AUT Journal of Civil Engineering

AUT J. Civil Eng., 6(4) (2022) 435-446
DOI: 10.22060/ajce.2023.21782.5811

A Two-Stage Procedure for Optimum Design of Plate Girders Using a Meta-heuristic Algorithm

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ABSTRACT: The main aim in the design of welded plate girders is to minimize the weight of the beam while satisfying design requirements which turns the design procedure into a nonlinear and complex optimization problem. In this paper, a two-stage optimization procedure is introduced to find the best design of welded plate girders in terms of both safety and economy. The total weight of the girder is considered as the objective function where some predefined constraints are applied using penalty functions to restrict the solution space. In the first stage, the theoretical optimum values of girder dimensions are obtained using the Artificial Bee Colony (ABC) algorithm based on the optimum use of steel material while satisfying restrictions on the flexural and shear strength, permissible deflection, and proportioning limits. Since the plate dimension values obtained by the first stage may not be available in the market, the second stage of optimization is carried out to reach a safe and economical design by considering available plate dimension values. The critical ratio of the designing procedure is obtained equal to one, which is the ideal design. To demonstrate the effectiveness of the proposed method, two examples are considered at the end of this article. The results show that the dimension of the plate girder calculated by the proposed approach is more economical and practical than that obtained by the traditional trial and error techniques.

Review History:

Received: Sep. 17, 2022
Revised: May, 18, 2023
Accepted: Jun. 13, 2023
Available Online: Jun. 24, 2023

Keywords:

Plate girder
Optimization
ABC algorithm
Penalty function
Market availability

1- Introduction

Using plate girders as flexural members is an alternative solution to resist the bending moment and shear force in beams when the standard rolled sections cannot satisfy the minimum requirements of the design criteria. Although other solutions are available in such situations, the advantages of plate girders have made them a desirable choice for long-span beams supporting considerable loads. One of the biggest advantages of plate girders is the more efficient use of steel material because of the placement of flanges far away from the centroid, increasing the section's modulus and moment of inertia. Other benefits include its ability to be used in non-prismatic and composite sections and the possibility of fabricating the plate girder at desired length and depth, reducing the material waste during construction. Although the high shear and flexural strengths and considerable rigidity of the plate girders may be attractive from an economic point of view, their design to achieve the minimum weight possible is usually a challenging subject for designers.

One of the first studies on this matter was carried out [1] in 1966 to optimize the design of plate girders with constant depth. Another research [2] investigated the optimum weight of a continuous beam with a non-uniform cross-section subject to a uniformly distributed load using the plastic method. The optimization procedure was carried out through a direct search method in which some design parameters sequentially varied in small steps. The results indicated the largest permissible depth of the web, based on the design requirements, that could be used in the interior support with the maximum bending moment. A generalized geometric programming technique for the optimum design of prismatic multi-span plate girders is utilized for highway bridges[3]. The weight of the beam was considered as the objective function. The study was extended by researchers [4] where the thickness of the plates was rounded up to the practical values that were available in the market. A direct search procedure was utilized to find the minimum weight of non-uniform built-up stiffened plate girders [5]. He compared the results of the direct search procedure in finding the minimum weight of the plate girder with those of the generalized gradient-based design procedure. He also showed that the results of the first method were more economical than those of the latter. This study [6] propose an optimization procedure developed using buckling and frequency constraints in addition to the traditional strength ones. An analytical model to predict the strength of plate girders with minimum weight based on a parametric study is developed [7]. The experimental results were used to validate the model. The results indicated that the lightest weight and maximum strength were obtained when the maximum web slenderness was used. A variable

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plate thickness for the girder along its length is studied [8]. Although using a variable thickness can reduce the total weight of the girder, the additional fabrication and welding required eventually increase the total cost. Saleem et al., 2019 studied the effect of the slenderness of the web and flange as well as the beam's unbraced length To find the minimum weight of plate girders. The selected slenderness values covered a broad range from compact to highly slender flange and web of plate girders.

In recent years, numerical methods have been widely used for designing steel structures [10] and [11]. Among numerical approaches, metaheuristic algorithms have been attracted a great deal of attention. One of the first studies utilizing such algorithms to find the optimum design of plate girders was carried out by Fu et al. They [12]used the Genetic Algorithm (GA) to optimally design highway bridge beams and utilized the weight of the beams as well as the fabrication welding needed and the space between two adjacent girders along the bridge as their objective functions. Three kinds of constraints were utilized in their study and the results showed that the GA method is able to successfully find the minimum cost of plate girders. used a solver code developed based on the Newton Conjugate method to find the minimum weight of built-up beams and used the load factor method to define the constraints and penalty functions [13]. In another research, [14] developed a mathematical model to predict the minimum self-weight of plate girders. The weight of the beam was obtained based on the minimum use of steel material by optimizing the weight as the objective function. The procedure was carried out for different span lengths and applied loads using the Generalized Reduced Gradient (GRG) technique. A different meta-heuristic approach, the Harmony Search algorithm, is used to optimize the cross-section of plate girders [15]. They found a smaller cross-section for their plate girder with the same load-carrying capacity using their optimization technique. Among the metaheuristic algorithm, the Artificial Bee Colony (ABC) algorithm [16] with three setting parameters, which is more flexible than other most known approaches, is used in this study.

Published papers on the optimum design of plate girders are scarce. Despite many parameters and design constraints affecting the dimension of plate girders, the optimum design of the plate girder when all constraints are considered has not been comprehensively studied yet. This study focuses on this gap and provides a solution geared towards the optimum and safe designing of plate girders. In this paper, the optimum design of I-shaped plate girders with equal flanges is investigated under bending about the beam's major axis based on the AISC360-16 design code requirements. The proposed optimization method is a two-stage procedure. In the first stage, the ABC algorithm is utilized to find the optimum weight of the beam including its web, flanges, and stiffeners. The problem restrictions include the depth of the web, the width of the flanges, and the thickness of plates, however, in practical problems, the main limit is usually triggered by the depth of the web. In the second stage, the obtained theoretical optimum dimensions have to be rounded to the available standard thicknesses in the market or other thickness values defined by the user. The height and width of the plates are also rounded to integer values for practical purposes. It should be noted that such changes in the dimensions of the plate girder may yield an unsafe or uneconomical design.

To obtain the best solution after modifying plate dimensions based on the market availability, some partial changes are applied to the web height and the flange width to restore a critical penalty function ratio close to one. For this purpose, three approaches are presented for the second optimization stage. In the first approach, two of the variables, namely the web height and the flange width of the beam section, are optimized simultaneously using the ABC algorithm while the thickness of the plates is kept unchanged. In the second and third approaches, only the height of the web or the width of the flange is changed. Here again, the thickness of plates remains constant. The final solution is obtained more quickly in the two last approaches since only one single parameter is considered to be variable. The results show that using the proposed techniques, the optimum design of plate girders can be obtained.

2- Methodology

One of the main advantages of plate girders is their flexibility in choosing the web and flange dimensions. Selecting the dimensions of the plate girder section is important from an economical point of view to minimize the use of steel material. Meanwhile, the plate girder should be designed to satisfy the strength criteria, serviceability requirements, and proportioning limits. Plate girders are usually categorized as flexural members based on their length-to-depth ratio, and occasionally as shear members. Therefore, they are primarily designed for flexural requirements and other requirements are examined for the sufficiency of the section.

The proposed procedure to find the optimum design of plate girders works as follows: first, the flexural strength of the I-shaped section bent about its major axis is determined according to the governing design code *(ANSI/AISC 360-16)*. The main aim is finding the height and thickness of the web and also the width and thickness of the bottom and top flanges. For simplification, designers prefer to design symmetric sections, therefore four unknown parameters are considered, i.e., the thickness and height of the web, thickness and width of the top or bottom flange. Since the weight of the beam is unknown at this stage, it needs to be estimated empirically To determine the total load that must be carried by the beam. It should be noted that under-estimating or over-estimating the plate girder's weight may lead to unsafe or uneconomical design, respectively. Then, the shear and bearing stiffeners are designed. The variable parameters in the design of stiffeners are their dimensions and the clear space between them. After completing the design, the weight of the plate girder is calculated and compared with its initial weight at the beginning of the optimization process. The new weight is hopefully less than the previous one. If the difference is comparatively large, it means that the assumed beam section is far from optimum and the design procedure must be repeated again to achieve a more economical design. The iterations will continue until satisfactory results are obtained.

Although theoretically optimum, the dimensions of the plate girder section at this stage most probably will not be practical considering the available dimensions of plates in the market or the user's predefined dimensions. Therefore, the reported dimensions at the end of the first optimization stage should be modified to be compatible with the market or predefined values. For this purpose, the reported plate thicknesses are rounded up to the available dimensions. Moreover, the width (or height) of the flange (or web) is also rounded up to the practical values, which are in increments of *cm*. Obviously, the new beam with modified dimensions does not satisfy the optimum design requirements. To obtain an optimum design for the beam with its new practical dimensions, a second stage of optimization is required.

In the second stage, it is assumed that the flange width and web height are variables while two remaining parameters (the flange and web thicknesses) are kept constant, equal to the modified values of the previous stage, and a second optimization is performed. The section obtained through the second optimization stage satisfies the necessary requirements of the design code as well as minimizing the use of the steel material. Note that at the end of this stage, the critical penalty function ratio will be next to zero and the thickness values are corresponding to the predefined ones.

To determine the flexural strength of a symmetric I-shaped section bent about its major axis, the design code *AISC360-16* presents equations based on the slenderness of the section's web and flange. The web is classified into three categories: compact, non-compact, and slender. To determine the nominal flexural strength, three limit states are used: yielding (M_{n_1}), flexural-torsional buckling (M_{n_2}), and local buckling of the compression flange (M_{n_3}). The yielding and lateral-torsional buckling formulas are used to define the flexural strength of a section with compact flanges. The flexural strength of a section with a non-compact or slender flange is controlled by the local buckling of the compression flange in addition to the first two limit states (see Appendix A).

A steel section should be designed to have enough shear strength capacity as well as flexural strength. Without stiffeners, plate girders usually do not have adequate strength to resist shear forces, so utilizing stiffeners is an indispensable part of the girder design. The shear design of the girder yields to determining the clear space between the dimensions of the stiffeners.

At the end of the optimization procedure, the weight of the plate girder, W, can be defined by Eq. (1) as:

$$W = (2b_f t_f + ht_w)L\gamma_s + W_s \tag{1}$$

where b_f and t_f are the width and thickness of the flanges, h and t_w denote the height and thickness of the web, respectively. W_s is the weight of the stiffeners. L indicates the length of the beam and γ_s is the specific weight of the steel. As stated earlier, the main aim of the study is to minimize the weight of the plate girder. This goal is accompanied by satisfying the requirements of the design code including strength, serviceability, and proportioning limit according to *AISC360-16*. The serviceability limitations are applied to prevent structural damages under service loads and are presented as:

$$\delta_{D+L} < L/240, \delta_L < L/360,, \tag{2}$$

where δ_L and δ_{D+L} are the maximum deflections under the live load and the sum of the live and dead loads, respectively.

Restrictions on the web height are also applied according to the design code. It is evident that the flexural strength of a plate girder grows by increasing the flanges' distance from the neutral axis, hence the use of a slender web may be justifiable and beneficial to the design procedure. The maximum slenderness of the web (λ_w) is limited by the code based on the following formula to prevent the compression buckling of the web:

$$\lambda_w = \begin{cases} 12\sqrt{E/F_y}, & a/h \le 1.5 \\ 0.4\sqrt{E/F_y}, & a/h > 1.5 \\ 260, & \textit{Unstiffned girder} \end{cases} \tag{3}$$

where a denotes the clear distance between transverse shear stiffeners E and F_y denote the modulus of elasticity and yield stress of the steel material, respectively.

3- Review of the Artificial Bee Colony Optimization

In finding an economical design of plate girders satisfying all the code requirements, the major variables are the beam's web and flange dimension. To find the economical dimensions of the beam among the structurally permissible ones that are determined by the code, it is necessary to utilize optimization techniques. The simultaneous economical and structural considerations in the design of plate girders increase the complexity of the optimization problem. Traditional algorithms based on the gradient approach are too tedious for solving such multidimensional and complicated problems, however, stochastic search methods are a powerful tool to find the optimum solution.

The stochastic algorithms explore the solution space by a logical decision. One of these metaheuristic algorithms is the Artificial Bee Colony (ABC) algorithm [16] which is developed based on the behavior of honeybees in finding nectars. Unlike most other algorithms, this method does not require initial setting parameters such as cross-over or mutation rate and only uses three control parameters: the number of population (*NP*), the number of iterations without fitness value improvement (*limit*), and the maximum cycle number of iterations (*MCN*). This algorithm has been successfully utilized to solve optimization problems even though it has fewer control parameters in comparison with most of other algorithms. In the ABC algorithm, bees in the hive are categorized into three types: employed bees, onlookers, and scouts. Half of the population in the hive, called employed bees, fly into the space to find possible food sources (i.e. solutions) based on Eq. (4):

$$\beta_{ij} = L_j + Rand.(U_i - U_j) \tag{4}$$

where U_j and L_j are the upper and lower bounds of parameters j, respectively. $Rand$ is a random number in the range $[0,1]$. β_{ij} is the value of $j-th$ the parameter of the $i-th$ *D*-dimensional solution vector, where $i \in [1,\ldots,NP/2]$, $j \in [1,\ldots,D]$ and D is the number of parameters to be optimized. A neighbor food source near the last found food source, β_{ij}^{old}, is searched to find a new food source, β_{ij}^{new}, with a better quality using this formula:

$$\beta_{ij}^{new} = \beta_{ij}^{old} + \phi_{ij}.(\beta_{ij}^{old} - \beta_{kj}), k \neq j \tag{5}$$

The $k-th$ solution vector is chosen randomly which is different from the $j-th$ solution. ϕ is a random number in the interval $[-1,1]$. The probability value associated with the $i-th$ food source can be defined by:

$$P(\boldsymbol{\beta}_i) = 0.1 + [0.9 Fit(\boldsymbol{\beta}_i) / \max Fit(\boldsymbol{\beta}_i)] \tag{6}$$

where $Fit(\boldsymbol{\beta}_i)$ is the fitness value of the $i-th$ food source, calculated by:

$$Fit(\boldsymbol{\beta}_i) = 1/[1 + F(\boldsymbol{\beta}_i)] \tag{7}$$

Here, $F(\boldsymbol{\beta}_i)$ is the value of the objective function of the $i-th$ food source. Artificial onlooker bees select a food source based on the calculated probability such that the superlative ones have greater chances to be selected in the next step. If a food quality does not enhance during a predefined number of trials (the *limit* value), then the food source is abandoned and will be replaced by a new food source found by scout bees using Eq. (4). The optimization algorithm will continue to meet the predefined termination condition, i.e. the maximum cycle number (*MCN*).

4- Objective function and constraints

In this study, the objective function is defined as the weight of the plate girder with the variables of flange width, web height, and stiffener dimensions and their corresponding clear space. The plate girder is designed to satisfy the requirements of safety and serviceability as well as the proportioning limits. Finding a design solution with a minimum weight of the plate girder while it satisfies the limitations can be looked at as a constrained optimization problem. So, the constrained objective function can be expressed as:

$$F'(\boldsymbol{\beta}) = F(\boldsymbol{\beta})(1 + rP),\ F(\boldsymbol{\beta}) = W \tag{8}$$

Where $\boldsymbol{\beta}$ denotes a vector defining the problem parameters and r is a problem-dependent parameter that affects the convergence rate. Utilizing a large or small value for a parameter r may result in an infeasible or premature solution. Based on some trial and error, the value of r equal to 5 seems to be appropriate in this study. Changing r can affect the convergence speed of the optimization procedure, however, a full investigation on this topic is out of the scope of this study. P is the penalty function related to the problem constraints, proposed as:

$$P = \sum_{i=1}^{c} V_i(\boldsymbol{\beta}) \tag{9}$$

where is the number of the problem constraints and $V_i(\boldsymbol{\beta})$ is expressed as:

$$V_i(\boldsymbol{\beta}) = \begin{cases} g(\boldsymbol{\beta}), & g(\boldsymbol{\beta}) > 0 \\ 0, & g(\boldsymbol{\beta}) < 0 \end{cases} \tag{10}$$

Utilizing a penalty function forces the candidate solution to be selected in the next step of the iteration when the defined constraints are not satisfied. In this study, the strength and serviceability requirements and proportioning limits of dimensions are defined as constraints as follows:

The constraint $g_1(\boldsymbol{\beta})$ is related to the flexural strength of the plate girder, defined by:

$$g_1(\boldsymbol{\beta}) = \frac{M_u}{\phi_b M_n} - 1 \tag{11}$$

where M_u and M_n are factored in bending moment and nominal flexural strength, respectively. As stated before, M_n can be calculated using the equations proposed in Appendix A. In the above equation, ϕ_b denotes the reduction factor for the flexural strength. The constraint $g_2(\boldsymbol{\beta})$ is defined based on the shear strength as:

$$g_2(\boldsymbol{\beta}) = \frac{V_u}{\phi_v V_n} - 1 \tag{12}$$

where V_u and V_n are the factored shear force and nominal shear strength of the plate girder, respectively, and ϕ_v denote the corresponding reduction factor. The serviceability constraints are defined as the common deflection limitations for floors subject to the live load alone and dead plus live loads as follows:

$$g_3(\boldsymbol{\beta}) = \begin{cases} \dfrac{\delta_{D+L}}{L/240} - 1 \\ \dfrac{\delta_L}{L/360} - 1 \end{cases} \tag{13}$$

Finally, the proportioning constraints are presented according to Eq. (14) which is defined based on Eq. (3):

$$g_4(\boldsymbol{\beta}) = \begin{cases} \dfrac{\lambda_w}{12\sqrt{E/F_y}} - 1, & a/h \le 1.5 \\ \dfrac{\lambda_w}{0.4\,E/F_y} - 1, & a/h > 1.5 \\ \dfrac{\lambda_w}{260} - 1, & \textit{Unstiffned girder} \end{cases} \tag{14}$$

Based on the initial limitations defined by the user, the ABC algorithm is utilized to minimize the weight of the plate girder satisfying the minimum design requirements of the *AISC360-16* design code. This procedure, which is called the first stage optimization, continues until the minimum weight of the beam is obtained as the critical penalty function approaches zero. Note that the reported dimension values based on the code limitations in the first stage will not coincide with the available market dimensions, so the results have to be modified for practical purposes.

5- Modifying the obtained dimensions of the plate girder

To be used in real engineering applications, it is important to find the lightest beam having practical plate dimensions while satisfying the defined constraints and initial user limitations. The depth of the plate girder is often limited due to the headroom constraints and house service requirements. The flange width of the beam is often limited to that of the column in the beam-column connection. Moreover, the solution space of the optimization procedure is defined as a continuous space, while the available plate thicknesses in the market are discrete values such as: 3, 4, 5, 6, 8, 10, 12, 15, 20, 25, 30 *mm*, etc. In order to be compatible with the market dimensions, the reported thicknesses of the web and flanges have to be rounded up to the available values. The limitations on the flange width or web height of the plate girder are not as serious since the required dimension can be achieved by longitudinally cutting or welding the available plate sizes.

Such changes in the dimensions of the theoretically optimum plate girder yield to new penalty functions which most probably will not satisfy the minimum design requirements or will not have the optimum weight. Since any change in the plate thicknesses results in significant changes in the section properties and defined ratios, in the second phase of optimization, the thickness of plates is kept unchanged and only the width of the flanges and the height of the web are varied to obtain the optimum solution, resulting in a new optimization problem with two parameters. Three approaches are proposed for solving this problem. In the first approach, both parameters are allowed to change. ABC algorithm is used to solve the new optimization problem from which optimum values for flange width and web height will be obtained. In the remaining approaches, only one of the two variables is allowed to change. Either the flange width or web height is assumed as the stochastic parameter and optimized through a classical approach. This is due to the fact that the stochastic approach is time-consuming and reducing the variable parameters to only one can lead to obtaining faster results from the ABC algorithm.

During the optimization process, the optimum and safe solution is obtained $g_i(\boldsymbol{\beta}) = 0$ for some $i = 1,2,3,4$. Suppose that $g_1(\boldsymbol{\beta}) = 0$ is the maximum value of $\{g_1(\boldsymbol{\beta}) = 0, g_2(\boldsymbol{\beta}) = 0, g_3(\boldsymbol{\beta}) = 0, g_4(\boldsymbol{\beta}) = 0\}$, i.e. the flexural strength of the section has the critical condition. The demand over the capacity ratio of the beam with rounded dimensions can be presented as:

$$M_u / \phi_b M_n^* = g_1(\boldsymbol{\beta}) + 1 \tag{15}$$

where M_n^* is the nominal flexural strength of the plate girder after rounding dimensions.

First, it is assumed that the height of the web is variable. After changing the height of the web to achieve the optimum design, the stress ratio takes a new value equal to one as:

$$M_u / \phi_b M_n^{**} = 1 \tag{16}$$

where M_n^{**} is the new flexural strength of the plate girder after changing the height of the web. Using Equations (15) and (16), the relation between M_n^* and M_n^{**} is obtained as follows:

$$M_n^{**} = \left(g_1(\boldsymbol{\beta}) + 1\right) M_n^* \tag{17}$$

Based on the *AISC360-16* design code, the nominal flexural strength is a function of the modulus of the section (S_x), the shape factor (R_p), and the critical stress (F_{cr}), but the effect of the two last parameters is relatively small so they are ignored in this study. Thus, the nominal flexural strength of the beam can be represented based on its section modulus with good accuracy as:

$$S_x^{**} = \left(g_1(\boldsymbol{\beta}) + 1\right) S_x^* \tag{18}$$

Table 1. Variation of the web height and flange width

Maximum ratio	Web height variation	Flange width variation
$g_1(\beta)$	$g_1(\beta)S_x^*\left(\frac{d}{2}\right)^2 \Big/ \left\{\left(0.25t_wh_w^2+b_ft_fh_0\right)\frac{d}{2}-\frac{I_x}{2}\right\}$	$g_1(\beta)S_x^*\frac{d}{2}\Big/\left(t_f^3/6+t_fh_0^2/2\right)$
$g_2(\beta)$	$h_w/\{g_2(\beta)+1\}$	*ineffective*
$g_3(\beta)$	$g_3(\beta)I_x^*/\left(0.25t_wh_w^2+b_ft_fh_0\right)$	$g_3(\beta)I_x^*/\left(t_f^3/6+t_fh_0^2/2\right)$
$g_4(\beta)$	$h_w/\{g_4(\beta)+1\}$	*ineffective*

where S_x^* and S_x^{**} are the section modulus after rounding and the partial change in the web height dimension, respectively. Differentiating the section modulus with respect to the height of the web yields to:

$$dS_x = S_x^{**} - S_x^* = \left(\frac{d}{2}\right)^{-2}\left\{\left(0.25t_wh_w^2+b_ft_fh_0\right)\frac{d}{2}-\frac{I_x}{2}\right\}dh_w \quad (19)$$

Where h_0 is the distance between the center of the flange and the section centroid. I_x is the moment of inertia of the section about its strong axis. Based on Eq. (19), the modified web height, h_w^{new}, can be derived as:

$$h_w^{new} = h_w + g_1(\beta)S_x^*\left(\frac{d}{2}\right)^2 \Big/ \left\{\left(0.25t_wh_w^2+b_ft_fh_0\right)\frac{d}{2}-\frac{I_x}{2}\right\} \quad (20)$$

Similarly, the web heights for other critical constraints can be calculated which are shown in Table 1 (second column).

Now, assume that the width of the flange, b_f, is the variable. The new flange width can be obtained with a similar approach as:

$$b_f^{new} = b_f + g_1(\beta)S_x^*\frac{d}{2}\Big/\left(t_f^3/6+t_fh_0^2/2\right) \quad (21)$$

The variations of the flange width for all cases obtained with the same method are also shown in Table 1 (third column).

The values in Table 1 are based on the partial change of the web height or flange width. When the changes are relatively large, more iterations are needed to find the optimal solution. Based on what is mentioned above, the two-stage optimization procedure for the optimum design of plate girders can be summarized as a flowchart in Figure 1.

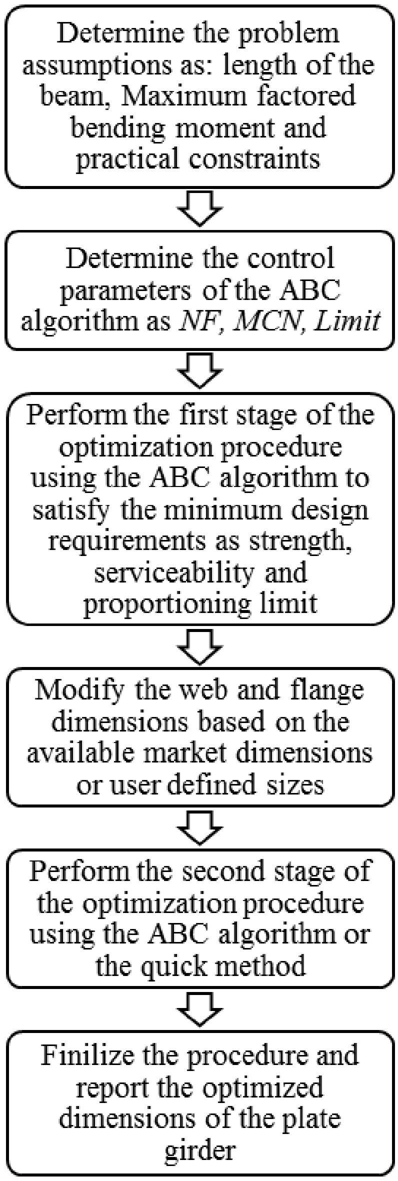

Fig. 1. The proposed procedure for the optimum design of plate girders

6- Numerical Examples

In this section, two numerical examples are examined to show the effectiveness of the proposed method in finding

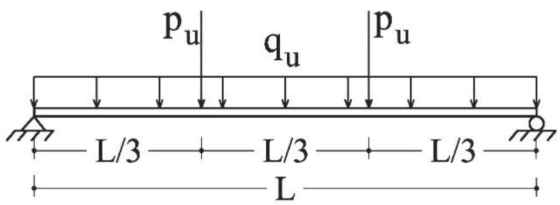

Fig. 2. Simply supported beam of the first example

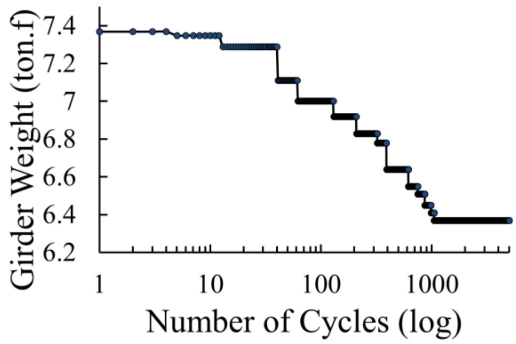

Fig. 3. Convergence of the first stage of the optimization procedure, Example 1

Table 2. Results of two-stage optimization procedure- Example 1

Section Properties & Defined Ratios	Theoretical (1st Stage)	Rounded (1st Stage)	Approach 1 (2nd Stage)	Approach 2 (2nd Stage)	Approach 3 (2nd Stage)
Total Weight (kg)	6369	7349	6697	6985	6727
Web Height (cm)	200.57	200.50	201.50	161.5	200.5
Web Thickness (cm)	0.59	0.60	0.60	0.60	0.60
Flange Width (cm)	70.62	70.50	62.00	70.50	62.50
Flange Thickness (cm)	2.06	2.50	2.50	2.50	2.50
Flexural Stress Ratio	1.000	0.829	1.000	1.000	0.993
Deflection Ratio (D+L)	0.172	0.145	0.160	0.226	0.161
Deflection Ratio (L)	0.053	0.044	0.049	0.069	0.049
Proportion Ratio	0.996	0.983	0.988	0.792	0.983

the optimum weight of a plate girder satisfying the design requirements of the *AISC360-16* design code. The first design example is a simply supported beam subjected to two-point loads and a uniformly distributed load. The other one is a two-span continuous beam carrying a distributed load.

Example 1: A simply supported beam as shown in Figure 2 with a span length of 18 m is supposed for this example which carries two third-point concentrated factored loads of 45 tons (dead and live loads of 17.5 tons and 15 tons, respectively) and a uniformly distributed load of 5 ton/m (dead and live loads of 1.5 ton/m and 2 ton/m, respectively). Lateral supports are provided only at the ends of the beam length and the steel material is *ST37*. The practical limitations of the problem for the thickness and height of the web are 1 cm and 300 cm, respectively. The thickness and width of the flange are also limited to 3 cm and 100 cm, respectively.

The maximum factored bending moment is calculated based on the load combinations for strength design (*ASCE7-16)* equal to 475.25 ton.m, acting at the middle of the span length. The lateral-torsional buckling modification factor, C_b , is equal to one. To optimize the girder weight, the optimization procedure presented in the last section is utilized with ABC algorithm control parameters of NP=2000, limit=500, and MCN=5000 for the first stage of the optimization. Figure 3 shows the weight of the plate girder during the optimization cycles. The convergence approximately occurs after 1000 trials.

The theoretical optimum dimensions of the beam section are presented in the second column of Table 2. As shown in this table, the optimization algorithm found the best solution where the flexural stress limit is equal to one. The deflection and proportioning limits are less than one which means these criteria do not control the dimensions of the optimum section.

The total weight of the plate girder is calculated to be 6369 kg, but the obtained thicknesses through the optimization procedure are not compatible with market availability, so the thicknesses have to be modified based on the available thicknesses in addition to specific limitations of this problem. The rounded dimensions and corresponding values are shown in the third column of Table 2. As expected, the weight of the beam increases (from 6369 kg to 7349 kg) while the critical ratio decreases (from 1.000 to 0.829,) which does not satisfy the optimum design.

Next, the second optimization procedure is performed which can be done in three different approaches as described previously. In the first approach, both flange width and web height are considered variables. Utilizing the setting parameters of the ABC algorithm as NP=2000, limit=500, and MCN=5000, the results of approach 1 are obtained which are shown in column 4 in Table 2. With fewer variables, the convergence of the second optimization occurs faster than the previous optimization stage having four variables. In this approach, the total weight of the beam is obtained as 6697 kg, which is greater than the theoretical optimum, but less than the one corresponding to the rounded values. Meanwhile, the critical ratio, which is related to flexural stress, is equal to one. This means that the proposed method has been able to efficiently find the minimum weight of the beam. In the second approach, the web height is selected as a variable, while the flange width obtained after rounding values is kept constant. As it is evident from Table 2, the constraint g_1 which is related to the flexural strength controls the optimization procedure, so the web height of the beam can be calculated using the closed form equation: $g_1(\beta)S_x^*\left(\frac{d}{2}\right)^2 \Big/ \left\{\left(0.25t_w h_w^{\ 2}+b_f t_f h_0\right)\frac{d}{2}-\frac{I_x}{2}\right\}$. The results of this approach are shown in column 5 of Table 2. The total weight is 6985 kg (about 4.5 % heavier than the first approach). A similar procedure can be utilized to find the optimum weight of the beam using the third approach where the web height is kept equal to the rounded value while the flange width is modified according to the closed-form equations in Table 1. The optimum weight of the beam resulting from this approach is 6727 kg (6^{th} column of Table 2) which is 0.4 % greater than the first approach. Similar to other approaches, the maximum ratio in this approach is also equal to one.

Considering the results of the three mentioned approaches in the second stage, the best solution is obtained using the first approach. As expected, when two parameters are optimized simultaneously, a better solution can be obtained compared to using just one variable parameter. Although the first approach demonstrated a good performance in finding the minimum weight of the plate girder, the two remaining ones may be preferred because of their convenience in usage and higher convergence speed.

To validate the calculations presented in Table 2, the obtained optimum dimensions of the plate girder are used to calculate the flexural and shear strengths, deflection, and proportioning limit and finally the total weight of the beam to be compared with the existing data.

The dimensions of the plate girder obtained from the first stage of the optimization procedure are b_f=70.62 cm, t_f=2.06 cm, h_w=200.57 cm, and t_w=0.59 cm. Hence, the slenderness ratios of the doubly symmetric I-shaped member bent about its major axis are calculated as: λ_w=339.4>λ_{rw}=166.2, λ_{pf}=11.08<λ_f=17.14<λ_{rf}=19.58. The web and flanges are classified as slender and noncompact, respectively. The flexural strength of the section can be calculated based on the three mentioned limit states. The nominal flexural strength is 543.89 tons. m governed by the lateral-torsional buckling limit state. To satisfy the shear strength requirement, 20 pairs of transverse stiffeners are needed between the support and the point load at both ends of the beam. In addition, the bearing stiffeners are required at supports and under-point loads. The weight of the plate girder is accounted for as a uniform distributed load of the weight added to the external load, therefore, the maximum factored bending moment, M_u, is obtained as 489.68 ton.m. Based on the service load combinations in *ASCE7-16*, the deflection ratio under the dead plus live loads and the live load alone is 0.172 and 0.053, respectively.

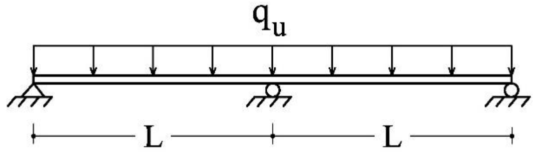

Fig. 4. Two spans continuous beam loaded with distributed load

Example 2: A continuous beam with two equal spans of 12 m is considered in this example subjected to uniformly distributed dead and live loads equal to 2.5 ton/m and 1.25 ton/m, respectively (Figure 4.) The steel material is ST37 and the compression flange is assumed to be fully braced along the beam length. The maximum factored bending moment is calculated at the middle support as:

$$q_u = \max\left(1.4q_D, 1.2q_D + 1.6q_L\right), \quad M_u = q_u L^2/8 = 90\,ton.m \tag{22}$$

The practical limitations on the web and flange dimensions are defined as follows: the maximum height and thickness of the web are 80 cm and 2 cm, respectively; The maximum width and thickness of the flanges are assumed to be 40 cm and 2 cm, respectively; The minimum practical thickness of the web and flanges is 0.5 cm based on the market availability. The setting parameters of the optimization algorithm are assumed to be the same as in the previous example.

The convergence of the first stage of optimization happens after about 1100 iterations ([Figure 5) and its results are shown in the second column of Table 3. In this example,

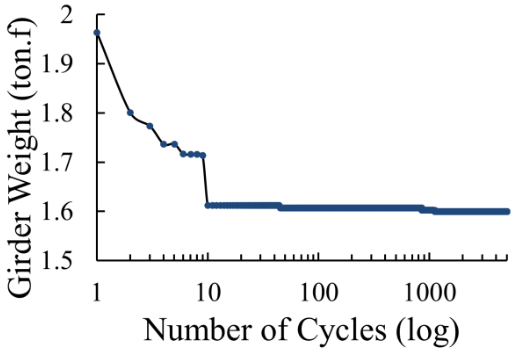

Fig. 4. Convergence of the first stage of the optimization procedure, Example 2

Table 3. Results of two-stage optimization procedure- Example 2

Section Properties & Defined Ratios	Theoretical (1st Stage)	Rounded (1st Stage)	Approach 1 (2nd Stage)	Approach 2-1 (2nd Stage)	Approach 2-2 (2nd Stage)
Total Weight (kg)	1606	1699	1658	1658	1658
Web Height (cm)	80.00	80.00	80.00	80.00	80.00
Web Thickness (cm)	0.51	0.60	0.60	0.60	0.60
Flange Width (cm)	28.67	29.00	28.00	28.00	28.00
Flange Thickness (cm)	1.99	2.00	2.00	2.00	2.00
Flexural Stress Ratio	0.834	0.780	0.803	0.803	0.803
Deflection Ratio (D+L)	1.000	0.969	1.000	1.000	1.000
Deflection Ratio (L)	0.322	0.311	0.321	0.321	0.321
Proportion Ratio	0.459	0.392	0.393	0.393	0.393

the governing criterion is the deflection ratio of the beam under the dead plus live loads. For the optimum beam, the deflection ratio is equal to one while other ratios are less than one. This means that the deflection limit controls the design dimension. In this stage, the total weight of the beam is 1606 kg. Again, although the obtained dimension values in the first step are theoretically the optimum ones, the calculated thickness values have to be rounded up because of the market availability of plate thicknesses or due to practical limitations. The new rounded dimensions are shown in the column 3 of Table 3. As expected, the strength of the section increases while the critical ratio decreases. Now, the weight of the beam is increased to 1699 kg and the deflection ratio is 0.829, which does not satisfy the expected target. In the optimum condition, this ratio should be equal to one, so the second stage of the optimization is performed. In this example, the results of the three proposed approaches are the same. The first approach is carried out considering two variables, flange width and web height, and two constant parameters, flange and web thicknesses. Utilizing the ABC algorithm, the optimum solution is obtained which is shown in column 4 of Table 3. The total weight in the second stage of optimization is 1658 kg, which is about 2.5 % less than the weight of the beam with rounded dimensions. Here, the second and third approaches yield similar results with the first approach, but with less computational effort and all proposed approaches can satisfy the serviceability and economic criteria as shown in columns 4 to 6 of Table 3.

7- Conclusion

In this study, a two-stage procedure for finding the optimum design of doubly symmetric I-shaped plate girders was proposed. The design procedure was defined as an optimization problem with four parameters: width and thickness of flanges and height and thickness of the web. In the first stage, the theoretical dimensions of the plate girder were calculated using the ABC algorithm. The critical ratio of the designing procedure was obtained equal to one, which is the ideal design. To account for the discrete values for plate thickness, the obtained thicknesses of the web and flanges were rounded up to the available market dimensions. Since this modification affects the economic aspect of the design, a second stage of optimization was utilized to find the new optimum dimensions using three different approaches. In this stage, the web height and flange width were defined as variables while the obtained thicknesses were considered as the given data. In the first approach using the ABC algorithm, the web height and flange width were optimized. In the other two approaches, just one of the two parameters was considered as a variable to achieve faster convergence. Finally, two examples were studied to show the effectiveness of the proposed technique. The results show that the dimension of the plate girder calculated by the proposed approach is more economical and practical than that obtained by the traditional trial and error techniques. The proposed procedure can be used for any beam design problem with different design codes and practical limitations.

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Appendix A.

Nominal flexural strength of I-shaped members bent about their major axis shall be calculated according to the limit states of yielding (M_{n_1}), lateral-torsional buckling (M_{n_2}) and compression flange local buckling (M_{n_3}).

The general parameters used in equation are defined as:

$\lambda_f = b_f/(2t_f)$, $\lambda_{pf} = 0.38\sqrt{E/F_y}$, $\lambda_{rf} = 0.95\sqrt{k_c E/F_L}$, $0.35 \le k_c = 4/\lambda_w^{0.5} \le 0.76$, $S_x = S_{xt} = S_{xc} \Rightarrow F_L = 0.7F_y$

$\lambda_w = h_w/t_w, \lambda_{pw} = 3.76\sqrt{E/F_y}, \lambda_{rw} = 5.70\sqrt{E/F_y}, r_{ts}^2 = \sqrt{I_y C_w}/S_x, r_t = b_f/\sqrt{12+2a_w}$ $a_w = A_w/A_f$

A-1: The section with compact webs and compact flanges. $M_n = (M_{n_1}, M_{n_2})$

Yielding:

$M_{n_1} = M_p = ZF_y$

Lateral-torsional buckling:

$L_b \le L_p \Rightarrow M_{n_2} = M_{n_1}$

$L_p < L_b \le L_r \Rightarrow M_{n_2} = C_b\left[M_p - \left(M_p - 0.7S_xF_y\right)\left(L_b - L_p\right)/\left(L_r - L_p\right)\right] \le M_p$

$L_b > L_r \Rightarrow M_{n_2} = S_xF_{cr} \le M_p$

The used parameters in above equations are defined as:

$L_p = 1.76r_y\sqrt{E/F_y}$ $L_r = \left\{1.95r_{ts}\,E/\left(0.7F_y\right)\right\}\sqrt{J/\left(S_xh_0\right) + \sqrt{\left[J/\left(S_xh_0\right)\right]^2 + 6.76\left[0.7F_y/E\right]^2}}$

$F_{cr} = \left\{C_b\,\pi^2E/\left(L_b/r_{ts}\right)^2\right\}\sqrt{1 + 0.078J\left(L_b/r_{ts}\right)^2/\left(S_xh_0\right)}$

A-2: The section with compact webs and noncompact or slender flanges. $M_n = \min(M_{n_1}, M_{n_2}, M_{n_3})$

The values of M_{n_1} and M_{n_2} are the same as calculated in section A-1.

Compression flange local buckling:

$\lambda_f \le \lambda_{pf} \Rightarrow M_{n_3} = M_{n_1}$

$\lambda_{pf} < \lambda_f \le \lambda_{rf} \Rightarrow M_{n_3} = M_p - \left(M_p - 0.7S_xF_y\right)\left(\lambda_f - \lambda_{pf}\right)/\left(\lambda_{rf} - \lambda_{pf}\right) \le M_p$

$\lambda_f > \lambda_{rf} \Rightarrow M_{n_3} = S_xF_{cr} \le M_p, F_{cr} = 0.9Ek_c/\lambda_f^2$

B-1: The section which have noncompact webs and compact flanges: $M_n = \min(M_{n_1}, M_{n_2})$

Yielding:

$M_{n_1} = R_pM_y = R_pS_xF_y$

Lateral-torsional buckling:

$L_b \le L_p \Rightarrow M_{n_2} = M_{n_1}$

$L_p < L_b \le L_r \Rightarrow M_{n_2} = C_b\left[R_pM_y - \left(R_pM_y - 0.7S_xF_y\right)\left(L_b - L_p\right)/\left(L_r - L_p\right)\right] \le R_pM_y$

$L_b > L_r \Rightarrow M_{n_2} = S_xF_{cr} \le R_pM_y$, $F_{cr} = \left\{C_b\,\pi^2E/\left(L_b/r_t\right)^2\right\}\sqrt{1 + 0.078J\left(L_b/r_{ts}\right)^2/\left(S_xh_0\right)}$

B-2: The section which have noncompact webs and noncompact or slender flanges:

$M_n = \min(M_{n_1}, M_{n_2}, M_{n_3})$ The values of M_{n_1} and M_{n_2} are the same as calculated in section B-1.

Compression flange local buckling:

$\lambda_f \le \lambda_{pf} \Rightarrow M_{n_3} = M_{n_1}$

$\lambda_{pf} < \lambda_f \le \lambda_{rf} \Rightarrow M_{n_3} = R_p M_y - \left(R_p M_y - 0.7 S_x F_y\right)\left(\lambda_f - \lambda_{pf}\right)/\left(\lambda_{rf} - \lambda_{pf}\right) \le R_p M_y$

$\lambda_f > \lambda_{rf} \Rightarrow M_{n_3} = S_x F_{cr} \le M_p, F_{cr} = 0.9 E k_c / \lambda_f^{\,2}$

The used parameters in above equations are defined as:

$R_p = M_p/M_y - \left(M_p/M_y - 1\right)\left(\lambda_w - \lambda_{pw}\right)/\left(\lambda_{rw} - \lambda_{pw}\right) \le M_p/M_y$, $M_{n_3} = \min\left(1.6 S_x F_y, Z_x F_y\right)$

$L_r = \left\{1.95 r_t\, E/\left(0.7 F_y\right)\right\}\sqrt{J/\left(S_x h_0\right) + \sqrt{\left[J/\left(S_x h_0\right)\right]^2 + 6.76\left[0.7 F_y/E\right]^2}}$, $L_p = 1.1 r_t \sqrt{E/F_y}$

C-1: The section with slender webs and compact flanges: $M_n = \min(M_{n_1}, M_{n_2})$

Yielding:

$M_{n_1} = R_{pg} M_y = R_{pg} S_x F_y$

Lateral-torsional buckling:

$L_b \le L_p \Rightarrow M_{n_2} = M_{n_1}$

$L_p < L_b \le L_r \Rightarrow M_{n_2} = R_{pg} S_x F_{cr}$ $F_{cr} = C_b\left[F_y - 0.3 F_y\left(L_b - L_p\right)/\left(L_r - L_p\right)\right] \le F_y$

$L_b > L_r \Rightarrow M_{n_2} = R_{pg} S_x F_{cr}$, $F_{cr} = C_b \pi^2 E/\left(L_b/r_t\right)^2 \le F_y$

C-2: The section with slender webs and noncompact or slender flanges: $M_n = \min(M_{n_1}, M_{n_2}, M_{n_3})$

The values of M_{n_1} and M_{n_2} are the same as calculated in section C-1.

Compression flange local buckling:

$\lambda_f \le \lambda_{pf} \Rightarrow M_{n_3} = M_{n_1}$

$\lambda_{pf} < \lambda_f \le \lambda_{rf} \Rightarrow M_{n_3} = R_{pg} S_x F_{cr}$, $F_{cr} = F_y - 0.3 F_y\left(\lambda_f - \lambda_{pf}\right)/\left(\lambda_{rf} - \lambda_{pf}\right) \le F_y$

$\lambda_f > \lambda_{rf} \Rightarrow M_{n_3} = R_{pg} S_x F_{cr}$, $F_{cr} = 0.9 E k_c / \lambda_f^{\,2} \le F_y$

The used parameters in above equations are defined as:

$R_{pg} = 1 - a_w\left(\lambda_w - \lambda_{rw}\right)\left(1200 + 300 a_w\right) \le 1$, $a_w \le 10$, $L_p = 1.1 r_t \sqrt{E/F_y}$, $L_r = \pi r_t \sqrt{E/\left(0.7 F_y\right)}$

HOW TO CITE THIS ARTICLE

S. A. Banimahd, M. A. Rahemi, A Two-Stage Procedure for Optimum Design of Plate Girders Using a Meta-heuristic Algorithm, AUT J. Civil Eng., 6(4) (2022) 435-446.

DOI: 10.22060/mej.2019.15465.6128