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Extended power series solution for Perkins-Kern-Nordgren model of hydraulic fracture

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ABSTRACT: The extended Power Series (XPS) method can be extremely useful for solving nonlinear equations with regular and irregular singular points. The extended power series is considered times a logarithm or times a fractional power of ζ , etc.). This research shows it is simple to solve approximately the Perkins-Kern-Nordgren (PKN) model of hydraulic fracture. To illustrate the effectiveness and convenience of the XPS method, we consider the two cases of dimensionless PKN equation containing the M-scaling and $\tilde{M}$-scaling. The results compared with available analytical results verified excellent agreements.

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1- Introduction

The classical Perkins-Kern-Nordgren (PKN) model is useful in hydraulic fracture [1, 2].This model is widely used in the oil and gas industry to assist in the design of hydraulic fracturing treatments that are engineered to enhance the recovery of hydrocarbons from underground reservoirs[3-5]. The PKN model is applicable for conditions when the vertical hydraulic fracture remains confined within the horizontal permeable or impermeable layer, on account of sufficiently high contrast in horizontal stress between the reservoir layer and the adjacent impermeable layers.

The original formulation of the PKN model is credited to Perkins and Kern (1961)[1]. Nordgren (1972)[2] modified the model and provided numerical solutions of the non-linear differential governing equation, PKN equation, as well as closed-form solutions.

Some ambiguity in the boundary condition at the moving front in Nordgren's article is discussed by Kemp (1990), who provided the correct asymptotic form of the fracture width in the tip region in the case of an impermeable layer. Also, approximated solutions are proposed by Economides et al. (2007)[6]. In the last decades, although, many efforts have been made by researchers to resolve and modify the PKN model, there also remains some ambiguity. The formulation of the PKN fracture model gives rise to a challenging mathematical problem, in contrast to the simplicity of the physical problem that it aims to capture[7]. Indeed, this problem is governed by a strongly non-linear differential equation and is characterized by the presence of a moving boundary and by a degeneracy of the governing equation near the moving tip due to the vanishing of the fracture aperture[7]. Because of the complexity of the underlying mathematical model and the presence of a singular point in the PKN equation, it is relatively difficult to be solved analytically, nevertheless, there are many analytical methods for singular boundary value problems [7-14].

In this paper, we present the Extended Power Series method which has a simple producer to obtain an analytical solution of the PKN equation. The PKN equation has a singular point that is not analytic, but this equation can still be solved by the proposed method. Indeed, the following simple theorem permits an extension of the power series method. The new method is called the XPS method.

2- Formulation

We consider the propagation of a PKN fracture of length $2\ell(t)$, emanating in a linear elastic rock characterized by Young's modulus E , and Poisson's ratio υ, (see Fig. 1). A Newtonian fluid with a viscosity of μ is injected via a constant volumetric flow, Q_0, from a straight-line source located at the center of the fracture, which is induced to internal fluid pressure $P_f(x,t)$ on the surfaces cracks. A far-

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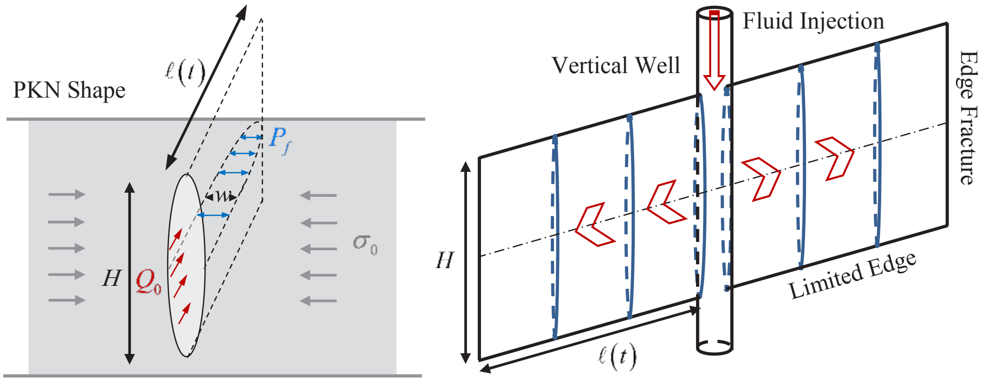

Fig. 1. Sketch of a PKN fracture.

field confining stress σ_0 acts perpendicular to the crack. The Linear Elastic Fracture Mechanics (LEFM) theory is adopted to obtain the net pressure in the fracture $P(x,t)=P_f(x,t)-\sigma_0$, the average aperture $w(x,t)$, as well as the fracture half-length $\ell(t)$, where t is the time and x is the position along the crack.

The analysis assumptions and boundary conditions were considered according to Ref. [7].

2- 1- Governing equations

The governing equations of the model consist of a propagation criterion, an elasticity equation, and the lubrication equation. These equations can be expressed in terms of the half of the crack, $0 \le x \le \ell$, the crack opening, the average fluid velocity, and the fluid net pressure by accounting for the problem symmetry as follows:

2- 1- 1- Fluid mass:

The fluid flow in the fracture is governed by continuity of mass and momentum. Global fluid continuity requires the injected fluid volume to be equal to the fracture volume; hence:

$$\frac{\partial q}{\partial x}+\frac{\partial w}{\partial t}=0, \tag{1}$$

Here, $q(x,t)$ is the average flow rate per unit height of the fracture, H.

2- 1- 2- Fluid momentum:

The unidirectional laminar flow of a Newtonian viscous fluid inside the elliptical cross-section crack is described by the momentum balance equation [2].

$$q=-\frac{w^3}{\mu'}\frac{\partial p}{\partial x}, \qquad \mu'=\pi^2\mu, \tag{2}$$

2- 1- 3- Elasticity equation

The average crack opening is related to the local net pressure on the crack[2].

$$w=\frac{H}{E'}p, \qquad E'=\frac{2}{\pi}\frac{E}{1-\upsilon^2}, \tag{3}$$

Where, E and υ are Young's modulus and Poisson's ratio of the rock, respectively.

The three governing Eqs. 1, 2, and 3 can be combined into one non-linear partial differential equation for the aperture $w(x,t)$,

$$\frac{\partial w}{\partial t}-\frac{E'}{4\mu' H}\frac{\partial^2 w^4}{\partial x^2}=0, \tag{4}$$

2- 1- 4- Initial/boundary conditions

The boundary conditions at the crack tip $x=\ell(t)$ *and* the fluid injection point $x=0$, are

$$w(x,t)=q=0 \text{ at } x=\ell(t) \tag{5}$$

$$q=\int_0^\ell w\,dx=\frac{Q_0 t}{2H}, \text{ at } x=0 \tag{6}$$

The initial conditions are formally given by

$$\ell = w = q = 0, \text{ at } t = 0. \tag{7}$$

2- 2- Dimensionless Formulation

To facilitate the solution of the set of equations (1- 4), let us introduce the following scaled and normalized quantities: the coordinate $\xi = x/\ell(t) \in [0,1]$, the crack opening, and the crack half-length, as follows:

$$w(x,t) = W(t)\Omega(\xi,t), \qquad \ell(t) = L(t)\gamma, \tag{8}$$
$$\bar{\Omega}(\xi,t) = \Omega(\xi,t)\gamma^{-\frac{2}{3}},$$

It is noted that the "bar sign" corresponds to the normalized quantities.

Using the above transformations, Eqs (4- 7) can be re-written in an alternative form as follows:

$$\mathrm{G}_v\left[\left(\frac{\dot{W}t}{W}+\frac{2}{3}\frac{\dot{\gamma}t}{\gamma}\right)\bar{\Omega}+\dot{\bar{\Omega}}t-\xi\left(\frac{\dot{\gamma}t}{\gamma}+\frac{\dot{L}t}{L}\right)\frac{\partial\bar{\Omega}}{\partial\xi}\right]=\frac{1}{\mathrm{G}_\mu}\frac{1}{4}\frac{\partial^2\bar{\Omega}^4}{\partial\xi^2}. \tag{9}$$
$$\frac{1}{2}=\gamma^{\frac{5}{3}}\mathrm{G}_v\int_0^1\bar{\Omega}d\xi, \qquad \left.\frac{\partial\bar{\Omega}^4}{\partial\xi}\right|_{\xi=0}=-2\gamma^{-\frac{5}{3}}\mathrm{G}_\mu,$$

Also, two dimensionless parameters G_i , and G_v in Eqs. (9) and (8) are expressed as:

$$\mathrm{G}_v=\frac{LWH}{Q_0t}, \mathrm{G}_\mu=\frac{\mu'}{E'}\frac{LQ_0}{W^4}, . \tag{10}$$

For more expressions of these dimensionless parameters in the two scales, identified as the viscosity scaling $\mathrm{G}_\mathrm{i}=1$, and the storage scaling $\mathrm{G}_v=1$, refer to[7]. In this way, the parameters for the M-scaling, Eq (10), are obtained by taking $\mathrm{G}_\mathrm{i}=1$ and $\mathrm{G}_v=1$. So, the quantities W, and L take the explicit forms:

$$W=\left(\frac{\mu'}{E'}\frac{Q_0^2}{H}\right)^{\frac{1}{5}}t^{\frac{1}{5}}, \quad L=\left(\frac{E'}{\mu'}\frac{Q_0^3}{H^4}\right)^{\frac{1}{5}}t^{\frac{4}{5}}, \tag{11}$$

Differentiating Eqs. (11) with respect to t, and substitution into Eq. 9, yields:

$$\left[\left(\frac{1}{5}+\frac{2}{3}\frac{\dot{\gamma}t}{\gamma}\right)\bar{\Omega}+\dot{\bar{\Omega}}t-\xi\left(\frac{\dot{\gamma}t}{\gamma}+\frac{4}{5}\right)\frac{\partial\bar{\Omega}}{\partial\xi}\right]=\frac{1}{4}\frac{\partial^2\bar{\Omega}^4}{\partial\xi^2}, \tag{12}$$
$$\frac{1}{2}=\gamma^{\frac{5}{3}}\int_0^1\bar{\Omega}d\xi, \qquad \left.\frac{\partial\bar{\Omega}^4}{\partial\xi}\right|_{\xi=0}=-2\gamma^{-\frac{5}{3}},$$

3- Extended power series solution to PKN equation

The PKN equation/model was solved by different analytical and numerical methods [7, 15-22] and its solution was applied in the references[11, 23]. In this work, the XPS method is improved and suggested to find a simple approach to the problem of PKN equation/model hydraulic fracture propagating in brittle rock. To illustrate the effectiveness and convenience of the XPS method, in this section, we consider the two cases of dimensionless PKN equation containing the M-scaling and -scaling [2, 7, 15].

3- 1- PKN equation in the M-scaling:

Formulation of the mathematical model in the M-scaling has already been presented in Eq 12. we seek

to compute the M-solution on the small-time solution, then Eq. 12 can be reduced to:

$$\bar{\Omega}_\mathrm{m}+A\,\xi\frac{\partial\bar{\Omega}_\mathrm{m}}{\partial\xi}+B\,\frac{\partial^2\bar{\Omega}_\mathrm{m}^4}{\partial\xi^2}=0, \quad A=-4, \quad B=-\frac{5}{4}, \tag{13}$$
$$\frac{1}{2}=\gamma_m^{\frac{5}{3}}\int_0^1\bar{\Omega}_\mathrm{m}d\xi. \quad \text{B.C.} \left.\frac{\partial\bar{\Omega}^4}{\partial\xi}\right|_{\xi=0}=$$
$$-2\gamma_m^{-\frac{5}{3}}, \quad \bar{\Omega}_\mathrm{m}(\xi_0)=0, \quad \xi_0=1$$

As before mentioned, Eq. (13) is determined using the scaling technique and combining three equations follow as: global continuity equation, momentum equation, and constitutive law for the hydraulic fracturing model. To show the simple solution process, we re-write Eq. (13) in the forms:

$$\begin{aligned}&\bar{\Omega}_m(\xi)+A\,\xi\bar{\Omega}_m{}'(\xi)\\&+12B\,\bar{\Omega}_m(\xi)^2\,\bar{\Omega}_m{}'(\xi)^2\\&+4B\,\bar{\Omega}_m(\xi)^3\,\bar{\Omega}_m{}''(\xi)=0\end{aligned} \tag{14}$$

The idea is to assume that the unknown function $\bar{\Omega}_m(\xi)$ can be expanded into a power series:

$$\bar{\Omega}_m(\xi)=(\xi-\xi_0)^r\,h(\xi-\xi_0), \tag{15}$$

Where,

$$h(\xi-\xi_0)=\sum_{n=0}^{\infty}a_n(\xi-\xi_0)^n,\qquad a_n=\frac{h^{(n)}(0)}{n!},\quad \xi_0=1 \tag{16}$$

According to the XPS method, fractional power, r, in Eq. (15) can be obtained by assuming $h(\xi-\xi_0)=1$. So, we have: $\bar{\Omega}_m(\xi)=(\xi-1)^r$, $\bar{\Omega}_m'(\xi)=r(\xi-1)^{r-1}$ and $\bar{\Omega}_m''(\xi)=(r^2-r)(\xi-1)^{r-2}$. Substitution of these equations into Eq. (2) yields:

$$\begin{aligned}&(\xi-1)^r+Ar\xi(\xi-1)^{r-1}\\&+B[12\left((\xi-1)^{2r}\right)\left(r^2(\xi-1)^{2r-2}\right)\\&+\left(4(\xi-1)^{3r}\right)r(r-1)(\xi-1)^{r-2}]=0\end{aligned} \tag{17}$$

By simplifying the above equation, we have:

$$\begin{aligned}&(\xi-1)^r+Ar\xi(\xi-1)^{r-1}\\&+4Br(4r-1)(\xi-1)^{4r-2}=0\end{aligned} \tag{18}$$

After further simplification, we obtain:

$$\begin{aligned}&(\xi-1)^{r-1}((\xi-1)+Ar\xi\\&+4Br(4r-1)(\xi-1)^{3r-1})=0\end{aligned} \tag{19}$$

$$\begin{aligned}&\left((\xi-1)+Ar\xi+4Br(4r-1)(\xi-1)^{3r-1}\right)=0\\&\to\quad r=\frac{1}{3}\end{aligned} \tag{20}$$

After substitution of $r=1/3$ into Eq. (15) and Eq. (14) yields:

$$\begin{aligned}&3\left(-3+(3+A)\xi\right)h+4Bh^4\\&+9A(-1+\xi)\xi h'+108B(-1+\xi)^2h^2h'^2\\&+12B(-1+\xi)h^3\left(8h'+3(-1+\xi)h''\right)=0\end{aligned} \tag{21}$$

For brevity, $h(\xi-1)$ have been replaced with h and Eq. (21) upon the substitution $\xi=\xi_0=1$ becomes:

$$\begin{aligned}&3Ah(0)+4Bh^4(0)=0\\&h(0)=-\frac{(-1)^{2/3}3^{1/3}A^{1/3}}{2^{2/3}B^{1/3}}\end{aligned} \tag{22}$$

Differentiating Eq. (21) with respect to ξ, we obtain:

$$\begin{aligned}&3(-3-3A+3\xi+7A\xi)h'\\&+3h\left(3+A+72B(-1+\xi)^2h'^3\right)\\&+9A(-1+\xi)\xi h''\\&+36B(-1+\xi)h^2h'\left(14h'+9(-1+\xi)h''\right)\\&+4Bh^3\left(28h'+42(-1+\xi)h''+9(-1+\xi)^2h^{(3)}\right)=0\end{aligned} \tag{23}$$

Setting $\xi=\xi_0=1$ in Eq. (23) results in:

$$\begin{aligned}&12Ah'(0)+3(3+A)h(0)\\&+112Bh^3(0)h'(0)=0\\&h'(0)=-\frac{(-1/6)^{2/3}(3+A)}{8A^{2/3}B^{1/3}}\end{aligned} \tag{24}$$

By a similar operation, we can obtain

$$\begin{aligned}&h''(0)=\frac{(-1)^{2/3}(3+A)(27+A)}{672\times6^{2/3}A^{5/3}B^{1/3}},\\&h^{(3)}(0)=\frac{(-1)^{2/3}(3+A)(-1341-102A+43A^2)}{40320\times6^{2/3}A^{8/3}B^{1/3}}\\&h^{(4)}(0)=-(-1)^{2/3}(3+A)\\&\times\frac{(-1)^{2/3}(3+A)(-2428245-314793A+173889A^2+21149A^3)}{55036800\times6^{2/3}A^{11/3}B^{1/3}}\end{aligned} \tag{25}$$

The extended power series solution is

$$\begin{aligned}\bar{\Omega}_m(\xi)&=(\xi-1)^{\frac{1}{3}}\sum_{n=0}^{\infty}\frac{h^{(n)}(0)}{n!}(\xi-1)^n\\&=\left\{-\frac{(-1)^{2/3}(3+A)}{13208832006^{2/3}A^{11/3}B^{1/3}}\right.\\&\times\left(-2428245-314793A+173889A^2+21149A^3\right)(\xi-1)^4\\&\quad+\frac{(-1)^{2/3}(3+A)(-1341-102A+43A^2)}{241920\times6^{2/3}A^{8/3}B^{1/3}}(\xi-1)^3\\&\quad+\frac{(-1)^{2/3}(3+A)(27+A)}{1344\times6^{2/3}A^{5/3}B^{1/3}}(\xi-1)^2\\&\quad\left.-\frac{\left(-\frac{1}{6}\right)^{2/3}(3+A)(-1+\xi)}{8A^{2/3}B^{1/3}}-\frac{(-1)^{2/3}3^{1/3}A^{1/3}}{2^{2/3}B^{1/3}}+-\ldots\right\}(\xi-1)^{\frac{1}{3}}\end{aligned} \tag{26}$$

Substitution of $A=-4$, and $B=-5/4$ into Eq. (26)

yields the final expression:

$$\begin{aligned}&\bar{\Omega}_m(\xi)=\left(\frac{3}{5}\right)^{1/3}2^{2/3}(1-\xi)^{1/3}\\&\times\left\{1+\frac{1}{96}(\xi-1)+\frac{23}{64512}(\xi-1)^2+\right.\\&\left.\frac{7}{1327104}(\xi-1)^3-\frac{51923}{202887659520}(\xi-1)^4+-\ldots\right\}\\&\gamma_m=0.660422,\ \Omega_m(0)=1.00514\end{aligned}\tag{27}$$

This result agrees with the solution obtained by Kemp (1990) [15] and Y. Kovalyshen and E. Detournay (2010)[7].

3- 2- PKN equation in the -scaling:

To determine PKN equations in the -scaling, Eq 1-7 should be rewritten considering the leak-off effect. For the sake of brevity, the details are not presented here. Dimensionless PKN equations in the -scaling has already been obtained by Y. Kovalyshen · E. Detournay [7, 15] and can be expressed as follows:

$$\begin{aligned}&\frac{1}{\sqrt{1-\xi^2}}-\frac{1}{4}\frac{\partial^2\bar{\Omega}_{\tilde{m}}^4}{\partial\xi^2}=0,\quad \text{B.C.}\\&\bar{\Omega}_{\tilde{m}}(\xi_0)=0,\quad \xi_0=1\\&\frac{1}{2}=2\gamma_{\tilde{m}}\int_0^1\sqrt{1-\xi^2}\,d\xi.\qquad \left.\frac{\partial\bar{\Omega}_{\tilde{m}}^4}{\partial\xi}\right|_{\xi=0}=-2\gamma_{\tilde{m}}^{-1}\end{aligned}\tag{28}$$

we extend Eq. (28) in the below form:

$$\begin{aligned}&\frac{1}{\sqrt{1-\xi^2}}-3\bar{\Omega}_{\tilde{m}}(\xi)^2\,\bar{\Omega}_{\tilde{m}}'(\xi)^2\\&-\bar{\Omega}_{\tilde{m}}(\xi)^3\,\bar{\Omega}_{\tilde{m}}''(\xi)=0\end{aligned}\tag{29}$$

The idea is to assume that the unknown function $\bar{\Omega}_{\tilde{m}}(\xi)$ can be expanded into a power series:

$$\bar{\Omega}_{\tilde{m}}(\xi)=\sum_{n=0}^{\infty}\frac{h^{(n)}(0)}{n!}(\xi-\xi_0)^{n+r},\qquad \xi_0=1\tag{30}$$

The fractional power, r, in Eq. (30) can be determined by the same process as in the previous section and is equal to 3/8. After substitution of $r=3/8$ into Eq. (30) and Eq. (29) yields:

$$\begin{aligned}&1-\sqrt{1+\xi}\,h^2\left(\frac{3}{16}h^2+3(-1+\xi)^2h'^2\right.\\&\left.+(-1+\xi)h\left(3h'+(-1+\xi)h''\right)\right)=0\end{aligned}\tag{31}$$

Substituting $\xi=1$ into Eq. (31) gives:

$$1-\frac{3h^4(0)}{8\sqrt{2}}=0\ \Rightarrow\ h(0)=\frac{2^{7/8}}{3^{1/4}}\tag{32}$$

Differentiating Eq. (31) with respect to ξ, we obtain:

$$\begin{aligned}&-\frac{1}{32\sqrt{1+\xi}}h[3h^3+192(-1+\xi)^2(1+\xi)h'^3\\&+48(-1+\xi)hh'\left((9+11\xi)h'+6(-1+\xi^2)h''\right)\\&+8h^2(3(3+7\xi)h'+2(-1+\xi)\\&\times\left((9+11\xi)h''+2(-1+\xi^2)h^{(3)}\right))=0\end{aligned}\tag{33}$$

Setting $\xi=\xi_0=1$ in Eq. (33) results in:

$$\begin{aligned}&\frac{3h^4(0)}{32\sqrt{2}}+\frac{15\sqrt{2}}{4}h^3(0)h'(0)=0\\&h'(0)=-\frac{h(0)}{80}\end{aligned}\tag{34}$$

By a similar operation, we can obtain

$$\begin{aligned}&h''(0)=\frac{159}{44800}h(0),\\&h^{(3)}(0)=-\frac{8527}{3584000}h(0),\\&h^{(4)}(0)=\frac{57774251}{22077440000}h(0),\end{aligned}\tag{35}$$

The extended power series solution is

$$\begin{aligned}&\bar{\Omega}_m(\xi)=(1-\xi)^{\frac{3}{8}}\sum_{n=1}^{\infty}\frac{h^{(n)}(0)}{n!}(\xi-1)^n\\&=(1-\frac{(-1+\xi)}{80}+\frac{159(-1+\xi)^2}{89600}\\&-\frac{8527(-1+\xi)^3}{21504000}+\frac{57774251(-1+\xi)^4}{529858560000})\frac{(2)^{7/8}}{3^{1/4}}(1-\xi)^{3/8}\\&\Omega_{\tilde{m}}(0)=0.797886,\quad \gamma_{\tilde{m}}=0.31831\end{aligned}\tag{36}$$

This result agrees with the exact solution, Eq.(37), obtained by Y. Kovalyshen and E. Detournay (2010)[7].

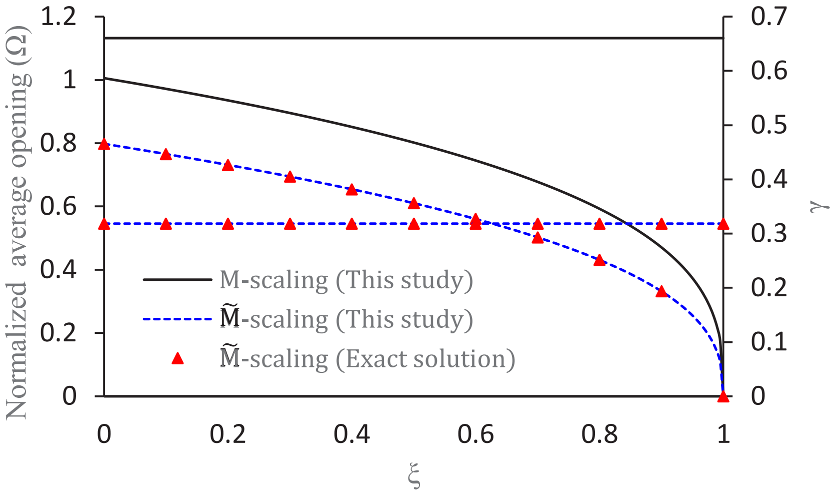

Fig. 2. Comparison of dimensionless fracture opening, Ω, and crack half-length, γ, from XPS method with exact solution.

$$\bar{\Omega}_m(\xi) = (2\pi)^{\frac{1}{4}} \left(\frac{2}{\pi} \xi \arcsin \xi + \frac{2}{\pi} \sqrt{1-\xi^2} - \xi \right)^{\frac{1}{4}}$$
$$\Omega_{\tilde{m}}(0) = \sqrt{\frac{2}{\pi}} = 0.797885, \quad \gamma_{\tilde{m}} = \frac{1}{\pi} = 0.31831 \tag{37}$$

The evolution of the crack length, γ, and normalized average opening, Ω, in various scaling are shown in Fig. 2. Comparison of the results obtained with the values in the exact solution indicates the acceptable ability of the XPS method in the solving the PKN model.

4- Conclusion

This research suggests a simple solution method, the extended power series method, to solve the PKN equation, the idea can be extended to all differential equations with moving boundary conditions. Compared with other analytical methods, the XPS method is straightforward with a simple solution process and accurate results. The most advantage of the XPS method is that the redundant terms will not be produced, and the series may converge to the exact solution.

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