Ric} = (n-1)\lambda F^2$, $\lambda = \lambda(x)$, with vanishing S-curvature is of isotropic projective Ricci curvature. It is remarkable that every Einstein Kropina metric has vanishing S-curvature. Thus an Einstein Kropina metric has isotropic projective Ricci curvature $\kappa = \lambda$.*

Let (M^n, F) and TM^n be an n-dimensional Finsler manifold, and its tangent bundle, respectively. Also, let (x^i, y^i) be the coordinates in a local chart on TM^n. In 1979, H. Shimada introduced a class of Finsler metric called m-th root Finsler metric, [16]. It has been introduced with the following form:

$$F = \sqrt[m]{A},$$

where $A := a_{i_1 \dots i_m}(x)y^{i_1}\dots y^{i_m}$ and $a_{i_1\dots i_m}$ symmetric in all its indices . It is easy to see that Riemannian metrics $F = \sqrt{a_{ij}(x)y^iy^j}$ are the simplest m-th root Finsler metrics. F is called cubic metric and quartic metric if m is equal to 3 and 4, respectively. Recent works show that the theory of m-th root Finsler metrics plays an essential role in physics, theory of space-time structures, gravitation, general relativity, and seismic ray theory [16, 12, 13]. For some new progress on these metrics, see [9, 17, 19, 18].

In [11], Matsumoto and Numata studied the cubic metrics and showed that a cubic Finsler metric on M^n,$(n \geq 3)$, $F = \sqrt[3]{a_{ijk}(x)y^iy^jy^k}$, can be written in the form of an (α,β)-metric:

$$F = \alpha\phi(s), \quad \phi(s) = \sqrt[3]{a_1 s + a_2 s^3},$$

by choosing suitable non-degenerate quadratic form $\alpha = \sqrt{a_{ij}(x)y^iy^j}$ and 1-form $\beta = b_i(x)y^i$, where a_1 and a_2 are real constants such that $a_1 + a_2 b^2 \neq 0$. Thus, a cubic metric is a special (α,β)-metric. On the other hand, Kim and Park obtained a fundamental function for the m-th root Finsler metric which admits an (α,β)-metric, [10]: $(m \geq 3)$

$$F = \sqrt[m]{\sum_{r=0}^{s} c_{m-2r}\alpha^{2r}\beta^{m-2r}}, \quad s \leq \frac{m}{2},$$

where c's are arbitrary constants and s is an integer. The rich class of (α,β)-metrics expressed by $F = \alpha\phi(s)$, $s = \frac{\beta}{\alpha}$, $\alpha := \alpha(x,y) = \sqrt{a_{ij}(x)y^iy^j}$. Here, α is a Riemannian metric, $\beta := \beta(y) = b_i(x)y^i$ is a 1-form, and $\phi(s) \in C^\infty$ is a positive function on some open interval. In the class of (α,β)-metrics with the form

$$F = \alpha + \varepsilon\beta + k\frac{\beta^2}{\alpha},$$

where ε and $k \neq 0$ are constants, there is a special kind of (α,β)-metric which has an interesting geometric properties. Let $\varepsilon = 2$ and $k = 1$, then the metric F becomes a square metric. A square metric is defined by

$$F = \alpha\phi(s), \quad \phi(s) = (1+s)^2.$$

In [3], we proved that a square metric F must be projectively Ricci-flat if F is of isotropic projective Ricci curvature. In this paper, we discuss the problem for a non-Riemannian m-th root metric. Then, we have the following theorem:

Theorem 1.2. *Let $F = \sqrt[m]{A}$, $A := a_{i_1\dots i_m}(x)y^{i_1}\dots y^{i_m}$ be a non-Riemannian m-th root metric on M^n ($n \geq 2$, $m \geq 3$). Suppose that F is of isotropic projective Ricci curvature, then it is projectively Ricci-flat.*

A Finsler metric F is of weakly isotropic projective Ricci curvature, namely ($WIPRC$): if there is a volume form dV on M^n, that is,

$$\mathbf{PRic}_{(\mathbf{G}_F,dV)} = (n-1)[\frac{3\theta}{F} + c]F^2, \tag{2}$$

where $\theta = \theta_i y^i$ is a $1-$form and $c = c(x)$ is scalar function on M^n.

Example 1.2. *For a constant number $a \in \boldsymbol{R}^n$, consider the Randers metirc $F = \alpha + \beta$ as follows:*

$$\alpha := \frac{\sqrt{(1-|a|^2|x|^4)|y|^2 + (|x|^2 < a,y > -2 < a,x >< x,y >)^2}}{1-|a|^2|x|^4},$$
$$\beta := \frac{|x|^2 < a,y > -2 < a,x >< x,y >}{1-|a|^2|x|^4}.$$

This Randers metrics satisfy the following equations

$$\boldsymbol{S} = (n+1)\kappa F,$$
$$\boldsymbol{Ric} = (n-1)(3\kappa_0 F + \delta F^2),$$

where

$$\kappa :=< a,x >, \quad \kappa_0 := \kappa_{x^m} y^m, \quad \delta := 3 < a,x >^2 -2|a|^2|x|^2.$$

For more details, see [2]. Then by (1) we can see

$$\boldsymbol{PRic} = (n-1)[\frac{4\kappa_0}{F} + \kappa^2 + \delta]F^2.$$

Therefore F is of $WIPRC$ with $\theta = \dfrac{4\kappa_0}{3}$ and $c = \kappa^2 + \delta$.

Theorem 1.3. *Let $F = \sqrt[m]{A}$, $A := a_{i_1\dots i_m}(x)y^{i_1}\dots y^{i_m}$ be a non-Riemannian m-th root metric on a M^n ($n \geq 2$, $m \geq 3$). Suppose that F is of WIPRC, then it is projectively Ricci-flat.*

2. Preliminary

Let F be a Finsler metric on M^n. It induces a spray [5]:

$$\mathbf{G} = y^i\frac{\partial}{\partial x^i} - 2G^i\frac{\partial}{\partial y^i},$$

where G^i are given by

$$G^i := \frac{1}{4}g^{il}\{[F^2]_{x^k y^l}y^k - [F^2]_{x^l}\},$$

g^{ij} is defined as the inverse of the fundamental tensor $g_{ij} := [\frac{1}{2}F^2]_{y^i y^j}$. If F is a Riemannian metric, then G^i can be expressed by the Christoffel symbols, $G^i(x,y) = \frac{1}{2}\Gamma^i_{jk}(x)y^j y^k$.

Let F be a Finsler metric defined by $F = \sqrt[m]{A}$, $A := a_{i_1\dots i_m}(x)y^{i_1}\dots y^{i_m}$, with $a_{i_1\dots i_m}$ symmetric in all its indices, [16]. Then F is called an m-th root Finsler metric. Clearly, A is homogeneous of degree m in y.

F is an m-th root Finsler metric on $U \subset \mathbf{R}^n$ where U is an open subset. For convenience, consider

$$A_k = \frac{\partial A}{\partial y^k}, \quad A_{kl} = \frac{\partial^2 A}{\partial y^k y^l}, \quad A_{x^k} = \frac{\partial A}{\partial x^k}, \quad A_0 = A_{x^k}y^k, \quad A_{0l} = A_{x^i y^l}y^i.$$

Suppose that the matrix (A_{kl}) defines a positive definite tensor and $(A_{kl})^{-1} = A^{kl}$. Then the following relations hold

$$g_{kl} = \frac{A^{\frac{2}{m}-2}}{m^2}[mAA_{kl} + (2-m)A_kA_l], \qquad g^{kl} = A^{-\frac{2}{m}}\left[mAAA^{kl} + \frac{m-2}{m-1}y^ky^l\right],$$

$$y^kA_k = mA, \qquad y^kA_{kl} = (m-1)A_l, \qquad y_k = \frac{1}{m}A^{\frac{2}{m}-1}A_k,$$

$$A^{kl}A_{li} = \delta^k_i, \qquad A^{kl}A_k = \frac{1}{m-1}y^l, \qquad A_kA_lA^{kl} = \frac{m}{m-1}A.$$

Then, the spray coefficients of an m-th root Finsler metric on an open subset $U \subset \mathbf{R}^n$ are given in [20] as follows:

$$G^k = \frac{1}{2}(A_{0l} - A_{x^l})A^{kl}. \tag{3}$$

Lemma 2.1. *[20] The spray coefficients of an m-th root Finsler metric on an open subset $U \subset \mathbf{R}^n$ are rational functions in y.*

The S-curvature is given by as follows:

$$\mathbf{S} = \frac{\partial G^i}{\partial y^i} - y^i\frac{\partial}{\partial x^i}\Big[\ln\ \sigma_{BH}\Big], \tag{4}$$

where $dV_F = \sigma_F(x)\ dx^1\ \wedge\cdots\wedge\ dx^n$ is the Busemann - Hausdorff volume form. A Finsler metric F is called of isotropic S-curvature if $\mathbf{S} = (n+1)cF$ for some scalar function $c = c(x)$ on M^n.

By (3) and (4), we have the following lemma:

Lemma 2.2. *The S-curvature of an m-th root Finsler metric on an open subset $U \subset \mathbf{R}^n$ is a rational function in y.*

Proposition 2.3. *Let $F = \sqrt[m]{A}$, $A := a_{k_1\dots k_m}(x)y^{k_1}\dots y^{k_m}$ be a non-Riemannian m-th root metric on a manifold M^n ($n \geq 2$, $m \geq 3$). Suppose that F is of isotropic S-curvature, then $\mathbf{S} = 0$.*

Proof. By Lemma 2.2, we have that $\mathbf{S}$ is a rational function in y. Since, F is of isotropic S-curvature, i.e., $\mathbf{S} = (n+1)cF$. Then, F Finsler metric is not a rational function, that is, F Finsler metric is not Riemannian. Therefore, $c = 0$. ☐

For any $x \in M^n$ and $y \in T_xM^n \setminus \{0\}$, the Riemann curvature $\mathbf{R}_y = R^i{}_j\frac{\partial}{\partial x^i}\otimes dx^j$ is defined by

$$R^i{}_j = 2\frac{\partial G^i}{\partial x^j} - \frac{\partial^2 G^i}{\partial x^k\partial y^j}y^k + 2G^k\frac{\partial^2 G^i}{\partial y^k\partial y^j} - \frac{\partial G^i}{\partial y^k}\frac{\partial G^k}{\partial y^j}. \tag{5}$$

Ric is the Ricci curvature and is defined as the trace of the Riemann curvature, i.e., [4]

$$\mathbf{Ric} = R^m{}_m.$$

Ric is defined as a scalar function on $TM^n \setminus \{\ 0\ \}$. If there is a scalar function $c = c(x)$ on M^n, then F is called an Einstein metric, namely, $\mathbf{Ric} = (n-1)cF^2$.

Using (5) and Lemma 2.1, one can give the following lemma:

Lemma 2.4. *$R^i{}_j$ and $\mathbf{Ric} = R^m{}_m$ are rational functions in y.*

To prove the main theorems, we need the following proposition:

Proposition 2.5. *[3] Let F be a Finsler metric on M^n and $\mathbf{G} = \mathbf{G}_F$ be the induced spray of F. The followings are equivalent:*

(1) F is of $WIPRC$,

(2) for any volume form (dV, M^n), there is a scalar function g on M^n, that is,

$$\mathbf{PRic}_{(\mathbf{G},dV)} = (n-1)\big\{g_{0|0} - g_0^2 + \frac{2}{n+1}g_0\mathbf{S}\big\} + (n-1)[\frac{3\theta}{F} + c]F^2, \tag{6}$$

(3) for any volume form (dV, M^n), there is a scalar function g on M^n, that is,

$$\mathbf{Ric}_G = -(n-1)\big\{\Xi_{|0} + \Xi^2\big\} + (n-1)[\frac{3\theta}{F} + c]F^2, \tag{7}$$

where " $|$ " is the horizontal covariant derivative with respect to $\mathbf{G}$, $g_0 := g_{x^m}(x)y^m, \Xi := \frac{\mathbf{S}}{n+1} - g_0$ and $\mathbf{S} = \mathbf{S}_{(G,dV)}, \theta = \theta_iy^i$ is a $1-$form and $c = c(x)$ is scalar function on M^n.

3. m-th root metrics of isotropic projective Ricci curvature

Proof of Theorem 1.2: Let $F = \sqrt[m]{A}$, $A := a_{k_1\ldots k_m}(x)y^{k_1}\ldots y^{k_m}$, be an m-th root Finsler metric. $U \subset \mathbf{R}^n$ is an open subset. Then F is introduced by the following spary coefficients, [20]:

$$G^k = \frac{1}{2}(A_{0l} - A_{x^l})A^{kl}, \tag{8}$$

where $(A_{kl}) = \frac{\partial^2 A}{\partial y^k \partial y^l}$ is a tensor that is positive definite, and (A^{kl}) denotes the inverse tensor of (A_{kl}). G^k are expressed as a rational function of y. Suppose that F is of isotropic projective Ricci curvature, then by Proposition 2.5, we have a scalar function g on M^n such that (when $\theta = 0$)

$$\mathbf{PRic}_{(\mathbf{G},dV)} - (n-1)\big\{g_{0|0} - g_0^2 + \frac{2}{(n+1)}g_0\mathbf{S}\big\} = (n-1)cF^2, \tag{9}$$

where $c = c(x)$ is scalar function on M^n. By (1), Lemma 2.2 and Lemma 2.4, it is easy to see that the left side of the above equation is a rational function of y and F^2 is not a rational function, and F is not Riemannian. Then, we have $c = 0$. Hence,

$$\mathbf{PRic}_{(\mathbf{G},dV)} = (n-1)\big\{g_{0|0} - g_0^2 + \frac{2}{(n+1)}g_0\mathbf{S}\big\}, \tag{10}$$

namely, F is projectively Ricci-flat, [15, Theorem 3.1]. □

4. m-th root metrics of $WIPRC$

Proof of Theorem 1.3: Let $F = \sqrt[m]{A}$, $A := a_{k_1\ldots k_m}(x)y^{k_1}\ldots y^{k_m}$ be an m-th root Finsler metric. $U \subset \mathbf{R}^n$ is an open subset. Note that, F Finsler metric's spray coefficients are rational functions of y. Assume that F is of $WIPRC$. Then by Proposition 2.5, there is a scalar function g on M^n,

$$\mathbf{PRic}_{(\mathbf{G},dV)} - (n-1)\big\{g_{0|0} - g_0^2 + \frac{2}{(n+1)}g_0\mathbf{S}\big\} = (n-1)(3\theta F + cF^2), \tag{11}$$

where $\theta = \theta_k y^k$ is a 1-form and $c = c(x)$ is scalar function on M^n. It is easy to see that the left side of the above equation is a rational function of y. Thus, we consider the following cases:

Case i. If $c \neq 0$, then we obtain the following equation by (4.1):

$$F = \frac{-3(n-1)\theta \pm \sqrt{9(n-1)^2\theta^2 + 4(n-1)c\big\{\mathbf{PRic} - (n-1)[g_{0|0} - g_0^2 + \frac{2}{(n+1)}g_0\mathbf{S}]\big\}}}{2\ (n-1)\ c}.$$

On the other hand, $F = \sqrt[m]{a_{k_1\ldots k_m}(x)y^{k_1}\ldots y^{k_m}}$. Thus, we get

$$\begin{aligned} F &= \sqrt[m]{a_{k_1\ldots k_m}(x)y^{k_1}\ldots y^{k_m}} \\ &= \frac{-3(n-1)\theta \pm \sqrt{9(n-1)^2\theta^2 + 4(n-1)c\big\{\mathbf{PRic} - (n-1)[g_{0|0} - g_0^2 + \frac{2}{n+1}g_0\mathbf{S}]\big\}}}{2(n-1)c}. \end{aligned}$$

Since $m \geq 3$, then we conclude that $\theta = 0$ or F is a 1-form. If $\theta = 0$, then F is an m-th root metric of isotropic projective Ricci curvature. By Theorem 2.3, we conclude that F is projectively Ricci-flat. If F is a 1-form, then F is not positive definite. But it is meaningless.

Case ii. If $\theta \neq 0$, we obtain

$$3(n-1)\theta F = \mathbf{PRic}_{(\mathbf{G},dV)} - (n-1)\big\{g_{0|0} - g_0^2 + \frac{2}{(n+1)}g_0\mathbf{S}\big\}.$$

Thus we get the result that F is a 1-form and F is not positive definite. But, it is meaningless.

Considering the cases we have examined above, we can conclude that $c = \theta = 0$. Therefore, F is projectively Ricci-flat.

□

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