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Original Article

Quasi-multipliers and quasi Jordan multipliers

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ABSTRACT: We show that every quasi-multiplier $\phi : L^1(G) \times L^1(G) \longrightarrow L^1(G)$, where G is a locally compact group, is of the form

$$\phi(f, g) = f \star \mu \star g, \quad f, g \in L^1(G),$$

for a unique measure $\mu \in M(G)$. As a consequence, we obtain a well-known result due to Wendel. We also prove the analogues result for C^*-algebras. Moreover, we introduce the notion of quasi Jordan multipliers and prove that each such map on a C^*-algebra, as well as group algebra $L^1(G)$, is a quasi-multiplier.

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1. Introduction

A linear map T from an algebra A into an A-bimodule X is called a left multiplier (right multiplier) if for all $a, b \in A$,

$$T(ab) = T(a)b, \quad \big(T(ab) = aT(b)\big),$$

and T is called a multiplier if it is both left and right multiplier. This notion (at least when $X = A$) is often called a centralizer in the literature [7].

The concept of a multiplier first introduced on commutative Banach algebras [9], and then various versions of multipliers such as quasi-multiplies and ϕ-multipliers on Banach algebras defined.

A double multiplier is a pair (L, R), where $L : A \longrightarrow X$ is a left multiplier, $R : A \longrightarrow X$ is a right multiplier and $aL(b) = R(a)b$ for all $a, b \in A$. The set of all double multipliers from A into X is denoted by $\mathfrak{M}(A, X)$. In particular, $\mathfrak{M}(A) = \mathfrak{M}(A, A)$.

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Each $x \in X$ induces a double multiplier (L_x, R_x) given by

$$L_x(a) = xa, \qquad R_x(a) = ax, \qquad (a \in A).$$

A map $\phi : A \times A \longrightarrow X$ is said to be quasi-multiplier if for every $a, b, x, y \in A$,

$$\phi(ax, yb) = a\phi(x, y)b. \tag{1}$$

The collection of all quasi-multipliers from $A \times A$ into X is denoted by $\mathfrak{QM}(A, X)$, and we write $\mathfrak{QM}(A)$ for $\mathfrak{QM}(A, A)$.

It is showed in [11] that $\mathfrak{QM}(A)$ is a Banach space with the following norm,

$$\|\phi\| = \sup\{\|\phi(a, b)\| : a, b \in S_A\},$$

where $S_A = \{a \in A : \ \|a\| = 1\}$.

The notion of a quasi-multiplier is a generalization of the notion of a left (right, double) multiplier on a Banach algebra and was introduced in [3] for C^*-algebras. The general theory of quasi-multipliers on a Banach algebra with a bounded approximate identity studied by McKennon in [11].

Since then many authors have been studied quasi-multipliers, and different results have been obtained; for instance, see [1, 2, 6, 10] and the references therein.

There exists another related concept of quasi-multipliers, called left (right) quasi-multiplier, which we introduce as follows. A left quasi-multiplier (right quasi-multiplier) is the map $\phi : A \times A \longrightarrow X$ for which

$$\phi(a, yb) = \phi(a, y)b, \quad \big(\phi(ax, b) = a\phi(x, b)\big),$$

holds for all $a, x, b, y \in A$, and ϕ is a quasi-multiplier if it is a left and right quasi-multiplier.

If ϕ is both left and right quasi-multiplier, then ϕ is a quasi-multiplier according to (1), but the converse is false, in general. The next example illustrates this fact.

Example 1.1. *Let*

$$A = \left\{ \begin{bmatrix} 0 & z_1 & z_2 & z_3 \\ 0 & 0 & z_4 & z_5 \\ 0 & 0 & 0 & z_6 \\ 0 & 0 & 0 & 0 \end{bmatrix} : \quad z_1, \ldots, z_6 \in \mathbb{C} \right\},$$

and define $\phi : A \times A \longrightarrow A$ via $\phi(a, b) = ba$. Then, for all $a, b, x, y \in A$,

$$\phi(ax, yb) = ybax = 0.$$

On the other hand, $a\phi(x, y)b = ayxb = 0$, hence ϕ satisfies in (1), but in general,

$$yax = \phi(ax, y) \neq a\phi(x, y) = ayx.$$

Thus, ϕ is not a right quasi-multiplier.

A net $\{e_\lambda\}_{\lambda \in I}$ in a Banach algebra A is a bounded approximate identity if $\sup_\lambda \|e_\lambda\| < \infty$ and $e_\lambda a \longrightarrow a$ and $ae_\lambda \longrightarrow a$ for every $a \in A$. C^*-algebras and the group algebras $L^1(G)$ for a locally compact group G are two important example of Banach algebras that have bounded approximate identity, see [4].

By a result of Johnson [7], any multiplier T on Banach algebra A with a bounded approximate identity is continuous. For generalization of this result to Banach A-bimodule X, see [12, Theorem 1].

The following result is [11, Theorem 1].

Theorem 1.1. *Suppose that A is a Banach algebra with a bounded approximate identity. If $\phi : A \times A \longrightarrow X$ is a quasi-multiplier of type (1), then*

(i) *ϕ is a left (right) quasi-multiplier,*
(ii) *ϕ is bilinear,*
(iii) *ϕ is jointly continuous.*

In this paper, we characterize quasi-multipliers on group algebras and C^*-algebras. We also introduce the notion of quasi Jordan multiplier as a generalization of Jordan multiplier, and prove that every quasi Jordan multiplier on mentioned algebras is a quasi-multiplier.

2. Quasi-Multipliers on group and C^*-algebras

Let G be a locally compact group. Then by Wendel's theorem [4, Theorem 3.3.40], we have $\mathfrak{M}(L^1(G)) = M(G)$, where $M(G)$ is the Banach algebra of all complex, regular Borel measures on G with respect to the convolution product $\star$, and this result extended to the quasi-multipliers in [11, Corollary of Theorem 22].

We mention that by [8, Theorem 6.3], the map $\nu \longrightarrow \mu \star \nu$ is w^*-continuous for each $\mu \in M(G)$ and $\mu \longrightarrow \mu \star \nu$ is w^*-continuous for each $\nu \in M(G)$.

Define $\phi : L^1(G) \times L^1(G) \longrightarrow L^1(G)$ by

$$\phi(f, g) = f \star \mu \star g, \quad f, g \in L^1(G), \tag{2}$$

where $\mu \in M(G)$ is a fixed element. Then clearly, ϕ is a quasi-multiplier.

Next we characterize quasi-multipliers on $L^1(G)$.

Theorem 2.1. *Let G be a locally compact group and $\phi : L^1(G) \times L^1(G) \longrightarrow L^1(G)$ be a quasi-multiplier. Then*

$$\phi(f, g) = f \star \mu \star g, \quad f, g \in L^1(G),$$

for some unique element $\mu \in M(G)$.

Proof. By Theorem 1.1, ϕ is jointly continuous and hence it is bounded. Let $\{e_\lambda\}_{\lambda \in I}$ be a bounded approximate identity in $L^1(G)$, then the net $\{\phi(e_\lambda, e_\lambda)\}_{\lambda \in I}$ is bounded and we may assume that it converges to $\mu \in M(G)$ with respect to the w^*-topology.

For each $f, g \in L^1(G)$, we have $\phi(f \star e_\lambda, e_\lambda \star g)$ tend to $\phi(f, g)$ in the norm topology because ϕ is continuous. From the separate w^*-continuity of convolution product in $M(G)$, we get

$$\phi(f \star e_\lambda, e_\lambda \star g) = f \star \phi(e_\lambda, e_\lambda) \star g \longrightarrow f \star \mu \star g,$$

in the w^*-topology. Consequently, $\phi(f, g) = f \star \mu \star g$ for every $f, g \in L^1(G)$. If $\mu_1 \in M(G)$ was another element such that $\phi(f, g) = f \star \mu_1 \star g$, then

$$f \star (\mu - \mu_1) \star g = 0,$$

for all $f, g \in L^1(G)$, thus also for all $f, g \in M(G)$. Since $M(G)$ is unital, we obtain $\mu = \mu_1$, and the proof is complete. □

As a consequence, we get a brief proof for the following remarkable result due to Wendel, [13, Theorem 1].

Corollary 2.2. *Let G be a locally compact group and $T : L^1(G) \longrightarrow L^1(G)$ be a multiplier. Then there exists a unique element $\mu \in M(G)$ such that*

$$T(f) = f \star \mu = \mu \star f, \quad f \in L^1(G).$$

Proof. Define $\phi : L^1(G) \times L^1(G) \longrightarrow L^1(G)$ via $\phi(f, g) = T(f \star g)$. Then ϕ is a quasi-multiplier and it is jointly continuous. By Theorem 2.1, there is a unique $\mu \in M(G)$ such that

$$T(f \star g) = \phi(f, g) = f \star \mu \star g, \quad f, g \in L^1(G).$$

Let $\{e_\lambda\}_{\lambda \in I}$ be a bounded approximate identity in $L^1(G)$. Replacing g by $\{e_\lambda\}_{\lambda \in I}$, we get

$$T(f) = f \star \mu, \quad f \in L^1(G).$$

Similarly, $T(f) = \mu \star f$ for all $f \in L^1(G)$. □

There are two naturally defined products on the second dual space A^{**} of a Banach algebra A, which we denote by $\Box$ and $\Diamond$, respectively. Thise products are defined by

$$\Phi \Box \Psi = \lim_i \lim_j a_i \cdot b_j, \quad \Psi \Diamond \Phi = \lim_j \lim_i a_i \cdot b_j, \quad \Phi, \Psi \in A^{**},$$

where $\{a_i,\}_{i \in I}$ and $\{b_i\}_{j \in I}$ are nets in A that converge in the w^*-topology, to Φ and Ψ, respectively. The Banach algebra A is said to be Arens regular if $\Phi \Box \Psi = \Psi \Diamond \Phi$ on the whole of A^{**}. It is shown [4] that every C^*-algebra A is Arens regular.

We regard A as a closed subalgebra of both $(A^{**}, \Box)$ and $(A^{**}, \Diamond)$, and A is w^*-dense in A^{**}. We refer the reader to [4] for a full discussion of these products.

Now we can formulate the following result.

Theorem 2.3. *Let A be a C^*-algebra and $\phi : A \times A \longrightarrow A$ be a quasi-multiplier. Then there is a unique element $\xi \in A^{**}$ such that*

$$\phi(a,b) = a\xi b, \quad a,b \in A.$$

Note that Theorem 2.3 shows that for each C^*-algebra A, $\mathfrak{QM}(A)$ may be embedded in the enveloping von Neumann algebra A^{**}.

From Theorem 2.3, we get the next result which characterize multipliers on C^*-algebras.

Corollary 2.4. *Let A be a C^*-algebra and $T : A \longrightarrow A$ be a multiplier. Then there exists a unique element $\xi \in A^{**}$ such that*

$$T(a) = a\xi = \xi a, \quad a \in A.$$

Let A be a Banach algebra with a bounded approximate identity $\{e_\lambda\}_{\lambda\in I}$, and let $\phi : A \times A \longrightarrow A$ be a quasi-multiplier.

(i) Define $S : A \longrightarrow A$ by $S(b) = \lim_\lambda \phi(e_\lambda, b)$, then we have

$$S(yb) = \lim_\lambda \phi(e_\lambda, yb) = \lim_\lambda \phi(e_\lambda, y)b = S(y)b, \quad b,y \in A,$$

hence S is a left multiplier.

(ii) Define $T : A \longrightarrow A$ by $T(a) = \lim_\lambda \phi(a, e_\lambda)$, then we get

$$T(ax) = \lim_\lambda \phi(ax, e_\lambda) = \lim_\lambda a\phi(x, e_\lambda) = aT(x), \quad a,x \in A,$$

so T is a right multiplier.

(iii) By letting $a = y = e_\lambda$ and $x = b = e_\lambda$ in (1), respectively, we arrive at

$$\phi(x,b) = \lim_\lambda \phi(x, e_\lambda)b, \quad \textit{and} \quad \phi(a,y) = \lim_\lambda a\phi(e_\lambda, y).$$

Therefore, $\lim_\lambda \phi(a, e_\lambda)b = \lim_\lambda a\phi(e_\lambda, b)$ for all $a, b \in A$, and hence

$$aS(b) = \lim_\lambda a\phi(e_\lambda, b) = \lim_\lambda \phi(a, e_\lambda)b = T(a)b,$$

Consequently, $(S,T) \in \mathfrak{M}(A)$. Therefore, we proved the following result.

Proposition 2.5. *Let A be a Banach algebra with a bounded approximate identity. Then $\Gamma : \mathfrak{QM}(A) \longrightarrow \mathfrak{M}(A)$ defined by $\Gamma(\phi) = (S,T)$ is linear and one to one.*

Next we show that Γ is continuous. To see this, first note that $\|(S,T)\| = \sup\{\|S\|, \|T\|\}$. On the other hand, by the above argument $S(b) = \lim_\lambda \phi(e_\lambda, b)$. Since e_λ is bounded and ϕ is jointly continuous by Theorem 1.1, we get

$$\|S(b)\| = \lim_\lambda \|\phi(e_\lambda, b)\| \le \|\phi\|\|e_\lambda\|\|b\| \le c\|\phi\|\|b\|.$$

Thus, $\|S\| \le c\|\phi\|$. Similarly, $\|T\| \le c\|\phi\|$, and hence

$$\|\Gamma(\phi)\| = \|(S,T)\| \le c\|\phi\|,$$

for all $\phi \in \mathfrak{QM}(A)$. Therefore, Γ is a continuous.

It should be pointed out that the map $f : A \longrightarrow \mathfrak{QM}(A)$ defined by $f(a)(x,y) = xay$ is linear and one to one. Moreover,

$$\|f(a)(x,y)\| = \|xay\| \le \|x\|\|a\|\|y\|,$$

therefore,

$$\|f(a)\| = \sup\{\|f(a)(x,y)\| : x,y \in S_A\} \le \|a\|.$$

So f is continuous. For each $a \in A$, $f(a) \in \mathfrak{QM}(A)$ and hence by Proposition 2.5, there exists $(S,T) \in \mathfrak{M}(A)$ such that $\Gamma(f(a)) = (S,T)$. Indeed,

$$S(y) = \lim_\lambda f(a)(e_\lambda, y) = \lim_\lambda e_\lambda a y, \quad y \in A.$$

Since $\{e_\lambda\}_{\lambda\in I}$ is a bounded approximate identity, we obtain $S(y) = ay = L_a(y)$.

Similarly, $T(x) = R_a(x)$ for all $x \in A$ and hence $\Gamma(f(a)) = (L_a, R_a)$. It is known that the map $\Delta : A \longrightarrow \mathfrak{M}(A)$ defined by $\Delta(a) = (L_a, R_a)$ for all $a \in A$ is a continuous homomorphism, and so we get the following result.

Proposition 2.6. *Let A be a Banach algebra with a bounded approximate identity, and Γ, f and Δ be as above. Then*

(i) $\Gamma \circ f = \Delta$,

(ii) $\Gamma \circ f$ *is a continuous homomorphism,*

(iii) Γ *is a continuous map.*

We do not know that under what condition the map $\Gamma : \mathfrak{QM}(A) \longrightarrow \mathfrak{M}(A)$ is surjective.

3. Quasi Jordan multipliers

A linear map T from an algebra A into an A-bimodule X is called a left Jordan multiplier (*right Jordan multiplier*) if for all $a \in A$,

$$T(a^2) = T(a)a, \quad \big(T(a^2) = aT(a)\big).$$

If T is both left as well right Jordan multiplier, then it is called a *Jordan multiplier.*

Clearly, every multiplier is a Jordan multiplier, however, there exists Jordan multipliers that are not multipliers, see [5, Example 2.6]. For more information of Jordan multiplier, see [14].

Next we introduce the concepts of quasi Jordan multiplier as a generalization of Jordan multiplier.

Definition 3.1. *The bilinear map $\phi : A \times A \longrightarrow X$ is called a right quasi Jordan multiplier (left quasi Jordan multiplier) if*

$$\phi(a^2, b) = a\phi(a, b), \quad \big(\phi(a, b^2) = \phi(a, b)b\big),$$

for all $a, b \in A$. If ϕ is left and right quasi Jordan multiplier, then it is natural to call ϕ a quasi Jordan multiplier.

Example 3.1. Let

$$A = \left\{ \begin{bmatrix} z_1 & z_2 \\ 0 & 0 \end{bmatrix} : \quad z_1, z_2 \in \mathbb{C} \right\}.$$

Then $X = \mathbb{C}$ is an A-bimodule with the following actions:

$$a\lambda = 0, \quad \lambda a = \lambda z_1, \quad \lambda \in \mathbb{C},\ a \in A.$$

Define $\phi : A \times A \longrightarrow X$ by

$$\phi\Big(\begin{bmatrix} z_1 & z_2 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} w_1 & w_2 \\ 0 & 0 \end{bmatrix} \Big) = z_2 w_2.$$

Then $\phi(a, b^2) = \phi(a, b)b$ for all $a, b \in A$, and so ϕ is a left quasi Jordan multiplier, but it is not a left (right) quasi-multiplier. Note that $\phi(a^2, b) \neq 0$, in general, while $a\phi(a, b) = 0$ for every $a, b \in A$.

Proposition 3.2. *If every Jordan multiplier $T : A \longrightarrow X$ is a multiplier, then every quasi Jordan multiplier $\phi : A \times A \longrightarrow X$ is a quasi-multiplier.*

Proof. Suppose that ϕ is a right quasi Jordan multiplier, then

$$\phi(a^2, b) = a\phi(a, b), \quad a, b \in A.$$

Let $a_0 \in A$ be a fixed element and $T : A \longrightarrow X$ be a linear map defined by $T(a) = \phi(a, a_0)$. Then

$$T(a^2) = \phi(a^2, a_0) = a\phi(a, a_0) = aT(a), \quad a \in A,$$

and thus, $T(ax) = aT(x)$ for all $a, x \in A$. This means that $\phi(ax, a_0) = a\phi(x, a_0)$. Since $a_0 \in A$ was assumed be an arbitrary, so ϕ is a right quasi-multiplier. Similarly, we can prove the left version. □

From Proposition 3.2, [15, Theorem 2.3] and [15, Theorem 2.11], we deduce the next results.

Corollary 3.3. *Let A be a C^*-algebra. Then each quasi Jordan multiplier $\phi : A \times A \longrightarrow X$ is a quasi-multiplier.*

Corollary 3.4. *Let G be an abelian locally compact group and $A = L^1(G)$. Then each quasi Jordan multiplier $\phi : A \times A \longrightarrow X$ is a quasi-multiplier.*

For $a, x, b \in A$, we set

$$\Delta_1(a, x, b) = a\phi(x, b) - x\phi(a, b).$$

We say that an element $w \in A$ is a left (right) separating point of A-bimodule X if the condition $wx = 0$ ($xw = 0$) for all $x \in X$ implies that $x = 0$.

If A is unital with unit e_A and X is unitary, i.e., $e_A x = x = x e_A$, then $w = e_A$ is a left (right) separating point of X.

Theorem 3.5. *Let A be a commutative algebra. If A has a left separating point w for A-bimodule X, then every right quasi Jordan multiplier $\phi : A \times A \longrightarrow X$ is a right quasi-multiplier.*

Proof. We intend to prove that $\Delta_1(a,x,b)=0$ for all $a,x,b\in A$. Let

$$\phi(a^2,b)=a\phi(a,b),\quad a,b\in A.$$

Replacing a by $a+x$ we get

$$2\phi(ax,b)=a\phi(x,b)+x\phi(a,b),\quad a,x,b\in A. \tag{3}$$

Interchanging x by xy in (3), we obtain

$$2\phi(axy,b)=a\phi(xy,b)+xy\phi(a,b). \tag{4}$$

Plugging (3) into (4) to get

$$4\phi(axy,b)=a\big(x\phi(y,b)+y\phi(x,b)\big)+2xy\phi(a,b). \tag{5}$$

Replacing a by x and x by a in (5), we have

$$4\phi(axy,b)=x\big(a\phi(y,b)+y\phi(a,b)\big)+2ay\phi(x,b). \tag{6}$$

Comparing (5) and (6), we arrive at

$$y\Delta_1(a,x,b)=0,\quad a,b,x,y\in A.$$

Interchanging y by w in the above equality, and since w is a left separating point of X, we get $\Delta_1(a,x,b)=0$. Hence, $a\phi(x,b)=x\phi(a,b)$ for all $a,x,b\in A$. Therefore, it follows from (3) that ϕ is a right quasi-multiplier. □

The next result follows from Theorem 3.5.

Corollary 3.6. *Let A be a commutative unital algebra. Then every quasi Jordan multiplier $\phi:A\times A\longrightarrow A$ is a quasi-multiplier.*

Lemma 3.7. *Let $\phi:A\times A\longrightarrow X$ be a right quasi Jordan multiplier. Then*

$$\phi(axb+bxa,y)=ax\phi(b,y)+bx\phi(a,y),$$

for all $a,b,x,y\in A$.

Proof. Suppose that $\phi(a^2,y)=a\phi(a,y)$ for all $a,y\in A$. Replacing a by $a+x$ we get

$$\phi(ax+xa,y)=a\phi(x,y)+x\phi(a,y),\quad a,x,y\in A. \tag{7}$$

Replacing x by $ax+xa$ in (7), we arrive at

$$\phi(a^2x+2axa+xa^2,y)=a\phi(ax+xa,y)+(ax+xa)\phi(a,y). \tag{8}$$

Using (7) and (8), we obtain

$$\phi(axa,y)=ax\phi(a,y),\quad a,x,y\in A.$$

Setting $a+b$ instead of a in the above equality, we get

$$\phi(axa+bxb+axb+bxa,y)=(ax+bx)(\phi(a,y)+\phi(b,y)),$$

for all $a,b,x,y\in A$. From the above two equality we reach the describe identity. □

As usual we write $[a,b]$ for the commutator $ab-ba$.

Theorem 3.8. *Assume that $\phi:A\times A\longrightarrow X$ is a right Jordan quasi-multiplier. Then*

$$cf(a,c,b)=[c,a]\phi(c,b)+\phi([a,c]c,b),\quad a,b,c\in A, \tag{9}$$

where $f(a,c,b)=\phi(ac,b)-a\phi(c,b)$.

Proof. By Lemma 3.7, we have

$$\begin{aligned}
cf(a,x,b)&=c\phi(ax,b)-ca\phi(x,b)\\
&=c\phi(ax,b)+xa\phi(c,b)-xa\phi(c,b)-ca\phi(x,b)\\
&=c\phi(ax,b)+xa\phi(c,b)-\phi(xac+cax,b)+\phi(acx,b)-\phi(acx,b)\\
&=c\phi(ax,b)+xa\phi(c,b)-\phi(xac+acx,b)+\phi(acx-cax,b)\\
&=c\phi(ax,b)+xa\phi(c,b)-x\phi(ac,b)-ac\phi(x,b)+\phi([a,c]x,b).
\end{aligned}$$

Replacing x by c, we get $cf(a,c,b)=[c,a]\phi(c,b)+\phi([a,c]c,b)$, for all $a,b,c\in A$. □

From Theorem 3.8, we have the next result.

Corollary 3.9. *Let $\phi:A\times A\longrightarrow X$ be a quasi Jordan multiplier. If A is unital and X is unitary, then ϕ is a quasi-multiplier.*

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References

[1] M. Adib, *Arens regularity of quasi-multipliers*, Politehn. Univ. Bucharest Sci. Bull. Ser. A. Appl. Math. Phys., 80 (2018), pp. 199–206.

[2] M. Adib, A. Riazi, and J. Bračič, *Quasi-multipliers of the dual of a Banach algebra*, Banach J. Math. Anal., 5 (2011), pp. 6–14.

[3] C. A. Akemann and G. K. Pedersen, *Complications of semicontinuity in C^*-algebra theory*, Duke Math. J., 40 (1973), pp. 785–795.

[4] H. G. Dales, *Banach algebras and automatic continuity*, vol. 24 of London Mathematical Society Monographs. New Series, The Clarendon Press, Oxford University Press, New York, 2000.

[5] H. Ghahramani, *On centralizers of Banach algebras*, Bull. Malays. Math. Sci. Soc., 38 (2015), pp. 155–164.

[6] M. Grosser, *Quasi-multipliers of the algebra of approximable operators and its duals*, Studia Math., 124 (1997), pp. 291–300.

[7] B. E. Johnson, *An introduction to the theory of centralizers*, Proc. London Math. Soc. (3), 14 (1964), pp. 299–320.

[8] C. E. Kenig and H. Porta, *Weak star and bounded weak star continuity of Banach algebra products*, Studia Math., 59 (1976/77), pp. 107–124.

[9] R. Larsen, *An introduction to the theory of multipliers*, Die Grundlehren der mathematischen Wissenschaften, Band 175, Springer-Verlag, New York-Heidelberg, 1971.

[10] H. X. Lin, *The structure of quasi-multipliers of C^*-algebras*, Trans. Amer. Math. Soc., 315 (1989), pp. 147–172.

[11] K. McKennon, *Quasi-multipliers*, Trans. Amer. Math. Soc., 233 (1977), pp. 105–123.

[12] M. A. Rieffel, *On the continuity of certain intertwining operators, centralizers, and positive linear functionals*, Proc. Amer. Math. Soc., 20 (1969), pp. 455–457.

[13] J. G. Wendel, *Left centralizers and isomorphisms of group algebras*, Pacific J. Math., 2 (1952), pp. 251–261.

[14] A. Zivari-Kazempour, *Characterization of n-Jordan multipliers*, Vietnam J. Math., 50 (2022), pp. 87–94.

[15] A. Zivari-Kazempour and A. Minapoor, *Jordan and local multipliers on certain Banach algebras are multipliers*, Sahand Commun. Math. Anal., 22 (2025), pp. 247–260.

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