## A Comprehensive Method for Expansion Planning of Active Distribution System Considering Reliability Assessment

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#### Abstract

Today the application of Distributed Energy Resources (DERs) in Distribution System expansion planning (DNEP) problems is more crucial than before. Despite of advantages, the presence of the resources considering Renewable Energy Sources (RESs) and Dispatchable Generation (DG) units in the distribution System expansion planning problems, brings more challenges, especially in reliability characteristics. This paper proposes a new distribution network expansion planning model embedded with a novel reliability assessment approach for Active Distribution Networks (ADNs). The proposed method aims to determine the optimal location and capacity of the new generation and distribution assets, responsible for providing power, in both the normal operation and contingency conditions. The load forecast significantly affects the results of the distribution network expansion planning. The K-means clustering method is used to address the uncertainty of load growth in the planning horizon which is coordinated with a Mixed Integer Linear Programming (MILP) optimization model. The proposed model is applied to the IEEE 33 bus test case, to guarantee its technical and economical effectiveness. The results verify that this model is cost-effective and can increase the robustness of the distribution network compared with recent similar works.

#### Keywords:

Distribution Network, Reliability, Expansion Planning, Expected Energy Not Supplied, Dispatchable Generation

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#### 1. Introduction

In recent years, the power system industry has been facing challenges related to sustainability, reliability, and stability issues as a result of the continuous increase in the world population and the electrification of many infrastructures [1]. To address these issues, two approaches have been suggested [2, 3]. The first is to add more generation units as a centralized approach to the network. This method is identified to be inefficient due to the several disadvantages consisting of the paramount amount of power loss, line power transaction limitations, a shortage of large-scale investors, etc [4, 5]. The second approach, known as the decentralized approach, can overcome these problems with less effort, losses, and monetary aspects in comparison with the previous one [6, 7]. In this approach, small-scale power generation units are installed through the Distribution Network (DN) and close to the loads [8]. Aside from decentralization, the decarburization strategy has collected remarkable attention in recent years from the global warming aspect[9]. Applying this rule over the power system leads to the utilization of a high share of Renewable Energy Sources (RESs) such as wind turbine (WT) and photovoltaic (PV) units as opposed to fossil fuelbased resources[10]. According to the Paris Agreement, greenhouse gas emissions should be limited in a way that the average temperature of the earth would be less than 2°C higher than the pre-industrial level[11]. As a result of the decision, the majority of industrial countries aim to increase the percentage of RES utilization in their upcoming roadmap [12, 13]. Despite the advantages of renewable generation units, many researchers today are grappling with the challenges related to the utilization of these resources consisting of fluctuations in power generation, voltage instability, uncertainty, etc. [14]. The challenges become more pronounced when considering the long-term planning of a distribution network. This process must incorporate several key factors, including load growth, voltage regulation, reliability, and costeffectiveness, with the main goal of ensuring a reliable and economical energy supply [15]. To obtain optimal control over a distribution network in a standard manner, there has been a requirement to identify the different topologies and operational rules governing such a network [16, 17]. In a distribution network, mesh and radial structures are two types of configuration that can be applied to this network to supply power

for all load nodes [16]. In radial structure, the supplying power for each load node depends on its upstream load node therefore each failure between the connection imposes power interruption for the downstream side whereas in the mesh structure, the aforementioned dependency is obliterated and as a result, the energy needed for each load node can be provided in two or more ways [18]. It is reasonably acceptable that the reliability, security, and other indices of a DN with a mesh structure would be higher than a radial one[19]. However, the utilization of a DN as a form of mesh structure forces undesirable results for the system consisting of high costs associated with investment operation and maintenance as well as the complexity of management and protection, etc [20]. Therefore, a tradeoff between these two approaches needs to be performed to reach the optimal objectives. Regarding this, a distributed network should be operated in a radial structure, while multiple lines should be reserved until the requirement of a reconfiguration for the network is created [21]. Among different network configurations, the selection of a suitable one depends on a range of factors, including varying load levels diverse contingency scenarios and availability of generation power, etc. [22]. According to [23] From flow direction perspective, a DN can be classified into two categories consisting of passive and active networks. In a passive DN, the power flows from the upstream to the downstream side exclusively, delivering electricity through distribution lines originating from substations. Therefore, the decisions are largely limited to the optimal selection of new substations and lines from the capacity and location perspectives. In contrast to the conventional model, an active distribution network (ADN) incorporates investment decisions that include the selection of optimal location and capacity of distributed generation units, such as WT, PV, and Dispatchable Generation (DG) systems [24]. The basic responsibility of these small-scale power plants in the DN is to compensate for power supply shortages, maintain grid stability, and enhance reliability in both normal operating and also under contingency conditions [25]. In the radial distribution systems, if a line is disconnected due to a contingency, all downstream nodes connected to that line will be deprived of electricity unless local generation sources or backup lines are available [26]. In such scenarios, reliability is brought into the Distribution Network Expansion Planning (DNEP) model through various indices that quantify the number

and volume of loads not supplied which is crucial for optimal placement and capacity determination of distributed generations or reserve lines [19].

	Different models and approaches																
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Ref	Publishing time	Test case	Planning duration	Load growth	Optimization Stages	Reactive power	Metaheuristic approach	Mathematical model	Reliability method	DNEP	Load shedding	Reconfiguration	SS	line	PV	WT	DG
[6]	2019	37	-	-	Single	-	-	MILP	-	$\checkmark$	-	-	$\checkmark$	$\checkmark$	-	-	$\checkmark$
[9]	2018	37	4	$\checkmark$	Bi-level	$\checkmark$	-	MILP	Classic	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$
[15]	2022	69	-	-	Single	$\checkmark$	MOEA	-	Classic	$\checkmark$	-	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$
[18]	2020	54	3	-	Single	$\checkmark$	-	MILP	New	$\checkmark$	-	$\checkmark$	$\checkmark$	$\checkmark$	-	-	-
[24]	2016	50	5	-	Single	$\checkmark$	-	MILNP	-	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	-	-	-
[27]	2018	31	10	$\checkmark$	Single	$\checkmark$	PSO	-	-	$\checkmark$		-	-	-	$\checkmark$	$\checkmark$	-
[28]	2022	24	-	-	Single	$\checkmark$	-	MILP	-	$\checkmark$	-	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$
[29]	2015	104	-	$\checkmark$	Single	$\checkmark$	$\checkmark$	-	Classic	$\checkmark$	-	-	$\checkmark$	$\checkmark$	-	-	$\checkmark$
[30]	2019	-	-	-	Single	$\checkmark$	-	MILP	-	$\checkmark$	-	-	$\checkmark$	$\checkmark$	-	-	$\checkmark$
[31]	2022	54	10	-	Single	$\checkmark$	-	MILP	New	$\checkmark$	-	$\checkmark$	$\checkmark$	$\checkmark$	-	-	-
[32]	2022	54	10	-	Bi-level	$\checkmark$	-	MILP	New	$\checkmark$	-	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$
Present pape	r 2024	33	10	$\checkmark$	single	$\checkmark$	Ē	MILP	New	$\checkmark$	-	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$

Table 1. Comparison of the proposed model with the existing approaches in the literature

There are many reliability indices such as expected energy not supplied (EENS), system average interruption duration index (SAIDI), and system average interruption frequency index (SAIFI) [33]. Table 1 presents the comparative features of several principal references concerning DNEP and aspects of reliability, which can be comparable with this work. Some references, such as [8], use a load-shedding strategy to enhance the capability of the system in response to the challenges related to the shortage of power under normal and contingency conditions. The authors of [8] propose a load switching method to deal with system contingencies. Similarly, the authors of [34], propose a load management method for both the gas and electric networks, in response to the contingencies of DN. The authors of [35] has made an investigation on the AC/DC DNEP considering investment strategy to deal with uncertainty related to variant generation units based on a multistage scenario tree method. However, they have neglected the reliability considerations. The nature of DNEP problems is non-linear. To deal with the non-linearity nature of DNEP, [36] proposed a Mixed-Integer Conic Programming model (MICP) to handle the long-term

DNEP problems. The authors of [30] apply the quadratic relaxation method to deal with the nonlinearity associated with the P-Q relationship. [28] proposes a Mixed Integer Linear Programming (MILP) model for a DNEP problem considering DG units, ESSs, and Electric Vehicle (EV) charging stations, alongside incorporating the environmental impact and uncertainties associated with demand and renewable generation. In the DNEP optimization problem, despite convex methods such as MILP, several researchers used iterative-based methods to solve their DNEP framework. Despite the apparent simplicity, these approaches are not efficient and do not provide the optimal output, resulting in inaccurate solutions for managing and controlling a network. In this context, investigators in [26, 29, 37], proposed a non-convex approach to the determination of the optimal solution from their reliability-based DNEP modeling. The remainder of the literature mainly examined the aspect of reliability in DNEP problems. In this regard, Authors in [31] proposed a MILP-based approach in their DNEP model including reliability assessment but their research is committed in a passive DN. According to [31, 38], for each line of a DN two fault parameters exist. The first, termed the 'switching-only interruption duration,' refers to the period wherein a fault on a line necessitates the disconnection of the corresponding substation from the rest of the feeder, achieved by tripping the associated circuit breaker. The duration of this interval is relatively short, and therefore it can be disregarded, as illustrated by investigation in [31, 38, 39]. The second term, designated as the 'switching due to repair interruption,' occurs when the line experiencing a fault is segregated from the remaining parts of the distribution network for repair purposes. In this duration, the disconnection occurs exclusively at the point of the fault's occurrence. The two aforementioned durations are incorporated into the EENS formulation for each line to measure the total reliability cost associated with the total amount of unsupplied load. In [40, 41] the reliability indices consisting of EENS, SAIDI, and SAIFI are used in the formulation for identifying the quantity of unsupplied energy, the duration, and the number of interruptions over the total planning horizon respectively. Upon all examinations and innovative research of the references, a particular deficiency becomes apparent. This shortfall relates to the approach employed in the reliability modeling through DNEP problem, especially in the presence of reserved lines and distributed

generations. This paper presents a method for formulating the reliability-based DNEP, and its main contributions are as the following:

- This paper introduces a novel formulation for system reliability, applicable to both passive and active distribution networks.
- The proposed reliability assessment model is capable to separately consider the sensitivity of the load points.
- This paper integrates the DNEP model with the developed reliability formulations to balance economic and technical aspects of the DNEP over a long-term horizon.
- To enhance the effectiveness of the proposed method under uncertain parameters like load forecasting, this paper employs clustering methods, specifically K-means, to explore various load growth scenarios.

The remainder of this paper is organized as follows: In section 2 the proposed methodology is illustrated and is compared with contemporary state-of-the-art methods for EENS calculation in detail. section 3 describes the problem formulation of the optimization framework. In section 4 the numerical results derived from the model are explained. Finally, in section 5 the conclusion and future research and development of the model are discussed.

### 2. Power distribution system expansion planning problem

As previously mentioned, DNEP problem is a long-term optimization model to determine the time, location and capacity of the new substations, feeders and distributed generation units. It aims to minimize the total cost during the planning period, including the installation and operation costs of the assets of power distribution system. The following sections introduce various parts of the mentioned optimization model.

### **2-1- Objective Function**

The objective function of the proposed model that is total cost includes the investment cost, operation cost and load interruption costs, and is depicted in Eq. (1). As can be observed, all the investment costs are annualized using recovery rate factor, and the net present value of the costs are considered in the objective function.

$$OF = \sum_{y} \frac{1}{(1 + IR)^{y}} \left[ \left( \frac{IR(1 + IR)^{y}}{(1 + IR)^{y} - 1} \right) C_{y}^{I} + C_{y}^{O} + C_{y}^{F} + C_{y}^{R} \right]$$
(1)

In accordance with Eq. (2), the investment cost is comprised of investment decisions regarding the construction of a new line, substation, WT, PV, and DG. Each of these components is represented by a set of binary decision variables, which determine the optimal time, location, and capacity for investing of each asset.

$$C_{y}^{I} = \sum_{i} \left[ \sum_{j=c_{l}}^{j} \sum_{c_{l}}^{L} D_{i,j}^{L} I C_{c_{l}}^{L} X_{i,j,c_{l},y}^{L} + \sum_{c_{s}}^{I} I C_{c_{s}}^{S} X_{i,c_{s},y}^{S} + \sum_{c_{w}}^{I} I C_{c_{w}}^{W} X_{i,c_{w},y}^{W} + \sum_{c_{dg}}^{I} I C_{c_{dg}}^{DG} X_{i,c_{dg},y}^{DG} \right], \forall y \in Y^{s} \quad (2)$$

In the DNEP model, the utilization variables are used to identify the on/off mode of each device within the system during the system operation period. The system operation cost includes two main terms. The first one is the operation cost of each asset that has been depicted in Eq. (3). As depicted in Eq. (4), the second part of system operation cost includes the power transaction cost with the upstream network and as well as the fuel cost of the DG units,

$$C_{y}^{O} = \sum_{b} N_{b}^{Y} \left[ \sum_{i,j} \sum_{c_{l}} D_{i,j}^{L} \times OC_{c_{l}}^{L} (U_{i,j,c_{l},y,b}^{LN} + U_{i,j,c_{l},y,b}^{LP}) + \sum_{i} \sum_{c_{s}} OC_{c_{s}}^{S} \times U_{i,c_{s},y,b}^{S} \\ + \sum_{i} \sum_{c_{w}} OC_{c_{w}}^{W} U_{i,c_{w},y,b}^{W} + \sum_{i} \sum_{c_{p}} OC_{c_{p}}^{P} U_{i,c_{p},y,b}^{P} + \sum_{i} \sum_{c_{dg}} OC_{c_{dg}}^{DG} \times U_{i,c_{dg},y,b}^{DG} \right], \forall y \in Y^{S}$$
(3)

$$C_{y}^{F} = \sum_{i} \sum_{b} N_{b}^{Y} \left[ \sum_{c_{s}} \mu_{b}^{E} P_{i,c_{s},y,b}^{S} + \sum_{c_{g}} \mu_{b}^{F} . \sigma^{DG} . P_{i,c_{dg},y,b}^{DG} \right], \forall y \in Y^{S}$$
(4)

The reliability cost which is introduced as a main aspect of the presented model is described by Eq. (5). In case of a line outage, its flow of power will be interrupted, and it is a measure of the load interruption in the down-stream side of the line. This measure has been utilized in some researches such as [41]. However, their proposed formulation may not be suitable for active distribution networks with bi-directional power flow through the lines. In such networks, DG units installed at downstream of a line can supply a portion of the demand if that line experiences an outage. The proposed reliability cost model presented in Eq. (5) is applicable to both passive and active distribution networks. In this model,  $\Delta_{i,j,y,b}$  denotes the fraction of demand that will be interrupted due to an outage on line *ij*. This variable is a measure for system reliability, that will be calculated through the reliability constraints in the next sections.

$$C_{y}^{R} = C^{ens} \left[ \sum_{b} \sum_{i,j} \sum_{c_{l}} N_{b}^{Y} \times FR_{c_{l}} \times RT_{c_{l}} \times \Delta_{i,j,y,b} \right], \forall y \in Y^{s}$$

$$(5)$$

#### 2-2- Model constraints

The previous section presented the objective function of the proposed reliability oriented DNEP model. Here the constraints are presented and explained.

#### 2-2-1- Investment and utilization constraints

As previously mentioned, distribution systems are designed in mesh structure and are operated radially. Reconfiguration techniques determine the best radial structure in various operation states considering aspects such as loss minimization, reliability maximization, voltage profile modification and etc. In the reconfiguration modeling, some utilization variables are introduced to show the activation state of the lines or other assets during the operation states. Eqs. (6-10) are used in the proposed model to show the relationship between utilization and investment decision variables used for lines, substations, WTs, PVs, and DGs. Here, the power flow for each line in each time slot is restricted to a specific direction. This

directionality constraint is captured by the variables on the left side of Eq. (6). As can be observed in the presented equations, the utilization variable can be determined for both existing and newly installed assets.

$$U_{i,j,c_{l},y,b}^{LN} + U_{j,i,c_{l},y,b}^{LN} \le U_{i,j}^{LI} + \sum_{\tau=1}^{y} X_{i,j,c_{l},y}^{L}, \forall (i,j) \in M^{s}, c_{l} \in C^{L}, y \in Y^{s} i \neq j, b \in B^{s}$$
(6)

$$U_{i,c_{s},y,b}^{S} \leq U_{i}^{SI} + \sum_{\tau=1}^{y} X_{i,c_{s},y}^{S}, \forall i \in M^{S}, c_{s} \in C^{S}, y \in Y^{S}, b \in B^{S}$$
(7)

$$U_{i,c_{w},y,b}^{W} \le U_{i}^{WI} + \sum_{\tau=1}^{y} \sum_{i,j} X_{i,c_{w},y}^{W}, \forall i \in M^{s}, c_{w} \in C^{W}, y \in Y^{s}, b \in B^{s}$$
(8)

$$U_{i,c_{p},y,b}^{P} \leq U_{i}^{PI} + \sum_{\tau=1}^{y} \sum_{i,j} X_{i,c_{p},y}^{P}, \forall i \in M^{s}, c_{p} \in C^{P}, y \in Y^{s}, b \in B^{s}$$
(9)

$$U_{i,c_{dg},y,b}^{DG} \le U_{i}^{DGI} + \sum_{\tau=1}^{y} X_{i,c_{dg},y}^{DG}, \forall i \in M^{s}, c_{dg} \in C^{DG}, y \in Y^{s}, b \in B^{s}$$
(10)

#### 2-2-2- Radiality constraints

For preserving the radial structure of the network, a specific defect is observed in several references which increases the potential for the creation of isolated networks within the larger network [36]. This issue can arise when the radial network relies on the presence of Distributed Energy Resources (DERs), such as local generators or energy storage systems, to maintain power supply to certain load nodes. In this regard, two additional constraints will be incorporated into the model. These constraints, represented by Eq. (11) and Eq. (12), respectively serve to ensure the radial operation of the distribution network and avoiding the isolated operation of substations.

$$\sum_{j} \sum_{c_{l}} U_{j,i,c_{l},y,b}^{LN} = 1 - \sum_{c_{s}} U_{i,c_{s},y,b}^{S}, \forall i \in M^{s}, y \in Y^{s}, b \in B^{s}$$
(11)

$$\sum_{j} \sum_{c_l} U_{i,j,c_l,y,b}^{LN} \ge \sum_{c_s} U_{i,c_s,y,b}^{S}, \forall i \in M^s, y \in Y^s, b \in B^s$$

$$\tag{12}$$

#### 2-2-3- Reliability constraints

As previously mentioned in the objective function,  $\Delta_{i,j,y,b}$  is a reliability measure. To calculate  $\Delta_{i,j,y,b}$ , the power demand ( $F_{i,j,y,b}^{LN}$ ) and DG capacity ( $H_{i,j,y,b}^{LN}$ ) located at downstream of line *ij* should be calculated. The load curtailment caused by the line outage can then be calculated using the difference between these two values as has been shown in Eq. (13)

$$\Delta_{i,j,y,b} \ge F_{i,j,y,b}^{LN} - H_{i,j,y,b}^{LN}$$

$$\Delta_{i,j,y,b} \ge 0$$
(13)

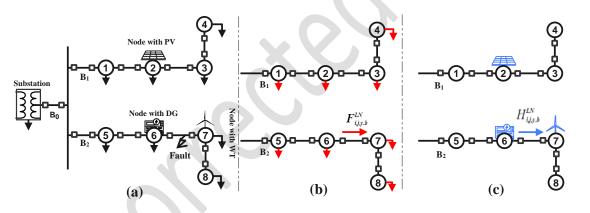


Fig. 1. Structure of a DN a) with DERs, b) considering only loads, c) considering DERs exclusively

Two virtual power flow processes are utilized to compute each of the aforementioned values, which are depicted in Fig. 1. Eqs (14-19) are virtual power flow formulation to calculate power demand at the downstream of line *ij* (see Fig.1b). In this formulation, all the DG units are neglected. Eq. (14) is the power balance in each load point, in which  $\kappa_i^D$  is a parameter utilized to explicitly define the relative importance of each load node in terms of system reliability. some load nodes possess greater significance in terms of reliability assessment, while for other nodes, this significance is comparatively lower. This parameter can be approximately calculated based on the average consumer damage function (CDF) through the network

as depicted in (15). Based on the Eqs. (16-19), only the activated assets are considered in the mentioned virtual power flow formulation.

$$F_{i,y,b}^{S} + \sum_{j,j\neq i} F_{j,i,y,b}^{LN} = \sum_{n,n\neq i} F_{i,n,y,b}^{LN} + \kappa_{i}^{D} \cdot P_{i,b}^{D}, \forall i \in M^{s}, y \in Y^{s}, b \in B^{s}$$
(14)

$$\kappa_i = \frac{C_i^{ens}}{C^{ens}} \tag{15}$$

$$F_{i,j,y,b}^{LN} \le \sum_{c_l} M \cdot [U_{i,j,c_l,y,b}^{LN} + U_{j,i,c_l,y,b}^{LN}], \forall (i,j) \in M^s, y \in Y^s, b \in B^s$$
(16)

$$F_{i,j,y,b}^{LN} \le M.B_{i,j,y,b}^{LN}, \forall (i,j) \in M^s, y \in Y^s, b \in B^s$$

$$\tag{17}$$

$$B_{i,j,y,b}^{LN} + B_{j,i,y,b}^{LN} \le 1, \forall (i,j) \in M^s, y \in Y^s, b \in B^s$$

$$\tag{18}$$

$$F_{i,y,b}^{S} \leq \sum_{c_{s}} M.U_{i,c_{s},y,b}^{S}, \forall i \in M^{s}, y \in Y^{s}, b \in B^{s}$$

$$\tag{19}$$

The Eqs. (20-24) are the virtual power flow formulation to calculate the installed capacity of DG units at the down-stream side of line *ij*. In this formulation, all the loads are removed and also the DG units are replaced by the loads with the demand equal to DG capacity, as depicted in Fig.3c. Eq. (20) is the power balance at each node. According to Eqs. (21-24), only the online DGs and lines are considered in this formulation.

$$H_{i,y,b}^{S} + \sum_{j,j \neq i} H_{j,i,y,b}^{LN} - \sum_{k,k \neq i} H_{i,k,y,b}^{LN} = H_{i,y,b}^{P} + H_{i,y,b}^{W} + H_{i,y,b}^{G}, \forall i \in M^{s}, y \in Y^{s}, b \in B^{s}$$
(20)

$$H_{i,j,y,b}^{LN} \leq Z_{i,j,y,b}^{L} \mathcal{M}, \forall (i,j) \in M^{s}, y \in Y^{s}, b \in B^{s}$$

$$\tag{21}$$

$$Z_{i,j,y,b}^{L} + Z_{j,i,y,b}^{L} \le 1, \forall (i,j) \in M^{s}, y \in Y^{s}, b \in B^{s}$$
(22)

$$H_{i,y,b}^{S} \leq \sum_{c_{s}} M.U_{i,c_{s},y,b}^{S}, \forall i \in M^{S}, y \in Y^{S}, b \in B^{S}$$

$$(23)$$

$$H_{i,j,y,b}^{LN} \leq \sum_{c_l} M \cdot [U_{i,j,c_l,y,b}^{LN} + U_{j,i,c_l,y,b}^{LN}], \forall (i,j) \in M^S, y \in Y^S, b \in B^S$$
(24)

Note that the two sets of aforementioned equations consist of Eqs. (14-24) considered line reconfiguration in their calculations. Also, the two virtual binary variables B and Z, are utilized within the formulation to limit the power directionality of each distributed line across each load level. Note that in the aforementioned set of formulations, the Big M method is employed to impose limitations on the utilization of both the distribution lines and substations.

#### 2-2-4- Power balance constraints

Eq. (25) is used to model the balance between generation and demand from an active term point of view. Similarly, Eq. (26) is used to explain the balance between demand and generation from a reactive power perspective. The parameter  $\lambda$  is used in Eq. (25) to model the linear form of the power loss of each line, which is functionally dependent on the distance of each pair of nodes. The loss of reactive power is assumed to be negligible in the formulation of the reactive power balance constraint.

$$\sum_{c_{s}} P_{i,c_{s},y,b}^{S} + \left[\sum_{c_{w}} P_{i,c_{w},y,b}^{W} + \sum_{c_{p}} P_{i,c_{p},y,b}^{P} + \sum_{c_{dg}} P_{i,c_{dg},y,b}^{DG}\right] +$$

$$(1-\lambda) \cdot \left[\sum_{j,i\neq j} \sum_{c_{l}} P_{j,i,c_{l},y,b}^{LN} - \sum_{n,i\neq n} \sum_{c_{l}} P_{i,n,c_{l},y,b}^{LN}\right] = P_{i,y,b}^{D}, \forall i \in M^{S}, y \in Y^{S}, b \in B^{S}$$

$$\sum_{c_{s}} Q_{i,c_{s},y,b}^{S} + \sum_{c_{w}} Q_{i,c_{w},y,b}^{W} + \sum_{c_{p}} Q_{i,c_{p},y,b}^{P} + \sum_{c_{dg}} Q_{i,c_{dg},y,b}^{DG} + \sum_{j,i\neq j} \sum_{c_{l}} Q_{j,i,c_{l},y,b}^{LN}$$

$$= Q_{i,c_{d},y,b}^{D} + \sum_{n,i\neq n} \sum_{c_{i}} Q_{i,j,c_{l},y,b}^{LN}, \forall i \in M^{S}, y \in Y^{S}, b \in B^{S}$$

$$(26)$$

#### **2-2-5-** Loading constraints

Considering  $S_a$ ,  $P_a$ ,  $Q_a$  respectively as the apparent, active, and reactive power of asset a, and  $S_a^{\text{max}}$  as its loading capability, Eq. (27) is the asset loading constraint. In order to linearize the presented non-linear constraint, the hexagon relaxation method is used.

$$S_a = P_a^2 + Q_a^2 \le S_a^{\max} \tag{27}$$

According to Fig. 2, the eight lines surrounded the circle to limit the amount of P, and Q based on the apparent power. The mathematical formulation of applying the hexagon relaxation method to the lines and generation units of the network is presented in Eqs. (28-31).

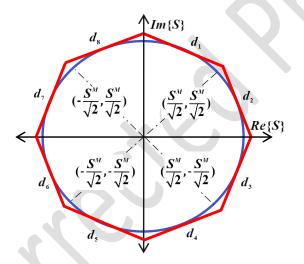


Fig. 2. Linear approximation of P-Q relationship

$$\alpha_{k} P_{i,j,c_{l},y,b}^{LN} + \beta_{k} Q_{i,j,c_{l},y,b}^{LN} \leq U_{i,j,c_{l},y,b}^{LN} S_{c_{l}}^{LM}, \forall (i,j) \in M^{s}, c_{l} \in C^{L}, y \in Y^{s}, b \in B^{s}, k \leq 4$$
(28)

$$\alpha_k P_{i,j,c_l,y,b}^{LN} + \beta_k Q_{i,j,c_l,y,b}^{LN} \ge -U_{i,j,c_l,y,b}^{LN} S_{c_l}^{LM}, \forall (i,j) \in M^s, c_l \in C^L, y \in Y^s, b \in B^s, k > 4$$
(29)

$$\alpha_{k} P_{i,c_{g},y,b}^{G} + \beta_{k} Q_{i,c_{g},y,b}^{G} \leq U_{i,c_{g},y,b}^{G} S_{c_{g}}^{GM}, \forall i \in M^{s}, c_{g} \in C^{G}, y \in Y^{s}, b \in B^{s}, k \leq 4,$$

$$G \in \{SS, WT, PV, DG\}$$
(30)

$$\alpha_{k}P_{i,c_{g},y,b}^{G} + \beta_{k}Q_{i,c_{g},y,b}^{G} \ge -U_{i,c_{g},y,b}^{G}S_{c_{g}}^{GM}, \forall i \in M^{s}, c_{g} \in C^{G}, y \in Y^{s}, b \in B^{s}, k > 4,$$

$$G \in \{SS, WT, PV, DG\}$$
(31)

#### **2-2-6- DER utilization constraint**

The limitation related to the penetration of DERs represents a significant obstacle that cannot be overlooked. The maximum permissible capacity of these local generation units is variable and depends on several factors, including the type and size of the network. According to the recommendation of researchers in [44-46], the maximum capacity occupied by these units varies between 30% to 45% of the total maximum demand per time slot. To illustrate this limitation, Eq. (32) is integrated into a model that coordinates the overall demand of the distribution system with the total substation capacity. The consideration of this restriction has the additional beneficial effect of reducing the size of the input decision pool for the model, thereby decreasing the time simulation of the system.

$$\sum_{i} \left[ P_{i,y,b}^{S} - 0.45 \times P_{i,y,b}^{D} \right] \ge 0, \forall y \in Y^{s}, b \in B^{s}$$

$$(32)$$

### 3. Uncertainty of load growth

The forecasted load significantly impacts the decisions of the system expansion. The power demand during the horizon years is an uncertain parameter that cannot be exactly forecasted. Generally, several scenarios are developed for the demand forecast using the historical data and applying the statistical and machine learning approaches. Here, the K-means clustering approach is employed to address the uncertainty, based on a dataset of 100,000 yearly initial load growth values. As can be seen in Algorithm 1, the presented approach is to iteratively determine the optimal solution for each of the nine identified centroids by coordinating the clustering and optimization steps. This process continues until an optimal set of solutions is obtained for each load growth scenario, encompassing the optimal placement, timing of investment, and capacity of the distribution assets. The proposed algorithm employs a nested iterative structure to coordinate the clustering and optimization components. The first loop focuses on accurately calculating the K-means clustering, partitioning the input data into the requisite number of representative centroids while the second loop, is dedicated to implement the optimization model for each centroid. In the context of the algorithm

provided, the  $\varepsilon$  represents a threshold value used for the convergence criterion. The results of the clustering

process yielded nine centroids, which are shown in Fig. 3.

```
Algorithm 1 Solution Algorithm of Load Growth
Uncertainty
           Inputs: Get data on load demand on the matrix \mathbf{A}_{\varpi_s \times Y_s}
    1:
           Initialize: \varphi \leftarrow 1
   2:
           While flag = false do
   3:
            (\Psi_{\varphi \times Y_{\epsilon}}, \mathbf{E}_{\sigma_{\epsilon} \times 1}) = calculate K-means (\varphi, \mathbf{A}_{\sigma_{\epsilon} \times Y_{\epsilon}})
   4:
                      for \omega = 1: \boldsymbol{\varpi}_s
   5:
                            if \max \left| \Psi_{[E_{(\omega,1)},:]} - \mathbf{A}_{[E_{(\omega,1)},:]} \right| \ge \varepsilon then
   6:
                             \varphi \leftarrow \varphi + 1
   7:
    8:
                             break
                            elseif max |\psi_{[E_{(\omega,1)},:]}|
                                                                      -\mathbf{A}_{\mathbb{E}(\omega,1)}
                                                                                              \leq \varepsilon
   9:
                             \omega = \sigma_s then
  10:
                            flag=true
                             else
  11:
                             continue the loop for next \omega
  12:
  13:
                       end for
  14:
           end while
           Initialize: \xi \leftarrow 1
  15:
           While \xi \leq \varphi do
  16:
           calculate the optimization block for \xi 'th centroid:
               \left(\mathbf{OF}_{\boldsymbol{\xi}}^{*}, \mathbf{X}_{\boldsymbol{\xi}}^{*}, \mathbf{U}_{\boldsymbol{\xi}}^{*}, \mathbf{P}_{\boldsymbol{\xi}}^{*}, \mathbf{Q}_{\boldsymbol{\xi}}^{*}\right) = min \ OF(\mathbf{X}, \mathbf{U}, \mathbf{P}, \mathbf{Q})
  17:
                                              s.t: Eq.(1-32)
            \xi \leftarrow \xi + 1
  18:
           end while
  19:
```

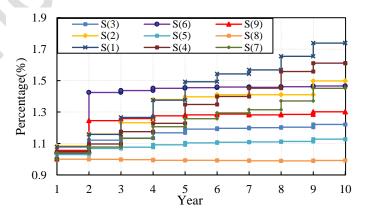


Fig. 3. The obtained centroids according to the proposed algorithm

#### **4-Numerical Results**

#### 4-1- Test System and Required Data

The proposed model has been applied to the IEEE 33 bus test case, which will be examined in detail in the following sections through four different simulation cases. All cases have been run to optimality, using GAMS version 25.1.2 on an Asus K55VM laptop computer equipped with an Intel Core i5 processor and 16 GB of RAM. Table 2 shows the investment and operation features of each component utilized in the system. As depicted in Fig. 4, three different load levels are considered, which categorize the demand into distinct parts consisting of on-peak, off-peak, and mid-peak, time slots. To reduce the simulation of the model, the case studies incorporate restrictions pertaining to the suitable locations for the installation of each component. The configuration and details of the nodal interconnections, as well as the candidate locations for installing new assets, have been depicted in Fig. 5. It is hypothesized that the upper portion of the under-study network exhibits an elevated potential for the installation of wind turbines while conversely, the lower segments of the system are deemed suitable for the implementation of photovoltaic generation units. Additionally, the restrictions on suitable locations for installing other components of the distribution network, such as substations, distributed generators, and lines, have also been considered.

		1 abic 2. 1	ine parameter of this	unbuted units in the model							
		Different Costs and features									
term	Investment cost (\$/kW)	Operation cost (\$/kWh)	Fuel cost (\$/m <sup>3</sup> )	Requirement Electricity cost (\$/kW)	Efficiency						
SS	350	0.08	-	0.3	-						
PV	600	0.01	-	-	-						
WT	700	0.01	-	-	-						
DG	250	0.01	0.25	-	0.4						

Table 2. The parameter of distributed units in the model

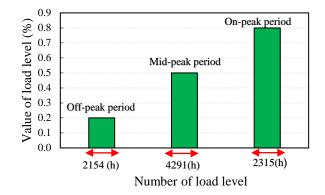


Fig. 4. The number of load levels and the corresponding duration

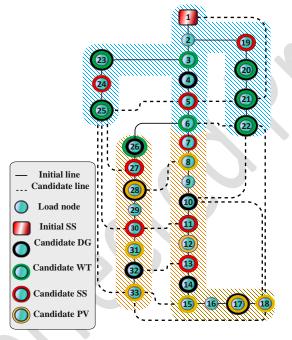


Fig. 5. The initial configuration of the investigated DN

In Table 3, A suite of case studies has been delineated to examine the impact of distribution system features on associated costs and other pertinent indices. These case studies will be subjected to comprehensive examination and analysis to provide a thorough understanding of system performance and design implications. In the context of case studies 1 and 3, the reliability formulation has been omitted to facilitate a comparative analysis with scenarios 2, and 4 in which the reliability assessment is incorporated. As a special work, the effectiveness of the proposed method in scenarios 2, and 4 will be compared to Ref [32] to validate the application of the model from different perspectives. It is important to note that all

simulations were run based on the proposed algorithm for various load growth trends. However, the results presented in this work are specified for the load growth of scenarios 1, and 9.

	Different models and approaches								
Case Study	Case (1)	Case (2)	Case (3)	Case (4)	Base case				
Substation investment	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$				
Line Investment	-	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$				
Reconfiguration	-	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$				
DG investment	-	-	$\checkmark$	$\checkmark$	$\checkmark$				
<b>RES</b> investment	-	-	$\checkmark$	$\checkmark$	$\checkmark$				
Reliability approach	-	$\checkmark$	-	Proposed model	Ref[32]				

Table 3. The details of feasures in the proposed case studies

By applying the various simulation cases to the test network, decision variables are calculated, and the optimal location, time and capacity to install new assets such as substation, DGs and lines are determined. A general comparison of the cost components across the different simulation cases is provided in Table (4. The comparative analysis demonstrates a similar behavioral pattern across the different scenarios of load growth and exhibits the cost-effectiveness of the proposed model from various perspectives, in comparison to the other cases. The findings from the proposed methodology in Case 4 indicate that the shift towards a more reliable and decentralized modeling approach yields greater cost-benefit advantages compared to the reference cases.

		Different models and approaches								
Load growth	Term (\$×10 <sup>3</sup> )		Case	(2)		Case	(4)			
scenario number		Case (1)	Benchmark [32]	Proposed model	Case (3)	Benchmark [32]	Proposed model			
	Investment Cost	6388.82	7031.41	7268.88	15965.34	17715.61	17975.62			
	O&M	55.71	57.58	54.69	96.3	124.98	126.88			
1	Power transaction	73284.3	72161.42	72032.16	46666.09	44425.927	43379.04			
	Reliability	8756.57	5616.31	5388.93	6127.57	4402.52	4186.21			
	Total	88485.4	84866.72	84744.66	68855.3	66669.037	65667.75			
	Investment Cost	6000	5812.3	5809.03	12146.1	11064.1	11152.6			
	O&M	55.2987	54.3452	53.5569	74.51	111.05	114.89			
9	Power transaction	62515.7	62472.1	62146.8	41472.2	37118.4	37027.9			
	Reliability	9312.68	5186.13	5062.05	7340.9	4073.87	3566.49			
	Total	77883.67	73524.87	73071.43	61033.71	52967.42	51861.88			

Table 4. Cost components of the different case studies in different scenarios

The details of investment decisions in each simulation case have been shown in Table 5. As can be observed, to achieve higher network reliability, in case 4, the number of investments in both strategies (proposed method and reference case) is relatively equal to each other but the location and time of investing the assets are different. The analysis of the data presented in the table suggests that the application of a reliability-based optimization method has significantly influenced the investment time of the distribution assets in the system under consideration. In contrast to case studies where the reliability factor was not incorporated, the implementation of the reliability method has led to optimal investment strategy, especially in time and location aspects.

Case study							Year					
	Case s	tudy -	1	2	3	4	5	6	7	8	9	10
	Case (1)		13,27	-	-	-	-	7	-	-	-	-
n of SS	Case (2)	Proposed model	13	-	30	-		-	5	-	-	-
	(2)	[32]	13	-	-		30	-	5	-	-	-
atio	C	ase (3)	-	13	-	-	-	-	-	-	-	-
Location of	Case	Proposed model	13	30	-		-	-	-	-	-	-
	(4)	[32]	13	-	30	-	-	-	-	-	-	-
Location of line	Case (1)		-	-	-	-	-	-	-	-	-	-
	Case	Proposed model	33,15 22,6			-	11,30 5,25	-	-	-	-	-
	(2)	[32]	13,32		25,27	-	22,6	-	-	-	-	-
ioi	Case (3)		25,26	-	-	-	-	-	-	33,15	-	-
Locat	Case	Proposed model	33,18	22-6	13-18	5,25	-	-	-	-	-	-
	(4)	[32]	18,13	-	6,22	30,11	25,26	-	-	-	-	-
	C	ase (1)		-	-	-	-	-	-	-	-	-
Location of DERs	Case	Proposed model	-	-	-	-	-	-	-	-	-	-
	(2)	[32]		-	-	-	-	-	-	-	-	-
	C	Case (3)		15	8	31	3,6,12	-	25		33	23
	Case	Proposed model	4,24	10,14	5,16	8,20	22,33	-	-	-	-	-
	(4)	[32]	4,32	15,10	3,6	20	5,25,12	-	-	-	-	-

Table 5. The placement status of key elements of the network in the first scenario of load growth

The following subsections will discuss different simulation cases.

#### 4-3- Case (1)

In case 1, substation investment is the primary alternative considered to address the demand requirements at each load node. It has been demonstrated that due to the limited availability of alternative options

consisting of line investment, reconfiguration technique, and DERs, the majority of the cost terms in the system would be higher than in comparison with the other cases. The optimal decision-making process pertaining to this case is depicted in Fig. 6. In this context, three distinct substations have been invested and located at nodes 7, 13, and 27.

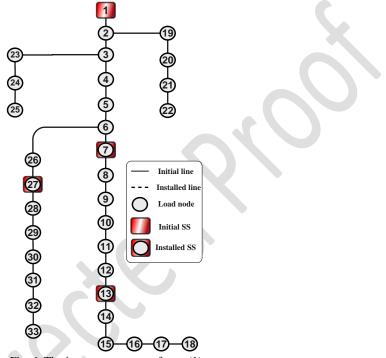
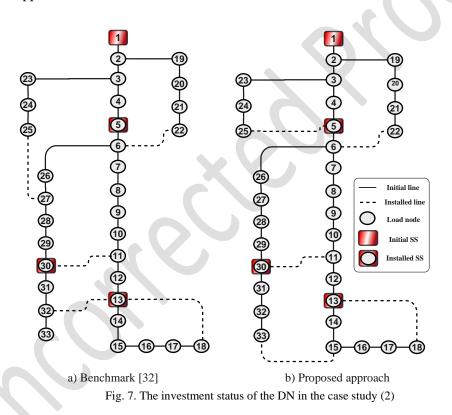


Fig. 6. The investment status of case (1)

In the absence of reliability assessment, owing to the reduction in total line losses, the new substations have been strategically placed in the central portions of the distribution network. This arrangement separates the network into multiple symmetrical feeders for each operating condition, thereby decreasing power losses. In the first load growth scenario, the total cost of case1 is 88485.4 (\$), which is higher than the total costs of the other simulation case. An investigation in cost terms of this case reveals that the reliability cost, which is incurred due to line failures, has the paramount value compared to the other cost components which is obtained to be 8756.57 (\$).

4-4- Case (2)

In this simulation case, it is assumed that the DN has the right to undertake possible and optimal reconfiguration through the existing and already invested lines. The proposed reliability calculation method is considered for this case to increase the security of power provision for all load nodes during the contingency conditions. The decision variables of this simulation case have been depicted in Fig. 7 and compared with the base case. As can be observed, the installed lines being strategically positioned near the feeder terminations, coordinated with the placement of the new substations. The results demonstrate similarity between the optimal output characteristics of the two models which leads to reduce cost terms for the presented approach.



The optimal outcome for the two models being described involves investment status for both of them similarly. The result of this case study shows that all load nodes can supported at least from two directions in which one of these streams is used in each load condition. The results in scenario 1 show that the proposed model can decrease the reliability cost by approximately 4% and 38% compared to the model in base case and case 1, respectively.

#### 4-5- Case (3)

Case 3, incorporates the contributions of DGs and DERs alongside the presence of line investment into the model and investigates the corresponding results. The proposed reliability method has been omitted from this case in order to examine the system's results and compare them to other cases. Without reliability assessment, the primary objective is to allocate the optimal capacity of DGs as well as WTs and PVs alongside the new and existing substation, in a way that the total energy transition from the utility grid is minimized. The results of the total investment in each asset are represented in Fig. 8. The proposed model installed four wind turbines as well as five photovoltaics in different locations of the DN. Additionally, two DG units are installed in nodes 4 and 32. Compared to the previous cases, the majority of the cost components reveal less value due to the presence of distributed generation units. In comparison with case 1, It can be concluded that the presence of these resources can enhance the system's robustness, especially during contingency events. According to the Table. 5, the total cost of the system is 68855.3 (\$), which is reduced by 22% and 19% compared to case 1 and case 2, respectively. Furthermore, the reliability cost term for this case is 6127.57 (\$), which has also been reduced in coordination with the total cost. It can be concluded that the presence of local generation units, such as the installed wind turbines and photovoltaic systems, as well as the lack of reliability considerations, has reduced the required number of substations as well as the number of reserve lines compared to the previous cases.

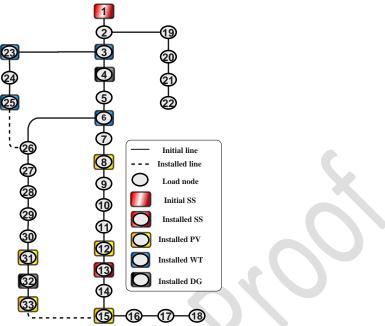


Fig. 8. The investment status of the DN in the case study (3)

#### 4-6- Case (4)

In Case 4, the reliability assessment method, combined with the accessibility to install the local generation units, has been applied to evaluate the proposed model and conduct a comprehensive comparison with the previous cases. Fig. 9 depicts a comparative analysis of the optimal locations for investment of the assets between the proposed model and the base case. The reliability cost in the proposed model is lower than the base case. The results show that the presented approach ensures the effective and optimal integration of DER technologies, while also considering the overall system reliability and performance. Similar to case 2, in this case, the result is compared with the result of the proposed model in Ref [32]. In comparison to the previous cases, the cost components are decreased, especially in reliability cost terms. As illustrated in Fig. 8, the benchmark and the presented model, yield different investment levels and capacity allocations for certain infrastructure components. From a cost optimization perspective, the methodology introduced in the current work demonstrates advantages over the reference model. The results in the first load growth scenario show that the proposed model can decrease the reliability cost by approximately 5%, 32%, and 22% respectively compared to the model in the base case, case 3, and case 2. The strategic placement of the local energy sources and reserve interconnections of nodes is one of the key findings of this case. This

interconnectivity provides additional pathways for power flow, thereby increasing the system's reliability against potential failures. This approach ensures that the distributed generation resources in coordination with the reserved lines and substations are strategically placed to effectively serve the loads, additionally improves the system's ability to maintain continuous power supply even in the event of localized disruptions. In this case study, two new substations have been added to the distribution network. The inclusion of these new substations has introduced additional feeders and power flow paths within the system.

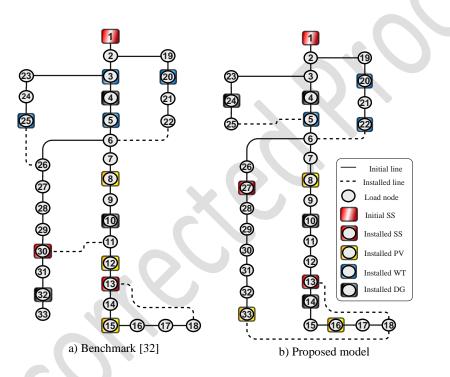


Fig.9. The investment status of the DN in the case study (4)

### 4-7- Sensitivity analysis

In this section, a sensitivity analysis was conducted to investigate the impact of increasing the reliability cost parameter. Four distinct values were considered for the cost of reliability consisting of 10, 30, 50, and 70. For each of these values, the total costs and the amount of unsupplied energy in the entire system were calculated. As illustrated in Fig. 10, the results indicate that as the reliability cost parameter increases, the total unsupplied energy of the system decreases. This finding suggests that prioritizing reliability in the system's

robustness during contingency events. The sensitivity analysis provides valuable insights into the trade-offs between reliability costs and other cost component, which can inform decision-making and optimization strategies. Therefore, more reliable structure of the system needs to perform more investment in the distribution network.

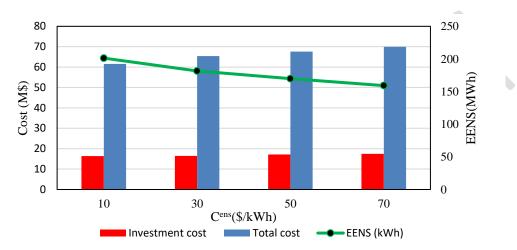


Fig.10. The sensitivity analysis of the model based on EENS cost variations

#### **5-** Conclusion

This paper presented a reliability-based DNEP approach for finding optimal alternatives for installing new assets in the distribution network. The uncertainty of load growth has been modeled by the K-means clustering method. The proposed DNEP model was evaluated on a 33-bus test case and the results have been compared with similar methods. The reliability-based DNEP approach aimed to enhance all-customer satisfaction through modeling reliability based on the EENS. The results indicated that the cost-related indices of a DN can be significantly improved by optimal reconfiguration and placement of the DERs. The case study demonstration on the 33-bus system validated the effectiveness of the reliability-driven DNEP in delivering satisfactory performance in terms of technical, economic, and reliability metrics compared with other approaches. Despite comprehensiveness analysis, this work needs to be expanded in order to consider more details of reliability based on the proposed approach. Adding more essential resources including energy storage systems can be regarded as another key feature that in the future should be examined by this approach.

### 6- Nomenclature

 $M^s$  Set of load nodes in the DN.

- $Y^{s}$  Set of planning horizon years.
- $B^{s}$  Set of load levels.
- $\psi_{\rho \times y}$  Centroid matrix.

### Subscript

*i*,*j*,*n* Index of number of load nodes.

- *y* Index of number of years.
- *b* Index of load level number.
- $c_s$  Type of substation.
- $c_l$  Type of line.
- $c_p$  Type of PV.
- $c_w$  Type of WT.
- $c_{dg}$  Type of DG.
- $\rho$  Number of centroids.

### **Parameters**

 $\sigma_s$  Number of sample data of load growth during planning.

 $\varphi$  Number of centroids.

- $N_b^{Y}$  Duration of load level b in each year, (h).
- $D_{i,j}^{L}$  The distance between nodes *i* and *j*, (km).

IR Interest rate.

$$IC_{c_s}^{S}$$
 Investment cost of substation for type  $c_s$ , (\$)

$$IC_{c_l}^L$$
 Investment cost of line for type  $c_l$ , (\$).

$$IC_{c_w}^W$$
 Investment cost of WT for type  $c_w$ , (\$).

 $IC_{c_p}^P$  Investment cost of PV for type  $c_p$ , (\$).

 $IC_{c_{dg}}^{DG}$  Investment cost of DG for type  $c_{dg}$ , (\$).

- $OC_{c_s}^s$  Operation cost of substation for type  $c_s$ , (\$).
- $OC_{c_l}^L$  Operation cost of line for type  $c_l$ , (\$).

 $OC_{c_w}^W$  Operation cost of WT for type  $c_w$ , (\$).

 $OC_{c_n}^P$  Operation cost of PV for type  $c_p$ , (\$).

 $OC_{c_{dg}}^{DG}$  Operation cost of DG for type  $c_{dg}$  , (\$).

$$\mu_b^F$$
 Fuel cost at load level b, (\$/m<sup>3</sup>).

 $\mu_b^E$  Electricity cost at load level b, (\$/kW).

 $C^{ens}$  Average cost coefficient for calculation of EENS, (\$/kWh)

 $FR_{c_l}$  Fault rate of line with type  $c_l$ , (1/h).

 $RT_{c_l}$  Repair time of line with type  $c_l$ , (h).

 $K_{i}^{s}, K_{i,i}^{L}$  Candid location for substation and line investment.

 $K_i^P, K_i^W, K_i^{DG}$  Candid location for PV, WT and DG investment.

- $\lambda$  Coefficient used for line loss modeling.
- $\alpha, \beta$  Coefficients used for apparent power linearization.

 $P_{i,y,b}^{D}$  The demand of node *i* at year *y* in load level *b*, (kW).

$$P_{c_l}^{LM}$$
 Maximum capacity of line with type  $c_l$ , (kW).

M Big M parameter.

 $S_{c_s}^{SM}$  Maximum capacity of SS with type  $c_s$ , (kVA).

 $P_{C_w}^{AW}$  Available generating power for WT with type  $c_w$ , (kW).

 $P_{C_p}^{AP}$  Available generating power for PV with type  $c_p$ , (kW).

 $S_{C_a}^{GM}$  Maximum capacity of each generation asset, (kVA).

 $E_{m1}$  index matrix which is utilized for determination of associated centrode for each data.

 $a_{\omega, \gamma}$  element of the load growth matrix  $(A_{\omega \times \gamma})$ , (kW).

#### Variables

$$X_{i,v,c}^{s}$$
 Binary variables for substation investment in node *i*.

 $X_{i,j,y,c_i}^L$  Binary variable for line investment between nodes *i*,*j*.

 $X_{i,y,c_w}^W$  Binary variables for WT investment in node *i*.

 $X_{i,y,c_p}^{P}$  Binary variables for PV investment in node *i*.

 $X_{i,y,c_{de}}^{DG}$  Binary variables for DG investment in node *i*.

 $U_{i,y,b,c_s}^{s}$  Utilization Status of SS in node *i* at year *y*.

 $U_{i,j,y,b,c_l}^{LN}$  Utilization Status of the line between nodes *i*,*j* at year *y*.

 $U_{i,y,b,c_p}^P$  Utilization Status of PV in node *i* at year y.

 $U_{i,y,b,c_w}^W$  Utilization Status of WT in node *i* at year *y*.

 $P_{i,y,b,c_s}^s$ ,  $Q_{i,y,b,c_s}^s$  Active and reactive power of SS in node *i* at year y and load level *b*, (kW, kVAR).

 $P_{i,y,b,c_{dg}}^{DG}$ ,  $Q_{i,y,b,c_{dg}}^{DG}$  Active and reactive power of DG in node *i* at year y and load level *b*, (kW, kVAR)

 $P_{i,j,y,b,c_{de}}^{LN}$ ,  $Q_{i,j,y,b,c_{de}}^{LN}$  Active and reactive power of the line in node *i* at year y and load level *b*, (kW, kVAR).

 $F_{i,j,v,b}^{LN}$ ,  $H_{i,j,v,b}^{LN}$  Virtual power, flowing through line *ij*, (kW).

 $F_{i,v,h}^{s}$ ,  $H_{i,v,h}^{s}$  Virtual generation of the substation in node *i*, (kW).

 $\Delta_{i,i,y,h}$  Unsupplied power of downside of line *ij*, (kW).

 $D_{i,j,y,b}^{LN}$ ,  $B_{i,j,y,b}^{LN}$  Binary variables for distinguishing flow direction.

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