AUT Journal of Electrical Engineering

AUT J. Electr. Eng., 56(2) (2024) 175-190
DOI: 10.22060/eej.2024.22754.5560

The Capacity Inner and Outer Bounds for the Wireless Interference Channel with a Cognitive Relay

Elaheh Asadollahi-Yazdi, Ghoushe Abed Hodtani*, Hossein Khoshbin- Ghomash

Department of Electrical Engineering , Ferdowsi University of Mashhad, Mashhad, Iran

ABSTRACT: Interference channel with a cognitive relay (IFC-CR), as one of the complex channels, consists of two transmitter-receiver pairs, where, one relay knowing some information of transmitted messages cooperates with transmitters to improve the achievable rate region and overall communication performance. The cognitive relays hold a pivotal role in the context of cognitive radio networks (CRN), where efficient spectrum utilization is a paramount concern. To study the impact of a cognitive relay in a wireless interference channel it is necessary to compute the rate region of wireless IFC-CR. In this paper, the capacity inner and outer bounds of IFC-CR known for discrete alphabet and memoryless channels are extended to the continuous alphabet wireless version. Due to High computational complexity, the gap between the outer and inner bounds is determined through Numerical Results. Various scenarios about transmitter power levels and noise variance are considered to encompass a diverse range of real-world conditions. The inner and outer bounds provided in this paper become valuable tools for various aspects of practical analysis, for example, the inner bound can be used to investigate the coverage region and the outer bound for the outage probability, thereby, facilitating practical decision-making in wireless communication system design and optimization.

Review History:

Received: Oct. 19, 2023
Revised: Jan. 27, 2024
Accepted: Jan. 27, 2024
Available Online: Mar. 01, 2024

Keywords:

Cognitive Relay
Interference Channel with a Cognitive Relay
Capacity region Inner and Outer Bounds
Cognitive Radio communications

1- Introduction

Interference channels (IFC) with a Cognitive relay (IFC-CR) is a promising solution for future wireless communication networks, particularly in scenarios with high interference and low signal-to-noise ratio (SNR). It can be applied in various applications, such as smart cities, the Internet of Things (IoT), and multimedia streaming. However, designing an efficient IFC-CR system is a challenging task due to the complex interference and dynamic wireless environment.

IFC-CR consists of an interference channel plus one more node known as the cognitive relay. This cognitive relay will detect all or part of the information from both transmitters and aids them to send messages, which consequently leads to improve data transmission rates.

The IFC-CR was primarily investigated by Sahin et al. [1]. Then, an achievable rate region for the Gaussian IFC-CR was presented with a combination of dirty paper coding (DPC) [2] and generalized beamforming [1], and was subsequently augmented via combining the Han-Kobayashi coding scheme for the general IFC with DPC [3]. In [4], two receiver and transmitter networks with the relay were also considered and the relay merely received the signal from one transmitter and forwarded it to one of the receivers. It was finally confirmed that recognition from even one transmitter could help both receivers.

*Corresponding author's email: ghodtani@gmail.com

Furthermore, an outer bound was initially obtained by [5] for a discrete memoryless (DM) IFC-CR. In, [6] interference was further deemed strong or very strong in one or two receivers and the outer bounds were consequently obtained. In [7], the deterministic DM IFC was introduced and in [8] its capacity region was developed and achieved within a constant gap to the cut-set bound. In [9, 10] the problem was generalized to a semi-deterministic IFC-CR and then the outer and inner bounds were achieved. The capacity region for the symmetric linear deterministic approximation (LDA) was obtained and an outer bound was derived for the general memoryless IFC-CR accordingly [11]. In [12] a new inner bound was further gained for a special case of IFC-CR, termed Gaussian parallel channels with a cognitive relay (PC-CR), but no IFC coefficients were reflected in it. New capacity inner and outer bounds for the IFC-CR have been similarly derived under various conditions [13-15]. In another attempt, the degrees of freedom region of the two-user Gaussian fading IFC-CR with delayed feedback were researched [16]. The classical definition of degrees of freedom (DoF) can be interpreted as the number of independent streams that can be sent in each communication channel in the high SNR regime. Also, a different superposition coding was employed to achieve the rate region in the causal cognitive interference relay channel (CCIRC). The CCIRC is the cognitive interference channel with a full-duplex relay with causal and noisy access to the transmitted signals that relay aids communication between the two users [17].

In [18], an Interference Alignment-based underlay Cognitive Relay Network with the Energy Harvest-enabled relay over Rayleigh fading channels was studied in perfect and imperfect Channel Side Information (CSI) cases. The capacity and Bit Error Rate (BER) performance of the Energy Harvesting-based Decode-and-Forward (EH-based DF) CRN were analyzed by deploying the Time-Switching Relaying (TSR) and Power-Splitting Relaying (PSR) protocols.

As mentioned earlier, there are numerous research studies on this topic, but in general, the capacity region is not known for IFC-CR channels. Various aspects of the analysis of wireless communication performance such as average rate, outage probability, coverage region, energy efficiency, etc. [19-22] need to compute the achievable rate region of wireless IFC-CR. In this paper the interference channel with cognitive relay is investigated, where a relay with some knowledge of the transmitted messages cooperates with the transmitters to improve the achievable rate region, furthermore, the capacity inner and outer bounds and the average achievable rate for the wireless IFC-CR are proved. The generality of the obtained results is then shown by deriving special cases.

Paper Organization: This paper consists of six sections. In Section II, a class of semi-deterministic IFC-CRs is explained, and the capacity inner and outer bounds of this channel are recalled. Next, Section III is devoted to the main work, i.e., proving the capacity inner and outer bounds for the wireless IFC-CR. Numerical results for capacity inner and outer bound and the gap between them are presented in section IV. The conclusion is then provided in Section V. Finally, the proofs are presented in Appendices.

Notations: In this paper, random variables are presented in uppercase letters with values in lowercase ones, and the following notations are employed:

$p(x)$ expresses the probability density function of a random variable X

R denotes the communication rate

$H(X)$ and $H(X|Y)$ respectively stand for the differential entropy of X and the differential entropy of X given Y

$I(0;0)$ represents the mutual information between X and Y and $C(x)=\frac{1}{2}\log(1+x)$

$E(0)$ implies the expectation of random variables $X\sim N(\mu,\sigma_X^2)$ shows the Gaussian distribution with mean μ and variance σ_X^2 and for $X\sim N(0,\sigma_X^2)$, $h(X)=\frac{1}{2}\log(2\pi e\sigma_X^2)$

2- Preliminaries and Study Motivations

In this section, the capacity inner and outer bounds for discrete alphabet and memoryless IFC-CR in [7] are reviewed, then, the study motivations are discussed. Fig 1 illustrates the simplified block diagram of a class of semi-deterministic IFC-CRs.

As shown in Fig 1, the inputs X_1 and X_2 pass through the independent and discrete memoryless channels, $P(S_1|X_1)$ and $P(S_2|X_2)$, generating the side information S_1 and S_2; then, the received signals are generated using two deterministic channels, $Y_1=g_1(X_1,X_c,S_2)$ and $Y_2=g_2(X_2,X_c,S_1)$ [10]. X_c is a relay signal that has some knowledge of the transmitted messages and cooperates with the transmitters.

For Fig 1, inner and outer bounds are derived in [10] and respectively presented in Theorems 1 and 2, where in Q refers to the time-sharing random variable, which is independent of all other variables, and is uniformly distributed on $[1,n]$, and then U_1 and U_2 are auxiliary random variables.

Theorem 1 [10]: The following (R_1,R_2) region in the form of (2a)-(2e) is achievable for the discrete alphabet and memoryless IFC-CR for any input distribution $p(q,x_1,x_2,x_c,u_1,u_2)$ in form (1):

$$p(q,x_1,x_2,x_c,u_1,u_2)= p(q)p(u_1,x_1|q)p(u_2,x_2|q)p(x_c|u_1,u_2,x_1,x_2,q) \tag{1}$$

$$\begin{aligned}
&R_1\le H(Y_1|U_2Q)-H(S_2|U_2X_cQ) && \text{(2a)}\\
&R_2\le H(Y_2|U_1Q)-H(S_1|U_1X_cQ) && \text{(2b)}\\
&R_1+R_2\le H(Y_1|U_1U_2Q)+H(Y_2|Q)- H(S_1|U_1X_cQ)-H(S_2|U_2X_cQ) && \text{(2c)}\\
&R_1+R_2\le H(Y_2|U_2U_1Q)+H(Y_1|Q)- H(S_2|U_2X_cQ)-H(S_1|U_1X_cQ) && \text{(2d)}\\
&R_1+R_2\le H(Y_1|U_1Q)+H(Y_2|U_2Q)- H(S_1|U_1X_cQ)-H(S_2|U_2X_cQ) && \text{(2e)}\\
&2R_1+R_2\le H(Y_1|U_1U_2Q)+H(Y_1|Q)+ H(Y_2|U_2Q)-H(S_1|U_1X_cQ)- 2H(S_2|U_2X_cQ) && \text{(2f)}\\
&R_1+2R_2\le H(Y_2|U_2U_1Q)+H(Y_2|Q)+ H(Y_1|U_1Q)-H(S_2|U_2X_cQ)- 2H(S_1|U_1X_cQ) && \text{(2g)}
\end{aligned} \tag{2}$$

and an outer bound is also derived as follows [10]:

Theorem 2 [10]: If (R_1,R_2) lies in the capacity region of the discrete alphabet and memoryless IFC-CR, then, for any input distribution in form (3):

$$p(q,x_1,x_2,x_c,u_1,u_2)= p(x_1|q)p(x_2|q)p(x_c|x_1,x_2,q)p(u_1,u_2|x_1,x_2,x_c,q) \tag{3}$$

Accordingly, we have:

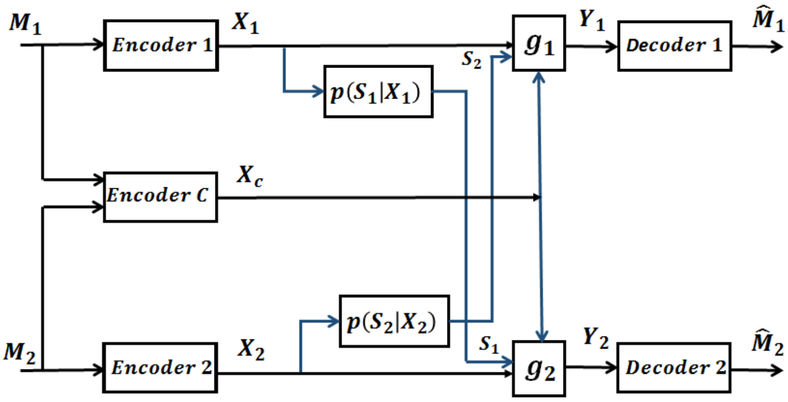

Fig. 1. A class of semi-deterministic IFC-CRs

$$R_1 \le H(Y_1|X_2Q) - H(S_2|X_2Q) \quad \text{(4a)}$$

$$R_2 \le H(Y_2|X_1Q) - H(S_1|X_1Q) \quad \text{(4b)}$$

$$R_1 + R_2 \le H(Y_1|U_1X_2Q) + H(Y_2|Q) - H(S_1|X_1Q) - H(S_2|X_2Q) + I(S_1;X_1|Q) \quad \text{(4c)}$$

$$R_1 + R_2 \le H(Y_2|U_2X_1Q) + H(Y_1|Q) - H(S_2|X_2Q) - H(S_1|X_1Q) + I(S_2;X_2|Q) \quad \text{(4d)}$$

$$R_1 + R_2 \le H(Y_1|U_1Q) + H(Y_2|U_2Q) - H(S_1|X_1Q) - H(S_2|X_2Q) + I(S_1;X_1|Q) + I(S_2;X_2|Q) \quad \text{(4e)}$$

$$2R_1 + R_2 \le H(Y_1|U_1X_2Q) + H(Y_1|Q) + H(Y_2|U_2Q) - H(S_1|X_1Q) - 2H(S_2|X_2Q) + I(S_1;X_1|Q) + I(S_2;X_2|Q) \quad \text{(4f)}$$

$$R_1 + 2R_2 \le H(Y_2|U_2X_1Q) + H(Y_2|Q) + H(Y_1|U_1Q) - H(S_2|X_2Q) - 2H(S_1|X_1Q) + I(S_2;X_2|Q) + I(S_1;X_1|Q) \quad \text{(4g)}$$

(4)

Motivations: Wireless (single-user or multi-user) channels have a continuous alphabet and random fading coefficients, where, the signal-to-noise ratios (SNRs), and hence, the capacity inner and outer bounds are all taken into account as random functions. Wireless communication performance criteria such as outage probability can be further evaluated by using these continuous alphabet capacity bounds. The aforementioned Theorems 1 and 2 give the capacity bounds for discrete alphabet and memoryless discrete alphabet and memoryless IFC-CR [10]. To explore the wireless communication problems, these bounds should be extended to continuous alphabet versions, and the capacity inner and outer bounds for the wireless IFC-CR will be proved in the next section.

3- Capacity Inner and Outer Bounds for the Wireless IFC-CR

In this section, the wireless IFC-CR is considered and the capacity inner and outer bounds are derived.

3- 1- Capacity Inner Bounds for the Wireless IFC-CR

In this sub-section, the discrete alphabet and memoryless results for the IFC-CR are extended to the continuous alphabet wireless version. This analytical endeavor is essential for comprehending the capabilities and limitations of such channels in practical communication scenarios.

The wireless block fading IFC-CR is shown in Fig 2. This model can be expressed in a standard form as follows [13] :

$$Y_1 = |h_{11}|X_1 + |h_{1c}|X_c + |h_{12}|X_2 + Z_1 \quad (5)$$

$$Y_2 = |h_{22}|X_2 + |h_{2c}|X_c + |h_{21}|X_1 + Z_2 \quad (6)$$

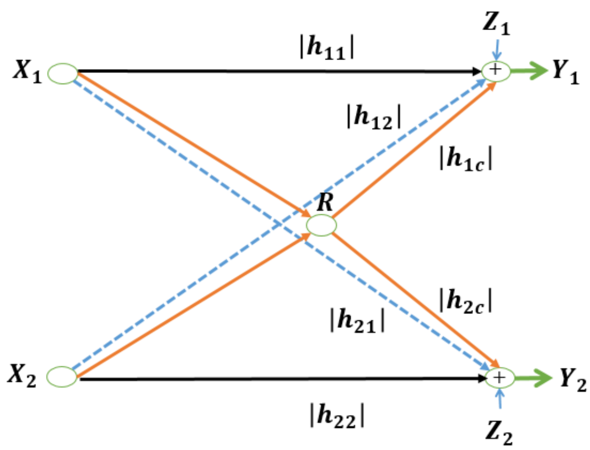

Fig. 2. Wireless IFC-CR

Where $\left|h_{ij}\right|, i, j \in \{1, 2, c, i \neq c\}$ refers to block fading coefficients which are constant in one block and vary randomly from one block to another, while they are known to the relay and the receivers. $Z_1 \sim N(0, N_1)$ and $Z_2 \sim N(0, N_2)$ also denote as the Gaussian noises. The side information, and , can be defined as:

$$S_2 = |h_{12}| X_2 + Z_1 \tag{7}$$

$$S_1 = |h_{21}| X_1 + Z_2 \tag{8}$$

Note: (i) For every wireless receiver y with complex random channel coefficient h known at the receiver, the input X and noise Z ($Y = hX + Z$), Y can be multiplied by $\frac{h^*}{|h|}$

and then, taking a real value of both sides results in real Y equals to $|h|X + Z$ or for real values of Y, X and Z, we have $Y = |h|X + Z$, and hence, all channel coefficients will be as $\left|h_i\right|$ and $\left|h_{i,j}\right|$ to be all real random variables known at the receivers and the relay, e.g., $\left|h_i\right|$ is Rayleigh (Nakagami) and $\left|h_i\right|^2$ is exponential (Gamma) and etc.

(ii) Due to the above explanation, all wireless channel coefficients in all of the following parts (relations 10a-10e, 26a-26e, and the appendices) are considered as variables $\left|h_i\right|$ and $\left|h_{i,j}\right|$, which are assumed to be constants for line of sight channels and random ones for fading or wireless channels resulting in random SNRs.

Theorem 3: The following (R_1, R_2) region is achievable for the wireless IFC-CR, in the form of (10a)-(10e), where the linear channel output-input relations are described by (5)-(8), and Q in (1) is chosen to be a constant in (9) ($p(q_1) = 1$).

$$\begin{aligned} &p(x_1, x_2, x_c, u_1, u_2) = p(u_1, x_1) p(u_2, x_2) p(x_c | u_1, u_2, x_1, x_2) \\ &= p(u_1) p(x_1 | u_1) p(u_2) p(x_2 | u_2) p(x_c | u_1, u_2, x_1, x_2) \end{aligned} \tag{9}$$

$$R_1 \leq \frac{1}{2} \log \left(\frac{\left(|h_{11}| + |h_{1c}|\right)^2 P_1 + 2|h_{1c}|\gamma_1 \left(|h_{11}| + |h_{1c}|\right)\overline{\alpha} P_1 \left(|h_{1c}|\gamma_3 + |h_{12}|\right)^2 \beta P_2 + N_1}{|h_{12}|^2 \beta P_2 + N_1 - \frac{|h_{12}|^2 \gamma_3^2 \beta^2 P_2^2}{\left(1 + \left(2\gamma_1 + \gamma_1^2\right)\overline{\alpha}\right) P_1 + \left(\gamma_3^2 + \left(2\gamma_2\gamma_3 + \gamma_2^2\right)\overline{\beta}\right) P_2}} \right) \tag{10a}$$

$$R_2 \leq \frac{1}{2} \log \left(\frac{\left(|h_{2c}| + |h_{21}|\right)^2 \alpha P_1 + \left(|h_{22}| + |h_{2c}|\gamma_3\right)^2 P_2 + \left(h_{2c}^2 \gamma_2^2 + 2\left(|h_{22}| + |h_{2c}|\gamma_3\right)\left(|h_{2c}|\right)\right)\overline{\beta} P_2 + N_2}{|h_{21}|^2 \alpha P_1 + N_2 - \frac{|h_{21}|^2 \alpha^2 P_1^2}{\left(1 + \left(2\gamma_1 + \gamma_1^2\right)\overline{\alpha}\right) P_1 + \left(\gamma_3^2 + \left(2\gamma_2\gamma_3 + \gamma_2^2\right)\overline{\beta}\right) P_2}} \right) \tag{10b}$$

$$R_1 + R_2 \leq \min\{\tau_1, \tau_2, \tau_3\}$$

$$\tau_1 \le \frac{1}{2}\log\left(\frac{\left(|h_{11}|+|h_{1c}|\right)^2 \alpha P_1 + (|h_{1c}|\gamma_3 + h_{12})^2 \beta P_2 + N_1}{|h_{12}|^2 \beta P_2 + N_1 - \frac{|h_{12}|^2 \gamma_3^2 \beta^2 P_2^2}{\left(1+\left(2\gamma_1+\gamma_1^2\right)\overline{\alpha}\right)P_1 + \left(\gamma_3^2 + \left(2\gamma_2\gamma_3+\gamma_2^2\right)\overline{\beta}\right)P_2}}\right) + \frac{1}{2}\log$$

$$\left(\frac{\left(|h_{2c}|+|h_{21}|\right)^2 P_1 + \left(\left(\gamma_1^2+2\gamma_1\right)|h_{2c}|^2 + 2\gamma_1|h_{21}||h_{2c}|\right)\overline{\alpha}P_1 + \left(|h_{22}|+\gamma_3|h_{2c}|\right)^2 P_2 + \left(\left(\gamma_2^2+2\gamma_2\gamma_3\right)|h_{2c}|^2 + 2|h_{22}||h_{2c}|\gamma_2\right)\overline{\beta}P_2 + N_2}{|h_{21}|^2 \alpha P_1 + N_2 - \frac{|h_{21}|^2 \alpha^2 P_1^2}{\left(1+\left(2\gamma_1+\gamma_1^2\right)\overline{\alpha}\right)P_1 + \left(\gamma_3^2 + \left(2\gamma_2\gamma_3+\gamma_2^2\right)\overline{\beta}\right)P_2}}\right)$$

$$\tau_2 = \frac{1}{2}\log\left(\frac{\left(|h_{11}|+|h_{1c}|\right)^2 P_1 + \left(2\gamma_1|h_{11}||h_{1c}| + \left(\gamma_1^2+2\gamma_1\right)h_{1c}^2\right)\overline{\alpha}P_1 + \left(|h_{12}|+\gamma_3|h_{1c}|\right)^2 P_2 + \left(h_{1c}^2\left(2\gamma_2\gamma_3+\gamma_2^2\right) + 2h_{12}h_{1c}\right)\overline{\beta}P_2 + N_1}{|h_{12}|^2 \beta P_2 + N_1 - \frac{|h_{12}|^2 \gamma_3^2 \beta^2 P_2^2}{\left(1+\left(2\gamma_1+\gamma_1^2\right)\overline{\alpha}\right)P_1 + \left(\gamma_3^2 + \left(2\gamma_2\gamma_3+\gamma_2^2\right)\overline{\beta}\right)P_2}}\right) +$$

$$\frac{1}{2}\log\left(\frac{\left(|h_{2c}|+|h_{21}|\right)^2 \alpha P_1 + (|h_{22}|+\gamma_3|h_{2c}|)^2 \beta P_2 + N_2}{|h_{21}|^2 \alpha P_1 + N_2 - \frac{|h_{21}|^2 \alpha^2 P_1^2}{\left(1+\left(2\gamma_1+\gamma_1^2\right)\overline{\alpha}\right)P_1 + \left(\gamma_3^2 + \left(2\gamma_2\gamma_3+\gamma_2^2\right)\overline{\beta}\right)P_2}}\right)$$

$$\tau_3 = \frac{1}{2}\log\left(\frac{\left(|h_{11}|+|h_{1c}|\right)^2 \alpha P_1 + \left((|h_{12}|+|h_{1c}|\gamma_3)^2 + h_{1c}^2\left(2\gamma_2\gamma_3+\gamma_2^2\right)\overline{\beta}\right)P_2 + N_1}{|h_{12}|^2 \beta P_2 + N_1 - \frac{|h_{12}|^2 \gamma_3^2 \beta^2 P_2^2}{\left(1+\left(2\gamma_1+\gamma_1^2\right)\overline{\alpha}\right)P_1 + \left(\gamma_3^2 + \left(2\gamma_2\gamma_3+\gamma_2^2\right)\overline{\beta}\right)P_2}}\right) + \tag{10c}$$

$$\frac{1}{2}\log\left(\frac{\left(\left(|h_{21}|+|h_{2c}|\right)^2 + |h_{2c}|^2\left(2\gamma_1+\gamma_1^2\right)\overline{\alpha}\right)P_1 + (|h_{22}|+|h_{2c}|\gamma_3)^2 \beta P_2 + N_2}{|h_{21}|^2 \alpha P_1 + N_2 - \frac{|h_{21}|^2 \alpha^2 P_1^2}{\left(1+\left(2\gamma_1+\gamma_1^2\right)\overline{\alpha}\right)P_1 + \left(\gamma_3^2 + \left(2\gamma_2\gamma_3+\gamma_2^2\right)\overline{\beta}\right)P_2}}\right)$$

$$2R_1 + R_2 \le \frac{1}{2}\log\left(\frac{\left(|h_{11}|+|h_{1c}|\right)^2 \alpha P_1 + (|h_{1c}|\gamma_3 + |h_{12}|)^2 \beta P_2 + N_1}{|h_{12}|^2 \beta P_2 + N_1 - \frac{|h_{12}|^2 \gamma_3^2 \beta^2 P_2^2}{\left(1+\left(2\gamma_1+\gamma_1^2\right)\overline{\alpha}\right)P_1 + \left(\gamma_3^2 + \left(2\gamma_2\gamma_3+\gamma_2^2\right)\overline{\beta}\right)P_2}}\right) + \frac{1}{2}\log$$

$$\left(\frac{\left(|h_{11}|+|h_{1c}|\right)^2 P_1 + \left(2\gamma_1|h_{11}||h_{1c}| + \left(\gamma_1^2+2\gamma_1\right)|h_{1c}|^2\right)\overline{\alpha}P_1 + \left(|h_{12}|+\gamma_3|h_{1c}|\right)^2 P_2 + \left(|h_{1c}|^2\left(2\gamma_2\gamma_3+\gamma_2^2\right) + 2|h_{12}||h_{1c}|\right)\overline{\beta}P_2 + N_1}{|h_{12}|^2 \beta P_2 + N_1 - \frac{|h_{12}|^2 \gamma_3^2 \beta^2 P_2^2}{\left(1+\left(2\gamma_1+\gamma_1^2\right)\overline{\alpha}\right)P_1 + \left(\gamma_3^2 + \left(2\gamma_2\gamma_3+\gamma_2^2\right)\overline{\beta}\right)P_2}}\right) +$$

$$\frac{1}{2}\log\left(\frac{\left(\left(|h_{21}|+|h_{2c}|\right)^2 + |h_{2c}|^2\left(2\gamma_1+\gamma_1^2\right)\overline{\alpha}\right)P_1 + (|h_{22}|+|h_{2c}|\gamma_3)^2 \beta P_2 + N_2}{|h_{21}|^2 \alpha P_1 + N_2 - \frac{|h_{21}|^2 \alpha^2 P_1^2}{\left(1+\left(2\gamma_1+\gamma_1^2\right)\overline{\alpha}\right)P_1 + \left(\gamma_3^2 + \left(2\gamma_2\gamma_3+\gamma_2^2\right)\overline{\beta}\right)P_2}}\right) \tag{10d}$$

$$R_1 + 2R_2 \le \frac{1}{2}\log\left(\frac{\left(|h_{2c}|+|h_{21}|\right)^2 \alpha P_1 + (|h_{22}|+\gamma_3|h_{2c}|)^2 \beta P_2 + N_2}{|h_{21}|^2 \alpha P_1 + N_2 - \frac{|h_{21}|^2 \alpha^2 P_1^2}{\left(1+\left(2\gamma_1+\gamma_1^2\right)\overline{\alpha}\right)P_1 + \left(\gamma_3^2 + \left(2\gamma_2\gamma_3+\gamma_2^2\right)\overline{\beta}\right)P_2}}\right) + \frac{1}{2}\log$$

$$\left(\frac{\left(|h_{2c}|+|h_{21}|\right)^2 P_1 + \left(\left(\gamma_1^2+2\gamma_1\right)h_{2c}^2 + 2\gamma_1|h_{21}||h_{2c}|\right)\overline{\alpha}P_1 + \left(|h_{22}|+\gamma_3|h_{2c}|\right)^2 P_2 + \left(\left(\gamma_2^2+2\gamma_2\gamma_3\right)|h_{2c}|^2 + 2|h_{22}||h_{2c}|\gamma_2\right)\overline{\beta}P_2 + N_2}{|h_{21}|^2 \alpha P_1 + N_2 - \frac{|h_{21}|^2 \alpha^2 P_1^2}{\left(1+\left(2\gamma_1+\gamma_1^2\right)\overline{\alpha}\right)P_1 + \left(\gamma_3^2 + \left(2\gamma_2\gamma_3+\gamma_2^2\right)\overline{\beta}\right)P_2}}\right) +$$

$$\frac{1}{2}\log\left(\frac{\left(|h_{11}|+|h_{1c}|\right)^2 \alpha P_1 + |h_{1c}|^2\left(2\gamma_2\gamma_3+\gamma_2^2\right)\overline{\beta}P_2 + (|h_{12}|+|h_{1c}|\gamma_3)^2 P_2 N_1}{|h_{12}|^2 \beta P_2 + N_1 - \frac{|h_{12}|^2 \gamma_3^2 \beta^2 P_2^2}{\left(1+\left(2\gamma_1+\gamma_1^2\right)\overline{\alpha}\right)P_1 + \left(\gamma_3^2 + \left(2\gamma_2\gamma_3+\gamma_2^2\right)\overline{\beta}\right)P_2}}\right) \tag{10e}$$

Inputs, and relay signals, are defined as linear combinations based on (9):

$$X_1 = U_1 + \sqrt{\frac{\alpha P_1}{P_{V_1}}} V_1 \tag{11}$$

$$X_2 = U_2 + \sqrt{\frac{\beta P_2}{P_{V_2}}} V_2 \tag{12}$$

$$X_c = X_1 + \gamma_1 U_1 + \gamma_2 U_2 + \gamma_3 X_2 \tag{13}$$

which U_1 V_1 , U_2 , and V_2 are auxiliary random variables.

There is also an inner bound for each input distribution which in this paper is obtained by the input Gaussian distribution in Theorem 3, $X_1 \sim N(0,P_1)$, $X_2 \sim N(0,P_2)$, $X_c \sim N(0,P_c)$, $U_1 \sim N(0,P_{U_1})$, $U_2 \sim N(0,P_{U_2})$, as well as $V_1 \sim N(0,P_{V_1})$ and $V_2 \sim N(0,P_{V_2})$.

Eq. (14) is obtained from (11) to (13).

$$\begin{aligned} &P_{U_1} = \bar{\alpha} P_1, P_{U_2} = \bar{\beta} P_2, P_{X_c} = \\ &(1 + (2\gamma_1 + \gamma_1^2)\bar{\alpha}) P_1 + \\ &(\gamma_3^2 + (2\gamma_2\gamma_3 + \gamma_2^2)\bar{\beta}) P_2, \\ &0 \le \alpha, \beta, \gamma_1, \gamma_2, \gamma_3 \le 1 \ and \\ &(\bar{\alpha} = 1 - \alpha \ and \ \bar{\beta} = 1 - \beta) \end{aligned} \tag{14}$$

Proof: The complete proof of Theorem 3 is presented in Appendix A.

3- 2- Capacity Outer Bound for the Wireless IFC-CR

In this subsection, the equations of the discrete alphabet and memoryless IFC-CR capacity outer bound (4a)-(4g) are extended to the continuous alphabet. The model is considered as Fig 2 and the new capacity outer bound is derived for the wireless IFC-CR.

To obtain the outer bound, the inequality that the entropy of all random variables with limited power is less than or equal to the entropy of the Gaussian variable is used (eq. (15)).

$$\begin{aligned} &\forall i, E\left[|X_i|^2\right] \le P_i, P_i \in R^+ we \ \ have: \\ &h(X_i) \le h\left(X \sim N\left(\mu, \sigma_X^2\right)\right) = \frac{1}{2}\log\left(2\pi e \sigma_X^2\right) \end{aligned} \tag{15}$$

Therefore, the input distribution is arbitrary for obtaining the outer bound and the Gaussian distribution is only utilized to calculate the maximum entropy.

According to the discrete alphabet, the input distribution $p(x_1, x_2, x_c, u_1, u_2)$ is as follows (16):

$$\begin{aligned} &p(x_1, x_2, x_c, u_1, u_2) = \\ &p(x_1)p(x_2)p(x_c|x_1, x_2) \\ &p(u_1|x_1, x_2, x_c)p(u_2|x_1, x_2, x_c, u_1) \end{aligned} \tag{16}$$

Eq. (16) can be extended to the continuous alphabet version as follows:

$$X_c = X_1 + a_1 X_2 \tag{17}$$

$$U_1 = X_1 + b_2 X_2 + b_c X_c + W_1 \tag{18}$$

$$U_2 = X_1 + c_1 U_1 + c_2 X_2 + c_c X_c + W_2 \tag{19}$$

Where, W_1 and W_2, are auxiliary random variables, which are defined as:

$$W_1 \sim N(0, P_{W_1}), W_2 \sim N(0, P_{W_2}) \tag{20}$$

By replacing X_c in equations (18)-(19), it can be rewritten as:

$$U_1 = \beta_1 X_1 + \beta_2 X_2 + W_1 \tag{21}$$

$$U_2 = \delta_1 X_1 + \delta_2 X_2 + W_2 \tag{22}$$

Where, $0 \le a_1, b_2, b_c, c_1, c_2, c_c \le 1$, $\beta_1 = 1 + b_c$, $\beta_2 = b_2 + b_c a_1$, $\delta_1 = 1 + c_1 + b_c c_1 + c_c$ and $\delta_2 = c_1 b_2 + c_1 b_c a_1 + c_2 + c_c a_1$.

From equations (17), (21), and (22), X_c, U_1, U_2 powers can be obtained as:

$$P_{X_c} = P_1 + a_1^2 P_2 \tag{23}$$

$$P_{U_1} = \beta_1^2 P_1 + \beta_2^2 P_2 + P_{W_1} \tag{24}$$

$$P_{U_2} = \delta_1^2 P_1 + \delta_2^2 P_2 + P_{W_2} \tag{25}$$

Theorem 4: By considering the (17)-(25), then (R_1, R_2) lies in the capacity region of the wireless IFC-CR if the (R_1, R_2) satisfies (26a)-(26e).

Proof: The complete proof is presented in Appendix B.

$$R_1 \le C\left(\frac{\left(|h_{11}|+|h_{1c}|\right)^2 P_1}{N_1}\right) \tag{26a}$$

$$R_2 \le C\left(\frac{\left(|h_{22}|+|h_{2c}|a_1\right)^2 P_2}{N_2}\right) \; R_1 + R_2 \le \min\left\{\vartheta_1, \vartheta_2, \vartheta_3\right\} \tag{26b}$$

$$\vartheta_1 = C\left(\frac{\left(|h_{11}|+|h_{1c}|\right)^2 P_1 - \frac{\left(|h_{11}|+|h_{1c}|\right)^2 \beta_1^2 P_1^2}{\beta_1^2 P_1 + \beta_2^2 P_2 + P_{W_1}}}{N_1}\right) +$$
$$C\left(\frac{\left(|h_{21}|+|h_{2c}|\right)^2 P_1 + \left(|h_{22}|+a_1|h_{2c}|\right)^2 P_2}{N_2}\right) + C\left(\frac{|h_{21}|^2 P_1}{N_2}\right)$$

$$\vartheta_2 = C\left(\frac{\left(|h_{22}|+a_1|h_{2c}|\right)^2 P_2 - \frac{\left(|h_{22}|+a_1|h_{2c}|\right)^2 \gamma_2^2 P_2^2}{\gamma_1^2 P_1 + \gamma_2^2 P_2 + P_{W_2}}}{N_2}\right) +$$
$$C\left(\frac{\left(|h_{11}|+|h_{1c}|\right)^2 P_1 + \left(|h_{12}|+a_1|h_{1c}|\right)^2 P_2}{N_1}\right) + C\left(\frac{|h_{12}|^2 P_2}{N_1}\right)$$

$$\vartheta_3 = C\left(\frac{\left(|h_{11}|+|h_{1c}|\right)^2 P_1 + \left(|h_{12}|+a_1|h_{1c}|\right)^2 P_2 - \frac{\left(\beta_1\left(|h_{11}|+|h_{1c}|\right)P_1 + \beta_2\left(|h_{12}|+a_1|h_{1c}|\right)P_2\right)^2}{\beta_1^2 P_1 + \beta_2^2 P_2 + P_{W_1}}}{N_1}\right) +$$
$$C\left(\frac{\left(|h_{21}|+|h_{2c}|\right)^2 P_1 + \left(|h_{22}|+a_1|h_{2c}|\right)^2 P_2 - \frac{\left(\gamma_1\left(|h_{21}|+|h_{2c}|\right)P_1 + \gamma_2\left(|h_{22}|+a_1|h_{2c}|\right)P_2\right)^2}{\gamma_1^2 P_1 + \gamma_2^2 P_2 + P_{W_2}}}{N_2}\right) + \tag{26c}$$
$$C\left(\frac{|h_{21}|^2 P_1}{N_2}\right) + C\left(\frac{|h_{12}|^2 P_2}{N_1}\right)$$

$$2R_1 + R_2 \le C\left(\frac{\left(|h_{11}|+|h_{1c}|\right)^2 P_1 - \frac{\beta_1^2\left(|h_{11}|+|h_{1c}|\right)^2 P_1^2}{\beta_1^2 P_1 + \beta_2^2 P_2 + P_{W_1}}}{N_1}\right) +$$
$$C\left(\frac{\left(|h_{11}|+|h_{1c}|\right)^2 P_1 + \left(|h_{12}|+a_1|h_{1c}|\right)^2 P_2}{N_1}\right) +$$
$$C\left(\frac{\left(|h_{21}|+|h_{2c}|\right)^2 P_1 + \left(|h_{22}|+a_1|h_{2c}|\right)^2 P_2 - \frac{\left(\gamma_1\left(|h_{21}|+|h_{2c}|\right)P_1 + \gamma_2\left(|h_{22}|+a_1|h_{2c}|\right)P_2\right)^2}{\gamma_1^2 P_1 + \gamma_2^2 P_2 + P_{W_2}}}{N_2}\right) + \tag{26d}$$
$$C\left(\frac{|h_{21}|^2 P_1}{N_2}\right) + C\left(\frac{|h_{12}|^2 P_2}{N_1}\right)$$

$$R_1+2R_2 \le C\left(\frac{\left(|h_{22}|+a_1|h_{2c}|\right)^2 P_2-\frac{\gamma_2^2\left(|h_{22}|+a_1|h_{2c}|\right)^2 P_2^2}{\gamma_1^2 P_1+\gamma_2^2 P_2+P_{W_2}}}{N_2}\right)+ C\left(\frac{\left(|h_{21}|+|h_{2c}|\right)^2 P_1+\left(|h_{22}|+a_1|h_{2c}|\right)^2 P_2}{N_2}\right)+ C\left(\frac{\left(|h_{11}|+|h_{1c}|\right)^2 P_1+\left(|h_{12}|+a_1|h_{1c}|\right)^2 P_2-\frac{\left(\beta_1\left(|h_{11}|+|h_{1c}|\right)P_1+\beta_2\left(|h_{12}|+a_1|h_{1c}|\right)P_2\right)^2}{\beta_1^2 P_1+\beta_2^2 P_2+P_{W_1}}}{N_1}\right)+ C\left(\frac{|h_{12}|^2 P_2}{N_1}\right)+C\left(\frac{|h_{21}|^2 P_2}{N_2}\right) \tag{26e}$$

4- Numerical Results

To investigate the gap between the inner and outer bounds (subtraction of Eq. (10c) and Eq. (26c)), we obtain Numerical Results using MATLAB and investigate the results with numerical assumptions for the different parameters used in Eqs (10a)-(10c) and Eqs (26a)-(26c) as explained in the Note after Eq. (8).

$$\begin{aligned} &P_1=10, P_2=10, N_1=2, N_2=2,\\ &\alpha=0.4, \beta=0.4, \gamma_1=0.4, \gamma_2=0.4, \gamma_3=0.4\\ &h_{11}=0.5, h_{1c}=0.5, h_{12}=0.5, h_{2c}=0.5, h_{22}=0.5\\ &P_{W_1}=1, P_{W_2}=1, a_1=0.4, b_c=0.4, c_1=0.4, c_c=0.4 \end{aligned} \tag{27}$$

In this section, with these above-mentioned parameters, we compute the inner and outer bounds. To perform the investigation we consider two different scenarios, variable transmitter powers scenario and variable noise variances. Because "SNR" is the ratio of transmitter average power to noise average power or noise variance (noise with zero mean). Therefore, it is possible to plot R in terms of SNR, where, the figure in terms of different values of SNR is equivalent to the figure for constant (varying) noise variance and varying (constant) power. In other words, $R(SNR)$ for constant noise variance N is the same as $R(P)$, $R(SNR)$ the figure is the same as $R(N)$ for constant P.

A.Variable Transmitter Power

To investigate the effect of transmitter power on the inner and outer bound, we considered the numerical assumptions of (Eq. (27)) constant and changed the transmitter power. In this situation, the inner and outer bound was computed in Fig 3.

Fig.3 shows that with the increase in transmitter power, the capacity of inner and outer bounds are increased.

B.Variable noise variance

To investigate the effect of noise variance on the inner and outer bound we considered the numerical assumptions of (Eq. (27)) constant and changed the noise variance. In this situation, the inner and outer bound was computed in Fig 4.

Fig 4 shows that with the increase in noise variance, the capacity inner and outer bounds are decreased.

5- Conclusion

In this paper, we have extended the discrete alphabet and memoryless results to derive the inner and outer capacity bounds for the wireless Interference Channel with Cognitive Relay (IFC-CR). Numerical Results using MATLAB, conducted under variable transmitter power and noise variance scenarios, showing the gap between capacity bounds. For future work, considering the channel coefficients as various correlated or independent random variables, wireless communications performances can be studied.

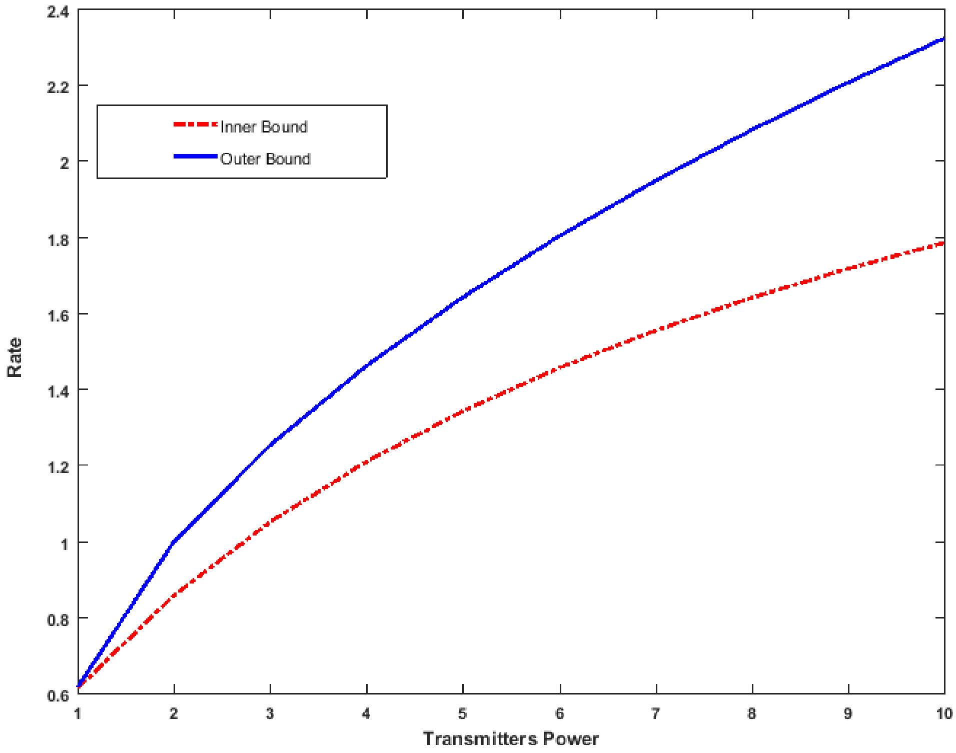

Fig. 3. Inner and outer bound for different transmitter power ($N_1 = 2, N_2 = 2$ **)**

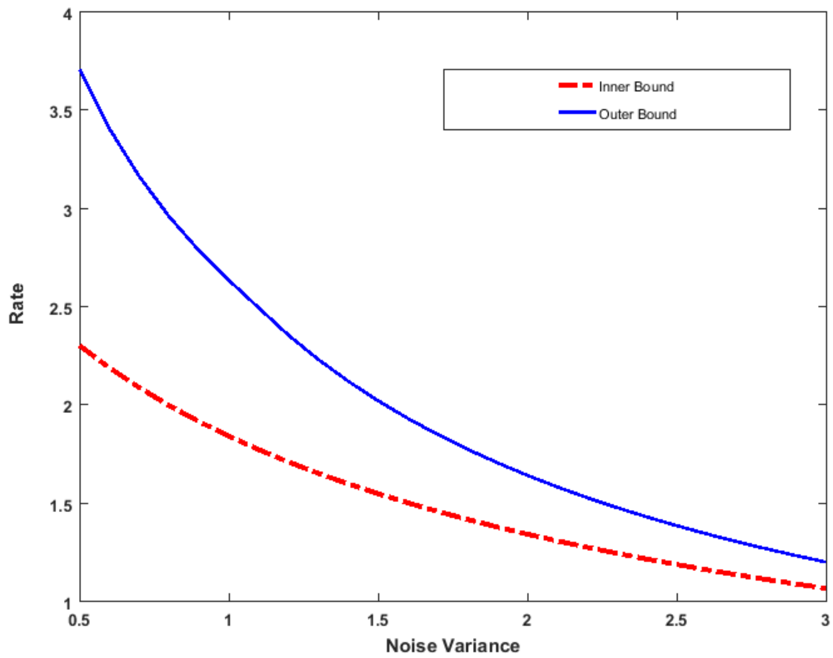

Fig. 4. Inner and outer bound for different noise variance ($P_1 = 10, P_2 = 10$ **)**

Appendices

APPENDIX A: PROOF OF THEOREM 3

Proof of (10a):

Here, Q is chosen to be constant. Considering the theorem assumption and substituting equation (13) in equation (5) as well as equations (5) and (7) in equation (2a), it yields:

$$R_1 \le h\left(Y_1|U_2\right) - h\left(S_2|U_2X_c\right) = h\left((|h_{11}|+|h_{1c}|)X_1 + |h_{1c}|\gamma_1 U_1 + \left(|h_{1c}|\gamma_3 + |h_{12}|\right)\sqrt{\frac{\beta P_2}{P_{V_2}}}V_2 + Z_1 \right) - h\left(|h_{12}|\sqrt{\frac{\beta P_2}{P_{V_2}}}V_2 + Z_1 | X_1 + \gamma_1 U_1 + \gamma_2 U_2 + \gamma_3 X_2 \right) \tag{28}$$

Since the inputs and noise distributions are Gaussian and the variables U and V are independent of each other, and knowing that the sum of some functions with normal distribution is a function with normal distribution. Accordingly, using $X \sim N(0,\sigma_X^2)$, $h(X) = \frac{1}{2}\log\left(2\pi e\sigma_X^2\right)$, there is:

$$R_1 \le \frac{1}{2}\log 2\pi e\left(((|h_{11}|+|h_{1c}|)^2 + 2(|h_{11}|+|h_{1c}|)|h_{1c}|\gamma_1\bar{\alpha}\right)P_1 + \left(|h_{1c}|\gamma_3 + |h_{12}|\right)^2 \beta P_2 + N_1 - \frac{1}{2}\log 2\pi e\sigma^2_{|h_{12}|\sqrt{\frac{\beta P_2}{P_{V_2}}}V_2 + Z_1 | X_1 + \gamma_1 U_1 + \gamma_2 U_2 + \gamma_3 X_2} \tag{29}$$

Noting that,

$$\sigma^2_{K_1|K_2} = E\left(K_1 - E\left(K_1|K_2\right)\right)^2, \hat{K}_1 = E\left(K_1|K_2\right) = \theta K_2, \theta = \frac{E\left[K_1K_2\right]}{E\left[K_2^2\right]} \tag{30}$$

And assuming $K_1 = |h_{12}|\sqrt{\frac{\beta P_2}{P_{V_2}}}V_2 + Z_1$, $K_2 = X_1 + \gamma_1 U_1 + \gamma_2 U_2 + \gamma_3 U_2 + \gamma_3\sqrt{\frac{\beta P_2}{P_{V_2}}}V_2$, there is:

$$\theta = \frac{E\left[K_1K_2\right]}{E\left[K_2^2\right]} = \frac{|h_{12}|\gamma_3\beta P_2}{\left(1+\left(2\gamma_1+\gamma_1^2\right)\bar{\alpha}\right)P_1 + \left(\gamma_3^2 + \left(2\gamma_2\gamma_3+\gamma_2^2\right)\bar{\beta}\right)P_2} \tag{31}$$

$$\sigma^2_{K_1|K_2} = E\left[K_1^2\right] + \frac{\left(|h_{12}|\gamma_3\beta P_2\right)^2}{\left(\left(1+\left(2\gamma_1+\gamma_1^2\right)\overline{\alpha}\right)P_1+\left(\gamma_3^2+\left(2\gamma_2\gamma_3+\gamma_2^2\right)\overline{\beta}\right)P_2\right)^2}E\left[K_2^2\right] - \frac{2|h_{12}|\gamma_3\beta P_2}{\left(1+\left(2\gamma_1+\gamma_1^2\right)\overline{\alpha}\right)P_1+\left(\gamma_3^2+\left(2\gamma_2\gamma_3+\gamma_2^2\right)\overline{\beta}\right)P_2}E\left[K_1K_2\right] \tag{32}$$

so,

$$\sigma^2_{|h_{12}|\sqrt{\frac{\beta P_2}{P_{V_2}}}V_2+Z_1|X_1+\gamma_1U_1+\gamma_2U_2+\gamma_3X_2} = |h_{12}|^2\beta P_2 + N_1 - \frac{|h_{12}|^2\gamma_3^2\beta^2P_2^2}{\left(1+\left(2\gamma_1+\gamma_1^2\right)\overline{\alpha}\right)P_1+\left(\gamma_3^2+\left(2\gamma_2\gamma_3+\gamma_2^2\right)\overline{\beta}\right))P_2} \tag{33}$$

by substituting $\sigma^2_{|h_{12}|\sqrt{\frac{\beta P_2}{P_{V_2}}}V_2+Z_1|X_1+\gamma_1U_1+\gamma_2U_2+\gamma_3X_2}$ into (39), it can be written as:

$$R_1 \le \frac{1}{2}\log\left(\frac{\left((|h_{11}|+|h_{1c}|)^2+2(|h_{11}|+|h_{1c}|)h_{1c}\gamma_1\overline{\alpha}\right)P_1+\left(|h_{1c}|\gamma_3+|h_{12}|\right)^2\beta P_2+N_1}{|h_{12}|^2\beta P_2+N_1-\frac{|h_{12}|^2\gamma_3^2\beta^2P_2^2}{\left(1+\left(2\gamma_1+\gamma_1^2\right)\overline{\alpha}\right)P_1+\left(\gamma_3^2+\left(2\gamma_2\gamma_3+\gamma_2^2\right)\overline{\beta}\right)P_2}}\right) \tag{34}$$

Proof of (10b): To prove (10b), the same strategy as proof of (10a) is implemented.

Proof of (10c):

Taking account of the theorem assumption and substituting equation (13) into equation (6) and equations (6), (7), (8) and Y_1 in terms of U_1, V_1, U_2, V_2 into equation (2c), there is:

$$\begin{aligned} &R_1+R_2 \le h\left(Y_1|U_1U_2\right)+h\left(Y_2\right)-h\left(S_1|U_1X_c\right)-h\left(S_2|U_2X_c\right)= \\ &= h\left((|h_{11}|+|h_{1c}|)\sqrt{\frac{\alpha P_1}{P_{V_1}}}V_1+\left(|h_{1c}|\gamma_3+h_{12}\right)\sqrt{\frac{\beta P_2}{P_{V_2}}}V_2+Z_1\right)+h\left(|h_{22}|X_2+|h_{2c}|X_c+|h_{21}|X_1+Z_2\right)- \\ &h\left(|h_{21}|X_1+Z_2|U_1X_c\right)-h(|h_{12}|X_2+Z_1|U_2X_c\right)= \end{aligned} \tag{35}$$

By using the entropy of normal distribution:

$$\begin{aligned} &=\frac{1}{2}\log 2\pi e((|h_{11}|+|h_{1c}|)^2\alpha P_1+\left(|h_{1c}|\gamma_3+|h_{12}|\right)^2\beta P_2+N_1)+\frac{1}{2}\log 2\pi e((|h_{2c}|+|h_{21}|)^2P_1+ \\ &(h_{2c}^2\left(2\gamma_1+\gamma_1^2\right)+2\gamma_1|h_{21}||h_{2c}|)\overline{\alpha}P_1+\left(|h_{22}|+\gamma_3|h_{2c}|\right)^2P_2+(|h_{2c}|^2(\gamma_2^2+2\gamma_2\gamma_3)+ \\ &2\gamma_2|h_{22}||h_{2c}|)\overline{\beta}P_2+N_2)-\frac{1}{2}\log 2\pi e\sigma^2_{|h_{21}|\sqrt{\frac{\alpha P_1}{P_{V_1}}}V_1+Z_2|X_1+\gamma_1U_1+\gamma_2U_2+\gamma_3X_2}-\frac{1}{2}\log 2\pi e\sigma^2_{|h_{12}|\sqrt{\frac{\beta P_2}{P_{V_2}}}V_2+Z_1|X_1+\gamma_1U_1+\gamma_2U_2+\gamma_3X_2} \end{aligned} \tag{36}$$

That

$$\sigma^2_{|h_{12}|\sqrt{\frac{\beta P_2}{P_{V_2}}}V_2+Z_1|X_1+\gamma_1U_1+\gamma_2U_2+\gamma_3X_2} = |h_{12}|^2 \beta P_2 + N_1 - \frac{|h_{12}|^2 \gamma_3^2 \beta^2 P_2^2}{\left(1+\left(2\gamma_1+\gamma_1^2\right)\overline{\alpha}\right)P_1+\left(\gamma_3^2+\left(2\gamma_2\gamma_3+\gamma_2^2\right)\overline{\beta}\right)P_2} \tag{37}$$

And

$$\sigma^2_{|h_{21}|\sqrt{\frac{\alpha P_1}{P_{V_1}}}V_1+Z_2|X_1+\gamma_1U_1+\gamma_2U_2+\gamma_3X_2} = |h_{21}|^2 \alpha P_1 + N_2 - \frac{|h_{21}|^2 \alpha^2 P_1^2}{\left(1+\left(2\gamma_1+\gamma_1^2\right)\overline{\alpha}\right)P_1+\left(\gamma_3^2+\left(2\gamma_2\gamma_3+\gamma_2^2\right)\overline{\beta}\right)P_2} \tag{38}$$

Upon substituting equations (48) and (49) into equation (47), it can be written as:

$$\begin{aligned}
&= \frac{1}{2}\log 2\pi e((|h_{11}|+|h_{1c}|)^2\alpha P_1 + \left(|h_{1c}|\gamma_3+|h_{12}|\right)^2 \beta P_2 + N_1) + \frac{1}{2}\log 2\pi e((|h_{2c}|+|h_{21}|)^2 P_1 + \\
&(|h_{2c}|^2\left(2\gamma_1+\gamma_1^2\right) + 2\gamma_1|h_{21}||h_{2c}|)\overline{\alpha}P_1 + \left(|h_{22}|+\gamma_3|h_{2c}|\right)^2 P_2 + (|h_{2c}|^2(\gamma_2^2+2\gamma_2\gamma_3) + \\
&2\gamma_2|h_{22}||h_{2c}|)\overline{\beta}P_2 + N_2) - \\
&\frac{1}{2}\log 2\pi e\left(|h_{21}|^2\alpha P_1 + N_2 - \frac{|h_{21}|^2\alpha^2P_1^2}{\left(1+\left(2\gamma_1+\gamma_1^2\right)\overline{\alpha}\right)P_1+\left(\gamma_3^2+\left(2\gamma_2\gamma_3+\gamma_2^2\right)\overline{\beta}\right)P_2}\right) - \\
&\frac{1}{2}\log 2\pi e\left(|h_{12}|^2\beta P_2 + N_1 - \frac{|h_{12}|^2\gamma_3^2\beta^2P_2^2}{\left(1+\left(2\gamma_1+\gamma_1^2\right)\overline{\alpha}\right)P_1+\left(\gamma_3^2+\left(2\gamma_2\gamma_3+\gamma_2^2\right)\overline{\beta}\right)P_2}\right)
\end{aligned} \tag{39}$$

After simplifications, there are:

$$\begin{aligned}
R_1+R_2 \le \frac{1}{2}\log&\left(\frac{(|h_{11}|+|h_{1c}|)^2\alpha P_1+\left(|h_{1c}|\gamma_3+|h_{12}|\right)^2\beta P_2+N_1}{|h_{12}|^2\beta P_2+N_1-\frac{|h_{12}|^2\gamma_3^2\beta^2P_2^2}{\left(1+\left(2\gamma_1+\gamma_1^2\right)\overline{\alpha}\right)P_1+\left(\gamma_3^2+\left(2\gamma_2\gamma_3+\gamma_2^2\right)\overline{\beta}\right)P_2}}\right)+\frac{1}{2}\log \\
&\left(\frac{(|h_{2c}|+|h_{21}|)^2P_1+\left(\left(\gamma_1^2+2\gamma_1\right)h_{2c}^2+2\gamma_1|h_{21}||h_{2c}|\right)\overline{\alpha}P_1+\left(|h_{22}|+\gamma_3|h_{2c}|\right)^2P_2+\left((\gamma_2^2+2\gamma_2\gamma_3)|h_{2c}|^2+2|h_{22}||h_{2c}|\gamma_2\right)\overline{\beta}P_2+N_2}{|h_{21}|^2\alpha P_1+N_2-\frac{|h_{21}|^2\alpha^2P_1^2}{\left(1+\left(2\gamma_1+\gamma_1^2\right)\overline{\alpha}\right)P_1+\left(\gamma_3^2+\left(2\gamma_2\gamma_3+\gamma_2^2\right)\overline{\beta}\right)P_2}}\right)
\end{aligned} \tag{40}$$

$$R_1 + R_2 \leq \frac{1}{2}\log$$

$$\left(\frac{(|h_{11}|+|h_{1c}|)^2 P_1 + \left(2\gamma_1|h_{11}||h_{1c}| + \left(\gamma_1^2+2\gamma_1\right)|h_{1c}|^2\right)\overline{\alpha}P_1 + \left(|h_{12}|+\gamma_3|h_{1c}|\right)^2 P_2 + \left(|h_{1c}|^2(2\gamma_2\gamma_3+\gamma_2^2)+2|h_{12}||h_{1c}|\right)\overline{\beta}P_2 + N_1}{|h_{12}|^2\beta P_2 + N_1 - \frac{|h_{12}|^2\gamma_3^2\beta^2P_2^2}{\left(1+\left(2\gamma_1+\gamma_1^2\right)\overline{\alpha}\right)P_1 + \left(\gamma_3^2+\left(2\gamma_2\gamma_3+\gamma_2^2\right)\overline{\beta}\right)P_2}} \right)$$

$$+\frac{1}{2}\log\left(\frac{(|h_{2c}|+|h_{21}|)^2\alpha P_1 + \left(|h_{22}|+\gamma_3|h_{2c}|\right)^2\beta P_2 + N_2}{|h_{21}|^2\alpha P_1 + N_2 - \frac{|h_{21}|^2\alpha^2P_1^2}{\left(1+\left(2\gamma_1+\gamma_1^2\right)\overline{\alpha}\right)P_1 + \left(\gamma_3^2+\left(2\gamma_2\gamma_3+\gamma_2^2\right)\overline{\beta}\right)P_2}} \right) \tag{41}$$

$$R_1 + R_2 \leq \frac{1}{2}\log\left(\frac{(|h_{11}|+|h_{1c}|)^2\alpha P_1 + \left(\left(|h_{12}|+|h_{1c}|\gamma_3\right)^2 + |h_{1c}|^2(2\gamma_2\gamma_3+\gamma_2^2)\overline{\beta}\right)P_2 + N_1}{h_{12}^2\beta P_2 + N_1 - \frac{h_{12}^2\gamma_3^2\beta^2P_2^2}{\left(1+\left(2\gamma_1+\gamma_1^2\right)\overline{\alpha}\right)P_1 + \left(\gamma_3^2+\left(2\gamma_2\gamma_3+\gamma_2^2\right)\overline{\beta}\right)P_2}} \right) +$$

$$\frac{1}{2}\log\left(\frac{\left((|h_{21}|+|h_{2c}|)^2 + |h_{2c}|^2\left(2\gamma_1+\gamma_1^2\right)\overline{\alpha}\right)P_1 + \left(|h_{22}|+|h_{2c}|\gamma_3\right)^2 P_2 + N_2}{|h_{21}|^2\alpha P_1 + N_2 - \frac{|h_{21}|^2\alpha^2P_1^2}{\left(1+\left(2\gamma_1+\gamma_1^2\right)\overline{\alpha}\right)P_1 + \left(\gamma_3^2+\left(2\gamma_2\gamma_3+\gamma_2^2\right)\overline{\beta}\right)P_2}} \right) \tag{42}$$

It is realized from (40), (41), and (42) that the sum of rates is less than the minimum of these terms.

The proofs of (10d) to (10e) are similar to the previous ones and they are omitted for brevity.

APPENDIX B: PROOF OF THEOREM 4

Proof of (26a) and proof of (26b) omitted for brevity.

Proof of (26c):

$$R_1 + R_2 \leq h\left(Y_1|U_1X_2\right) + h\left(Y_2\right) - h\left(S_1|X_1\right) - h\left(S_2|X_2\right) + I\left(S_1;X_1\right) \tag{43}$$

Y_1, Y_2 is defined in terms of U_1, U_2, V_2 and it is inserted in equation (4d):

$$\begin{aligned} &h\left(Y_1|U_1X_2\right) + h\left(Y_2\right) - h\left(S_1|X_1\right) - h\left(S_2|X_2\right) + I\left(S_1;X_1\right) = \\ &h\left(|h_{11}|X_1 + |h_{1c}|X_c + |h_{12}|X_2 + Z_1 \| U_1X_2\right) \\ &+h\left(|h_{22}|+|h_{2c}|\right))X_1 + \left(|h_{21}|+a_1|h_{2c}|\right)X_2 + Z_2) - \\ &h\left(|h_{21}|X_1 + Z_2|X_1)\right) - h(|h_{12}|X_2 + Z_1|X_2) + I\ (|h_{21}|X_1 + Z_2;X_1) \end{aligned} \tag{44}$$

On the other hand,

$$I(|h_{12}|X_1+Z_2;X_1)=h\left(|h_{21}|X_1+Z_2\right)-h\left(|h_{21}|X_1+Z_2|X_1\right)=C\left(\frac{|h_{21}|^2P_1}{N_2}\right) \tag{45}$$

So,

$$\begin{aligned}R_1+R_2\le&\frac{1}{2}\log\sigma^2_{(|h_{11}|+|h_{1c}|)X_1+Z_1|U_1}+\\&C\left(\frac{\left(|h_{21}|+|h_{2c}|\right)^2P_1+\left(|h_{22}|+a_1|h_{2c}|\right)^2P_2}{N_2}\right)-h\left(Z_1\right)+C\left(\frac{|h_{21}|^2P_1}{N_2}\right)\end{aligned} \tag{46}$$

According to (40), assuming $K_1=(|h_{11}|+|h_{1c}|)X_1+Z_1$, $K_2=\beta_1X_1+\beta_2X_2+W_1$

$$\sigma^2_{K_1|K_2}=(|h_{11}|+|h_{1c}|)^2P_1+N_1-\frac{(|h_{11}|+|h_{1c}|)^2\beta_1^2P_1^2}{\beta_1^2P_1+\beta_2^2P_2+P_{W1}} \tag{47}$$

So,

$$\begin{aligned}R_1+R_2\le&C\left(\frac{(|h_{11}|+|h_{1c}|)^2P_1-\frac{(|h_{11}|+|h_{1c}|)^2\beta_1^2P_1^2}{\beta_1^2P_1+\beta_2^2P_2+P_{W1}}}{N_1}\right)+\\&C\left(\frac{\left(|h_{21}|+|h_{2c}|\right)^2P_1+\left(|h_{22}|+a_1|h_{2c}|\right)^2P_2}{N_2}\right)+C\left(\frac{|h_{21}|^2P_1}{N_2}\right)\end{aligned} \tag{48}$$

From (2d), it is obtained as follows:

$$\begin{aligned}R_1+R_2\le&C\left(\frac{\left(|h_{22}|+a_1|h_{2c}|\right)^2P_2-\frac{\left(|h_{22}|+a_1|h_{2c}|\right)^2\gamma_2^2P_2^2}{\gamma_1^2P_1+\gamma_2^2P_2+P_{W2}}}{N_2}\right)+\\&C\left(\frac{\left(|h_{11}|+|h_{1c}|\right)^2P_1+\left(|h_{12}|+a_1|h_{1c}|\right)^2P_2}{N_1}\right)+C\left(\frac{|h_{12}|^2P_2}{N_1}\right)\end{aligned} \tag{49}$$

Similarly from (4e):

$$R_1 + R_2 \le C\left(\frac{\left(|h_{11}|+|h_{1c}|\right)^2 P_1 + \left(|h_{12}|+a_1|h_{1c}|\right)^2 P_2 - \frac{\left(\beta_1\left(|h_{11}|+|h_{1c}|\right)P_1 + \beta_1\left(|h_{12}|+a_1|h_{1c}|\right)P_2\right)^2}{\beta_1^2 P_1 + \beta_2^2 P_2 + P_{W1}}}{N_1}\right) +$$
$$C\left(\frac{\left(|h_{21}|+|h_{2c}|\right)^2 P_1 + \left(|h_{22}|+a_1|h_{2c}|\right)^2 P_2 - \frac{\left(\gamma_1\left(|h_{21}|+|h_{2c}|\right)P_1 + \gamma_2\left(|h_{22}|+a_1|h_{2c}|\right)P_2\right)^2}{\gamma_1^2 P_1 + \gamma_2^2 P_2 + P_{W2}}}{N_2}\right) + \quad (50)$$
$$C\left(\frac{|h_{21}|^2 P_1}{N_2}\right) + C\left(\frac{|h_{12}|^2 P_2}{N_1}\right)$$

It is realized from (48)-(50) that the sum of the rates is less than the minimum of the above terms.

The proofs of (26d) to (26e) are also similar to the previous ones and they are omitted for brevity.

References

[1] O. Sahin, E. Erkip, On achievable rates for interference relay channel with interference cancelation, in: Signals, Systems and Computers, 2007. ACSSC 2007. Conference Record of the Forty-First Asilomar Conference on, IEEE, 2007, pp. 805-809.

[2] M. Costa, Writing on dirty paper (corresp.), IEEE transactions on information theory, 29(3) (1983) 439-441.

[3] S. Sridharan, S. Vishwanath, S.A. Jafar, S. Shamai, On the capacity of cognitive relay assisted Gaussian interference channel, in: Information Theory, 2008. ISIT 2008. IEEE International Symposium on, IEEE, 2008, pp. 549-553.

[4] I. Maric, R. Dabora, A. Goldsmith, On the capacity of the interference channel with a relay, in: Information Theory, 2008. ISIT 2008. IEEE International Symposium on, IEEE, 2008, pp. 554-558.

[5] S. Rini, D. Tuninetti, N. Devroye, Outer bounds for the interference channel with a cognitive relay, in: Information Theory Workshop (ITW), 2010 IEEE, IEEE, 2010, pp. 1-5.

[6] S. Rini, D. Tuninetti, N. Devroye, Capacity to within 3 bits for a class of Gaussian interference channels with a cognitive relay, in: Information Theory Proceedings (ISIT), 2011 IEEE International Symposium on, IEEE, 2011, pp. 2627-2631.

[7] A. Gamal, M. Costa, The capacity region of a class of deterministic interference channels (corresp.), IEEE Transactions on information Theory, 28(2) (1982) 343-346.

[8] A.S. Avestimehr, S.N. Diggavi, N. David, Wireless network information flow: A deterministic approach, IEEE Transactions on Information theory, 57(4) (2011) 1872-1905.

[9] S. Rini, D. Tuninetti, N. Devroye, A. Goldsmith, The capacity of the interference channel with a cognitive relay in strong interference, in: Information Theory Proceedings (ISIT), 2011 IEEE International Symposium on, IEEE, 2011, pp. 2632-2636.

[10] H. Charmchi, G.A. Hodtani, M. Nasiri-Kenari, A new outer bound for a class of interference channels with a cognitive relay and a certain capacity result, IEEE Communications Letters, 17(2) (2013) 241-244.

[11] A. Dytso, S. Rini, N. Devroye, D. Tuninetti, On the capacity region of the two-user interference channel with a cognitive relay, IEEE Transactions on Wireless Communications, 13(12) (2014) 6824-6838.

[12] E.A. Yazdi, G.A. Hodtani, H.K. Ghomash, New Inner Bounds for the Gaussian Interference Channel with a Cognitive Relay, in: 2019 Iran Workshop on Communication and Information Theory (IWCIT), IEEE, 2019, pp. 1-4.

[13] S. Rini, D. Tuninetti, N. Devroye, A.J. Goldsmith, On the capacity of the interference channel with a cognitive relay, IEEE Transactions on information theory, 60(4) (2014) 2148-2179.

[14] S. Rini, D. Tuninetti, N. Devroye, New inner and outer bounds for the memoryless cognitive interference channel and some new capacity results, IEEE Transactions on Information Theory, 57(7) (2011) 4087-4109.

[15] S. Rini, D. Tuninetti, N. Devroye, Inner and outer bounds for the Gaussian cognitive interference channel and new capacity results, IEEE Transactions on Information Theory, 58(2) (2012) 820-848.

[16] H.S. Kang, M.G. Kang, A. Nosratinia, W. Choi, The Degrees of Freedom of the Interference Channel with a Cognitive Relay under Delayed Feedback, IEEE Transactions on Information Theory, 63(8) (2017) 5299-5313.

[17] Z. Al-qudah, A. Musa, On the capacity of the state-dependent interference relay channel, International Journal of Communication Systems, 32(14) (2019) e4079.

[18] S. Arzykulov, G. Nauryzbayev, T.A. Tsiftsis, M. Abdallah, On the performance of wireless powered cognitive relay network with interference alignment, IEEE Transactions on Communications, 66(9) (2018) 3825-3836.

[19] Z. Al-qudah, K.A. Darabkh, A simple Encoding Scheme to Achieve the Capacity of Half-Duplex Relay Channel, Advances in Electrical and Electronic Engineering, 20(1) (2022) 33-42.

[20] H. Tran, V.-H. Dang, D. Niyato, D.N. Cuong, N.C. Luong, C. So-In, Outage Probability Minimization in Secure NOMA Cognitive Radio Systems with UAV Relay: A Machine Learning Approach, IEEE Transactions on Cognitive Communications and Networking, 9(2) (2022) 435-451.

[21] P. Yang, L. Yang, W. Kuang, S. Wang, Outage performance of cognitive radio networks with a coverage-limited RIS for interference elimination, IEEE Wireless Communications Letters, 11(8) (2022) 1694-1698.

[22] D. Sahu, S. Maurya, M. Bansal, D. Kumar, Data-driven approach to design energy-efficient joint precoders at source and relay using deep learning in MIMO-CRNs, Transactions on Emerging Telecommunications Technologies, 33(5) (2022) e4454.

HOW TO CITE THIS ARTICLE

E. Asadollahi-Yazdi, G. A. Hodtani, H. Khoshbin- Ghomash. The Capacity Inner and Outer Bounds for the Wireless Interference Channel with a Cognitive Relay. AUT J Electr Eng, 56(2) (2024) 175-190.

DOI: 10.22060/eej.2024.22754.5560