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Adaptive Output-Feedback Control for Switched Nonlinear Systems with Unknown Control Directions

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ABSTRACT: This article deals with the design of an adaptive controller for switched non-strict feedback nonlinear systems. In the studied system, the switching signal is arbitrary, the states are not measurable, and the signs of the control gain functions that describe the control directions are completely unknown. First, the unknown nonlinear functions in the switched system are approximated using the universal approximation theorem. Then, the unmeasured states are estimated using the linear state observer, and the controller is designed through an adaptive back-stepping design procedure. Due to the appropriate change of coordinates, 1) neither fuzzy nor radial basis function is used in the design of the controller, 2) only one adaptation law is designed to estimate the unknown parameters in the switched non-strict feedback nonlinear system, and 3) there is no Nussbaum function in the proposed adaptive controller so, the large control signal in the initial stages and the consequent damage to the actuators can be prevented. These features can lead to the simplicity of controller design and the reduction of computational burden. Therefore, the proposed method can be used for practical systems. The stability of the closed-loop system is proved using Lyapunov stability theory. It is shown that, in addition to the semi-globally uniformly ultimately boundedness of all closed loop signals, the tracking error converges to a small neighborhood around zero. In the end, the efficiency of the proposed control method is confirmed through the simulation results of an example.

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1- Introduction

Many technological systems can be modeled by switched systems due to their multi-mode property. Hence, the motivation for studying the switched systems derived from the fact that some practical systems such as networked control systems, power systems, and chemical processes are inherently multimodal in the sense that several dynamical systems are needed to explain their behavior. A switched system as a typical hybrid system consists of a limited number of subsystems, with a switching law arranging the switching between them [1, 2]. The classification of switched systems leads to two general categories: event-driven and time-driven switched systems. The switching strategy separates these two categories [3]. In the event-driven system, the switching law is affected by system states satisfying particular predefined conditions, while the time-driven systems describe the system that is switched according to a time sequence.

In many nonlinear systems, detailed information on nonlinear functions describing the structure of the system is not available. Hence, the use of an adaptive control strategy for systems with unknown nonlinear functions has attracted much attention [4]. Adaptive back-stepping design procedure is one of the most useable adaptive control strategies for nonlinear systems. However, in this method, the problem of "explosion of complexity" caused by the repeated differentiations of the virtual control signal is not negligible. Dynamic surface control (DSC) method has been introduced to eliminate this problem [5].

Recently, much attention has been paid to controller design for non-strict feedback nonlinear systems, because many practical systems, such as ball and beam systems, hyperchaotic oscillation circuit systems, and motor-driven single-link manipulator models, are in non-strict feedback form [6]. It should be noted that nonlinear functions in strict feedback nonlinear systems, are functions of partial state vector. But, in the non-strict feedback form, nonlinear functions contain all the state variables. If the control method applied to strict feedback systems is also used for non-strict feedback systems, the algebraic loop problem can appear [7]. It means that the virtual control signal can be the function of the whole state vector.

Many adaptive back-stepping control methods have investigated various non-strict feedback nonlinear systems [8, 9]. In [8], a class of non-strict feedback nonlinear systems with input delay has been studied, and by applying the adaptive back-stepping method, a state-feedback controller has been developed. An adaptive neural tracking control problem has been studied in [9] for non-strict feedback

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nonlinear systems with full-state constraints. On the other hand, practical systems often encounter switching behavior. So, many researchers have been interested in designing adaptive controllers for switched non-strict feedback nonlinear systems [10, 11]. An adaptive controller for switched non-strict feedback nonlinear systems with time-varying full state constraints and unmeasurable states has been designed in [10]. For a class of switched non-strict feedback nonlinear systems under arbitrary switching, an adaptive fuzzy output-feedback control method has been studied in [11].

For the unknown nonlinear functions that are not linearly parameterized, employing the universal approximation theorem [12] can be useful to approximate these functions. So, the adaptive back-stepping design procedure incorporated with fuzzy logic systems (FLSs) or neural networks (NNs) has received more attention [13-15]. In [14], an adaptive neural-networked control method is investigated to solve the problem of adaptive control for a class of switched pure-feedback nonlinear systems under arbitrary switching. The adaptive fuzzy state feedback and observer-based output feedback control design techniques have been considered in [15] for single-input-single-output (SISO) non-strict feedback nonlinear systems. For the switched nonlinear systems with unknown external disturbance and performance requirements, a composite adaptive fuzzy finite-time controller has been investigated in [16]. Also, with the help of this control method, the tracking error converges to a preassigned area with a finite time. An adaptive finite-time tracking control issue has been studied in [17] for switched nonlinear systems with time-varying delay under average dwell time switching. An event-triggered fixed-time adaptive fuzzy controller has been designed for a class of switched non-strict feedback nonlinear systems in [18], where, different event-triggered adaptive fuzzy controllers for different subsystems have been constructed.

When the adaptive control procedure is utilized, several adaptive parameters will be produced that should be estimated. To tackle this issue, the approximation-based adaptive control methods were proposed in [19-21], in which the number of adaptation parameters was greatly reduced. On the other hand, the global Lipschitz conditions as a severe constraint for nonlinear systems are required in most systems to obtain global stability. However, this condition has been relaxed in [22, 23].

For many applied nonlinear systems, a priori knowledge about the sign of the gain multiplying the control input or the sign of the control gain function is unknown. The motion direction of the system is determined through this sign, which is called control direction. For nonlinear systems with unknown control directions, designing the adaptive controllers can be challenging. To deal with unknown control directions, Nussbaum functions have been widely employed [24-26]. In [26], to deal with the problem of state-feedback regulation for a class of switched nonlinear systems with unknown control directions, a control method has been studied using the Nussbaum functions. An adaptive fault-tolerant control scheme is considered in [27] for a class of switched nonlinear systems in which the control directions are unknown. An adaptive control method using the Nussbaum function has been introduced in [28] for a class of nonlinear systems with unknown control directions, in which a command filter is used to resolve the explosion of complexity problem. Based on the fuzzy back-stepping procedure and using the Nussbaum gain technique, an adaptive dynamic surface controller has been proposed in [29] to settle the issue of unknown control direction for a class of non-strict feedback systems. In [30], an adaptive control procedure has been developed to guarantee global exponential stability of parameter-varying nonlinear systems with unknown control direction. To overcome difficulties associated with unknown control directions in nonlinear interconnected high-order systems, an adaptive fixed-time control method utilizing the Nussbaum gain functions has been proposed in [31].

However, the mentioned literature deals with the problem of unknown control directions, there is not enough attention to eliminate the very large control signals at the primary stage. It is necessary to state that many actuators are affected by these large control signals and are hurt. In this paper, an adaptive control design procedure is proposed for a class of switched non-strict feedback nonlinear systems, which prevents the increase of the control signals in the initial stages. The main innovations are stated here.

Compared to the previous results, the method presented in this paper does not involve the use of any Nussbaum function. Hence, it prevents the increase of the control signal in the early stages and the consequent damage to the actuators. Therefore, the proposed method can be used for practical systems.

Despite the existence of unknown nonlinear functions, unmeasured states of the system, and unknown control directions, in the proposed method, only one adaptive law needs to be designed. Also, there isn't any fuzzy or radial basis function in the controller design. These significant features are achieved due to the use of the proposed design procedure and can lead to the reduction of computational burden and simplicity of controller in design and construction.

This paper is categorized as follows. The plant model and some preliminaries are introduced in section II. The next section presents control design and stability analysis. In section IV, simulation results are given. Section V includes the conclusions of this paper.

2- Problem Statement

A class of switched non-strict feedback nonlinear systems is modeled as

$$\begin{cases} \dot{x}_i = f_i^{\sigma(t)}(x) + g_i^{\sigma(t)}(x)\, x_{i+1}, & i = 1,\ldots,n-1 \\ \dot{x}_n = f_n^{\sigma(t)}(x) + g_n^{\sigma(t)}(x)\, u, & \\ y = x_1, & \end{cases} \tag{1}$$

where $x = [x_1, \ldots, x_n]^T \in R^n$ is the state vector of the system, $u \in R$ and $y \in R$ represents the system input and the system output, respectively. The functions $f_i^{\sigma(t)}(x)$ and $g_i^{\sigma(t)}(x), i=1,\ldots,n$ are unknown smooth nonlinear functions. Also, the signs of the control gain functions $g_i^{\sigma(t)}(x), i=1,\ldots,n$ are unknown. The signal $\sigma(t):[0,+\infty)\rightarrow\mathbb{M}=\{1,2,\ldots,s\}$ specifies the switching signal, which is a piecewise right continuous function. When s th subsystem is active, the switching signal is $\sigma(t)=s\in\mathbb{M}$.

Control objective. The purpose of the control followed in this article is to design an adaptive output-feedback control method for switched nonlinear system (1) with unknown control directions such that all the signals in the closed-loop system are semi-globally uniformly ultimately bounded, and the output of the system y tracks the reference signal y_r well.

Definition 1 [32]. The solution of system (1) is semi-globally uniformly ultimately bounded (SGUUB), if for any compact subset of R^n such as X and all $x(t_0)=x_0\in\mathrm{X}$, there exist a $\mu>0$ and a number $N(\mu,x_0)$ such that $x(t)<\mu$ for all $t>t_0+T$.

To achieve the goal of control, it is necessary to consider two assumptions as follows.

Assumption 1. The reference signal y_r and its first and second-time derivatives are continuous and bounded.

Assumption 2. The nonlinear functions $g_i^{\sigma(t)}(x), 1\le i\le n$ and their signs are unknown, and there exist two constants $0<g_{i,m}<g_{i,M}$ such that $g_{i,m}\le\left|g_i^{\sigma(t)}(x)\right|\le g_{i,M}, \forall x\in R^i$.

Lemma 1 [33]. Consider that $f(x)$ is a continuous function defined on a compact set Ω. Then, for a defined level of accuracy $\varepsilon>0$, there exists a fuzzy logic system $\theta^T\varphi(x)$ such that

$$\sup_{x\in\Omega}\left|f(x)-\theta^T\varphi(x)\right|\le\varepsilon.$$

3- Main results

For the th active subsystem of the switched nonlinear system (1), it can be rewritten

$$\begin{cases}\dot{x}_i=F_i^s(x)+x_{i+1}, & i=1,\ldots,n-1\\ \dot{x}_n=F_n^s(x)+g_n^s(x)\,u, & \\ y=x_1, & \end{cases}\tag{2}$$

where $F_i^s(x)=f_i^s(x)-x_{i+1}+g_i^s(x)x_{i+1}, i=1,\ldots,n-1$ and $F_n^s(x)=f_n^s(x)$. In accordance with the universal approximation theorem and lemma 1, the nonlinear functions $F_i^s(x), i=1,\ldots,n$ can be approximated by a FLS as $\hat{F}_i^s(x|\eta_i)=\eta_i^T\varphi_i(x), i=1,\ldots,n$ The ideal weight vector η_i^* is defined as

$$\eta_i^*=\arg\min_{\eta_i\in U}\left[\sup_{x\in\Omega}\left|\hat{F}_i^s(x|\eta_i)-F_i^s(x)\right|\right],\tag{3}$$

where U and Ω denote the compact regions for η_i and x, respectively. The approximation error is determined as

$$\varepsilon_i(x)=F_i^s(x)-\hat{F}_i^s(x|\eta_i^*),\ \left|\varepsilon_i(x)\right|\le\varepsilon_i^*\tag{4}$$

where ε_i^* is a positive design parameter.

In this paper, all the system states except $x_1(t)$ are unmeasured. Therefore, the following linear state observer is created to have a feedback control strategy. One can get

$$\begin{cases}\dot{\hat{x}}_i=\hat{x}_{i+1}+k_i(x_1-\hat{x}_1), & i=1,\ldots,n-1\\ \dot{\hat{x}}_n=k_n(x_1-\hat{x}_1)+u, & \\ \hat{y}=\hat{x}_1, & \end{cases}\tag{5}$$

where $k_i, i=1,\ldots,n$ are positive design parameters. Define vector $e=x-\hat{x}$ as the observer error vector, where vector $\hat{x}=[\hat{x}_1,\ldots,\hat{x}_n]^T$ is the estimation of vector $x=[x_1,\ldots,x_n]^T$. Combining (2) and (5), we have

$$\dot{e}=Ae+F^s(x)+G^s(x)u,\tag{6}$$

where

$$A=\begin{bmatrix}-k_1 & 1 & 0 & 0 & \cdots & 0\\ -k_2 & 0 & 1 & 0 & \cdots & 0\\ \vdots & \vdots & & \ddots & & \vdots\\ -k_{n-1} & 0 & \cdots & & 0 & 1\\ -k_n & 0 & \cdots & & 0 & 0\end{bmatrix},$$

$$F^s(x)=\begin{bmatrix}F_1^s(x)\\ \vdots\\ F_n^s(x)\end{bmatrix}, G^s(x)=\begin{bmatrix}0\\ \vdots\\ g_n^s(x)-1\end{bmatrix}$$

Vector $K=[k_1,\dots,k_n]^T$ is selected such as matrix A is Hurwitz. Then, for any symmetric positive definite matrix Q, there is a symmetric positive definite matrix $P=P^T>0$ such that .

$$A^T P + PA = -Q$$

Remark 1. In the following, it becomes clear that the global Lipschitz condition as a restrictive assumption can be removed due to the use of the linear state observer (5).

The Lyapunov function for the error system (6) is selected as

$$V_o = e^T Pe \quad . \tag{7}$$

The time derivative of V_o is obtained through (6) and (7) as

$$\dot{V}_0 = -e^T Qe + 2e^T P M(x), \tag{8}$$

where $M(x)=F^s(x)+G^s(x)u$. Using the completion of the square, leads to

$$2e^T PM(x) \le e^2 + P^2 \sum_{i=1}^{n} \left(M_i(x)\right)^2 . \tag{9}$$

The Unknown term $P^2\sum_{i=1}^{n}(M_i(x))^2$ can be approximated through a FLS as $\hat{\Xi}(x|\omega)=\omega^T\psi(x)$, where the ideal parameter vector is as follows

$$\omega^* = \arg\min_{\omega\in\bar{U}_s}\left[\sup_{x\in\Omega}\left|\hat{\Xi}(x|\omega) - P^2\sum_{i=1}^{n}\left(M_i(x)\right)^2\right|\right]. \tag{10}$$

Also, $\bar{U}_s$ is the compact set for ω. The approximation error is

$$\begin{aligned}\epsilon(x) = {} & P^2\sum_{i=1}^{n}\left(M_i(x)\right)^2 - \\ & \hat{\Xi}(x|\omega^*),\ |\epsilon(x)| \le \epsilon^*\end{aligned} \tag{11}$$

where ϵ^* is a positive parameter. Through (8)-(11), it is obtained

$$\dot{V}_0 \le -\left(\lambda_m(Q)-1\right)e^2 + \omega^{*T}\psi(x) + \epsilon^* , \tag{12}$$

In which, $\lambda_m(Q)$ is the indication of the minimal eigenvalue of the matrix Q. In accordance to Young's inequality [34], one gets

$$\omega^{*T}\psi(x) \le \frac{\vartheta^*}{4l} + l , \tag{13}$$

where l is a positive constant, $0<\psi(x)^T\psi(x)\le 1$, and $\vartheta^* = \omega^{*2}$.

In the following, the adaptive back-stepping design procedure is presented, in which the change of coordinates as

$$z_1 = y - y_r , \tag{14}$$

$$\begin{cases} z_i = \hat{x}_i - \beta_i^d , \\ \lambda_i = \beta_i^d - \beta_{i-1}, \end{cases} \quad i = 2,\dots,n, \tag{15}$$

is employed, where z_1 and z_i determine the tracking error and the surface error, respectively. Also, λ_i denotes the error between β_i^d and β_{i-1}. The definition of the variables β_i^d and β_{i-1} will be presented later.

3- 1- Adaptive back-stepping design procedure

The back-stepping design scheme is extended as follows.

Step1. Consider the Lyapunov function V_1 as

$$V_1 = V_o + \frac{1}{2}z_1^2 + \frac{1}{2\gamma_1}\tilde{\theta}_1^2 , \tag{16}$$

where γ_1 is a positive design parameter, $\tilde{\theta}_1 = \theta_1^* - \hat{\theta}_1$ and $\hat{\theta}_1$ is the estimate of $\theta_1^* = \eta_1^{*2}$. Considering (2)-(5), (14) and (15), the time derivative of the Lyapunov function V_1 is

$$\begin{aligned}\dot{V}_1 = {} & \dot{V}_o + z_1(\varepsilon_1(x) + \eta_1^{*T}\varphi_1(x) + \\ & z_2 + \lambda_2 + \beta_1 + e_2 - \dot{y}_r) - \frac{1}{\gamma_1}\tilde{\theta}_1\dot{\hat{\theta}}_1\end{aligned} \tag{17}$$

In accordance to the Young's inequality, one can obtain

$$z_1\left(\varepsilon_1(x)+\eta_1^{*T}\varphi_1(x)+z_2+\lambda_2+e_2\right)\leq 2z_1^2+\frac{(\varepsilon_1^*)^2}{2}+\alpha_1 z_1^2\theta_1^*+\frac{1}{4\alpha_1}+\frac{z_2^2}{2}+\frac{\lambda_2^2}{2}+\frac{\|e\|^2}{2}, \tag{18}$$

where $0<\varphi_1(x)^T\varphi_1(x)\leq 1$, and α_1 is a positive design parameter. From (12), (13), (17) and (18), we have

$$\begin{aligned}\dot{V}_1 \leq & -(\lambda_m(Q)-\frac{3}{2})\|e\|^2+\frac{\vartheta^*}{4l}+l+\\ & \epsilon^*+z_1(2z_1+\alpha_1 z_1\hat{\theta}_1+\beta_1-\dot{y}_r)\\ & +\alpha_1 z_1^2\tilde{\theta}_1+\frac{(\varepsilon_1^*)^2}{2}+\frac{1}{4\alpha_1}+\frac{z_2^2}{2}+\frac{\lambda_2^2}{2}-\frac{1}{\gamma_1}\tilde{\theta}_1\dot{\hat{\theta}}_1\end{aligned} \tag{19}$$

Now, the first virtual control signal β_1 and the only adaptive law $\hat{\theta}_1$ are selected as follows

$$\beta_1=-c_1z_1-2z_1-\alpha_1 z_1\hat{\theta}_1+\dot{y}_r\ , \tag{20}$$

$$\dot{\hat{\theta}}_1=\gamma_1\alpha_1 z_1^2-\delta_1\hat{\theta}_1, \tag{21}$$

where c_1 and δ_1 are positive design constants. Substituting (20) and (21) into (19) yields

$$\begin{aligned}\dot{V}_1 \leq & -(\lambda_m(Q)-\frac{3}{2})\|e\|^2-c_1z_1^2+\\ & \frac{z_2^2}{2}+\frac{\lambda_2^2}{2}+\frac{\delta_1}{\gamma_1}\tilde{\theta}_1\hat{\theta}_1+\pi_1\ ,\end{aligned} \tag{22}$$

where

$$\pi_1=\frac{\vartheta^*}{4l}+l+\epsilon^*+\frac{(\varepsilon_1^*)^2}{2}+\frac{1}{4\alpha_1}$$

.

step $i\,(2\leq i\leq n-1)$. To eliminate the repeated differentiation β_{i-1}, a low-pass filter with positive time constant τ_i is employed as follows

$$\tau_i\dot{\beta}_i^d+\beta_i^d=\beta_{i-1}\ ,\qquad \beta_i^d(0)=\beta_{i-1}(0). \tag{23}$$

Let β_{i-1} pass through this filter to obtain a new state variable β_i^d. From (15) and (23), the time derivative of λ_i can be obtained as

$$\dot{\lambda}_i=\dot{\beta}_i^d-\dot{\beta}_{i-1}=-\frac{\lambda_i}{\tau_i}+N_i(.)\ , \tag{24}$$

where $N_i(.)$ is a continuous function equals to $-\dot{\beta}_{i-1}$. Now, the Lyapunov function V_i is candidate as

$$V_i=V_{i-1}+\frac{1}{2}z_i^2+\frac{1}{2}\lambda_i^2\ . \tag{25}$$

Considering (5) and (15), the time derivative of the variable z_i can be rewritten as

$$\begin{aligned}\dot{z}_i&=\dot{\hat{x}}_i-\dot{\beta}_i^d=z_{i+1}+\\ &\lambda_{i+1}+\beta_i+k_i y-k_i\hat{x}_1-\dot{\beta}_i^d\ .\end{aligned} \tag{26}$$

Invoke (26), differentiating V_i results in

$$\begin{aligned}\dot{V}_i&=\dot{V}_{i-1}+z_i\dot{z}_i+\lambda_i\dot{\lambda}_i\\ &=\dot{V}_{i-1}+\lambda_i\dot{\lambda}_i\\ &+z_i(z_{i+1}+\lambda_{i+1}+\beta_i+k_i y-k_i\hat{x}_1-\dot{\beta}_i^d),\end{aligned} \tag{27}$$

Using Young's inequality, we can write

$$z_i\left(z_{i+1}+\lambda_{i+1}\right)\leq z_i^2+\frac{1}{2}z_{i+1}^2+\frac{1}{2}\lambda_{i+1}^2\ . \tag{28}$$

Substituting (28) into (27), we have

$$\begin{aligned}\dot{V}_i\leq&\dot{V}_{i-1}+\lambda_i\dot{\lambda}_i+\frac{1}{2}z_{i+1}^2+\frac{1}{2}\lambda_{i+1}^2+\\ & z_i(z_i+\beta_i+k_i y-k_i\hat{x}_1-\dot{\beta}_i^d)\ ,\end{aligned} \tag{29}$$

According to (29), the i th virtual control signal can be obtained as

$$\beta_i = -c_i z_i - z_i - k_i y + k_i \hat{x}_1 + \dot{\beta}_i^d \ , \tag{30}$$

where c_i is a positive design parameter. Substituting (22), (24) and (30) into (29) yields

$$\begin{aligned} \dot{V}_i \le & -(\lambda_m(Q) - \frac{3}{2})\|e\|^2 - \\ & \sum_{q=1}^{i} c_q z_q^2 + \sum_{q=2}^{i+1} \frac{z_q^2}{2} + \sum_{q=2}^{i+1} \frac{\lambda_q^2}{2} \\ & + \sum_{q=2}^{i} (-\frac{\lambda_q^2}{\tau_q} + \lambda_q N_q(.)) + \frac{\delta_1}{\gamma_1} \tilde{\theta}_1 \hat{\theta}_1 + \pi_1 \ . \end{aligned} \tag{31}$$

Similar to (23) and (24), we define a new variable β_{i+1}^d which is obtained by pass β_i through a low-pass filter as

$$\begin{aligned} & \tau_{i+1} \dot{\beta}_{i+1}^d + \beta_{i+1}^d = \beta_i \ , \\ & \beta_{i+1}^d(0) = \beta_i(0), \end{aligned} \tag{32}$$

where τ_{i+1} is a positive time constant of this filter. From (15) and (23), the time derivative of λ_{i+1} can be obtained as

$$\dot{\lambda}_{i+1} = \dot{\beta}_{i+1}^d - \dot{\beta}_i = -\frac{\lambda_{i+1}}{\tau_{i+1}} + N_{i+1}(.) \ , \tag{33}$$

where $N_{i+1}(.)$ is a continuous function equals to $-\dot{\beta}_i$.
step n . From (5) and (15), we have

$$\dot{z}_n = \dot{\hat{x}}_n - \dot{\beta}_n^d = k_n y - k_n \hat{x}_1 + u - \dot{\beta}_n^d \ . \tag{34}$$

Select the whole Lyapunov function candidate as

$$V_n = V_{n-1} + \frac{1}{2} z_n^2 + \frac{1}{2} \lambda_n^2 \ . \tag{35}$$

Consider (34), the time derivative of V_n can be written as

$$\begin{aligned} & \dot{V}_n = \dot{V}_{n-1} + z_n \dot{z}_n + \lambda_n \dot{\lambda}_n = \\ & \dot{V}_{n-1} + \lambda_n \dot{\lambda}_n + z_n (k_n y - k_n \hat{x}_1 + u - \dot{\beta}_n^d) \ . \end{aligned} \tag{36}$$

Now, the adaptive control signal u can be designed as

$$u = -c_n z_n - k_n y + k_n \hat{x}_1 + \dot{\beta}_n^d \ , \tag{37}$$

where c_n is a positive design parameter.

Remark 2. As observed from (20), (21), (30), and (37), there is no Nussbaum function, no fuzzy basis function, and no radial basis function in the virtual control signals, the adaptive control law, and especially the control signal. These significant features are achieved due to the use of the proposed design procedure and can lead to the simplicity of adaptive controller design.

Substituting (31), (33) and (37) into (36) yields

$$\begin{aligned} \dot{V}_n \le & -(\lambda_m(Q) - \frac{3}{2})\|e\|^2 - \sum_{q=1}^{n} \bar{c}_q z_q^2 + \\ & \sum_{q=2}^{n} (-\frac{\lambda_q^2}{\tau_q} + \frac{\lambda_q^2}{2} + \lambda_q N_q(.)) + \frac{\delta_1}{\gamma_1} \tilde{\theta}_1 \hat{\theta}_1 + \pi_1 \ , \end{aligned} \tag{38}$$

where $\bar{c}_1 = c_1$ and $\bar{c}_q = c_q + \frac{1}{2}, q = 2, \ldots, n$. In accordance to Young's inequality, one can get

$$\lambda_q N_q(.) \le \frac{(\lambda_q N_q(.))^2}{2\delta_2} + \frac{\delta_2}{2} \ , \tag{39}$$

$$\frac{\delta_1}{\gamma_1} \tilde{\theta}_1 \hat{\theta}_1 \le -\frac{\delta_1}{2\gamma_1} \tilde{\theta}_1^2 + \frac{\delta_1}{2\gamma_1} \theta_1^{*2} \ , \tag{40}$$

where δ_2 is a positive constant. Consider (39) and (40), we can rewrite (38) as

$$\begin{aligned} \dot{V}_n \le & -(\lambda_m(Q) - \frac{3}{2})\|e\|^2 - \sum_{q=1}^{n} \bar{c}_q z_q^2 - \\ & \sum_{q=2}^{n} (\frac{1}{\tau_q} - \frac{1}{2} - \frac{N_q^2(.)}{2\delta_2}) \lambda_q^2 - \frac{\delta_1}{2\gamma_1} \tilde{\theta}_1^2 + \pi_n \ , \end{aligned} \tag{41}$$

where

$$\pi_n = \pi_1 + \frac{\delta_1}{2\gamma_1} \theta_1^{*2} + \frac{\delta_2}{2}$$

Based on Assumption 1, $\Omega_I := \{(y_r, \dot{y}_r, \ddot{y}_r) : y_r^2 + \dot{y}_r^2 + \ddot{y}_r^2 \le M\}$ is a compact set for any

$M > 0$. On the other hand, for any $\rho > 0$, the set

$$\Omega_d := \left\{ 2e^T P\ e + \sum_{k=1}^{q} z_k^2 + \sum_{k=2}^{q} \lambda_k^2 + \frac{\tilde{\theta}_1^2}{\gamma_1} \le 2\rho \right\},\quad (q = 1,...,n).$$

is compact. So, the set $\Omega_l \times \Omega_d$ will be compact. Hence, $|N_q(.)|$ has a maximum as $\bar{N}_q > 0$ on $\Omega_l \times \Omega_d$, and we get $|N_q(.)| \le \bar{N}_q$. Now, (41) can be rewritten as follows

$$\dot{V}_n \le -(\lambda_m(Q) - \frac{3}{2})\|e\|^2 - \sum_{q=1}^{n} \bar{c}_q z_q^2 - \sum_{q=2}^{n} (\frac{1}{\tau_q} - \frac{1}{2} - \frac{\bar{N}_q^2}{2\delta_2})\lambda_q^2 - \frac{\delta_1}{2\gamma_1}\tilde{\theta}_1^2 + \pi_n . \quad (42)$$

The parameters Q, τ_q and δ_2 are chosen so that $\left(\lambda_m(Q) - \frac{3}{2}\right) > 0$ and $\left(\frac{1}{\tau_q} - \frac{1}{2} - \frac{\bar{N}_q^2}{2\delta_2}\right) > 0$. Define

$$\beta = \min\left\{ \frac{\lambda_m(Q)\text{-}1}{\lambda_{\max}(P)},\ 2\bar{c}_q,\ \delta_1,\ 2(\frac{1}{\tau_q} - \frac{1}{2} - \frac{\bar{N}_q^2}{2\delta_2}) \right\}. \quad (43)$$

From (35), (42) and (43), it can be achieved

$$\dot{V}_n \le -\beta V_n + \pi_n . \quad (44)$$

It should be considered that π_n is bounded due to the small selection of the positive design parameters. Also, (44) states that

$$0 \le V_n(t) \le \frac{\pi_n}{\beta} + \left(V_n(0) - \frac{\pi_n}{\beta}\right)e^{-\beta t}, \quad (45)$$

which implies that all the closed-loop signals are semi-globally uniformly ultimately bounded (SGUUB). The design procedure and analysis proposed in this paper can be summarized as the following theorem.

Theorem. Consider the switched non-strict feedback nonlinear system (1) with the simplified form (2), in which the control directions are unknown. With the help of the state observer (5), the virtual control signals (20) and (30), the adaptive control signal (37), the adaptation law (21), and Assumptions 1 and 2, all the closed-loop signals are SGUUB, while the tracking error converges to zero.

4- Simulation Results

The effectiveness of the proposed adaptive control scheme is illustrated through the following example.

Example. Consider the switched non-strict feedback nonlinear system as follows

$$\begin{cases} \dot{x}_1 = f_1^{\sigma(t)}(x) + g_1^{\sigma(t)}(x)\,x_2, \\ \dot{x}_2 = f_2^{\sigma(t)}(x) + g_2^{\sigma(t)}(x)\,u, \\ y = x_1, \end{cases} \quad (46)$$

where $f_1^1(x) = (0.3 + x_1 + x_2)$, $f_1^2(x) = -0.1(x_1 + x_2)$, $f_2^1(x) = x_1 x_2^2$, $f_2^2(x) = x_1 x_2$, $g_1^1(x) = 1.5 + 0.5\sin x_1$, $g_1^2(x) = 1.5 + 0.1\sin(x_1 x_2)$, $g_2^1(x) = 6 + 0.1\sin x_1$, and $g_2^2(x) = 5 + 0.3\sin(x_1 x_2)$. The initial conditions are selected as $[x_1(0), x_2(0)]^T = [0.3, -0.1]^T$, $[\hat{x}_1(0), \hat{x}_2(0)]^T = [0,0]^T$, $\hat{\theta}_1(0) = 0.1$, $\beta_2^d(0) = 0.01$. The purpose of the controller design is to track the reference signal $y_r = \sin(t) + \sin(2t)$ by the output signal y. In this simulation, the design parameters are chosen as $k_1 = 4, k_2 = 1, c_1 = 8, \alpha_1 = 0.1, \gamma_1 = 0.7, \delta_1 = 0.5, \tau_2 = 0.01, c_2 = 0.3$. As we know, with the help of this control method, there is no need to design any parameters related to fuzzy basis functions or Nussbaum functions. Also, only one adaptive law needs to be designed. Hence, fewer parameters are used in the design of the proposed controller. The simulation results are presented in Figs. 1-6. As observed from Fig. 1, the reference signal is well-tracked by the system output, and the tracking error illustrated in Fig. 2 has been converged to zero. The arbitrary switching signal is applied to the system according to Fig. 3. The boundedness of the adaptive law i.e. signal $\hat{\theta}_1$ is well shown in Fig. 4. The response of the proposed control signal without using the Nussbaum function is shown in Fig. 5. As can be seen from Fig. 5, the control signal at the initial step is not extremely large. The simulations are repeated for the case where the controller is designed with the help of Nussbaum functions. The result can be seen in Fig. 6. From Fig. 6, it is clear that the control signal designed through the Nussbaum function method is extremely large in the initial stage, which can be destructive for actuators or equipment.

Remark 3. Some literature such as [35, 36] have studied the adaptive tracking control problem for a class of switched and non-switched nonlinear systems subject to unknown control direction. The Nussbaum-type functions are used to handle the problem of unknown control directions in these studies. In both of them, the control signal is extremely large in the initial stage due to the use of Nussbaum functions. Different from these literatures, the proposed adaptive control scheme solves the problem of unknown control directions in switched non-strict feedback nonlinear systems without using Nussbaum functions, meanwhile, the control objective is well achieved.

Remark 4. As can be seen from Figs 5 and 6, the control

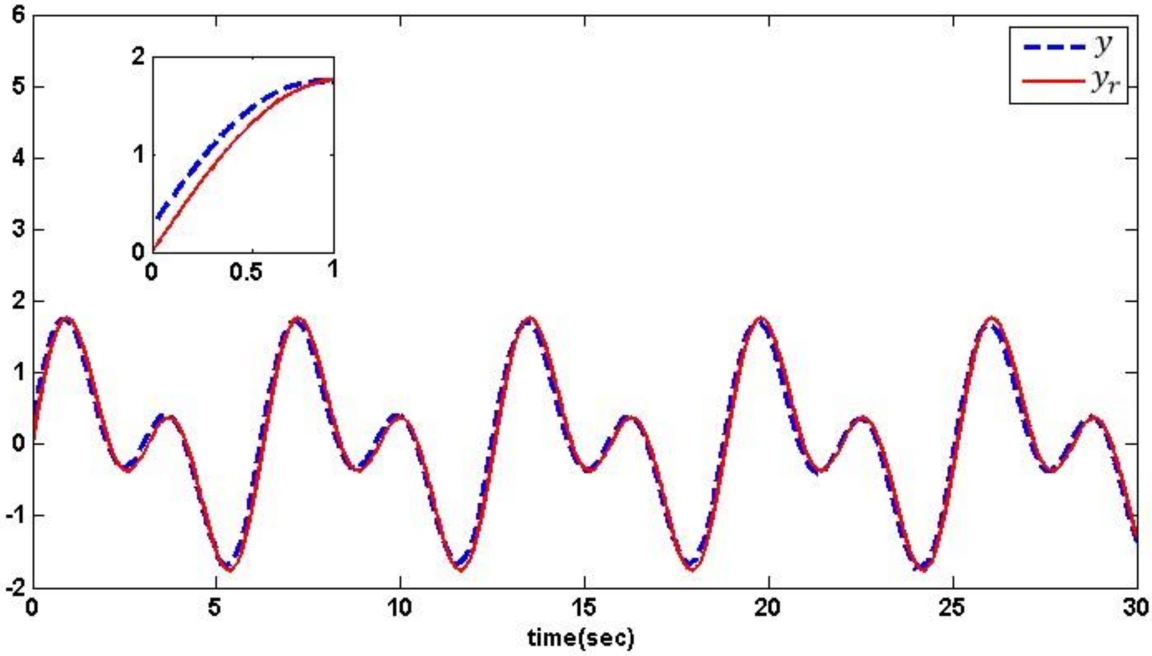

Fig. 1. The system output y and the reference signal y_r

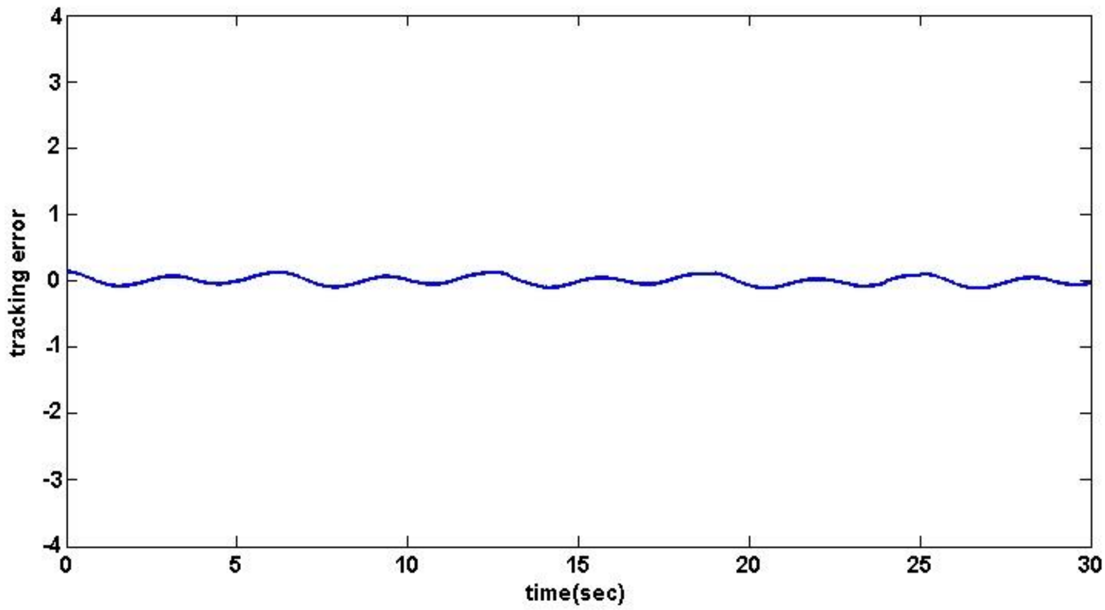

Fig. 2. The trajectory of the tracking error z_1

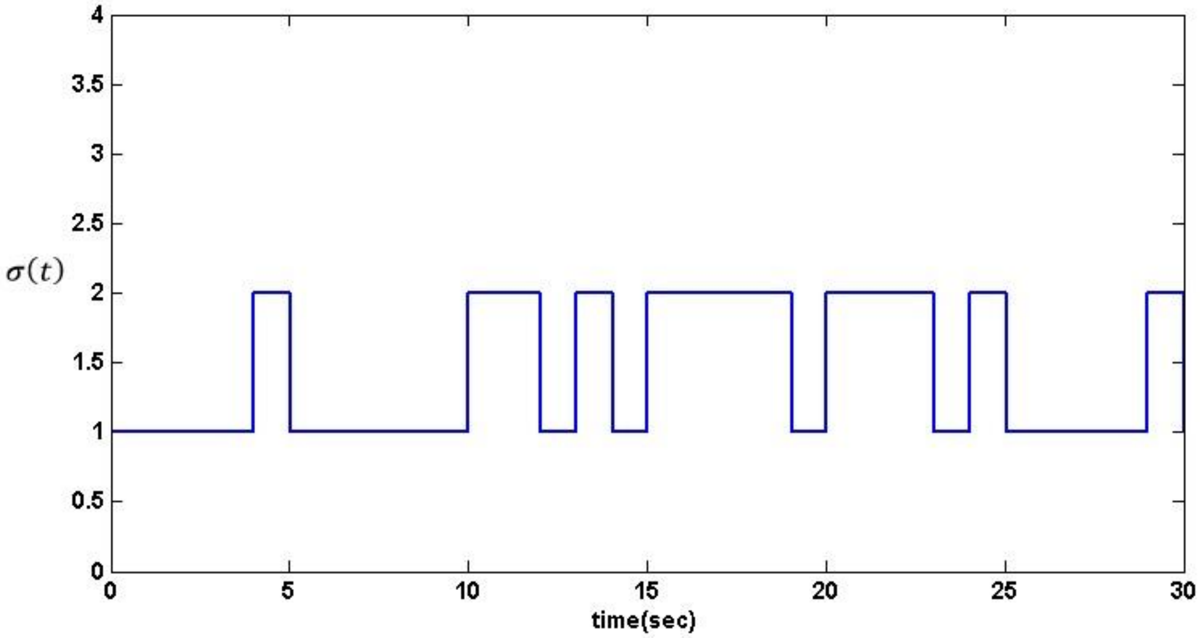

Fig. 3. The switching signal $\sigma(t)$

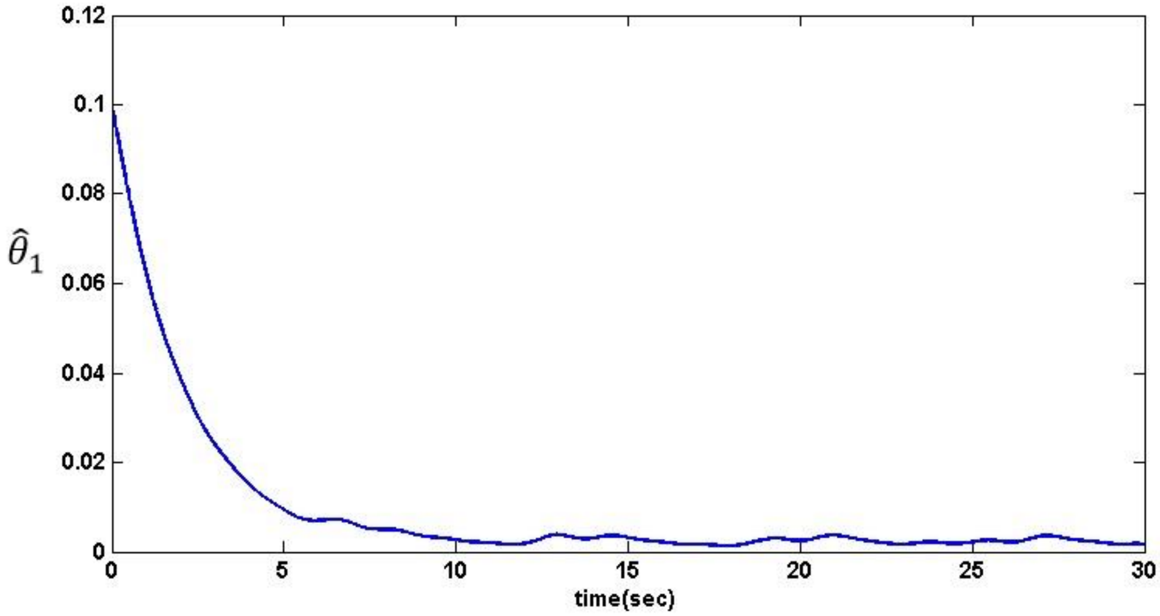

Fig. 4. The trajectory of the signal $\hat{\theta}_1$

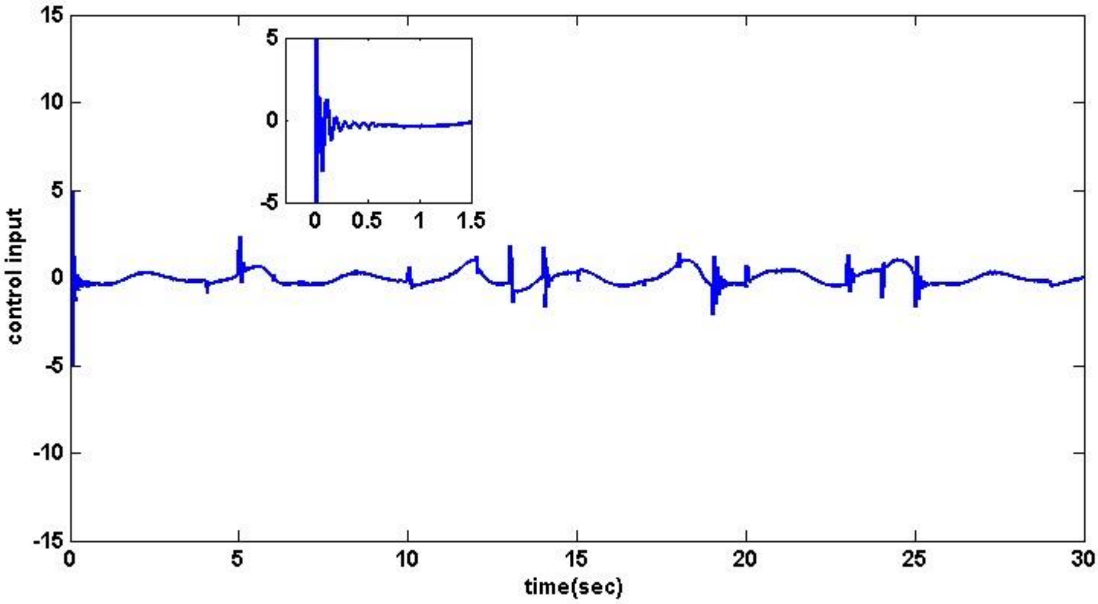

Fig. 5. The proposed control signal u

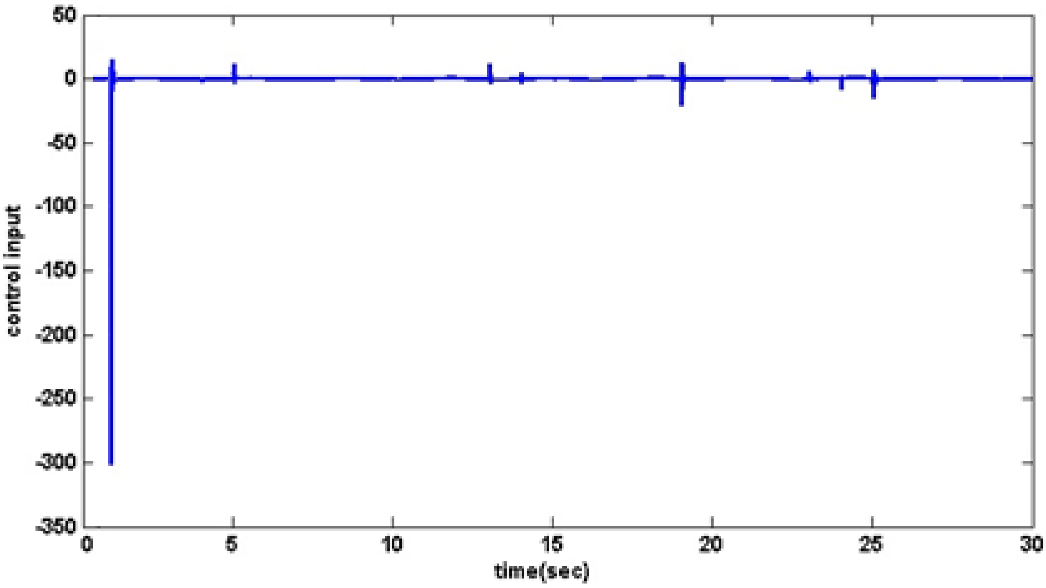

Fig. 6. The trajectory of the control signal with Nussbaum function approach

signal changes at the moments of switching suddenly. Because in these moments, the dynamics of the system have changed and the adaptive controller has to adapt itself to the new conditions to fulfill the control objective. Therefore, it can be said that the adaptive controller is an appropriate candidate for such systems.

5- Conclusion

A non-Nussbaum function approach has been provided to solve the problem of adaptive output-feedback control for a class of switched non-strict feedback nonlinear systems, which guarantees that the control signal does not become very large in the initial stages. In the presented system, the switching signal is arbitrary; the control directions are unknown, and the states are unmeasurable. The unknown nonlinear functions have been approximated with the help of the universal approximation theorem, but there is neither fuzzy nor radial basis function in the adaptive control signal. On the other hand, only one adaptation law has been designed to estimate the unknown parameters. It has been demonstrated that using the proposed controller, all the signals in the closed-loop system are SGUUB. Finally, the simulation results illustrate the features of the proposed control scheme.

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