AUT Journal of Mathematics and Computing

AUT J. Math. Comput., 6(1) (2025) 55-66
https://doi.org/10.22060/AJMC.2023.22698.1184

Original Article

Two-stage stochastic capacitated Lot-Sizing problem by Lot-Size adaptation approach

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ABSTRACT: In this paper, a two-stage stochastic capacitated lot-sizing problem with random demand, and service level constraints under static and static-dynamic uncertainty strategies is introduced. A static strategy determines the setup period and lot sizing at the beginning of planning period, whereas a static-dynamic strategy allows the lot size to be adjusted during the planning period. A new model formulation of the demand differential adjustment policy in a multi-stage production system is proposed. Lot-sizing adjustments depend on the difference in demand between actual and expected demand. To quantify the economics of our uncertainty strategy for multi-level lot size the problems, the number of test instances with different parameter settings is evaluated. Computational experiments show that the additional costs of semi-finished products, and the lack of storage capacity in the downstream processes reduce the potential for cost savings via multi-volume reform. Also a robust model is developed and as the robust model under study is NP-hard, it solved by a hybrid heuristic using the proposed stochastic model, a robust model is developed, which is solved by a hybrid heuristic algorithm based on Lagrangian relaxation and Bender's decomposition algorithms. To evaluate the convergence rate and solution quality, the method is applied to some random test instances generated in the literature. The computational results indicate that the proposed method is capable of efficiently solving the model.

Review History:

Received:23 September 2023
Revised:03 November 2023
Accepted:11 November 2023
Available Online:01 December 2024

Keywords:

Two-stage stochastic programming
Productive planning
Inventory systems
Robust model
Lagrangian relaxation
Benders decomposition

MSC (2020):

90-xx

1. Introduction

Production planning is the placement of machines, equipment, workers, and labor on a production line to meet customer needs. The objective is to fulfill demand during a time of constrained production capacity by allocating production resources as efficiently as possible. Work schedules, shifts, inventory levels, and delay rules all need decisions. Production planning is often done by forecasting potential demand in response to previous customer orders. This means that planners should make production planning decisions even if orders are not fulfilled when planning begins. Such decisions are necessary here and now. When actual demand is lower or higher than expected demand, this often results in excess inventory, and insufficient production capacity. Further uncertainty modeling

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plays a key role in production planning to address these issues. One way to achieve this is to use scenario trees to model demand uncertainty based on a combination of prior understanding of demand, and associated probabilities. Scenario models help companies implement realistic plans [9]. Besides, decision-makers can make flexible and specific decisions to implement scenarios that require wait-and-see measures.

Lot-sizing decisions are made to economically balance inventory to compensate for the fluctuations in demand, and planning decisions define the sequence of production orders for different products within the planning period [15]. Karimi et al. [8] categorized lot size problems by planning horizon, number of levels, number of items, capacity or resource constraints, perishability, demand structure, construction structure, and backlog. Uncertainty in demand poses a significant challenge to production scheduling. The goal of capacitated lot size choices is to fulfill unpredictable demand with limited capacity by determining the best time and amount of goods to manufacture at the lowest possible cost. In general, demand is known, and determined using forecasting techniques. However, there may be many uncertain events that can affect demand, such as seasonality and customer behavior [4]. Brandimarte [3] discusses a multistage mixed-integer stochastic programming model for the classic multidimensional capacity problem. In this model, demand uncertainty is modeled through a scenario tree. Hu and Hu [6] propose a two-stage stochastic programming model to minimize the total cost of production, storage, and backlog under demand uncertainty. This is illustrated by the scene tree. In another study, Hu and Hu [7] model stochastic lot size and scheduling under demand uncertainty using scenario trees.

Zanjani et. al. [20] focused on multi-period, multi-product stochastic production planning problems for the case of cart production planning with uncertain yield and demand, and used scenario trees to deal with uncertainty. Tunc et al. [18] presents an extended mixed integer programming formulation of the lot size problem that can solve variants of the lot-sizing problem characterized by penalty costs and service level constraints, backlogs, and revenue losses. A multi-element, multi-level, zero-capacity stochastic integer programming technique based on scenario trees with unknown parameters is presented by Quezadaet. al. [12]. The challenge of creating a lot-sizing of one item at capacity in a stage with a minimum lot-size limitation under a static-dynamic uncertainty strategy is solved by a modified basic inventory policy, as shown by Randa et al. [14]. show that a modified basic inventory policy solves the problem of producing a lot-sizing of one item at capacity in a stage with a minimum lot-size constraint under a static-dynamic uncertainty strategy. In another study, Ramaraj et al. [13] present a multidimensional mixed integer stochastic problem with uncertain demand, and costs. In this problem, the scenario tree represents the evolution of the uncertain parameters. Numerical results show that the proposed algorithm is significantly better than SDDiP algorithm in terms of performing the objective function values within the computational time. Tempelmeier and Hilger [16] present a linear programming model for multi-scale dynamic problems with limited capacity for multi-objective. In this problem, the nonlinear functions of expected backlog, and expected inventory are approximated by piecewise linear functions. Thevenin et al. [17] Two-Stage and Multi-Stage Models for Static-static and Static-dynamic decision framework processing using stochastic optimization techniques for MRP systems under demand uncertainty. As the leaders of this category Bookbinder and Tan [2] introduce static, dynamic, and static-dynamic uncertainty strategies for lot-sizing according to stochastic demand. At the beginning of the planning phase, the static strategy determines the settlement periods, and the lot size. A dynamic strategy allows configuration and lot size decisions to be adjusted over the planning horizon. The static-dynamic strategy defines the installation steps for the entire planning scope but allows for later lot size adjustments A static strategy specifies a robust production schedule, while a static-dynamic strategy provides a flexible schedule that requires more planning. However, the static-dynamic strategy achieves lower costs than the static strategy for the single-stage lot-sizing problem. Nonetheless, a sufficient supply of semi-finished goods is necessary to allow for flexibility in the final product's lot-sizing. As a result, earlier production stages must to be included. Applying a demand-oriented demand variance adjustment policy with restricted modifications is necessary since the popular inventory-oriented level-up order strategy is only appropriate for capacity shortfall situations without material needs [11]. Lot-sizing adjustments depend on the difference in requirements between realized and expected demands. In this article, we present a model formulation for the two-stage stochastic capacitated lot-sizing problem (2S-SCLSP) for an adaptive policy with limited demand differences.

To the best of our knowledge, there is no paper on harmony search for integrated lot-sizing and by lot-size adaptation in stochastic environment. This study quantifies the economics of applying a static-dynamic strategy to a multistage production system under the additional cost of semi-finished products and lack of storage capacity in upstream processes. In this paper we proposed a hybrid Lagrangianrelaxation and benders decomposition algorithm. For small-size problems, we evaluate and compare exact solutions with those derived from hybrid method. For large problems, the results from hybrid method are compared to those of literature based on benchmark instances and the results indicate a good performance. The remainder of the paper is organized as follows. Section 2 gives the problem description. The mathematical optimization models are presented in Section 3. The robust model corresponding to the non-deterministic model is presented in Section 4. Section 5 presents the numerical experiments. The conclusion and future research directions are given in Section 6.

2. Problem Description

The two-stage lot-sizing problem under consideration is subject to demand uncertainty. There are several different scenarios s for the demands, which are denoted by$S$. The production problem includes periods and $|J|$ products with the index and$j$. The multilevel bill of material J_j defines the predecessor and successor relationship between finished and semi-finished products $\hat{J}$. Two assumptions about the stages of production were made. First, semi-finished products follow a static strategy, and finished products follow a static or static-dynamic strategy. Applying static strategies to semi-finished products smoothed out fluctuations outside the production system, and avoids bullwhip effects. Second, the demand for finished goods can be reordered taking into account the γ service level, while the demand for semi-finished goods must be met immediately. To ensure production capacity for finished products, it is necessary to meet the demand for semi-finished products.

The classical lot size problem determines the setup period X_{tj}, the lot size $Q_{t,j}$ and the stock level I_{stj}. Due to back counting, net inventory consists of backlog I^-_{stj} and physical inventory I^+_{stj}. There is a restriction to achieve a specific γ-service level. The objective function uses maintenance cost h and setup costs v to minimize expected maintenance and setup costs. Capacityc_r limits the production time according to the fixed production time of the product J_r assigned to the r. The production factor f_{ji} determines the quantity of product i required to produce one unit of product j.

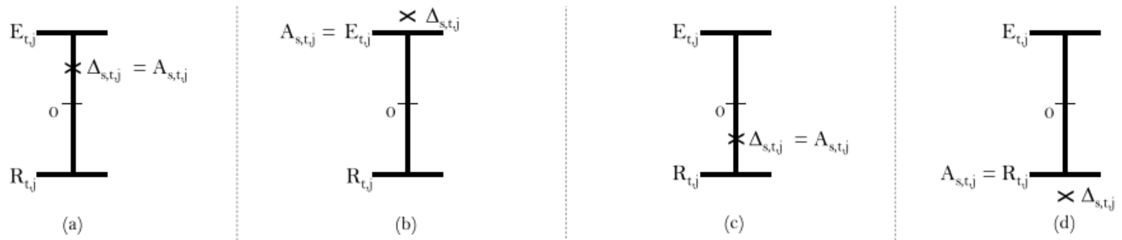

Figure 1: The lot-sizing mechanism approach

Allow lot size adjustments A_{stj} based on the cumulative demand difference Δ_{stj} between the scenario demand d_{stj} and the expected demand $\bar{d}_{stj}$. To avoid double counting, previous adjustments should be taken into account.

$$\Delta_{stj} = \sum_{\tau<t} d_{s\tau j} - \bar{d}_{\tau j} \qquad \forall\, s,t,j \tag{1}$$

$$A'_{stj} = \Delta_{stj} - \sum_{\tau<t} A_{s\tau j} \qquad \forall\, s,t,j|X_{tj}=1 \tag{2}$$

Lot size adaptation is limited by the maximum expansion and the maximum reduction R_{tj}. The binary decision variables and Z_{stj}, represent the adaptation for the maximum amount of extension and the amount of reduction, preventing them from taking arbitrary values. The Y_{stj} variable is equal to 1 if the theoretical adaptation is greater than the maximum extension. Similarly, Z_{stj} is equal to 1 if the theoretical adaptation is less than the maximum reduction value.

$$A_{stj} = \min\left\{E_{tj}, A'_{stj}\right\} \qquad \forall\, s,t,j\,\middle|\, A'_{stj} > 0 \tag{3}$$

$$A_{stj} = \max\left\{-R_{tj}, A'_{stj}\right\} \qquad \forall\, s,t,j\,\middle|\, A'_{stj} < 0 \tag{4}$$

The mechanics of the lot-sizing strategy are shown in Figure 1. It is important to acknowledge that the sample provided illustrates the initial manufacturing cycle without any preceding modifications. In Figure 1(a), the depicted scenario illustrates the situation in which the lot-size adaptation aligns with a positive variation in aggregate demand. Figure 1(b) shows that the positive cumulative demand difference exceeds the maximum extension. Figure 1(c) shows the case in which the lot-size adaptations correspond to a negative change in aggregate demand. Figure 1(d) shows that the negative variance of aggregate demand is less than the maximum reduction.

3. Model Formulation

Below is the design of the 2S-SCLSP model based on [5]. The original design is based on service level γ, considers a multi-stage production system, and applies a static-dynamic uncertainty strategy.

$$P_1 : \min\ F = \frac{1}{|S|}\sum_s\sum_t\sum_j hI^+_{stj} + \sum_t\sum_j vX_{tj} \tag{5}$$

$$\text{s.t. } I_{0j} + Q_{tj} + A_{stj} - d_{stj} - \sum_{i\in J_j} f_{ij}\left(Q_{ti} + A_{sti}\right) = I_{stj} \qquad \forall\, s, t = 1, j \tag{6}$$

$$I_{st-1j} + Q_{tj} + A_{stj} - d_{stj} - \sum_{i\in J_j} f_{ij}\left(Q_{ti} + A_{sti}\right) = I_{stj} \qquad \forall\, s, t > 1, j \tag{7}$$

$$I^+_{stj} \geq I_{stj} \qquad \forall\, s, t, j \tag{8}$$

$$I^-_{stj} \geq -I_{stj} \qquad \forall\, s, t, j \tag{9}$$

$$Q_{tj} \leq mX_{tj} \qquad \forall\, t, j \tag{10}$$

$$\sum_t Q_{tj} \geq t\sum_t \bar{d}_{tj} + \sum_t\sum_{i\in J_j} f_{ij}\bar{d}_{tj} \qquad \forall\, j \tag{11}$$

$$\sum_{i\in J_j} p\left(Q_{ti} + E_{tj}\right) \leq c_r \qquad \forall\, t, r \tag{12}$$

$$\sum_t\sum_s I^-_{stj} \leq (1-\gamma)\,|S|\sum_t \bar{d}_{tj} \qquad \forall\, j \tag{13}$$

$$A_{stj} \leq \Delta_{stj} - \sum_{\tau|\tau<t} A_{s\tau j} + m(1 - X_{tj} + \mathrm{Z}_{stj}) \qquad \forall\, s, t, j \tag{14}$$

$$A_{stj} \geq \Delta_{stj} - \sum_{\tau|\tau<t} A_{s\tau j} + m(1 - X_{tj} + Y_{stj}) \qquad \forall\, s, t, j \tag{15}$$

$$A_{stj} \leq E_{tj} \qquad \forall\, s, t, j \tag{16}$$

$$-A_{stj} \leq R_{tj} \qquad \forall\, s, t, j \tag{17}$$

$$A_{stj} \geq E_{tj} - m(1 - Y_{stj}) \qquad \forall\, s, t, j \tag{18}$$

$$-A_{stj} \geq R_{tj} - m(1 - \mathrm{Z}_{stj}) \qquad \forall\, s, t, j \tag{19}$$

$$E_{tj} \leq mX_{tj} \qquad \forall\, s, t, j \tag{20}$$

$$R_{tj} \leq Q_{tj} \qquad \forall\, t, j \tag{21}$$

$$\sum_{j\in\hat{J}}\Big(I_{0j} + \sum_{\tau=0}^{t} Q_{j\tau}\Big) - \sum_{j\notin\hat{J}}\sum_{\tau=0}^{t}(Q_{j\tau} - R_{j\tau}) \leq w \qquad \forall\, t \tag{22}$$

$$X_{tj}, Y_{stj}, \mathrm{Z}_{stj} \in \{0, 1\} \qquad \forall\, t, j \tag{23}$$

$$I^+_{stj}, I^-_{stj}, I_{0j}, Q_{tj}, E_{tj}, R_{tj} \geq 0 \qquad \forall\, s, i, t, j \tag{24}$$

The objective function (5) minimizes the expected holding and adjustment costs. Period 1 inventory is demonstrated by (6) and the planning period is shown by constraint (7) which includes net inventory, current production, finished products demand, and semi-finished products demand. Constraint (8) defines physical inventory based on positive net inventory, and constraint (9) defines the backlog based on negative net inventory. Equation (10) shows the situation of production adjustment. Equation (11) ensures that the expected demand is produced within the planning period, thus ensuring production capacity outside the planning period. Capacity constraints (12) limit production time to available capacity. The service level equation (13) limits credits based on service level. The demand gap adaptation policy is implemented through constraints (16) to (21) . Note that lot sizing and backlog for semi-finished products are set to zero because the immediate demand should be met and the static uncertainty strategy is applied.

Equations (14) and (15) define the lot-size adaptation based on the cumulative difference in demand and previous adjustments. Equations (16) and (17) limit the lot size adaptation to the maximum amount of extension and reduction. Equation (18) defines a lot size adaptation when the cumulative demand and previous adjustments exceed the maximum extension. Equation (19) is used to ascertain the lot-size adaptation in cases where the cumulative demand and previously decreased modifications are lower than the maximum threshold. Equations (20) and (21) provide the adjustment condition that pertains to production quantities that are both constant and non-negative. Equation (22) considers memory constraints for upstream processes. These storage capacity constraints include the initial inventory and production quantities of semi-finished products and the minimum requirements for finished production. It represents the approximate maximum capacity of the semi-finished product and is limited by the storage capacity w.

The proposed model is different from other related Lot Sizing models and their extension due to the demand difference adaptation policy. However, the proposed model is adaptable to other relevant areas. The proposed model will be extended to a more robust model in the following section.

4. The Robust Model

The robustness of solutions minimizes changes in uncertainty while increasing the cost of the entire system. There is a trade-off between robustness against changes and cost savings. In this study, a robust programming approach developed by Yu and Li [19] and Leung et al. [10] is applied. In this method, the average absolute deviation is used instead of the standard one. Furthermore, coefficients (which depend on the decision maker's opinion) are assigned to each scenario-dependent term in the objective function. Hence, for the first term, parameters ε is considered. So, a robust mathematical model of P_1 is introduced as the following model $P_1(R)$:

$$
\begin{aligned}
P_1(R): \min\ F = &\frac{1}{|S|}\sum_s\sum_t\sum_j hI^+_{stj} + \sum_t\sum_j vX_{tj} \\
&+ \varepsilon\Big(\frac{1}{|S|}\sum_s p^s\Big(\sum_t\sum_j hI^+_{stj} - \sum_s p^s\sum_t\sum_j hI^+_{stj}\Big) + 2\theta^s\Big) \qquad (25)\\
\text{s.t. } &(6)-(23)\\
&\sum_t\sum_j hI^+_{stj} - \sum_{s'} p^{s'}\sum_t\sum_j hI^+_{stj} + \theta^s \geq 0 \quad \forall\, s \qquad (26)\\
&I^+_{stj}, I^-_{stj}, I_{0j}, Q_{tj}, E_{tj}, R_{tj}, \theta \geq 0 \qquad \forall\, s, i, t, j \qquad (27)
\end{aligned}
$$

5. Lagrangian Relaxation algorithm

From the mathematical modeling of 2S-SCLSP, equations (12) and (22) are the only constraints connecting the levels of the product structure. These equations are relaxed using Lagrangianmethods to solve the two-stage problem. Relax the constraint (12), (22) and (26) with the non-negative Lagrange multiplier λ^{12}_{rt}, λ^{22}_t and λ^{26}_s respectively. Problem (P2) for 2S-SCLSP takes the following form:

$$
\begin{aligned}
P_2: \max_{\lambda}\ \min\ F(\lambda) = &\frac{1}{|S|}\sum_s\sum_t\sum_j hI^+_{stj} + \sum_t\sum_j vX_{tj} \\
&+ \varepsilon\Big(\frac{1}{|S|}\sum_s p^s\Big(\sum_t\sum_j hI^+_{stj} - \sum_s p^s\sum_t\sum_j hI^+_{stj}\Big) + 2\theta^s\Big) \\
&- \lambda^{12}_{rt}\Big(\sum_{i\in J_j} p\,(Q_{ti}+E_{tj}) - c_r\Big) \\
&- \lambda^{22}_t\left(\sum_{j\in\hat{J}}\Big(I_{0j} + \sum_{\tau=0}^{t} Q_{j\tau}\Big) - \sum_{j\notin\hat{J}}\sum_{\tau=0}^{t}(Q_{j\tau}-R_{j\tau}) - w\right) \\
&- \lambda^{26}_s\Big(-\sum_t\sum_j hI^+_{stj} + \sum_{s'} p^{s'}\sum_t\sum_j hI^+_{stj} - \theta^s\Big) \qquad (28)\\
\text{s.t. } &(6)-(11), (13)-(21), (23)-(25)
\end{aligned}
$$

As can be seen, by assuming that the Lagrange coefficients are constant $\bar{\lambda}$, the problem P_2 is decomposed into $|J|$ subproblems (named $P_2(j,\bar{\lambda})$) for every $j \in J$. Therefore, instead of solving a problem $P_2(\bar{\lambda})$, $|J|$ sub-problems with smaller sizes are solved.

The models $P_2(j,\bar{\lambda})$ include both continuous $(I^+_{stj}, I^-_{stj}, I_{0j}, Q_{tj}, E_{tj}, R_{tj}, \theta^s)$ and binary $(X_{tj}, Y_{stj}, Z_{stj})$ variables, for which the Benders decomposition method is recommended. The procedure of the Benders decomposition method on each $P_2(j,\bar{\lambda})$ is described below.

Benders Decomposition Phase

In the Benders decomposition method, the mixed integer programming problem is divided into two sub-problems, which are called the master problem and the sub-problem. Besides, each sub-problem is solved iteratively until an optimal solution is obtained [1]. A sub-problem is a continuous linear model containing the continuous variables, and their constraints in the main problem. The master problem is a mixed integer model that contains the integer variables of the master problem and their constraints, and a constant variable that connects the two sub-problems. The optimal solution to the basic problem is the lower bound of the basic problem. Furthermore, by using this approach and utilizing the integer variables' values derived from the issue's sub-problem, together with the solution of the dual problem associated with said sub-problem, it becomes feasible to determine the upper limit of the master problem. In the subsequent cycle, one or more incisions are produced. These cuts are added to the master problem, which is again solved with new constraints to achieve new and better lower bounds. This process continues until the high and low gaps are set to an acceptable value or zero. Benders decomposition algorithm finds the optimal solution in a finite number of iterations. Next, the master problem $P_2(j,\bar{\lambda})$ used to create the master problem, as well as the sub-problem, is presented:

$$P_2'(j,\bar{\lambda}): \min\ F'(j,\bar{\lambda}) = \sum_t vX_{tj} + H(I^+_{stj}, I^-_{stj}, I_{0j}, Q_{tj}, E_{tj}, R_{tj}, \theta^s | X_{tj}, Y_{stj}, Z_{stj}) \tag{29}$$

$$\text{s.t. } A_{stj} \le \Delta_{stj} - \sum_{\tau|\tau<t} A_{s\tau j} + m(1 - X_{tj} + \mathrm{Z}_{stj}) \qquad \forall\, s,t \tag{30}$$

$$A_{stj} \ge \Delta_{stj} - \sum_{\tau|\tau<t} A_{s\tau j} + m(1 - X_{tj} + Y_{stj}) \qquad \forall\, s,t \tag{31}$$

$$X_{tj}, Y_{stj}, \mathrm{Z}_{stj} \in \{0,1\} \qquad \forall\, s,t \tag{32}$$

Where $H(I^+_{stj}, I^-_{stj}, I_{0j}, Q_{tj}, E_{tj}, R_{tj}, \theta^s | X_{tj}, Y_{stj}, \mathrm{Z}_{stj})$ is the benders sub-problem, the development process of which is presented in the next section.

Benders Sub-problem

The benders sub-problem $(H(I^+_{stj}, I^-_{stj}, I_{0j}, Q_{tj}, E_{tj}, R_{tj}, \theta^s | X_{tj}, Y_{stj}, \mathrm{Z}_{stj}))$ includes continuous variables $(I^+_{stj}, I^-_{stj}, I_{0j}, Q_{tj}, E_{tj}, R_{tj}, \theta^s)$, optimal values of which are obtained by solving the sub-problem for the fixed values $(\bar{X}_{tj}, \bar{Y}_{stj}, \bar{Z}_{stj})$. This problem can be represented as:

$$\begin{aligned} P_2''(j,\bar{\lambda}): \min\ F''(j,\bar{\lambda}) = {} & \frac{1}{|S|}\sum_s\sum_t hI^+_{stj} + \varepsilon\Big(\frac{1}{|S|}\sum_s p^s\Big(\sum_t\sum_j hI^+_{stj} - \sum_s p^s \sum_t\sum_j hI^+_{stj}\Big) + 2\theta^s\Big) \\ & - \bar{\lambda}^{12}_{rt}\Big(\sum_{i\in J_j} p\,(Q_{ti} + E_{tj}) - c_r\Big) - \bar{\lambda}^{22}_t\Big((I_{0j} + \sum_{\tau=0}^{t} Q_{j\tau}) - \sum_{\tau=0}^{t}(Q_{j'\tau} - R_{j'\tau}) - w\Big) \\ & - \bar{\lambda}^{26}_s\Big(-\sum_t\sum_j hI^+_{stj} + \sum_{s'} p^{s'}\sum_t\sum_j hI^+_{stj} - \theta^s\Big) \end{aligned} \tag{33}$$

$$\text{s.t. } I_{0j} + Q_{tj} + A_{stj} - d_{stj} - \sum_{i\in J_j} f_{ij}\,(Q_{ti} + A_{sti}) = I_{stj} \qquad \forall\, s, t=1 \tag{34}$$

$$I_{st-1j} + Q_{tj} + A_{stj} - d_{stj} - \sum_{i\in J_j} f_{ij}\,(Q_{ti} + A_{sti}) = I_{stj} \qquad \forall\, s, t>1 \tag{35}$$

$$I^+_{stj} \ge I_{stj} \qquad \forall\, s,t \tag{36}$$

$$I^-_{stj} \ge -I_{stj} \qquad \forall\, s,t \tag{37}$$

$$Q_{tj} \le m\bar{X}_{tj} \qquad \forall\, t \tag{38}$$

$$\sum_t Q_{tj} \ge t\sum_t \bar{d}_{tj} + \sum_t\sum_{i\in J_j} f_{ij}\bar{d}_{tj} \tag{39}$$

$$\sum_t\sum_s I^-_{stj} \le (1-\gamma)\,|S|\sum_t \bar{d}_{tj} \tag{40}$$

$$A_{stj} \le E_{tj} \qquad \forall\, s,t \tag{41}$$

$$-A_{stj} \le R_{tj} \qquad \forall\, s,t \tag{42}$$

$$A_{stj} \ge E_{tj} - m(1 - \bar{Y}_{stj}) \qquad \forall\, s,t \tag{43}$$

$$-A_{stj} \ge R_{tj} - m(1 - \bar{Z}_{stj}) \qquad \forall\, s,t \tag{44}$$

$$E_{tj} \le m\bar{X}_{tj} \qquad \forall\, s,t \tag{45}$$

$$R_{tj} \le Q_{tj} \qquad \forall\, t \tag{46}$$

$$I^+_{stj}, I^-_{stj}, I_{0j}, Q_{tj}, E_{tj}, R_{tj}, \theta \ge 0 \qquad \forall\, s,i,t \tag{47}$$

Sub-problem $(P_2''(j,\bar{\lambda}))$ is also used to generate cuts to add to the master problem.
For this purpose, a dual sub-problem is used.

Dual Sub-problem

Let $(\pi_{s1}^6, \pi_{st}^7, \pi_{st}^8, \pi_{st}^9, \pi_t^{10}, \pi^{11}, \pi^{13}, \pi_{st}^{14}, \pi_{st}^{16}, \pi_{st}^{17}, \pi_{st}^{18}, \pi_{st}^{19}, \pi_{st}^{20}, \pi_t^{21})$ be the dual variables of constraints from the sub-problem $(P_2''(j,\bar{\lambda}))$. Then, the dual sub-problem, $H(\pi_{s1}^6, \pi_{st}^7, \pi_{st}^8, \pi_{st}^9, \pi_t^{10}, \pi^{11}, \pi^{13}, \pi_{st}^{14}, \pi_{st}^{16}, \pi_{st}^{17}, \pi_{st}^{18}, \pi_{st}^{19}, \pi_{st}^{20}, \pi_t^{21})$ is presented as:

$$\begin{aligned}
P_3(\bar{\lambda}):\ \max W(\bar{\lambda}) = &\sum_s \pi_{s1}^6 \Big(I_{0j} + Q_{tj} + A_{stj} - d_{stj} - \sum_{i\in J_j} f_{ij}\left(Q_{ti} + A_{sti}\right) - I_{stj}\Big) \\
&+ \sum_s \sum_t \pi_{st}^7 \Big(I_{st-1j} + Q_{tj} + A_{stj} - d_{stj} - \sum_{i\in J_j} f_{ij}\left(Q_{ti} + A_{sti}\right) - I_{stj}\Big) \\
&- \sum_s \sum_t \pi_{st}^8 (I_{stj} - I_{stj}^+) - \sum_s \sum_t \pi_{st}^9 (-I_{stj} - I_{stj}^-) + \sum_t \pi_t^{10} (Q_{tj} - m\bar{X}_{tj}) \\
&- \pi^{11} \Big(t \sum_t \bar{d}_{tj} + \sum_t \sum_{i\in J_j} f_{ij} \bar{d}_{tj} - \sum_t Q_{tj}\Big) - \pi^{11} \Big(\sum_t \sum_s I_{stj}^- - (1-\gamma)\,|S| \sum_t \bar{d}_{tj}\Big) \\
&- \sum_s \sum_t \pi_{st}^{16} (A_{stj} - E_{tj}) - \sum_s \sum_t \pi_{st}^{17} (-A_{stj} - R_{tj}) \\
&- \sum_s \sum_t \pi_{st}^{18} \Big(E_{tj} - m(1 - \bar{Y}_{stj}) - A_{stj}\Big) \\
&- \sum_s \sum_t \pi_{st}^{19} \Big(R_{tj} - m(1 - \bar{Z}_{stj}) + A_{stj}\Big) \\
&- \sum_s \sum_t \pi_{st}^{20} (E_{tj} - m\bar{X}_{tj}) - \sum_t \pi_t^{21} (R_{tj} - Q_{tj}) \qquad (48)
\end{aligned}$$

$$\text{s.t. } \pi_{s1}^6, \pi_{st}^7, \pi_{st}^8, \pi_{st}^9, \pi_t^{10}, \pi^{11}, \pi^{13}, \pi_{st}^{14}, \pi_{st}^{16}, \pi_{st}^{17}, \pi_{st}^{18}, \pi_{st}^{19}, \pi_{st}^{20}, \pi_t^{21} \in \prod \qquad \forall\, s,t \qquad (49)$$

Where $\prod$ is the feasible dual space of $(P_2''(j,\bar{\lambda}))$. Hence, the benders master problem can be formulated as follows.

Benders Master Problem

Let $(\tilde{\pi}_{s1}^6, \tilde{\pi}_{st}^7, \tilde{\pi}_{st}^8, \tilde{\pi}_{st}^9, \tilde{\pi}_t^{10}, \tilde{\pi}^{11}, \tilde{\pi}^{13}, \tilde{\pi}_{st}^{14}, \tilde{\pi}_{st}^{16}, \tilde{\pi}_{st}^{17}, , \tilde{\pi}_{st}^{18}, \tilde{\pi}_{st}^{19}, \tilde{\pi}_{st}^{20}, \tilde{\pi}_t^{21})$ be the optimal value of (P_3), which can be obtained by solving the Benders dual sub-problem $(P_2''(j,\bar{\lambda}))$. Then, the Benders master problem is presented as:

$$P_4 : \min V(\bar{\lambda}) = \phi(\bar{\lambda}) \qquad (50)$$

$$\begin{aligned}
\text{s.t. } &\sum_s \tilde{\pi}_{s1}^6 \Big(I_{0j} + Q_{tj} + A_{stj} - d_{stj} - \sum_{i\in J_j} f_{ij}\left(Q_{ti} + A_{sti}\right) - I_{stj}\Big) \\
&+ \sum_s \sum_t \tilde{\pi}_{st}^7 \Big(I_{st-1j} + Q_{tj} + A_{stj} - d_{stj} - \sum_{i\in J_j} f_{ij}\left(Q_{ti} + A_{sti}\right) - I_{stj}\Big) \\
&- \sum_s \sum_t \tilde{\pi}_{st}^8 (I_{stj} - I_{stj}^+) - \sum_s \sum_t \tilde{\pi}_{st}^9 (-I_{stj} - I_{stj}^-) \\
&+ \sum_t \tilde{\pi}_t^{10} (Q_{tj} - mX_{tj}) - \tilde{\pi}^{11} \Big(t \sum_t \bar{d}_{tj} + \sum_t \sum_{i\in J_j} f_{ij} \bar{d}_{tj} - \sum_t Q_{tj}\Big) \\
&- \tilde{\pi}^{11} \Big(\sum_t \sum_s I_{stj}^- - (1-\gamma)\,|S| \sum_t \bar{d}_{tj}\Big) - \sum_s \sum_t \tilde{\pi}_{st}^{16} (A_{stj} - E_{tj}) \\
&- \sum_s \sum_t \tilde{\pi}_{st}^{17} (-A_{stj} - R_{tj}) - \sum_s \sum_t \tilde{\pi}_{st}^{18} \Big(E_{tj} - m(1 - Y_{stj}) - A_{stj}\Big) \\
&- \sum_s \sum_t \tilde{\pi}_{st}^{19} \Big(R_{tj} - m(1 - Z_{stj}) + A_{stj}\Big) - \sum_s \sum_t \tilde{\pi}_{st}^{20} (E_{tj} - mX_{tj}) \\
&- \sum_t \tilde{\pi}_{st}^{18} (R_{tj} - Q_{tj}) + \sum_t vX_{tj} \le \phi(\bar{\lambda}) \qquad (51)
\end{aligned}$$

$$\sum_s \tilde{\pi}^6_{s1}\Big(I_{0j} + Q_{tj} + A_{stj} - d_{stj} - \sum_{i\in J_j} f_{ij}\left(Q_{ti} + A_{sti}\right) - I_{stj}\Big)$$
$$+ \sum_s \sum_t \tilde{\pi}^7_{st}\Big(I_{st-1j} + Q_{tj} + A_{stj} - d_{stj} - \sum_{i\in J_j} f_{ij}\left(Q_{ti} + A_{sti}\right) - I_{stj}\Big)$$
$$- \sum_s \sum_t \tilde{\pi}^8_{st}(I_{stj} - I^+_{stj}) - \sum_s \sum_t \tilde{\pi}^9_{st}(-I_{stj} - I^-_{stj})$$
$$+ \sum_t \tilde{\pi}^{10}_t(Q_{tj} - mX_{tj}) - \tilde{\pi}^{11}\Big(t\sum_t \bar{d}_{tj} + \sum_t \sum_{i\in J_j} f_{ij}\bar{d}_{tj} - \sum_t Q_{tj}\Big)$$
$$- \tilde{\pi}^{11}(\sum_t \sum_s I^-_{stj} - (1-\gamma)\,|S|\sum_t \bar{d}_{tj}) - \sum_s \sum_t \tilde{\pi}^{16}_{st}(A_{stj} - E_{tj})$$
$$- \sum_s \sum_t \tilde{\pi}^{17}_{st}(-A_{stj} - R_{tj}) - \sum_s \sum_t \tilde{\pi}^{18}_{st}(E_{tj} - m(1 - Y_{stj}) - A_{stj})$$
$$- \sum_s \sum_t \tilde{\pi}^{19}_{st}(R_{tj} - m(1 - Z_{stj}) + A_{stj}) - \sum_s \sum_t \tilde{\pi}^{20}_{st}(E_{tj} - mX_{tj})$$
$$- \sum_t \tilde{\pi}^{18}_{st}(R_{tj} - Q_{tj}) \le 0 \tag{52}$$

s.t.

$$A_{stj} \le \Delta_{stj} - \sum_{\tau|\tau<t} A_{s\tau j} + m(1 - X_{tj} + Z_{stj}) \qquad \forall s,t \tag{53}$$

$$A_{stj} \ge \Delta_{stj} - \sum_{\tau|\tau<t} A_{s\tau j} + m(1 - X_{tj} + Y_{stj}) \qquad \forall s,t \tag{54}$$

$$X_{tj}, Y_{stj}, Z_{stj} \in \{0,1\} \qquad \forall s,t \tag{55}$$

Equation (50) shows the objective function of Benders Master problem. The optimal cut is demonstrated by inequality (51) which is added to the master one when solving the sub-problem to reach the optimality. Therefore, the feasibility cut of the sub-problem is denoted by constraint (52) which is added to the master problem.

Upper Bound Computation

The Lagrangian relaxation heuristic usually provides a lower bound and an upper bound for the MIP problem at each iteration. The solution (per λ) obtained by the Lagrangian problem provides the lower bound. Also, for each optimal implementation of the sub-gradient method, one can find an upper bound of the problem (which must be feasible). For this, let $(\tilde{X}_{tj}(\bar{\lambda}), \tilde{Y}_{stj}(\bar{\lambda}), \tilde{Z}_{stj}(\bar{\lambda})$ (for all scenarios)) be the best optimal solutions for the master problems (P_4) corresponding to the Lagrangian multiplier $(\bar{\lambda})$. Then by putting these values into $P_1(R)$ and solving for these values, the upper bound is reached.

As an overall Procedure of the Hybrid Method, the method starts with the initial value of the Lagrange multiplier (λ). Then, according to the number of scenarios, Lagrangian problems are generated, which are solved by the Benders decomposition method. Benders begins with an initial feasible solution to the Master Benders problem. In order to generate a feasible solution, the problem is solved without any supplementary cuts. This solution to the master problem is given to the sub-problem. Based on the resolution of the sub-issue, an extra cut is formulated and included into the main problem for further iterations. Therefore, in the event that the sub-problem is not feasible, the dual sub-problem becomes unbounded, and an unbounded ray is used to create the infeasibility cut. Furthermore, if the sub-problem is feasible and optimal, a cut is generated based on the optimality. Also, when the upper bound is better, it is updated. This process is repeated for each scenario until the gap between the lower and upper bounds is reduced to an acceptable level. Then the sub-gradient method is used to update the Lagrange multiplier (λ). This process is repeated until the Lagrangian gap between lower and upper bounds is less than a specified value.

6. Computational Results

This section evaluates the computational results of some test problems using the proposed algorithm. Calculations are performed on a computer with Core$^{\text{TM}}$ i5, 3.19 GHz processor, 4.0 GB RAM, and Windows 10 operating system. Additionally, GAMMS mathematical modeling language with ILOG CPLEX 12.6 solver is used to code the hybrid algorithm.

6.1. Analysis of the Model

144 test cases are evaluated to determine the economic viability of uncertainty strategies in different parameter settings. The test cases vary based on service level (SLV= $0.90, 0.98$), demand variation coefficient (VCO = $0.1, 0.3$), time among orders (TBO = $1, 5$), and target capacity utilization (TCU = $0.80, 0.95$), cost ratio between semi-finished and finished products (CPL = $0.1, 0.2, 0.5$) and lack of storage capacity in downstream processes (SSC = $1.00, 0.50, 0.25$). In this study, we examine an alternative manufacturing framework including four fully manufactured items and two partially manufactured products. The distribution of expected demand is uniform throughout time, but the distribution of scenario demand is often Gaussian for different scenarios. To generate samples of scenarios that more accurately satisfy the statistical properties of the distribution, factors of production, production time, and holding costs are the same. Production capacity and storage depend on certain parameters such as demand, time among orders, production time, target capacity utilization and estimated scarcities.

Table 1 shows the objective values of static, and static-dynamic strategies related to the additional costs of semi-finished products and the lack of storage capacities in upstream processes. All test samples meet the required service level for out-of-sample simulations. Notwithstanding the supplementary expenses and limitations in capability, the inherent worth of the static method stays mostly unaltered. The reason for this is that static techniques do not need the retention of partially completed goods. On the other hand, the static-dynamic technique necessitates the implementation of saving in order to accommodate adjustments to the lot-size. Therefore, additional costs and capacity constraints affect the target cost. Additional costs for semi-finished products reduce savings through a static-dynamic strategy. At low costs (CPL = 0.1), a cost reduction of 0.6% is achieved, while at high costs (CPL = 0.5) the cost reduction is reduced to 0.1%. This reduction was in terms of higher costs and a 55% reduction in lot-sizing quantity. A static-dynamic strategy results in cost savings due to the limited maintenance capacity of downstream processes. Even with moderate capacity constraints (SSC = 0.50), cost savings drop to 0.3%. This is a 30% reduction compared to the lower capacity limit (SSC = 1.00). Such a decrease in the capacity of the depot to store semi-finished products reduces the lot-sizing quantity by 48%.

Table 1: Objective values regarding to additional costs of semi-finished products and lack of upstream storage.

	Static\Static 2L-SCLSP			Static-dynamic\Static 2L-SCLSP		
F(SSC)	CPL = 0.1	CPL = 0.2	CPL = 0.5	CPL = 0.1	CPL = 0.2	CPL = 0.5
F(1.00)	42009	42029	42034	41763	41860	41989
F(0.50)	42029	42029	42033	41858	41886	42016
F(0.25)	42027	42032	42034	41865	41917	42009

6.2. Comparison and Validation of the Proposed Method

This section attempts to verify the performance of proposed heuristic algorithm. Based on the number of products and the period, each example is generated based on the method created in the previous section, each of which contains five instances. Thus, 45 test cases are used to test the effectiveness of the algorithm. The results of the calculation of cases are shown in Table 2. 5 different scenarios with probabilities of 0.15, 0.2, 0.3, 0.2 and 0.15 are available for uncertain parameters.

The first column of the table shows the name of each test. $\mathrm{Z_{CP}}$ represents the best objective value corresponding to the feasible solution obtained by the branch and bound method using CPLEX solver for 7200 seconds. CPLEX solver can obtain optimal solutions for small instances. During medium-sized tests, CPLEX did not complete its solution process in time; therefore, the best value obtained from the objective function is given as the best solution found. Also, for large data sets, CPLEX cannot find a feasible solution $P_1(R)$. As, the problem grows, the CPU time of CPLEX increases very quickly. Therefore, a comparison is made with a hybrid metaheuristic based on the imperial competition algorithm (ICA) and the variable neighborhood search (VNS) method recently introduced in [22] for the same model.

Labels $\mathrm{Z_{IV}}$ and $\mathrm{Z_{LB}}$ indicate the best objective value solutions obtained by the hybrid metaheuristic based on ICA-VNS and the proposed heuristic method based on the Lagrange-Benders decomposition method.

Also, $\mathrm{Gap_{IV}}$ ($\mathrm{Gap_{LB}}$) shows the gap between $\mathrm{Z_{CP}}$ and $\mathrm{Z_{IV}}$ ($\mathrm{Z_{LB}}$), calculated by $\frac{\mathrm{Z_{IV}}(\mathrm{Z_{LB}})-\mathrm{Z_{CP}}}{\mathrm{Z_{CP}}}$. Labels $\mathrm{T_{CP}}$, $\mathrm{T_{IV}}$, and $\mathrm{T_{LB}}$ represent the time required to solve the proposed robust model $P_1(R)$ by CPLEX, the ICA-VNS-based metaheuristic approach, and the Lagrangean-Benders-based algorithm, respectively.

Table 2: Computational Results Obtained from Comparison of Methods.

Data set	Z_{CP}	Z_{IV}	Z_{LB}	Gap_{IV}	Gap_{LB}	T_{CP}	T_{IV}	T_{LB}
5*5	354.77	403.56	354.77	0.13	0	68	20	29
5*10	407.53	444.21	407.53	0.09	0	174	31	42
10*10	609.64	660.36	609.64	0.08	0	1081	54	75
10*15	700.26	740.17	708.25	0.05	0.01	3359	77	109
10*20	890.31	872.08	861.39	-0.02	-0.03	> 7200	105	148
20*20	1305.42	1283.56	1243.05	-0.01	-0.04	> 7200	176	323
20*30	-	1569.33	1390.50	-	-	> 7200	201	417
20*40	-	1988.71	1751.82	-	-	> 7200	265	443
20*50	-	2373.82	2016.47	-	-	> 7200	312	641

Based on the results in Table 2, the proposed algorithm can obtain the optimal solution for all small cases, proving its effectiveness to obtain the optimal solution for the cases.

For other cases, no possible solution is available from CPLEX in the specified time. In most cases, the proposed algorithm, and ICA-VNS obtain better solutions than the upper bounds provided by CPLEX. This fact, of course, is confirmed by non-positive gaps. Since the proposed method and ICA-VNS can obtain at least a feasible solution for large cases, their computational time and efficiency are given. Computational results show that the time solutions of the methods are acceptable and increase slightly with the problem size.

Based on the comparative analysis of the Lagrangean-Benders decomposition technique and the ICA-VNS method, it is seen that the ICA-VNS approach exhibits comparatively higher computational efficiency, while the Lagrangean-Benders decomposition method yields a solution of greater precision. Moreover, the present time difference (albeit small) is related to the master sub-problem in the Lagrange-Benders decomposition method. In general, the time required increases as expected, but is still reasonable for such a challenge. These comparisons are shown in Figures 2 and 3.

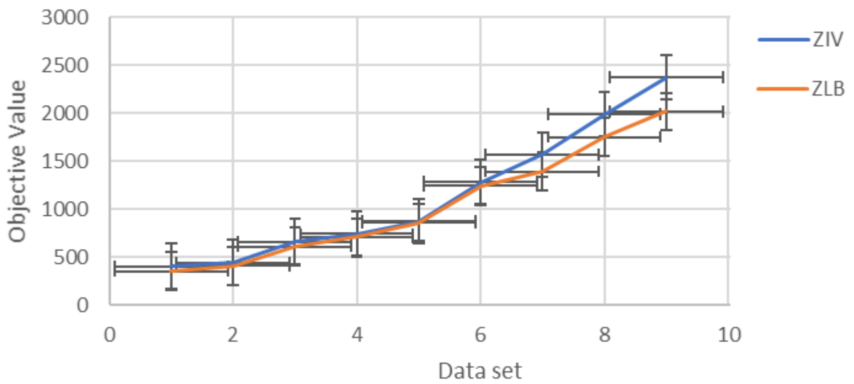

Figure 2: Comparison of the Final Objective Values of the Methods

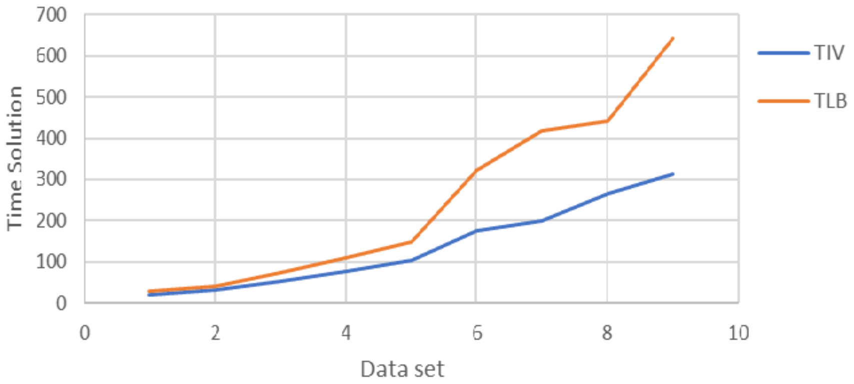

Figure 3: Comparison of the Time Solution of the Methods

7. Conclusion

This study presents a two-stage stochastic capacitated lot sizing problem under static, and static-dynamic uncertainty strategies. Computational analysis shows that the cost of semi-finished products, and the lack of upstream process storage capacity reduce the economic viability of static-dynamic strategy. This result suggests that the multilevel problem formulation, and its associated limitations reduce the usefulness of possible adaptations. However, other reasons, such as production system flexibility and planning efforts should be considered when choosing an uncertainty strategy. Furthermore, as the coefficient of variation significantly affects economic efficiency, more in-depth prediction mistakes should be the focus of future study. This paper provides a robust technique for the suggested stochastic model. Combining two useful techniques, Benders decomposition algorithms and Lagrangean relaxation, is one of the other primary components of the study. Lot-sizing issues are often Np-hard problems.

Lagrangian relaxation of the material some constraints leave the problem structure into decomposable issues. The sub-problems are solved using Benders decomposition. The advantage of applying Benders decomposition is that, using basic algebraic computations, we can solve a well-known complex mixed-integer linear programming problem close to optimality. The sub-gradient optimization procedure is used to solve the Lagrangian relaxation. After obtaining a lower bound through this Lagrangean procedure, the upper bound is obtained as a feasible solution. The procedure is verified through limited empirical investigations on some different problem sizes.

Computational time taken by the developed procedure is noted to be less than that of the commercial solver CPLEX. Furthermore, for the largest-sized problem considered, the branch and bound of CPLEX could not solve the problem and went out of memory, while through the developed procedure, the same problem could be solved in around a reasonable time. This clearly highlights the efficacy of the developed procedure. Also, to show the efficiency of the algorithm, for the higher size of the problem a comparison of the proposed method with an applicable ICA-VNS method in terms of solution quality and time was provided. Computational results confirm the superiority of proposed method for different sizes of the problem. The solution technique applies to any problem, the structure of which is amenable or reducible to such structures.

In a future research, we propose to apply the procedure developed in this work to bi-level problems, which are much closer to reality. Also, while improving the solution by implicit Bender's decomposition procedure, branch and cut may be used instead of it.

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Please cite this article using:

Arsalan Rahmani, Meysam Hosseini, Amir Sahami, Two-stage stochastic capacitated Lot-Sizing problem by Lot-Size adaptation approach, AUT J. Math. Comput., 6(1) (2025) 55-66
https://doi.org/10.22060/AJMC.2023.22698.1184