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Distributed Nash Equilibrium Seeking of Residential Energy Grids over Unreliable Communication Networks Using Predictive Control

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ABSTRACT: In this article, we manage the energy consumption of numerous intelligent homes and charging stations including several electric vehicles in real-time using a computationally efficient predictive controller. The studied scenario is made up of a couple of primary layers. At the low level of the hierarchical framework, users are clustered into different groups based on their vehicles' departure times. Meanwhile, the energy consumption of subordinate users is controlled by multiple aggregators, interaction among which is modeled as an aggregative game. The high-level interactive and distributed control problem can be solved by a predictive controller, wherein the terminal constraints related to the reference energy of each cluster's storage capacity are transferred to the end of the prediction horizon. Additionally, each aggregator can only exchange local data with some neighboring aggregators through an untrustable communication network. As a result of denial of service attacks on the aggregators' network, the strong connectivity of the communication graph may be directly destroyed, leading to performance degradation. To address such an issue, each aggregator reconstructs the attacked information of its neighbors using a linear combination of received data in the last two iterations. Furthermore, a time-of-use pricing tariff whose value is small for faithful households is investigated so that convergence time remains unaltered. Practical examples are simulated to assess the usefulness of the proposed iterative algorithm.

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1- Introduction

Renewable Energy Resources (RESs), for instance, Wind Turbine (WT) along with photovoltaic (PV) systems, are utilized by microgrids to provide power for controllable and uncontrollable electric loads [1]. To consume RESs in time, Electric Vehicles (EVs) are reported as good alternatives to auxiliary power supply equipment capable of storing produced energy for later use [1]. The Demand Side Management (DSM) describes a change in Residential Households' (RHs)' energy consumption, including the aforementioned devices, from their desirable demand profiles to the micro-grid's optimal profile. The primary goal of the DSM is to decline the costs of electricity and boost grid reliability and safety [2]. Surging the number of RHs, resulting in charging innumerable EVs, has a detrimental impact on the electrical grid. Many studies have been conducted on obtaining coordinated policies for the charging and discharging of energy storage devices [e.g., 3, 4]. However, in scenarios wherein many EVs access the power grid, considering an explicit single EV model in optimization may lead to a high computational burden and information transmission [5].

The micro-grid control strategies constitute distributed, decentralized, and centralized strategies. In centralized approaches, a central manager's responsibility is to optimize a centralized objective function by collecting information from all subordinate RHs. In such a structure, significant computation load, system failure when the central controller is unable to work, and privacy violations are reported as disadvantages [3]. To cope with these drawbacks in energy management programs, decentralized and distributed schemes have been investigated, in which game theory as an analytical tool has been employed to model the interaction among self-interested decision-makers in micro-grids [3]. In aggregative games, as one of the well-known game theoretic methods, the objective function of each competitor is influenced by the aggregative impacts of all other competitors in the population. The noncooperative games in this class are claimed to have the decisive advantages of being independent of the number of selfish decision-makers as well as requiring less computational effort [6].

What each RH does in decentralized methods is to determine the optimal energy profile by finding a solution to the optimization problem. In such approaches, RHs do not directly communicate with each other yet try to learn the aggregated term by only relying on a central coordinator gathering the local decisions of subordinate RHs and broadcasting the updated value [7]. The idea has been extended to the multi-population hierarchical scenario in [4],

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where a group of populations exists, each of which is managed by its relevant local coordinator. At the bottom level of the hierarchical structure, the individual selfish policy-makers do not establish communication with one another yet only with their local coordinator to learn local aggregate information. At the higher level, each coordinator directly exchanges the local aggregate information with its neighboring coordinators through a communication network to cooperatively estimate and learn the overall aggregative term. The motivation for proposing distributed methods is that reaching all RHs with the updated value broadcasted by a central coordinator may not be possible, especially in large-scale and geographically distributed systems [4]. In [6], a distributed Nash Equilibrium (NE)-seeking algorithm under an incomplete information setting has been studied where each self-interested policy-maker, reliant only on the received data from some neighbors, estimates all the other players' policies.

Although the utilization of EVs in a micro-grid can lead to cutting the peak and filling the valley in the desired load demand of the RHs, their optimal charging/discharging power is highly related to the arbitrarily driving tendencies of EV owners, which may have unforeseen impacts on the system [8]. In [8], a robust, multi-objective optimization problem comprising such EVs and considering the uncertainty of RES devices' output power has been investigated for minimizing operation cost and carbon emissions. The variation range of uncertain parameters is described by a polyhedral uncertainty set. To deal with uncertainties stemming from RESs, a distributionally robust chance-constrained optimization combining distributionally robust optimization and chance-constrained programming is investigated in [9] for managing the energy consumption of islanded microgrids. In such approaches, a low possibility of constraint violation along with the robustness of the micro-grid operation can be guaranteed.

As another alternative, the Model Predictive Control (MPC)'s receding horizon property has been employed in [10] to solve the optimization based on the most up-to-date accessible data at the starting point of each time slot. In MPC literature [e.g., 10, 11], an MPC method was proposed to provide a feedback mechanism, making the microgrid anti-fragile against volatility. In [12], an MPC strategy has been adopted to re-optimize the objectives based on real-time estimation of the system's states and/or sensors' measurements to dispel the effects of uncertainties such as RES power output. Also, a distributed MPC algorithm has been designed in [13] to compensate for uncertainty and handle constraints. The latter study considered a prediction horizon smaller than the length of the planning horizon. The investigated controller has been computationally efficient since the predictive model has not been required to predict the microgrid's behavior until the end of the planning horizon. However, in some cases, ignoring the terminal constraints may lead to performance degradation. In the present study, an MPC framework is used to tackle the high stochasticity of EVs' behavior, wherein in each time slot the optimization problem is successively optimized with the accessible knowledge of new entrances. Also, the terminal constraints are transferred to the end of the prediction horizon.

Additionally, the present work assumes the price of electricity is calculated according to the time and amount of energy consumption [14]. When energy consumers decide to be in a management program more than analogous consumers and as a result have a longer time of departure, then the policy-maker can impose smaller values of the Time-of-Use (ToU) pricing tariff for these groups. By doing so, the number of iterations will be depicted to remain constant. The shorter the length of time energy purchasers engage in DSM, the higher fee they must pay. In other words, purchasers participating less in DSM should be penalized more. A similar penalizing policy has also been done in [15] by altering the generation cost coefficient. However, by doing so, the number of iterations goes up [14].

Strongly relying on telecommunication networks to pursue a goal of monitoring and control, multi-agent systems are extremely vulnerable to deliberate as well as adversarial cyber-attacks [16]. Denial-of-Services (DoS) attacks among other types of cyber-attacks like man-in-the-middle, false data injection, etc., [17], can disrupt information flow and prevent communication over networks [18]. It is even possible that multi-agent systems cannot reach NE when DoS attacks occur, as self-interested policy-makers cannot exchange information between themselves. In terms of wireless communication, as a common means of communicating and exchanging information in many emerging applications, such attacks can be easily found. Nevertheless, it is mostly healthy communication which is the base of existing work, and has fruitful results on distributed NE-seeking algorithms in aggregative games (e.g., [6]). When distributed NE-seeking algorithms are adversely influenced by DoS attacks, they become less useful, and their work may even fail.

In [19], authors have developed a resilient, adaptive algorithm to cope with multiple cyber-attacks where the weights of the communication graph's edges automatically decline whenever adversarial attackers manipulate communicated information through this link. A signal-to-interference-plus-noise ratio-based dynamic along with a proactive event-triggered communication scheme has been investigated in [20] to avoid occupying communication resources and reduce the detrimental impact of DoS attacks on decentralized secondary energy management of storage devices. In [21], a unified notion of Persistency-of-Data-Flow is proposed to assess the destructive effect of DoS attacks which are multi-layer. Then, as opposed to existing literature reliant on synchronous communication, a local controller with the capability of collecting and processing data in asynchronous settings for each microgrid is designed which can ensure reaching consensus in networked microgrids.

The present study addresses the problem of distributed MPC-based NE-seeking algorithms over communication channels interrupted by DoS attacks. The major contributions of the present study are outlined as follows.

Energy consumers taking part in DSM less than their counterparts are penalized by making them pay a higher cost.

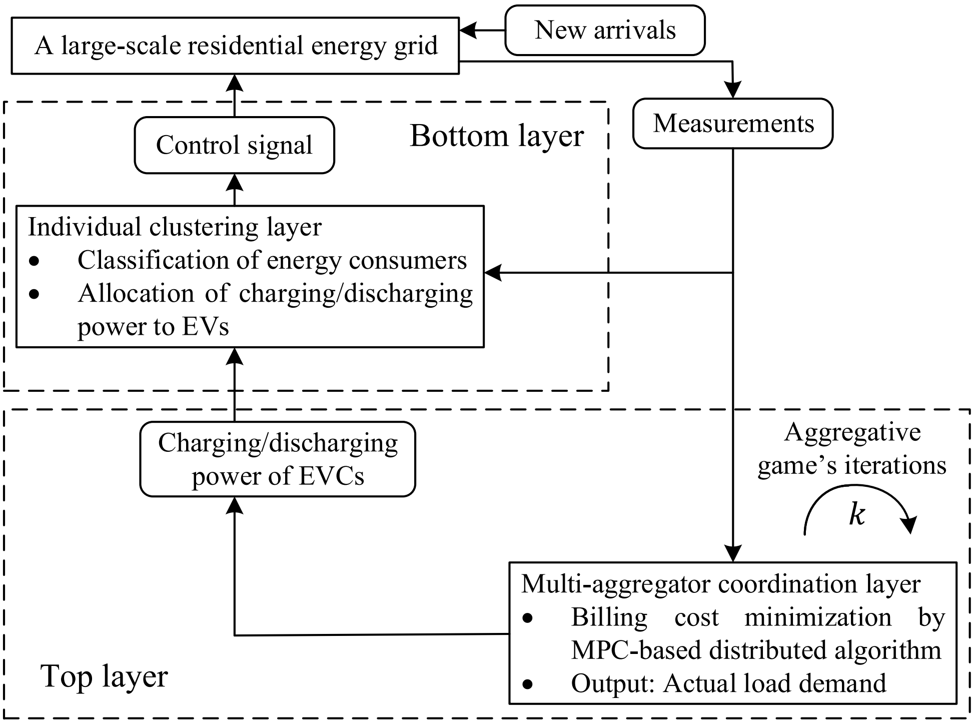

Fig. 1. Hierarchical structure of real-time energy management.

This is done by adding a ToU price tariff to the price function, which has the advantage of unchanged convergence time. We have proven that the iterations required for the convergence of the proposed algorithm do not depend on the ToU price tariff.

The MPC's receding horizon concept is employed as RHs can go into or depart the DSM program all the time, wherein the optimization is resolved at each time slot based on the latest available information. In the paper, different from existing work, a computationally efficient MPC is designed wherein terminal constraints are transferred to the end of the prediction horizon.

Communication links are assumed to be affected by DoS attackers being able to block the communication channel and destroy the validity of the data. To cope with this issue, a linear combination of received data in the last two iterations is considered instead of absent information. In real-world applications, a secure network environment is hardly guaranteed because of the network's openness, which may be filled with various network attacks. It is observed that despite the simplicity of this method, the reconstructed information can act satisfactorily in the absence of blocked transmitted data.

Following is the remainder of the paper's organizations. The mathematical model of the problem and design of the computationally efficient controller is studied in Section 2. Then, Section 3 investigates distributed algorithms over unreliable networks. To demonstrate the efficiency of the proposed method, Section 4 presents simulation results. As a final conclusion, Section 5 provides a summary of the paper.

2- Problem Formulation

The energy management problem for $\mathcal{P}=\{1,2,\ldots,P\}$ aggregators and $\mathcal{N}=\{1,2,\ldots,N\}$ RHs and charging stations is investigated (Fig. 1). The studied scenario is comprised of a couple of levels. In the higher level, by designing an MPC-based distributed method, we manage the energy consumption among multiple aggregators in real-time, and in the bottom layer, a kind of reallocating of power from each Electric Vehicle Cluster (EVC) to its subordinate EVs is performed by a priority-based reallocation algorithm [5]. It is worth noting that some aggregators constitute RHs and others are made up of charging stations including only EVs. At time slot t, it is the set $\mathcal{T}_{i,n}^{t}$ which includes that section of the prediction horizon in which the $EV_{i,n}$ is connected to the charging station and is defined as

$$\mathcal{T}_{i,n}^{t}=\{k_{i,n}^{arr},k_{i,n}^{arr}+1,\ldots,k_{i,n}^{dep}\}. \quad (1)$$

If user n connects the $EV_{i,n}$ to supply equipment prior to the current time slot t , then $k_{i,n}^{arr}=1$ and if this EV is connected until the end of the prediction horizon or even beyond, then $k_{i,n}^{dep}=N_P$.

2- 1- Electric vehicle's dynamic

Mathematically, the $EV_{i,n}$ can be modeled as a discrete-time first-order system with different charging and discharging efficiencies and a couple of decision variables for

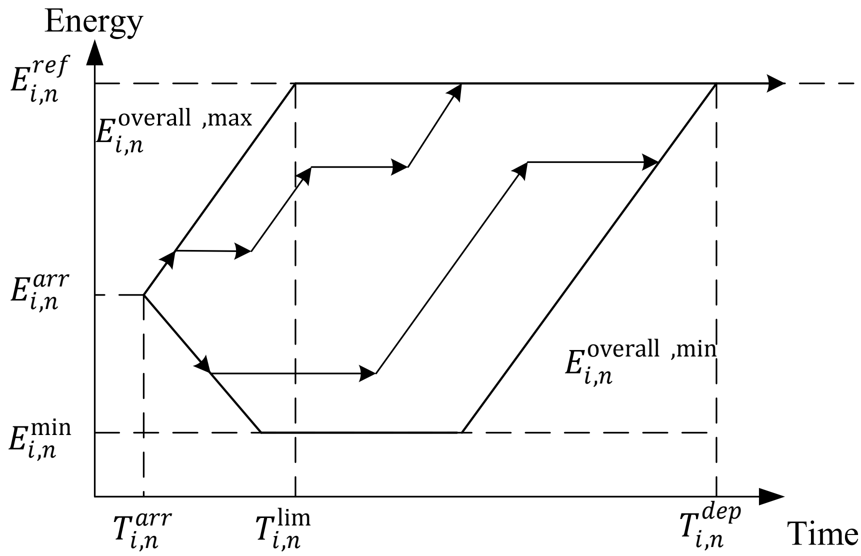

Fig. 2. Cumulative energy boundaries modelling of *EV(i,n)*.

charging and discharging power as,

$$E_{i,n}(t+1) = E_{i,n}(t)$$

$$+t\left(\eta_{i,n}^{ch}u_{i,n}^{ch}(t) - \frac{1}{\eta_{i,n}^{dch}}u_{i,n}^{dch}(t)\right), \forall t \in \mathcal{T}_{i,n}^{t}, \tag{2}$$

where $u_{i,n}^{ch}(t)$ and $u_{i,n}^{dch}(t)$ satisfy the following constraints.

$$\underline{u}_{i,n}^{ch} \le u_{i,n}^{ch}(t) \le \overline{u}_{i,n}^{ch}, \underline{u}_{i,n}^{dch} \le u_{i,n}^{dch}(t) \le \overline{u}_{i,n}^{dch},$$

$$u_{i,n}^{ch}(t)u_{i,n}^{dch}(t) = 0, \forall t \in \mathcal{T}_{i,n}^{t}. \tag{3}$$

According to [22], we can indicate the $EV_{i,n}$' flexibility by defining boundaries for its cumulative energy. As we have illustrated in Fig. 2, the upper bound, $E_{i,n}^{\text{overall, max}}$, happens when EV charging provider instantly charges the $EV_{i,n}$ at maximum charging power as long as $E_{i,n}(t)$ has cumulative charged power $E_{i,n}^{\text{ref}}$ at $T_{i,n}^{\lim}$. Based on the maximum charging power, we can define the boundary time as,

$$T_{i,n}^{\lim} = T_{i,n}^{arr} + \frac{E_{i,n}^{ref} - E_{i,n}^{arr}}{\eta_{i,n}^{ch}\overline{u}_{i,n}^{ch}} \tag{4}$$

Note that if the $EV_{i,n}$ the owner decides to depart the charging station before $T_{i,n}^{\lim}$, it cannot be scheduled. On the other hand, $E_{i,n}^{\text{overall,min}}$ as the lower bound of stored energy is obtained when $EV_{i,n}$ owner discharges the vehicle's battery until $E_{i,n}^{\min}$, where $E_{i,n}(t)$ remains unchanged. This trend is set to continue as long as $EV_{i,n}$ owner get obligated to charge the vehicle's battery to reach $E_{i,n}^{ref}$ at $T_{i,n}^{dep}$. There is also a distinction between an ideal and a more realistic setting in (2). These dynamics can therefore be used to define the stored energy envelope for MPC. For the top of the envelope, starting at $E_{i,n}^{0}$, the energy flexibility grows towards $E_{i,n}^{\max}$ with $E_{i,n}^{\text{MPC,max,slope}}(t+1)$ until it reaches $E_{i,n}^{\text{MPC},ref}$. It then leveled off until $k_{i,n}^{dep}$. At the bottom of the envelope, $E_{i,n}^{\text{MPC,min,slope}}(t+1)$ starts at $E_{i,n}^{0}$ and decreases until $E_{i,n}^{\min}$ is reached. In the bottom right, $E_{i,n}^{\text{MPC},ref\text{,slope}}(t+1)$ denotes the slope by which the stored energy needs to rise, so that $E_{i,n}^{\text{MPC},ref}$ is met at $k_{i,n}^{dep}$. Based on the current value of $E_{i,n}^{0}$, $E_{i,n}^{\text{MPC,min}}$, $E_{i,n}^{\text{MPC},ref}$, and $E_{i,n}^{\text{MPC,max}}$ are formulated as

$$\begin{aligned}&E_{i,n}^{\text{MPC,min,slope}}(t+1)=\\&E_{i,n}^{0}-(t+1-k_{i,n}^{arr})u_{i,n}^{dch}\frac{1}{\eta_{i,n}^{dch}}\\&E_{i,n}^{\text{MPC},ref,\text{slope}}(t+1)=\\&E_{i,n}^{\text{MPC},ref}+(t+1-k_{i,n}^{dep})u_{i,n}^{ch}\eta_{i,n}^{ch}\\&E_{i,n}^{\text{MPC,max,slope}}(t+1)=\\&E_{i,n}^{0}+(t+1-k_{i,n}^{arr})u_{i,n}^{ch}\eta_{i,n}^{ch}\end{aligned}\tag{5}$$

These slopes can be used to mathematically define the upper and lower bounds of $EV_{i,n}$'s energy flexibility within the horizon is as follows,

$$\begin{aligned}&E_{i,n}(t+1)\geq E_{i,n}^{\text{MPC,min}}(t+1)=\\&\max\left(E_{i,n}^{\text{MPC,min,slope}}(t+1),E_{i,n}^{\min},E_{i,n}^{\text{MPC},ref,\text{slope}}(t+1)\right),\end{aligned}\tag{6}$$

$$\begin{aligned}&E_{i,n}(t+1)\leq E_{i,n}^{\text{MPC,max}}(t+1)=\\&\min\left(E_{i,n}^{\text{MPC,max,slope}}(t+1),E_{i,n}^{\max}\right).\end{aligned}\tag{7}$$

2- 2- Electric vehicle cluster equivalent model

To deal with the "curse of dimensionality" which is the result of a large number of users, each of them has an EV, the equivalent model of the l^{th} EVC belonging to the i^{th} aggregator, $EVC_{i,l}$, is described as

$$\begin{aligned}&E_{i,l}(t+1)=E_{i,l}(t)+t(\eta_{i,l}^{ch}p_{i,l}^{ch}(t)-\tfrac{1}{\eta_{i,l}^{dch}}p_{i,l}^{dch}(t))\\&E_{i,l}^{\text{MPC,min}}(t+1)=\textstyle\sum_{n=1}^{n_{i,l}^{t}}E_{i,n}^{\text{MPC,min}}(t+1)\\&E_{i,l}^{\text{MPC,max}}(t+1)=\textstyle\sum_{n=1}^{n_{i,l}^{t}}E_{i,n}^{\text{MPC,max}}(t+1)\\&p_{i,l}^{ch}(t)=\textstyle\sum_{n=1}^{n_{i,l}^{t}}u_{i,n}^{ch}(t),p_{i,l}^{dch}(t)=\textstyle\sum_{n=1}^{n_{i,l}^{t}}u_{i,n}^{dch}(t)\\&\underline{p}_{i,l}^{ch}\leq p_{i,l}^{ch}(t)\leq\overline{p}_{i,l}^{ch},\underline{p}_{i,l}^{dch}\leq p_{i,l}^{dch}(t)\leq\overline{p}_{i,l}^{dch}\\&E_{i,l}^{\text{MPC,min}}(t+1)\leq E_{i,l}(t+1)\leq E_{i,l}^{\text{MPC,max}}(t+1)\end{aligned},\tag{8}$$

$$E_{i,l}(T_{i,l}^{\text{dep}})=E_{i,l}^{\text{ref}}.\tag{9}$$

Remark 1: As EVCs are believed to equivalently model the behavior of numerous EVs if EVCs' charging/discharging powers satisfy (8) and (9), a distinct policy for charging/discharging power which can satisfy (3), (6), and (7) always exists (See [5] and references therein for proof).

2- 3- Model Predictive Control

One of the crucial issues in charging/discharging EVC storage devices is satisfying their expected energy when they leave the charging stations. To satisfy this constraint, their dynamic behaviors have to be predicted until their departure times. This can impose a high computational burden on the predictive controller. In this section, we aim to transfer the terminal constraints (9) to the end of the prediction horizon. To do so, considering the dynamic model of an EVC as (8), one can obtain

$$\begin{aligned}&E_{i,l}(0)+t(\eta_{i,l}^{ch}\sum_{\kappa=1}^{T_{i,l}^{dep}}p_{i,l}^{ch}(\kappa)\\&-\tfrac{1}{\eta_{i,l}^{dch}}\textstyle\sum_{\kappa=1}^{T_{i,l}^{dep}}p_{i,l}^{dch}(\kappa))=E_{i,l}^{ref}.\end{aligned}\tag{10}$$

At the first step of the planning, (10) can be rewritten as,

$$\begin{aligned}&t\left(\eta_{i,l}^{ch}\sum_{\kappa=1}^{N_P}p_{i,l}^{ch}(\kappa)-\frac{1}{\eta_{i,l}^{dch}}\sum_{\kappa=1}^{N_P}p_{i,l}^{dch}(\kappa)\right)=\\&E_{i,l}^{ref}-E_{i,l}(0)-\\&t\left(\eta_{i,l}^{ch}\textstyle\sum_{\kappa=N_P+1}^{T_{i,l}^{dep}}p_{i,l}^{ch}(\kappa)-\tfrac{1}{\eta_{i,l}^{dch}}\textstyle\sum_{\kappa=N_P+1}^{T_{i,l}^{dep}}p_{i,l}^{dch}(\kappa)\right),\end{aligned}\tag{11}$$

where $p_{i,l}^{ch}=\left[p_{i,l}^{ch}(\kappa)\right]$, $p_{i,l}^{dch}=\left[p_{i,l}^{dch}(\kappa)\right]$, and $\kappa\in\mathcal{N}_P$ are the decision variables of the optimization problem. Since $p_{i,l}^{ch}=\left[p_{i,l}^{ch}(\kappa)\right]$ and $p_{i,l}^{dch}=\left[p_{i,l}^{dch}(\kappa)\right]$, $\kappa\in\left[N_P+1,N_P+2,\ldots,T_{i,l}^{dep}\right]$ are outside the prediction horizon, they are replaced by nominal values calculated on the previous day. This can be considered as a weakness of this method, but as planning approaches the end of the horizon, the number of incomputable constants decreases and thus the accuracy of the method increases. Eventually, at time-slot t, the terminal constraints transferred to the end of the prediction horizon can be derived as,

$$t\left(\eta_{i,l}^{ch}\sum_{\kappa=t}^{t+N_P-1}p_{i,l}^{ch}(\kappa)-\frac{1}{\eta_{i,l}^{dch}}\sum_{\kappa=t}^{t+N_P-1}p_{i,l}^{dch}(\kappa)\right)=$$

$$E_{i,l}^{ref}-E_{i,l}(0)-t\left(\eta_{i,l}^{ch}\sum_{\kappa=1}^{t-1}p_{i,l}^{ch}(\kappa)-\frac{1}{\eta_{i,l}^{dch}}\sum_{\kappa=1}^{t-1}p_{i,l}^{dch}(\kappa)+\right. \tag{12}$$

$$\left.\eta_{i,l}^{ch}\sum_{\kappa=t+N_P}^{T_{i,l}^{dep}}p_{i,l}^{ch,\mathrm{nom}}(\kappa)-\frac{1}{\eta_{i,l}^{dch}}\sum_{\kappa=t+N_P}^{T_{i,l}^{dep}}p_{i,l}^{dch,\mathrm{nom}}(\kappa)\right).$$

It is worth noting $p_{i,l}^{ch}=\left[p_{i,l}^{ch}(\kappa)\right]$ and $p_{i,l}^{dch}=\left[p_{i,l}^{dch}(\kappa)\right]$, $\kappa\in[1,2,\ldots,t-1]$ should be replaced with the result of prior hours.

2- 4- Electric load

The electric loads in smart homes can either be controlled or not. The time of those capable of being managed (such as schedulable and reducible) can be altered while unmanageable loads are required to be energized in every time slot; thus, for billing expenditure minimization, they cannot be reduced or omitted [2]. All that energy consumers are expected to do is to report their profile $d_{i,n}^{des}(t)$ to the aggregator which is deciding to shift or reduce controllable loads to minimize the energy cost while satisfying the following constraints,

$$d_i^{UNC}(t)\le d_i^{\mathrm{real}}(t)\le d_i^{des}(t)+d_i^{SH}(t) \qquad \text{(a)}$$

$$d_i^{\mathrm{real}}(t)=p_i^{g}(t)+p_i^{ch}(t) \qquad \text{(b)}$$

$$-p_i^{dch}(t)+p_i^{PV}(t)+p_i^{WP}(t) \tag{13}$$

$$\sum_t\left(d_i^{des}(t)-d_i^{\mathrm{RED}}(t)\right)=\sum_t d_i^{\mathrm{real}}(t) \qquad \text{(c)}$$

Additionally, for stand-alone charging stations, we have,

$$p_i^{b}(t)=p_i^{PV}(t)+p_i^{WP}(t)+p_i^{g}(t). \tag{14}$$

where $p_i^b(t)$ is the power exchanged with the storage unit and is negative for discharging.

2- 5- Price function

The total price of electricity production as well as the local expenditure of an agent such as ToU pricing tariff, are traded off by a local aggregator. It is worth noting the ToU is not necessarily the same for all hours of the day. The total price of generation (contributed by all aggregators) is written below

$$\Pi_G\left(p^g(t)\right)=P\left(\tfrac{1}{P}\textstyle\sum_{i=1}^{P}\pi_i^g(t)\right), \tag{15}$$

where $\pi_i^g=k_c p_i^g$ and k_c is the constant for the cost of electricity, which is adjusted based on the policies adopted by governments or the grid's practical experiences. In addition, there should be some rational reasons, such as profit making to motivate RHs and charging stations to join the DSM program. To do so, ToU tariff is provided to RHs and charging stations. Thus, overall price function of each aggregator can be formulated as

$$\sigma_i^{\mathrm{buy}}\left(p^g(t)\right)=\Pi_G\left(p^g(t)\right)+k_i^{ToU}(t), \tag{16}$$

$$k_{i,l}^{ToU}=\frac{1}{T_{i,l}^{dep}-T_{i,l}^{arr}}\begin{cases}\mathrm{price}_1 & \text{if}\quad T_{i,l}^{arr}\le t<t_1\\ \mathrm{price}_2 & \text{if}\quad t_1\le t<t_2\\ \mathrm{price}_3 & \text{if}\quad t_2\le t<t_3\\ \mathrm{price}_4 & \text{if}\quad t_3\le t\le T_{i,l}^{dep}\end{cases}, \tag{17}$$

$$k_i^{ToU}=\frac{1}{n_i^t}\textstyle\sum_{l\in M_i}n_{i,l}^t k_{i,l}^{ToU}. \tag{18}$$

Note that loyal users should be treated more fairly and charged a lower ToU tariff.

2- 6- The game

The main purpose of each local controller at each time slot t is to diminish its own users' total energy costs. The cost function of the i^{th} aggregator, f_i, which is comprised of users' discomfort because of altering their desired loads and the cost resulting from energy generation, could be written as

$$f_i(x_i,X_{-i})=\sum_{k'\in\mathcal{N}_P}\sigma_i^{\mathrm{buy}}(p^g)(t+k'|t)p_i^g(t+k'|t)$$

$$+\lambda_i\left(d_i^{\mathrm{real}}(t+k'|t)-d_i^{des}(t+k'|t)\right)^2, \tag{19}$$

where $x_i=\left(p_{i,l}^{ch},p_{i,l}^{dch},p_i^g,p_i^b\right)$, $X_{-i}\triangleq\mathrm{col}\left(\left(x_j\right)_{j\in\mathcal{P}\setminus\{i\}}\right)$, $p_{i,l}^{ch}=\left[p_{i,l}^{ch}(\kappa)\right]$, $p_{i,l}^{dch}=\left[p_{i,l}^{dch}(\kappa)\right]$, $p_i^g=\left[p_i^g(\kappa)\right]$, and $p_i^b=\left[p_i^b(\kappa)\right]$, $\kappa\in\mathcal{N}_P$. Energy consumers' dissatisfaction is measured by $\lambda_i\left(\text{cents}/\text{kWh}^2\right)$, which is a weighting parameter.

Eventually, the feasible set of solutions is given by

$$\Omega_i = \{(p_{i,l}^{ch}, p_{i,l}^{dch}, p_i^g, p_i^b) | (8), (12), (13), (14) \text{ are satisfied}\}.$$

Given the fact that the objective of each aggregator (19) depends on the local strategy and the aggregative effects of all other aggregators, such interaction among them is modelled as an aggregative game. The following P inter-dependent optimization problems describe the game.

$$\forall i \in \mathcal{P}: \quad \underset{x_i \in \Omega_i}{\operatorname{argmin}} f_i(x_i, X_{-i}), \tag{20}$$

In the following, the NE of the game, a set of concurrent policies for all problems in (20) will be distributedly computed.

Definition 1. An NE is a collection of policies $X^* = \operatorname{col}\left(\left(x_i^*\right)_{i\in\mathcal{P}}\right)$ such that, for all $i \in \mathcal{P}$,

$$\forall i \in \mathcal{P}: \quad x_i^* \in \underset{x_i \in \Omega_i}{\operatorname{argmin}} f_i(x_i, X_{-i}). \tag{21}$$

Our attention is devoted to convex games. To establish regularity conditions, we make the following assumptions.

Assumption 1. For each $i \in \mathcal{P}$, set Ω_i is convex, closed, and nonempty; for any X_{-i}, $f_i(.,X_{-i})$ is convex and f_i is continuous functions.

In addition, the existence of a solution is assumed as follows.

Assumption 2. In (20), at least one NE is admissible.

There are sufficient conditions in literature for example, in [23] to support the existence of an NE (e.g., compactness of $\Omega = \Omega_1 \times \Omega_2 \times \ldots \times \Omega_P$).

Lemma 1. The optimization problems' solutions in (21) are Lipschitz, which means

$$\|x_i^*(z_i') - x_i^*(z_i'')\| \le L\|z_i' - z_i''\|, \tag{22}$$

where $L = \dfrac{P}{2\lambda_i}\sqrt{\dfrac{\lambda_{\max}}{\lambda_{\min}}}$ for all $i \in \mathcal{P}$.

Proof: See the appendix.

To achieve a feedback control policy, only the first element of the optimal sequences, $p_{i,l}^{ch}$, $p_{i,l}^{dch}$, p_i^g, and p_i^b is implemented according to the receding horizon concept, and the optimization problem is resolvable at time slot $t+1$ as soon as new entrance information is provided.

3- Proposed method

The competitive aggregators are assumed to exchange information over an unreliable network suffering from DoS attacks. Such attacks may destroy the communication graph's connectivity, which is a necessary assumption in proving the convergence of existing iterative algorithms.

3- 1- Communication network

Our algorithm to iteratively find the game' NE will be proposed in this section. In the investigated scenario, each aggregator i has full knowledge of its objective function f_i and feasible set Ω_i. Also, aggregator i lacks knowledge of other aggregators' strategies and the aggregate term and are reliant only on data communicated locally between neighbors over communication graph $\mathcal{G}(\mathcal{I},\varepsilon)$. A pair of ordered edges (i,j) can only belong to the group of links ε if agent i receives data from agent j. The adjacency matrix of communication graph $\mathcal{G}$ is denoted by $W \in \mathbb{R}^{N\times N}$, where in $\omega_{i,j} \triangleq [W]_{i,j}$ with $\omega_{i,j} > 0$ if $(i,j) \in \varepsilon$ and $\omega_{i,j} = 0$, otherwise. Let $D = \operatorname{diag}\left((d_i)_{i\in\mathcal{I}}\right)$ and $L = D - W$ respectively denote the in-degree and Laplacian matrices of $\mathcal{G}$, with $d_i = |\mathcal{N}_i^{in}|$, where $\mathcal{N}_i^{in} = \{j \,|\, (i,j) \in \varepsilon\}$ is the set of in-neighbors of agent i. Furthermore, the communicated message that agent j transmits to agent i via graph $\mathcal{G}$ is denoted by $y_{i,j}^k$.

Assumption 3. The communication graph $\mathcal{G}(\mathcal{I},\varepsilon)$ is strongly connected.

The malicious adversaries are reported to interrupt the communication channel among aggregators. There are multiple adversaries in this work, each of which has separate attack patterns and their active periods are not necessarily the same. Suppose that NE is attainable at iteration k^{NE} and $\tilde{n}_{i,j}^q \triangleq \left[k_{i,j}^q, k_{i,j}^q + \tau_{i,j}^q\right)$ denote the iteration interval of the q^{th} DoS attack on link (i,j), where $k_{i,j}^q$ and $\tau_{i,j}^q$ are starting point and attack length, respectively. During the period, exchanging local data on link (i,j) is interrupted. Consider $k^{\text{NE}} > 1$ and denote $\vartheta_{ij}\left(1,k^{\text{NE}}\right) \triangleq \bigcup_{q\in Q} \tilde{n}_{i,j}^q \cap \left[1,k^{\text{NE}}\right]$ as the series of iteration instants, where communication link (i,j) is denied. To further describe the characteristics of the DoS attack, we make the following assumptions.

Assumption 4. Let $n\left(1,k^{\text{NE}}\right)$ denote the total number of DoS attacks active transmission in $\left[1,k^{\text{NE}}\right]$. There exist scalars N_f and $\tau_f > 0$ such that $n\left(1,k^{\text{NE}}\right) = N_f + \left(k^{\text{NE}} - 1\right)/\tau_f$ is satisfied.

Assumption 5. There exist scalars N_d and $\tau_d > 0$ that satisfy $\left|\vartheta_{ij}\left(1,k^{\text{NE}}\right)\right| \le N_d + \left(k^{\text{NE}} - 1\right)/\tau_d$ [18].

It follows that assumptions 4 and 5 guarantee a limit to the duration and frequency of DoS attacks, which means they cannot occur all the time and at an infinite rate.

3- 2- Distributed NE seeking algorithm

In order for each aggregator to independently minimize its

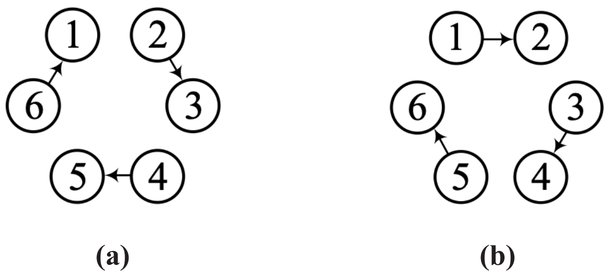

Fig. 2. Multi-aggregator network under DoS attacks. (a) In odd iterations. (b) In even iterations.

objective, it should estimate the aggregative term to relax the interdependencies. To this end, a local estimation of the term has to be appropriately provided by each local aggregator. Let z_i^k denotes the estimation of $\text{avg}(\pi_G)$ whose value is calculated by the i^{th} aggregator. By placing z_i^k in (19) instead of $\text{avg}(\pi_G)$, the game in (21) is then changed to

$$\forall i \in \mathcal{P}: \quad x_i^*(z_i^k) \in \underset{x_i \in \Omega_i}{\operatorname{argmin}} f_i(x_i, z_i^k). \tag{23}$$

Presented below is a summary of the mechanism used in the proposed iterative optimization and estimation algorithm. Firstly, at each iteration k, a local estimation signal $y_{i,j}^k$ is received by each aggregator from its in-neighbors. It is also required to transmit its local estimation, $y_{j,i}^k$ to the out-neighbors which need the local estimate. Then, for each aggregator to optimally respond to the messages received from in-neighbors, they solve the relaxed and independent optimization problem. The Krasnosel'skii-Mann formula is then used by aggregators to calculate the local estimation signal to be used in iteration $k+1$,

$$z_i^{k+1} = \left(1-\alpha^k\right)\left(\sum_{j \in \mathcal{N}_i^{in}} \omega_{i,j}^k \, y_{i,j}^k\right) + \alpha^k \pi_i^{g,*,k}, \tag{24}$$

where $\alpha^k \in (0,1)$, $\forall k \geq 0$ are step sizes. The iterative algorithm continues until the game's NE point is achieved ($\left\| z_i^{k+1} - z_i^k \right\| \leq \varepsilon_{\text{stop}}$).

In the presence of DoS attacks, each aggregator, receiving no information, is supposed to utilize the linear combination of the received information at the last two iterations. More precisely, assume $y_{i,j}^{k-1}$ and $y_{i,j}^{k-2}$ are the information of agent j at iterations $(k-1)$ and $(k-2)$, respectively. Agent i reconstructs the blocked information at iteration k as follows.

$$y_{i,j}^k = \theta y_{i,j}^{k-1} + (1-\theta) y_{i,j}^{k-2} = y_{i,j}^{k-2} + \theta(y_{i,j}^{k-1} - y_{i,j}^{k-2}), \tag{25}$$

where $\theta \in \mathbb{R}$. The range $0 < \theta < 1$ results in the points between $y_{i,j}^{k-1}$ and $y_{i,j}^{k-2}$. However, for $\theta > 1$, the point $y_{i,j}^k$ lies on the line beyond $y_{i,j}^{k-1}$. The proposed distributed NE seeking of residential energy grids over unreliable communication networks is outlined in Algorithm 1.

4- Simulation results

Simulation examinations are conducted to evaluate the successfulness of the proposed MPC-based distributed NE-seeking algorithm in peak shaving when the communication network is interrupted by DoS attacks. A scenario is considered wherein there are 3×10^6 residential households and 3×10^6 stand-alone EVs. There are also six aggregators; three of them are responsible for managing stand-alone charging stations, while others are responsible for managing the power consumption of RHs. To reduce the level of complexity, EVs' participation requests with only two different departure times are accepted by each aggregator. The communication topology under DoS attacks is depicted in Fig. 3.

The charging/discharging efficiencies of each EV are set to 0.95 and $k_c = 10^{-3}$ cents/MWh. The experimental datasets of wind turbines and PV panels investigated in [24] are employed in this simulation. Moreover, the battery capacity of each EV is randomly selected from [9, 11kW] and the initial energy stored in batteries is randomly chosen from [0.9-2kW]. It is also assumed that $\alpha^k = 1/(1+k)$. The algorithm is run on a 2.30 GHz core $i5$ process with 8-GB RAM.

4- 1- Attack-resilient feature of the algorithm

We apply the proposed strategy in (25) to handle the DoS attack depicted in Fig. 3 for several values of θ. The main goal here is to determine the best range for θ for which the

Algorithm 1 Distributed algorithm over an unreliable network

Iteration t

According to the new EV entrances' information, each EVC's boundary information is initialized by aggregators.

Initialization:

Arbitrarily initialize $z_i^1, k \leftarrow 1 \ \forall i \in \mathcal{P}$

Iteration k**:**

Optimization: for each $i \in \mathcal{P}$

$\left(p_{i,l}^{\mathrm{ch},*}(z_i^k), p_{i,l}^{\mathrm{dch},*}(z_i^k), p_i^{\mathrm{g},*}(z_i^k), p_i^{b,*}(z_i^k)\right) \leftarrow \arg\min f_i\left(p_{i,l}^{\mathrm{ch}}, p_{i,l}^{\mathrm{dch}}, p_i^{g}, z_i^k\right),$

Communication and Update: for each $j \in \mathcal{P}$

if the information of neighbor j is interrupted **then**

$y_{i,j}^k = \theta y_{i,j}^{k-1} + (1-\theta) y_{i,j}^{k-2}$

end if

$z_i^{k+1} = (1-\alpha^k)\left(\sum_{j \in \mathcal{N}_i^{in}} \omega_{i,j}^k \, y_{i,j}^k\right) + \alpha^k \pi_i^{g,*,k},$

$k \leftarrow k+1$

Until $\|z_i^{k+1} - z_i^k\| \le \varepsilon_{\mathrm{stop}}$

$\left(p_{i,l}^{\mathrm{ch},*}(t), p_{i,l}^{\mathrm{dch},*}(t), p_i^{g,*}(t), p_i^{b,*}(t)\right) = \left(p_{i,l}^{\mathrm{ch},*}(t|t), p_{i,l}^{\mathrm{dch},*}(t|t), p_i^{g,*}(t|t), p_i^{b,*}(t|t)\right)$

Until $t = T$

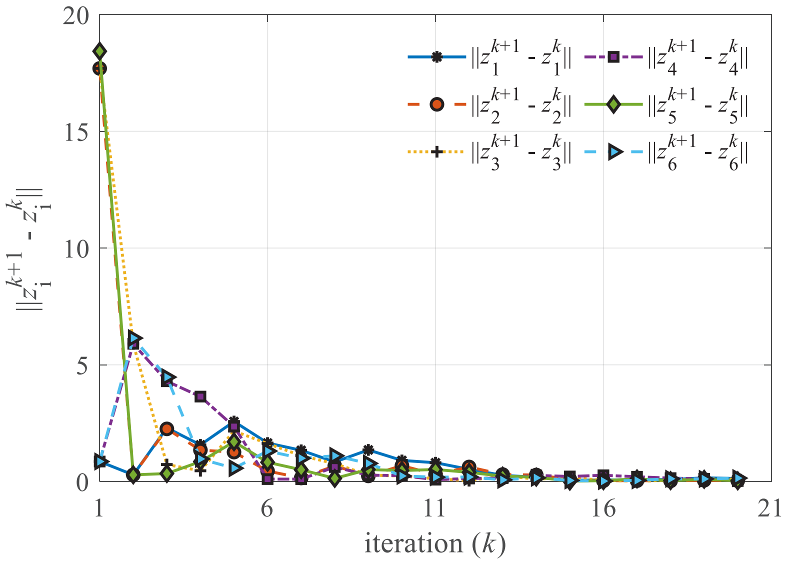

Fig. 4. Convergence of local estimated signals for θ=1.2.

detrimental impacts of DoS attacks are well compensated. The simulation results are illustrated in Table 1 and Fig. 4. Table 1 indicates that the smallest convergence errors is achieved for $\theta = 1$, 1.2, and 1.4. Also, the local estimated signals of all aggregators are depicted in Fig. 4.

As a comparison with existing works [e.g., 4], if DoS attacks occur and the designed controller takes no action, the value of $\max_i z_i^{k+1} - z_i^k$ will be 2.17. This means that in cases where $\theta = 0$ and $\theta = 1.8$, the convergence error is larger than that obtained when the controller takes no action. Therefore, the acceptable range for the parameter θ could be [0.2, 1.6].

As explained earlier, the reason for proposing the ToU tariff idea is to motivate users to participate in the management program longer and penalize those departing the stations sooner than others. Another alternative studied in [15] was the multi-level price policy that defined a different generation cost (k_c) coefficient for different energy consumption levels. In the present study, a multi-level price policy was simulated and results are summarized in Table 1. As concluded from

Table 1. The convergence error for two different scenarios.

θ	$\max_i \lVert z_i^{k+1} - z_i^k \rVert$	
	After increasing k_c	After increasing k_{ToU}
0	**19.3**	**3.91**
0.2	6.47	1.231
0.4	3.20	0.68
0.6	1.26	0.25
0.8	0.87	0.18
1	0.63	**0.136**
1.2	0.73	**0.14**
1.4	0.79	**0.14**
1.6	2.02	0.32
1.8	**23.8**	**3.68**

simulation results, for a more considerable amount of k_c , a high number of iterations are required for convergence. Still, in the proposed method, by changing k^{ToU} , the number of iterations remains constant, and as a result, this method could be much better from a real-time energy management point of view.

4- 2- The effectiveness of the algorithm in peak-shaving

The best power utilization profiles of intelligent homes are ascertained by their relevant aggregators by arranging or eliminating the loads that are controllable. This could also be done by charging or discharging the EVs' storage devices. As described earlier, proposing k^{ToU} is for encouraging aggregators to alter their desirable power consumption hours to different times, and its value is higher in time periods when the consumption of electricity is at its maximum level. More importantly, its value should be adjusted higher to behave fairly towards users participating in the proposed program less than they should.

As illustrated in Fig. 5 and as expected, a successful transfer of schedulable loads from desired peak consumption hours to the time period [11-19] has been achieved. The reason why this happens is that EVs' batteries are mostly discharged in the mentioned period. Moreover, as planning approaches the end of the planning horizon, the accuracy of the algorithm increases. This can be seen in Fig. 5 that the actual load demand for $N_p = 21$ has equaled that for $N_p = 24$ after 11 AM. Furthermore, in the present study, aggregators of stand-alone stations purchase a maximum value of 5.25 MW to satisfy their subordinate consumers' demand, whereas this value is computed as 8.25 MW for the investigated scenario in [13].

The bar charts in Fig. 6 depict that EVCs are not charged and discharged simultaneously. Once the charging or discharging power of clusters at each time slot is computed, they are allocated to the subordinate EVs in the lower level.

5- Conclusion

This paper investigates aggregators that can communicate with a few neighbors through a communication network. Given that the objective function of each aggregator depends on the local strategy and the aggregative impacts of others, aggregative games have been utilized in which agents are cooperatively trying to learn the aggregative term. Moreover, terminal constraints are transferred to the end of the prediction horizon to propose a computationally efficient predictive controller. Then, a linear combination of the received data in the last two iterations is proposed as a promising substitution for the blocked data. It was also illustrated that the destructive impact of DoS attacks is compensated for a specific combination of prior information. Additionally, the ToU pricing tariff is proposed to penalize some energy users leaving the charging station sooner than their counterparts. By doing so, simulation results have indicated that the convergence time remains unchanged.

Nomenclature

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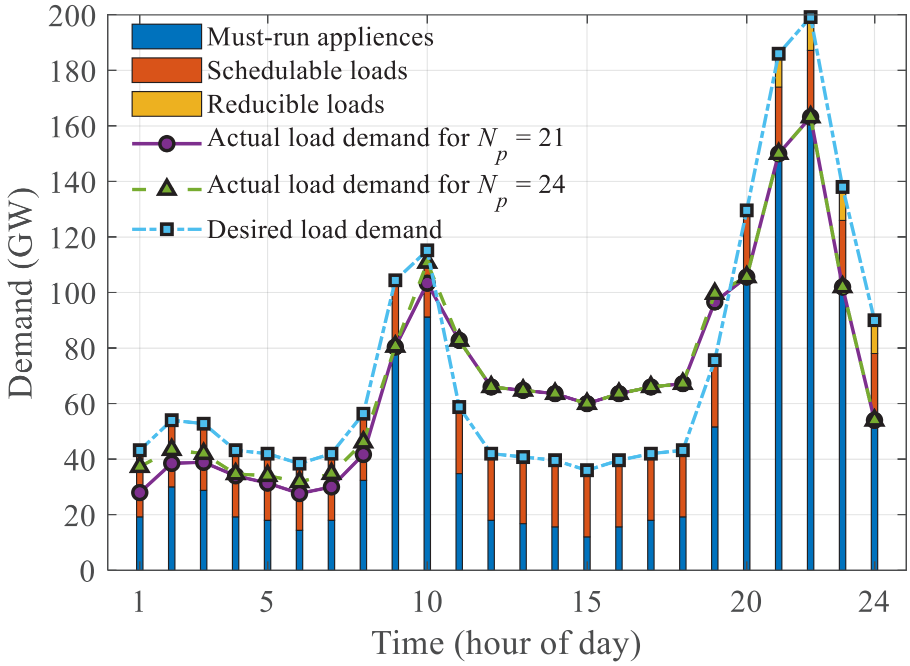

Fig. 5. Residential households' daily electricity demand profiles.

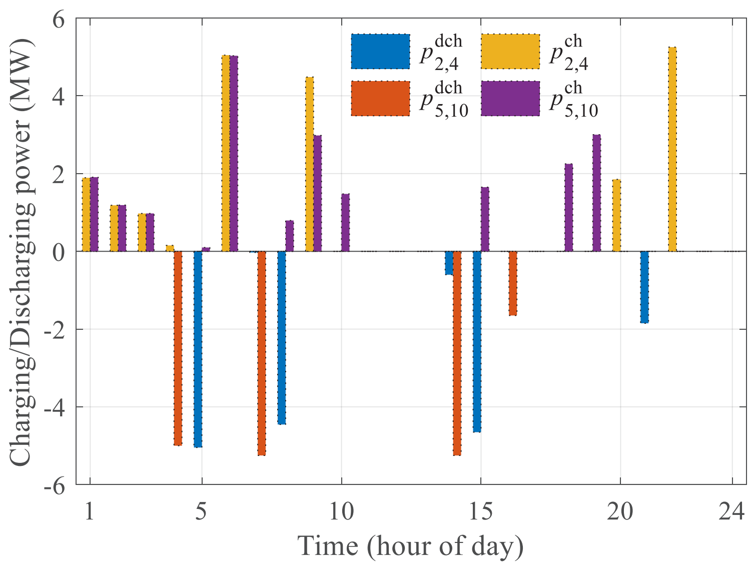

Fig. 6. Charging/discharging power of two random EVC.

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$\mathcal{N}$	Group of residential households ($\lvert\mathcal{N}\rvert = N$).
$\mathcal{P}$	Group of aggregators ($\lvert\mathcal{P}\rvert = P$).
$\mathcal{T}, t$	Set of time-slots $\mathcal{T} = \{1,2,\dots,T\}$ in a day and sample time.
$\mathcal{N}_P$	Group of time slots within the prediction horizon $\mathcal{N}_P = \{1,2,\dots,N_P\}$.
$\mathcal{T}_{i,n}^{t}$	Part of the prediction horizon at time-slot t.
$T_{i,n}^{arr}/T_{i,n}^{dep}$	Arrival and departure time of $EV_{i,n}$.
$n_{i,l}^{t}/n_{i}^{t}$	Number of the residential households at the l^{th} EVC/i^{th} aggregator at time-slot t.
$E_{i,n}/E_{i,l}$	Energy trajectory of the n^{th} EV/l^{th} EVC connected to the i^{th} aggregator.
$E_{i,n}^{\min}/E_{i,n}^{\max}$	Minimum/Maximum attainable energy of $EV_{i,n}$.
$E_{i,n}^{arr}/E_{i,n}^{ref}$	Initial/ expected energy demand of $EV_{i,n}$.
$u_{i,n}^{ch}/u_{i,n}^{dch}$	Charging/discharging power of $EV_{i,n}$.
$\eta_{i,n}^{ch}/\eta_{i,n}^{dch}$	Charging/discharging efficiency of $EV_{i,n}$.
$\overline{u}_{i,n}^{ch}/\overline{u}_{i,n}^{dch}$	Maximum charging/discharging power of $EV_{i,n}$.
$\underline{u}_{i,n}^{ch}/\underline{u}_{i,n}^{dch}$	Minimum charging/discharging power of $EV_{i,n}$.
$E_{i,l}^{\min}/E_{i,l}^{\max}$	Lower/Upper cumulative energy boundary of $EVC_{i,l}$.
$E_{i,l}^{arr}/E_{i,l}^{ref}$	Initial/Expected energy of $EVC_{i,l}$.
$T_{i,l}^{arr}/T_{i,l}^{dep}$	Arrival and departure time of $EVC_{i,l}$.
$p_{i,l}^{ch}/p_{i,l}^{dch}$	Battery charging/discharging power of $EVC_{i,l}$.
$\eta_{i,l}^{ch}/\eta_{i,l}^{dch}$	Battery charging/discharging efficiency of $EVC_{i,l}$.
$\overline{p}_{i,l}^{ch}/\overline{p}_{i,l}^{dch}$	Maximum charging/discharging power of $EVC_{i,l}$.
$\underline{p}_{i,l}^{ch}/\underline{p}_{i,l}^{dch}$	Minimum charging/discharging power of $EVC_{i,l}$.
$p_{i}^{PV}/\mathrm{p}_{\boldsymbol{i}}^{WP}$	PV/WT power generation of the i^{th} aggregator.
p_{i}^{g}/π_{i}^{g}	Power consumption/ local cost of i^{th} aggregator.
d_{i}^{unc}/d_{i}^{SH}	Uncontrollable/schedulable demands the i^{th} aggregator.
$d_{i}^{\mathrm{real}}/d_{i}^{des}$	Actual/desired demand the i^{th} aggregator.
d_{i}^{RED}	Reducible demand the i^{th} aggregator.

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Appendix

Let us consider that the optimization problem (19) is unconstrained and one can conclude that,

$$\tilde{p}_i^g(z_i) = -\frac{Pz_i+k_i^{ToU}}{2\lambda_i} - \tilde{p}_i^R - p_i^{PV} - p_i^{WP} + d_i^{des}, \tag{A.1}$$

Changing the input variable of $\tilde{p}_i^g$ from z_i' to z_i'' we have,

$$\|\tilde{p}_i^g(z_i') - \tilde{p}_i^g(z_i'')\| \leq \frac{P}{2\lambda_i}\|z_i' - z_i''\|, \tag{A.2}$$

Therefore the unconstrained solution of the (19) is $P/2\lambda_i$-Lipshitz. Since objective function $f_i(x_i, X_{-i})$ is a quadratic, the solution of constrained problem (19) is obtained by the projection of the solution of unconstrained problem on the feasible set of the constraints set $x_i \in \Omega_i$. Hence, there exists a matrix Q such that,

$$x_i^* = \mathrm{proj}_{\Omega_i}^Q(\tilde{x}_i) = \underset{x_i\in\Omega_i}{\mathrm{argmin}}\|x_i - \tilde{x}_i\|_Q^2, \tag{A.3}$$

Since the feasible set Ω_i is nonempty, closed, and convex, the projection operator $\mathrm{proj}_{\Omega_i}^Q$ is Lipschitz with constant 1. This means, Changing the input variable of the x_i^* from z_i' to z_i'' we have,

$$\|x_i^*(z_i') - x_i^*(z_i'')\|_Q \leq \|\tilde{x}_i(z_i') - \tilde{x}_i(z_i'')\|_Q. \tag{A.4}$$

Using the property $\lambda_{\min}(\|y\|^2) \leq \|y\|_Q^2 \leq \lambda_{\max}(\|y\|^2)$, one can conclude,

$$\|x_i^*(z_i') - x_i^*(z_i'')\| \leq \sqrt{\frac{\lambda_{\max}}{\lambda_{\min}}}\|\tilde{x}_i(z_i') - \tilde{x}_i(z_i'')\|. \tag{A.5}$$

Then by using $\|a+b\| \leq \|a\| + \|b\|$ and (A.2) we have,

$$\|x_i^*(z_i') - x_i^*(z_i'')\| \leq \frac{P}{2\lambda_i}\sqrt{\frac{\lambda_{\max}}{\lambda_{\min}}}\|z_i' - z_i''\|. \tag{A.6}$$

Finally by assuming $\Lambda_i(z_i)$ as the estimation of the agent i from local agent cost $\pi_i^g = k_c p_i^g$, we have,

$$\|\Lambda_i(z_i') - \Lambda_i(z_i'')\| \leq L\|z_i' - z_i''\|. \tag{A.7}$$

where $L = (Pk_c/2\lambda_i)\sqrt{\lambda_{\max}/\lambda_{\min}}$. Therefore, the mapping Λ_i is both contractive and continuous for the aggregative game for $0 < L < 1$. As a consequence, it can be concluded from the contraction mapping theorems that the proposed iterative algorithm would converge to an exclusive fixed point of the mapping Λ_i ($\Lambda_i(z_i) = z_i$) from any initial conditions. As it can be seen L does not depend on k_i^{ToU} and does depend on k_c.

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