AUT Journal of Mathematics and Computing

AUT J. Math. Comput., 7(2) (2026) 189-200
https://doi.org/10.22060/AJMC.2025.23674.1284

Original Article

New general location models for mixed responses

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ABSTRACT: In this paper, we introduce new general location model for mixed responses including correlated nominal, ordinal and continuous outcomes by using latent variable approach. We discuss regression methods for jointly analysis of continuous and categorical (nominal and ordinal) responses. After presenting the Leon and Carrière general location model [7], new general location model is introduced. A full likelihood-based approach is used to obtain maximum likelihood estimations of the models parameters. The proposed model is applied to BMI, Steatosis and Osteoporosis data.

Review History:

Received:14 November 2024
Revised:07 January 2025
Accepted:22 January 2025
Available Online:01 May 2026

Keywords:

Mixed-data
Latent variable
Multivariate normal distribution
General location model

MSC (2020):

62J02; 62J05; 62H10

1. Introduction

One of the important topics to researchers in other sciences, including education (learning of language, mathematics, etc.), medical sciences, etc. is the investigating of association between continuous and discrete variables. Finding a model which operates on the mixed responses, including discrete and continuous data is the main problem that takes researchers to challenge. For example, consider the data of medical study in which the correlated responses are as ordinal and continuous where categorical responses are osteoporosis of the spine, steatosis and continuous response is the body mass index (BMI). Separate analysis of each response gives biased estimates of parameters and misleading inference (de Leon and Carrière [6], Bahrami Samani and Ganjali [2], and Mogouie et al. [11]). The way is to use methods that simultaneously allow joint modelling.

The first, Olkin and Tate [13] describe General Location Model (GLOM) by multinomial model for discrete outcomes and a multivariate Gaussian model for continuous outcomes given discrete outcomes. In this model, the joint distribution of the continuous and categorical variables are factorized to a marginal multinomial distribution for categorical variables and a conditional multivariate normal distribution for continuous variables given categorical

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variables. The nominal variables form a contingency table by intersection of their levels. In each cell of the table, continuous variables are distributed as multivariate normal with constant covariance matrix and changeable mean vector without considering any covariate effects on the responses, simultaneously. Alternatively, Cox [4] describes the model in which marginal distribution of the continuous response is Gaussian and considers a logistic regression for conditional distribution of the binary response given continuous response. More recently, Cox and Wermuth [5] compare and contrast a number of different models based on these two factorizations of the joint distribution. The analysis of mixed response data—comprising correlated nominal, ordinal, and continuous outcomes—has long posed significant challenges to statisticians and researchers. Traditional regression methods often struggled to accommodate the complexities arising from the interplay of different types of data (Onwuegbuzie et al. [14]; Zhang et al. [19]). The need for a more flexible statistical framework to address these challenges led to the development of general location models (Leon and Carrière [7]; Sen et al. [17]). In 2007, de Leon and Carrière [7] introduced a general location model that laid the groundwork for analyzing mixed responses. Their approach utilized latent variable frameworks to simultaneously model various types of outcomes, allowing for a more nuanced understanding of the relationships between variables (de Leon and Carrière [7]). This model was instrumental in paving the way for further advancements in the field, particularly in its ability to handle correlated responses (López-Fidalgo et al. [10]). Building on the foundational work of Leon and Carrière, the present paper introduces a new general location model that enhances the existing framework. This model is designed to integrate correlated nominal, ordinal, and continuous outcomes effectively. By employing a latent variable approach, the new model expands the analytical capabilities available to researchers, enabling them to derive insights from complex datasets that were previously difficult to analyze jointly (Zhou et al. [20]). A significant aspect of this paper is the adoption of a full likelihood-based approach for parameter estimation. This methodology not only facilitates the computation of maximum likelihood estimates but also ensures that the derived estimates are robust and reliable (Stroup et al., [18]). By focusing on a full likelihood framework, the proposed model enhances the statistical rigor of the analysis, providing researchers with a powerful tool for understanding mixed responses. The practical application of the new general location model is demonstrated through its implementation on data related to Body Mass Index (BMI), Steatosis, and Osteoporosis. These applications highlight the model's versatility and effectiveness in real-world scenarios, showcasing its potential to contribute to fields such as epidemiology, health sciences, and social sciences (Berg [3]). In summary, the evolution of general location models has significantly advanced the analysis of mixed response data. The introduction of the new general location model represents a crucial step forward, offering enhanced methodologies for researchers to jointly analyze and interpret complex datasets. As the field continues to develop, such innovative approaches are essential for addressing the intricate challenges posed by mixed responses in statistical modeling.

Joint model of mixed ordinal and continuous responses based on a latent variable approach, in both univariate and multivariate cases, are introduced by Poon and Lee [15] and Bahrami Samani and Ganjali [1]. Using latent variables, several models have been proposed to analyze multiple non-commensurate outcomes as function of covariates. Sammel et al. [16] discuss the model where the outcomes are assumed to be a physical manifestation of a latent variable and conditioned on this latent variable. Another approach, based on latent variable, is proposed by Dunson [8]. A major difference between this approach and sammel's approach is association between responses and covariates.

de Leon and Carrière [7] introduce general mixed data model in which variables have different levels of measurement, i.e., models for joint distribution of data with a mixture of nominal, ordinal and continuous variables with no covariate effects on the responses, simultaneously. An obvious but indifferent approach to handle such data is to convert one of the variables to another type. For example, if nominal variables can be subjected as some scoring scheme, all the discrete variables can be treated as ordinal variables. Alternatively, the ordinal scale may be treated as interval, and then ordinal variables can be considered as continuous variables.

In this paper, we propose new general location model with latent variable approach for nominal, ordinal and continuous responses. In this model, the joint distribution of nominal, ordinal and continuous responses is factorized to a marginal multinomial distribution for nominal variables, a conditional multivariate normal distribution for ordinal variables given nominal variables and a conditional multivariate normal distribution for continuous variables given ordinal and nominal variables. The nominal and ordinal variables jointly form a contingency table by intersection of their levels, jointly. In each cell of the table, continuous variables are distributed as multivariate normal with constant covariance matrix and changeable mean vector by considering any covariate effects on the responses, simultaneously.

This paper is organized as follow in which Section 2, first shows de Leon and Carrière's model [7], then new general location model for mixed nominal, ordinal and continuous responses that is proposed by authors, is explained. Section 3, reports the result of the simulations regarding their finite sample performance. Section 4, applies the model for medical study data which takes account the association between steatosis, osteoporosis and body mass index (BMI). Section 5, includes conclusion remarks and possibilities for further studies.

2. General Mixed-data model (GMDM)

Suppose $\mathbf{x} = (X_{i1}, X_{i2}, \dots, X_{iP})^\top$, $\mathbf{y} = (Y_{i1}, Y_{i2}, \dots, Y_{iC})^\top$ and $\mathbf{z} = (Z_{i1}, Z_{i2}, \dots, Z_{iQ})^\top$ are nominal, continuous and ordinal vectors, respectively. Also, suppose $\mathbf{y}^* = \left(Y^*_{i1}, Y^*_{i2}, \dots, Y^*_{iQ}\right)^\top$ is latent variables vector corresponding to ordinal variables vector. The joint distribution of $\mathbf{x}$, $\mathbf{y}$ and $\mathbf{z}$ is factorized as

$$[\mathbf{x}, \mathbf{y}, \mathbf{z}] = [\mathbf{x}]\,[\mathbf{z}|\mathbf{x}]\,[\mathbf{y}|\mathbf{x}, \mathbf{z}]\,, \tag{1}$$

where $[\mathbf{x}]$,$[\mathbf{z}|\mathbf{x}]$ and $[\mathbf{y}|\mathbf{x}, \mathbf{z}]$ are respectively marginal distribution of $\mathbf{x}$, conditional distribution of $\mathbf{z}$ given $\mathbf{x}$ and conditional distribution of $\mathbf{y}$ given $\mathbf{x}$ and $\mathbf{z}$. In this model, nominal and ordinal variables form jointly a contingency table in which continuous variables sit within cells of the table. Suppose $\mathbf{u}$ is a vector of nominal variables with dimension $L \times 1$ where the l-th element of $\mathbf{u}$ has s'_l states, $l = 1, 2, \dots, L$, and $\mathbf{v}$ is the vector of ordinal variables with dimension $K \times 1$ where the k-th element of $\mathbf{v}$ has s''_k states, $k = 1, 2, \dots, K$. Vector $\mathbf{w} = (\mathbf{u}, \mathbf{v})$ defines a contingency table with

$$S = \prod_{d=1}^{D} s_d = \prod_{l=1}^{L}\prod_{k=1}^{K} s^{'}_l s^{''}_k,$$

possible states for values of $\mathbf{w}$. We define $\mathbf{w} = (W_1, W_2, \dots, W_S)^\top$ with dimension $S \times 1$ where if $\mathbf{u}$ and $\mathbf{v}$ are in state s, then w_s is equal to 1 and otherwise 0. Then, any row of $\mathbf{w}$ includes only one 1. Under GLOM, $\mathbf{w}$ is distributed as multinomial

$$[\mathbf{w}, \boldsymbol{\pi}] \propto \prod_{s=1}^{S} \pi_s^{W_s},$$

where $\boldsymbol{\pi} = (\pi_s; s = 1, 2, \dots, S)^\top$ is a probabilities vector of the elements of $\mathbf{w}$. Association between latent variables and ordinal variables is defined through threshold model, i.e.,

$$Z_{ij} = \begin{cases} 1 & Y^*_{ij} \le \theta_{1j} \\ \vdots & \vdots \\ h+1 & \theta_{hj} \le Y^*_{ij} \le \theta_{(h+1)j} \\ \vdots & \vdots \\ C & Y^*_{ij} \ge \theta_{(C-1)j}, \end{cases}$$

where $h = 1, 2, \dots, C-2$, $j = 1, 2, \dots, Q$ and $i = 1, 2, \dots, n$. Also, $\theta_{1j}, \theta_{2j}, \dots, \theta_{(C-1)j}$ are cutpoints. Indicator variables $I_{y_{ij}}$, $I_{y^*_{ij}}$ and I_{x_i} are defined as

$$I_{y_{ij}} = \begin{cases} 1 & Y_{ij} \le y_{ij} \\ 0 & \text{o.w.} \end{cases}, \qquad I_{y^*_{ij}} = \begin{cases} 1 & \theta_{z_{ij}-1} \le Y^*_{ij} \le \theta_{z_{ij}} \\ 0 & \text{o.w.} \end{cases} \qquad \text{and} \qquad I_{x_i} = \begin{cases} 1 & X_i = x_i \\ 0 & \text{o.w.} \end{cases}$$

The general location model of nominal, ordinal and continuous variables given covariates are defined as $\mathbf{y}^* = \boldsymbol{\beta}_1^\top \mathbf{T}_1 + \boldsymbol{\varepsilon}_1$ and $\mathbf{y} = \boldsymbol{\beta}_2^\top \mathbf{T}_2 + \boldsymbol{\varepsilon}_2$ in which $\mathbf{T} = (\mathbf{T}_1, \mathbf{T}_2)^\top$ is a covariate vectors where none of the variables has missing values and $\boldsymbol{\varepsilon} = (\boldsymbol{\varepsilon}_1, \boldsymbol{\varepsilon}_2)^\top$ is a vector of errors. Also, suppose $E\left(\boldsymbol{\varepsilon}|\mathbf{T}\right) = 0$ and

$$\boldsymbol{\Sigma} = \begin{pmatrix} \boldsymbol{\Sigma}_{11} & \boldsymbol{\Sigma}_{12} \\ \boldsymbol{\Sigma}_{21} & \boldsymbol{\Sigma}_{22} \end{pmatrix},$$

is the covariance matrix of errors vector.

One of the key assumptions in this model is the issue of parameter identifiability. Given that the ordinal response variables are defined based on a latent variable, ensuring model parameter identifiability becomes crucial. Consequently, a necessary assumption for this model is to maintain a constant variance for the latent variables associated with the ordinal variables, while also excluding any source of randomness in the models related to these responses. So, for the identifiable in the covariance matrix we let $\Sigma_{11} = I$, $\Sigma_{22} = I$, where I is the identity matrix.

The vector of model parameters is as $\boldsymbol{\psi} = \left(\boldsymbol{\beta}_1^\top, \boldsymbol{\beta}_2^\top, \theta_1, \theta_2, \dots, \theta_{C-1}, \boldsymbol{\Sigma}\right)$. Using Joe's approximation (1995), the maximum likelihood function of GLOM for complete data of nominal, ordinal and continuous is as

$$\begin{aligned} L\left(\boldsymbol{\psi}|\mathbf{y}, \mathbf{z}, \mathbf{x}\right) = \prod_{i=1}^{n} \Bigg[&(2\pi)^{-\frac{1}{2}} |\boldsymbol{\Sigma}|^{-\frac{1}{2}} \exp\left\{ -\frac{1}{2} \left(\mathbf{y}_i - \boldsymbol{\mu}_{\mathbf{y}_i}\right)^\top \boldsymbol{\Sigma}^{-1} \left(\mathbf{y}_i - \boldsymbol{\mu}_{\mathbf{y}_i}\right) \right\} \\ &+ \frac{\partial}{\partial y_i} \left(\boldsymbol{\Omega}_{21} \boldsymbol{\Omega}_{11}^{-1} \left(1 - E\left(I_{y^*_i}\right), 1 - E\left(I_{x_i}\right)\right)^\top \right) \Bigg] \pi_d, \end{aligned} \tag{2}$$

where $\boldsymbol{\mu}_{\mathbf{y}_i} = \boldsymbol{\beta}_1^\top \mathbf{T}_2$, $\boldsymbol{\Omega}_{21}$ is a row vector which includes member of $\left(\boldsymbol{\Sigma}_{I_{y_i^*}, I_{y_i}}, \boldsymbol{\Sigma}_{I_{y_i}, I_{x_i}}\right)$ and $\boldsymbol{\Omega}_{11}$ is a matrix whose members are as

$$\begin{pmatrix} \boldsymbol{\Sigma}_{I_{y_i^*}, I_{y_i^*}} & \boldsymbol{\Sigma}_{I_{y_i^*}, I_{x_i}} \\ & \boldsymbol{\Sigma}_{I_{x_i}, I_{x_i}} \end{pmatrix}.$$

Also, the probability related to any cell of contingency table is as $\Pr\left(\theta_{h-1} < Y^* < \theta_h | X = x\right) \Pr(X = x)$ where θ_h is cutpoint, $\theta_0 = -\infty$ and $\theta_{l+1} = +\infty$. Now, by getting likelihood function of the model, maximum likelihood estimates of the model parameters are obtained using "nlminb" code in **R** which is used to minimize the non-linear functions and to specify the covariates which are significant in model.

In This model, particularly with likelihood-based methods like maximum likelihood estimation (MLE) or Bayesian inference, the choice of initial parameter estimates is critical for achieving global convergence. Key points regarding initial estimates include: Initial values are often based on prior knowledge or theoretical expectations. Estimates can come from analyzing data trends like means or variances. Fitting simpler models can provide initial estimates for more complex models. Multiple random starts may be used in complex models to find the best fit. Evaluating sensitivity to initial estimates is vital for model robustness: Running the optimization multiple times with varied starts to check for consistent convergence. Observing how convergence behavior varies with different initial values. Simulating data with known parameters to test the model's ability to recover true values.

3. Simulation

In this section, we provide a simulation study to generate mixed-data from finite GMDM to estimate parameters of the de Leon and Carrière's general location model (2007) and our new general location model and then comparing them. In simulation studies, the conditions for the identifiability of model parameters include the constancy of the variances of the latent variables of the ordinal variables.

Here, we assume that $C = 1$ and $Q = S = 2$. In this model, there are a continuous variable Y, two nominal variables X_1 and X_2, by two levels for each of them, and two ordinal variables Z_1 and Z_2 by two levels for each of them. Note that, the ordinal variables are defined as

$$Z_1 = \begin{cases} 1 & Y_1^* \leq \theta_1 \\ 2 & \theta_1 < Y_1^* \leq \theta_2 \\ 3 & Y_1^* > \theta_2 \end{cases} \qquad \text{and} \qquad Z_2 = \begin{cases} 1 & Y_2^* \leq \eta_1 \\ 2 & \eta_1 < Y_2^* \leq \eta_2 \\ 3 & Y_2^* > \eta_2, \end{cases}$$

where Y_1^* and Y_2^* are latent variables corresponding to Z_1 and Z_2, respectively. Also, the continuous variables Y, Y_1^* and Y_2^* are generated from multivariate normal distribution with mean $\mathbf{0}$ and covariance matrix as,

$$\boldsymbol{\Sigma} = \begin{pmatrix} 1 & 0.5 & 0.5 \\ & 1 & 0.5 \\ & & 1 \end{pmatrix}.$$

Suppose $\theta_1 = \eta_1 = -1$ and $\theta_2 = \eta_2 = 1$ and for simulation, consider samples with sizes 50, 1000 and 10000 from GMDM by 2000 repeated times. It should be noted that the general location model of nominal, ordinal and continuous variables are considered as $Y = \mu_y + \varepsilon_1$, $Y_1^* = \varepsilon_2$ and $Y_2^* = \varepsilon_3$.

3.1. de Leon and Carrière's GLOM [7]

In this part, to access the precision of approximation and by using the joint distribution of the de Leon and Carrère's GLOM, we perform a simulation study. The joint distribution of X_1, X_2, Z_1, Z_2 and Y, based on model (1) is factorized as

$$[X_1, X_2, Z_1, Z_2, Y] = [Z_1, Z_2 | Y, X_1, X_2] \, [Y | X_1, X_2] \, [X_1, X_2],$$

where $[X_1, X_2]$, $[Y|X_1, X_2]$ and $[Z_1, Z_2|Y, X_1, X_2]$ are respectively marginal distribution of X_1 and X_2, conditional distribution of Y given X_1 and X_2, and conditional distribution of Z_1 and Z_2 given X_1, X_2 and Y. Because of existing four cells of contingency table, the parameter vector is as $\boldsymbol{\psi} = \left(\boldsymbol{\pi}, \boldsymbol{\mu}, \sigma^2, \boldsymbol{\rho}, \theta_1, \theta_2, \eta_1, \eta_2\right)$ where $\boldsymbol{\mu} = \left(\mu_1, \mu_2, \mu_3, \mu_4\right)^\top$ is as $E\left(Y | \mathbf{x} = \mathbf{x}_{(s)}\right)$, $s = 1, 2, 3, 4$, σ^2 is variance of Y, $\boldsymbol{\rho} = \left(\rho_{y, y_1^*}, \rho_{y, y_2^*}\right)$ in which ρ_{y, y_1^*} is correlation between Y and Y_1^* and ρ_{y, y_2^*} is correlation between Y and Y_2^*, θ_1 and θ_2 are cutpoints associated to Z_1, and η_1 and η_2 are cutpoints associated to Z_2. The results of simulation have displayed in Table 1. According to Table 1, we see slight improvements in parameters estimates along their standard deviation while the sample size is increased.

Table 1: Maximum likelihood estimates of the de Loan and Carrière's GLOM (2007) based on 2000 simulated data sets from GMDM with $C = 1$ and $S = 2$.

Parameter	**True value**	$n = 500$		$n = 1000$		$n = 10000$	
		Est.	**S.D.**	**Est.**	**S.D.**	**Est.**	**S.D.**
$\mu_y^{(1)}$	0.000	0.072	0.871	0.064	0.772	0.048	0.234
π_1	0.25	0.43	0.787	0.12	0.624	0.53	0.425
$\mu_y^{(2)}$	0.000	0.089	0.625	0.076	0.423	0.063	0.363
π_2	0.25	0.40	0.623	0.530	0.566	0.12	0.345
$\mu_y^{(3)}$	0.000	0.086	0.821	0.079	0.763	0.051	0.463
π_3	0.25	0.16	0.763	0.30	0.601	0.175	0.445
$\mu_y^{(4)}$	0.000	0.053	0.623	0.042	0.524	0.032	0.321
π_4	0.25	0.01	0.569	0.05	0.431	0.175	0.223
θ_1	-1	-1.672	0.422	-1.521	0.378	-1.422	0.522
θ_2	1	1.829	0.526	1.734	0.437	1.626	0.327
η_1	-1	-1.723	0.531	-1.635	0.528	-1.428	0.329
η_2	1	1.629	0.628	1.522	0.598	1.334	0.432
σ^2	1	0.625	0.210	0.699	0.200	0.838	0.142
$\rho_{y_1^*,y_2^*}$	0.5	0.328	0.235	0.345	0.201	0.449	0.198
ρ_{y,y_1^*}	0.5	0.301	0.199	0.385	0.187	0.438	0.156
ρ_{y,y_2^*}	0.5	0.348	0.183	0.399	0.179	0.441	0.148

3.2. New general location model

Now, According to the model, the joint distribution of X_1, X_2, Z_1, Z_1 and Y is factorized as

$$[X_1, X_2, Z_1, Z_2, Y] = [Y|Z_1, Z_2, X_1, X_2]\,[Z_1, Z_2|X_1, X_2]\,[X_1, X_2]\,,$$

where $[X_1, X_2]$, $[Z_1, Z_2|X_1, X_2]$ and $[Y|Z_1, Z_2, X_1, X_2]$ are respectively marginal distribution of X_1 and X_2, conditional distribution of Z_1 and Z_2 given X_1 and X_2, and conditional distribution of Y given Z_1, Z_2, X_1 and X_2. Note that, as we mentioned in Section 2, in this case, the number of cells of the contingency table will be 36 and the parameters which should be estimated are along their estimations displayed in Table 2. According to the simulation results of new general location model, we see that the estimates are closer to true value and their standard deviation have been decreased in contrast to the de Leon and Carrière [7]'s general location model while increasing the sample sizes. So, we can conclude that the new general location model has better performance to estimate parameters than the de Leon and Carrière's general location model [7]. It should be mentioned that the estimates of parameters of both models are optimized through "nlminb" code and estimates of the covariance matrix are obtained using "fdHess" code in **R**.

Table 2: Maximum likelihood estimates of the new general location model based on 2000 simulated data sets from GMDM with $C = 1$ and $S = 2$.

	Parameter	**True value**	$n = 500$		$n = 1000$		$n = 10000$	
			S.D.	**Est.**	**S.D.**	**Est.**	**S.D.**	**Est.**
(1,1)(1,1)	$\mu_y^{(1)}$	0.000	-0.027	0.138	-0.025	0.095	-0.020	0.025
	π_1	0.028	0.024	0.139	0.025	0.102	0.026	0.055
(1,1)(1,2)	$\mu_y^{(2)}$	0.000	-0.025	0.215	-0.019	0.138	-0.017	0.035
	π_2	0.028	0.029	0.142	0.026	0.123	0.027	0.079
(1,1)(1,3)	$\mu_y^{(3)}$	0.000	-0.022	0.110	-0.020	0.092	-0.020	0.045
	π_3	0.028	0.032	0.109	0.030	0.087	0.029	0.031
(1,1)(2,1)	$\mu_y^{(4)}$	0.000	-0.032	0.105	-0.028	0.088	-0.025	0.056
	π_4	0.028	0.030	0.132	0.029	0.102	0.026	0.078
(1,1)(2,2)	$\mu_y^{(5)}$	0.000	-0.027	0.148	-0.023	0.106	-0.020	0.076

Continued on next page

Table 2: Maximum likelihood estimates of the new general location model based on 2000 simulated data sets from GMDM with $C = 1$ and $S = 2$. (Continued)

	π_5	0.028	0.026	0.146	0.030	0.105	0.027	0.068
(1,1)(2,3)	$\mu_y^{(6)}$	0.000	-0.029	0.166	-0.026	0.125	-0.024	0.098
	π_6	0.028	0.021	0.187	0.025	0.141	0.029	0.083
(1,1)(3,1)	$\mu_y^{(7)}$	0.000	-0.023	0.178	-0.022	0.125	-0.020	0.083
	π_7	0.028	0.032	0.132	0.031	0.099	0.030	0.047
(1,1)(3,2)	$\mu_y^{(8)}$	0.000	-0.030	0.121	-0.029	0.102	-0.024	0.053
	π_8	0.028	0.031	0.154	0.030	0.113	0.029	0.083
(1,1)(3,3)	$\mu_y^{(9)}$	0.000	-0.032	0.147	-0.030	0.121	-0.029	0.063
	π_9	0.028	0.022	0.198	0.025	0.132	0.026	0.078
(1,2)(1,1)	$\mu_y^{(10)}$	0.000	-0.028	0.115	-0.025	0.103	-0.022	0.073
	π_{10}	0.028	0.034	0.219	0.030	0.153	0.026	0.099
(1,2)(1,2)	$\mu_y^{(11)}$	0.000	-0.026	0.189	-0.023	0.134	-0.021	0.087
	π_{11}	0.028	0.021	0.183	0.026	0.151	0.027	0.093
(1,2)(1,3)	$\mu_y^{(12)}$	0.000	-0.023	0.234	-0.019	0.115	-0.015	0.078
	π_{12}	0.028	0.032	0.197	0.030	0.121	0.029	0.073
(2,1)(2,1)	$\mu_y^{(13)}$	0.000	-0.027	0.157	-0.025	0.108	-0.023	0.075
	π_{13}	0.028	0.021	0.201	0.024	0.134	0.026	0.067
(2,1)(2,2)	$\mu_y^{(14)}$	0.000	-0.029	0.185	-0.027	0.152	-0.024	0.078
	π_{14}	0.028	0.024	0.117	0.029	0.109	0.027	0.069
(2,1)(2,3)	$\mu_y^{(15)}$	0.000	-0.030	0.206	-0.026	0.178	-0.022	0.093
	π_{15}	0.028	0.032	0.198	0.031	0.131	0.029	0.077
(2,1)(3,1)	$\mu_y^{(16)}$	0.000	-0.032	0.135	-0.030	0.113	-0.026	0.087
	π_{16}	0.028	0.034	0.185	0.032	0.132	0.030	0.105
(2,1)(3,2)	$\mu_y^{(17)}$	0.000	-0.029	0.201	-0.025	0.114	-0.021	0.043
	π_{17}	0.028	0.024	0.198	0.026	0.135	0.027	0.069
(2,1)(3,3)	$\mu_y^{(18)}$	0.000	-0.025	0.198	-0.022	0.135	-0.021	0.093
	π_{18}	0.028	0.033	0.191	0.030	0.115	0.029	0.073
(2,1)(1,1)	$\mu_y^{(19)}$	0.000	-0.021	0.255	-0.020	0.178	-0.018	0.092
	π_{19}	0.028	0.024	0.185	0.025	0.105	0.026	0.073
(2,1)(1,2)	$\mu_y^{(20)}$	0.000	-0.024	0.221	-0.023	0.119	-0.021	0.083
	π_{20}	0.028	0.025	0.113	0.026	0.098	0.027	0.043
(2,1)(1,3)	$\mu_y^{(21)}$	0.000	-0.020	0.156	-0.019	0.108	-0.014	0.097
	π_{21}	0.028	0.035	0.223	0.032	0.187	0.031	0.073
$(2,1)(2,1)$	$\mu_y^{(22)}$	0.000	-0.030	0.178	-0.028	0.104	-0.024	0.045
	π_{22}	0.028	0.023	0.213	0.024	0.132	0.026	0.053
(2,1)(2,2)	$\mu_y^{(23)}$	0.000	-0.032	0.191	-0.030	0126	-0.028	0.089
	π_{23}	0.028	0.033	0.184	0.031	0.134	0.029	0.093
(2,1)(2,3)	$\mu_y^{(24)}$	0.000	-0.028	0.215	-0.023	0.138	-0.020	0.073
	π_{24}	0.028	0.024	0.173	0.025	0.135	0.027	0.062
(2,1)(3,1)	$\mu_y^{(25)}$	0.000	-0.025	0.213	-0.022	0.188	-0.019	0.112
	π_{25}	0.028	0.025	0.158	0.026	0.110	0.027	0.081
(2,1)(3,2)	$\mu_y^{(26)}$	0.000	-0.020	0.132	-0.018	0.106	-0.017	0.091
	π_{26}	0.028	0.033	0.169	0.032	0.119	0.030	0.081
(2,1)(3,3)	$\mu_y^{(27)}$	0.000	-0.023	0.196	-0.021	0.163	-0.019	0.101
	π_{27}	0.028	0.020	0.154	0.021	0.107	0.027	0.047
(2,2)(1,1)	$\mu_y^{(28)}$	0.000	-0.027	0.196	-0.024	0.163	-0.023	0.100

Continued on next page

Table 2: Maximum likelihood estimates of the new general location model based on 2000 simulated data sets from GMDM with $C = 1$ and $S = 2$. (Continued)

	π_{28}	0.028	0.023	0.187	0.022	0.123	0.026	0.093
(2,2)(1,2)	$\mu_y^{(29)}$	0.000	-0.030	0.169	-0.028	0.141	-0.024	0.078
	π_{29}	0.028	0.034	0.218	0.032	0.105	0.030	0.061
(2,2)(1,3)	$\mu_y^{(30)}$	0.000	-0.023	0.135	-0.021	0.117	-0.019	0.071
	π_{30}	0.028	0.032	0.118	0.031	0.108	0.027	0.053
(2,2)(2,1)	$\mu_y^{(31)}$	0.000	-0.026	0.242	-0.025	0.121	-0.022	0.093
	π_{31}	0.028	0.021	0.153	0.022	0.113	0.026	0.091
(2,2)(2,2)	$\mu_y^{(32)}$	0.000	-0.032	0.185	-0.031	0.133	-0.027	0.107
	π_{32}	0.028	0.032	0.167	0.030	0.109	0.029	0.063
(2,2)(2,3)	$\mu_y^{(33)}$	0.000	-0.027	0.212	-0.024	0.165	-0.021	0.063
	π_{33}	0.028	0.025	0.187	0.026	0.162	0.027	0.112
(2,2)(3,1)	$\mu_y^{(34)}$	0.000	-0.029	0.221	-0.026	0.138	-0.023	0.053
	π_{34}	0.028	0.033	0.158	0.031	0.108	0.030	0.086
(2,2)(3,2)	$\mu_y^{(35)}$	0.000	-0.028	0.205	-0.026	0.114	-0.023	0.093
	π_{35}	0.028	0.023	0.193	0.024	0.168	0.026	0.067
(2,2)(3,3)	$\mu_y^{(36)}$	0.000	-0.024	0.188	-0.022	0.142	-0.021	0.079
	π_{36}	0.028	0.034	0.121	0.031	0.105	0.030	0.078
	θ_1	-1.000	-1.280	0.131	-1.277	0.105	-1.265	0.045
	θ_2	1.000	0.973	0.187	0.980	0.153	0.990	0.110
	η_1	-1.000	-1.129	0.176	-1.125	0.138	-1.122	0.097
	η_2	1.000	0.963	0.181	0.968	0.143	0.977	0.075
	$\sigma^2_{y_1^*}$	1.000	0.865	0.161	0.871	0.128	0.890	0.062
	$\sigma^2_{y_2^*}$	1.000	0.859	0.142	0.863	0.112	0.872	0.033
	σ^2_y	1.000	0.901	0.121	0.913	0.102	0.921	0.052
	$\rho_{y_1^*,y_2^*}$	0.500	0.471	0.115	0.488	0.089	0.494	0.065
	ρ_{y,y_1^*}	0.500	0.462	0.108	0.475	0.097	0.482	0.054
	ρ_{y,y_2^*}	0.500	0.453	0.124	0.461	0.106	0.478	0.071

4. Application to Osteoporosis, Steatosis and BMI data

In this section, we present an application to investigate association between Osteoporosis, Steatosis and body mass index (BMI). The data of the study recorded from an observational study on 8163 patients in Taleghani hospital, Tehran, Iran. Osteoporosis, which literally means "porous bone", is a disease in which the density and quality of bone are reduced. As the bones become more porous and fragile, the risk of fracture is greatly increased. The loss of bone occurs silently and progressively. Often, there are no symptoms until the first fracture occurs. The most common fractures associated with osteoporosis occur at the hip, spine and wrist. The incidence of these fractures, particularly at the hip and spine, increases with age in both women and men. It is often seen in post menopausal women, particularly light-skinned, small-framed women with a family history of osteoporosis. The loss of calcium from bones is the major effect of ageing on the skeletal system. The body mass index is defined as the individual's body weight divided by the square of his/her height. The formulae universally used in medicine produces a unit measure of $BMI = weight\,(kg) \Big/ (height\,(m))^2$. Steatosis is fatty infiltration of the liver. When inflammation is associated with the fatty change, the term steatohepatitis is used. Steatosis is often but not exclusively an early histological feature of alcoholic liver disease (alcohol-related fatty liver) leading to alcohol-related steatohepatitis. The non alcohol-related cases are known as non-alcoholic fatty liver disease (NAFLD) and non-alcoholic steatohepatitis (NASH).The osteoporosis and steatosis are defined as two ordinal variables (Z_1 and Z_2) with three levels as

$$Z_1 = \begin{cases} 1 & \text{the person doesn't have osteoporosis} \\ 2 & \text{the person has mediocre osteoporosis} \\ 3 & \text{the person has many osteoporosis,} \end{cases}$$

and

$$Z_2 = \begin{cases} 1 & \text{the person doesn't have steatosis} \\ 2 & \text{the person has mediocre steatosis} \\ 3 & \text{the person has many steatosis.} \end{cases}$$

So, the main problem, which we are interested in, is to investigate association between the osteoporosis, steatosis and BMI by considering of existing covariate effects on the responses, simultaneously. Explanatory variables which effects these responses are as amount of total body calcium (Ca), job status as employee or housekeeper (Job), type of accommodation as house or apartment (Ta) and the systolic blood pressure (SBP).

Numerous studies have established a complex relationship between BMI and bone health. Generally, a higher BMI, particularly when due to increased fat mass, is associated with lower osteoporosis risk, while lower BMI is often linked to higher fracture risk and lower bone mineral density (BMD). However, this relationship can vary significantly based on factors such as age, sex, and body composition (fat vs. lean mass). In this paper findings suggest that certain BMI categories have unexpected relationships with osteoporosis incidence or severity, this could offer new insights, particularly if they are specific to certain populations (e.g., postmenopausal women or specific ethnic groups). Obesity (high BMI) is a well-established risk factor for hepatic steatosis (fatty liver disease). Studies show that increased body fat, particularly visceral fat, contributes to the development of steatosis through mechanisms involving insulin resistance and inflammation. Our paper findings indicate that certain BMI thresholds do not correlate with expected levels of steatosis, or if they suggest protective factors in specific BMI ranges, this could challenge existing paradigms and warrant further investigation. The connection between osteoporosis and steatosis is less direct but can be inferred through shared risk factors, such as age, lifestyle, and metabolic syndrome components. Some studies suggest that non-alcoholic fatty liver disease (NAFLD) may impact bone health negatively due to inflammatory processes and altered metabolism. In this paper, we results indicate a direct relationship between osteoporosis and steatosis that has not been previously reported, it could point to new pathways linking metabolic health with bone density and strength. (via Ensrud and Crandall [9] and Nomura et al. [12]).

Descriptive statistics (mean and standard deviation of BMI and percentages of osteoporosis and steatosis) are given in Table 3.

Table 3: Descriptive statistics of the Osteoporosis, Steatosis and BMI data.

	No.	**Mean**	**S.E.**
BMI	8163	29.357	10.806
	Levels	No.	Precentage
	None	2955	0.362
Y_1^* :Osteoporosis	Mild	3257	0.399
	Severe	1951	0.239
	None	2040	0.258
Y_2^* :Steatosis	Mild	3404	0.417
	Severe	2719	0.325

As the Table 3 shows, the mean of BMI is 29.357 with standard deviation of 10.806, the percentage of osteoporosis in "severe" level is less than its other levels, and the percentage of steatosis in "none" level is less than its other levels. Let $J = (J_1, J_2)$, $J_d = 1, 2$ and $L = (L_1, L_2)$, $L_Q = 1, 2, 3$, we consider joint models related to responses along with covariate effects as follow:

(I) $BMI_{(J,L)}|(Ta, Job) = \gamma_{0(J,L)} + \gamma_{11(J,L)}SBP + \gamma_{12(J,L)}Ca + \lambda_1^{(1)}Job + \lambda_2^{(1)}Ta + \varepsilon_{1(J,L)}$,

(II) ${Y_1^*}_{(J)}|(Ta, Job) = \gamma_{21(J)}SBP + \gamma_{22(J)}Ca + \lambda_1^{(2)}Job + \lambda_2^{(2)}Ta + \varepsilon_{2(J)}$,

(III) ${Y_2^*}_{(J)}|Ta, Job = \gamma_{31(J)}SBP + \gamma_{32(J)}Ca + \lambda_1^{(3)}Job + \lambda_2^{(3)}Ta + \varepsilon_{3(J)}$,

where $\gamma_{0(J,L)}$, $\gamma_{11(J,L)}$ and $\gamma_{12(J,L)}$ are regression coefficients related to the BMI in the cell (J, L) and $\gamma_{uv(J)}$, $u = 2, 3$, $v = 1, 2$ are regression coefficients related to the ordinal responses (Y_1^*, Y_2^*) in the cell J. Also, $\lambda_1^{(v)}$ and $\lambda_2^{(v)}$, $v = 1, 2$ are regression coefficients.

Figure 1 shows the GMDM model contingency table for the data. This contingency table is derived from the levels of the nominal variables of household type and occupation, and the variables of severity of fatty liver disease and severity of osteoporosis, and the body mass index variable is placed in each cell of this table.

GMDM	TA							
			0			1		
			Y1*			Y1*		
Job	0	Y2*	BMI	BMI	BMI	BMI	BMI	BMI
			BMI	BMI	BMI	BMI	BMI	BMI
			BMI	BMI	BMI	BMI	BMI	BMI
	1	Y2*	BMI	BMI	BMI	BMI	BMI	BMI
			BMI	BMI	BMI	BMI	BMI	BMI
			BMI	BMI	BMI	BMI	BMI	BMI

Figure 1: The form of the GMDM model's antecedent table for the data

The covariance matrix of the errors vector $(\varepsilon_{1(J)}, \varepsilon_{2(J)}, \varepsilon_{3(J)})^\top$ is as

$$\mathbf{\Sigma} = \begin{pmatrix} 1 & \sigma\rho_{12} & \sigma\rho_{13} \\ & 1 & \sigma\rho_{23} \\ & & 1 \end{pmatrix},$$

where σ is standard deviation of BMI, ρ_{12} is correlation between BMI and osteoporosis, ρ_{13} is correlation between BMI and steatosis and ρ_{23} is correlation between osteoporosis and steatosis.

Note that a multivariate normal distribution with correlation between errors ρ_{12}, ρ_{13} and ρ_{23} are assumed and should be estimated. The results of parameters estimations are given in Table 4. Table 4 shows significant effect of SBP, job status and type of the accommodation (Ta) on BMI, significant effect of SBP on the probability of low value of steatosis and a significant effect of Ca and job status on the probability of low value of osteoporosis of the spine. From these effects we can inferred that the people who live in apartment have more BMI than of people who live in house. Also, we see that correlation parameters ρ_{12}, ρ_{13} and ρ_{23} are significant. It shows a negative correlation between BMI and osteoporosis of the spine ($\hat{\rho}_{12} = -0.212$), a positive correlation between BMI and steatosis ($\hat{\rho}_{13} = 0.559$) and a positive correlation between BMI.

5. Conclusion

In this article, we proposed new general location models for mixed nominal, ordinal and continuous responses. This procedure should be useful when the specification of a general location models of all the variables are difficult. Two likelihood estimation strategies are proposed which use the full likelihood function to estimate parameters jointly. In the first model, the nominal variables make contingency table by intersection of their levels, but in the second model, the table is formed by intersection levels of both nominal and ordinal variables. Simulation results indicate that both estimation methods of model Carri'ere's GLOM and New GLOM , are able to record parameters reasonably well which their standard deviations reflect true sampling variability. Work on extending our methodology to allow for data clustering is on-going. We are also exploring generalizations of the model to the multivariate case of more than one discrete and continuous outcomes, including the incorporation of random effects. The challenge, here, lies on defining models that allow for different levels of association among outcomes, as in longitudinal studies, and always guarantee proper joint distributions. In future research, the models presented in this article will be considered with a Bayesian approach and a machine learning algorithm approach for big data.

Table 4: Results of parameters estimations of our model

Part (I) Model			**Part (II) Model**			**Part (III) Model**		
Parameter	**Est.**	**S.E.**	**Parameter**	**Est.**	**S.E.**	**Parameter**	**Est.**	**S.E.**
σ^2	12.643	0.251	$\lambda_2^{(2)}$	0.005	0.153	η_1	-2.643	1.304
$\lambda_1^{(1)}$	1.654	0.118	$\lambda_2^{(3)}$	0.006	0.181	η_2	-1.787	1.295
$\lambda_1^{(2)}$	0.570	0.115	ρ_{12}	-0.212	0.086	θ_1	1.205	1.475
$\lambda_1^{(3)}$	-0.548	0.515	ρ_{13}	0.559	0.113	θ_2	2.283	1.470
$\lambda_2^{(1)}$	3.003	1.742	ρ_{23}	0.019	0.011			
$\gamma_{0(1,1),(1,1)}$	21.386	4.218	$\gamma_{0(1,1),(1,2)}$	22.423	4.528	$\gamma_{0(1,1),(1,3)}$	20.564	3.899
$\gamma_{11(1,1),(1,1)}$	0.037	0.024	$\gamma_{11(1,1),(1,2)}$	0.043	0.027	$\gamma_{11(1,1),(1,3)}$	0.039	0.021
$\gamma_{12(1,1),(1,1)}$	-0.252	0.514	$\gamma_{12(1,1),(1,2)}$	-0.276	0.443	$\gamma_{12(1,1),(1,3)}$	-0.294	0.442

Continued on next page

Table 4: Results of parameters estimations of our model (Continued)

$\gamma_{21(1,1),(1,1)}$	-0.001	0.005	$\gamma_{21(1,1),(1,2)}$	-0.006	0.008	$\gamma_{21(1,1),(1,3)}$	-0.007	0.006
$\gamma_{22(1,1),(1,1)}$	0.213	0.123	$\gamma_{22(1,1),(1,2)}$	0.312	0.143	$\gamma_{22(1,1),(1,3)}$	0.229	0.186
$\gamma_{31(1,1),(1,1)}$	0.044	0.020	$\gamma_{31(1,1),(1,2)}$	0.054	0.025	$\gamma_{31(1,1),(1,3)}$	0.063	0.123
$\gamma_{32(1,1),(1,1)}$	0.010	0.173	$\gamma_{32(1,1),(1,2)}$	0.012	0.154	$\gamma_{32(1,1),(1,3)}$	0.023	0.169
$\gamma_{0(1,1),(2,1)}$	23.678	5.321	$\gamma_{0(1,1),(2,2)}$	19.789	4.860	$\gamma_{0(1,1),(2,3)}$	21.765	3.990
$\gamma_{11(1,1),(2,1)}$	0.035	0.028	$\gamma_{11(1,1),(2,2)}$	0.041	0.028	$\gamma_{11(1,1),(2,3)}$	0.057	0.041
$\gamma_{12(1,1),(2,1)}$	-0.243	0.421	$\gamma_{12(1,1),(2,2)}$	-0.287	0.423	$\gamma_{12(1,1),(2,3)}$	-0.321	0.478
$\gamma_{21(1,1),(2,1)}$	-0.007	0.009	$\gamma_{21(1,1),(2,2)}$	-0.004	0.006	$\gamma_{21(1,1),(2,3)}$	-0.003	0.005
$\gamma_{22(1,1),(2,1)}$	0.341	0.142	$\gamma_{22(1,1),(2,2)}$	0.231	0.421	$\gamma_{22(1,1),(2,3)}$	0.452	0.267
$\gamma_{31(1,1),(2,1)}$	0.067	0.059	$\gamma_{31(1,1),(2,2)}$	0.078	0.032	$\gamma_{31(1,1),(2,3)}$	0.034	0.019
$\gamma_{32(1,1),(2,1)}$	0.018	0.154	$\gamma_{32(1,1),(2,2)}$	0.012	0.0149	$\gamma_{32(1,1),(2,3)}$	0.019	0.169
$\gamma_{0(1,1),(3,1)}$	22.476	4.678	$\gamma_{0(1,1),(3,2)}$	27.403	4.670	$\gamma_{0(1,1),(3,3)}$	21.504	3.670
$\gamma_{11(1,1),(3,1)}$	0.048	0.033	$\gamma_{11(1,1),(3,2)}$	0.049	0.036	$\gamma_{11(1,1),(3,3)}$	0.051	0.041
$\gamma_{12(1,1),(3,1)}$	-0.321	0.430	$\gamma_{12(1,1),(3,2)}$	-0.387	0.326	$\gamma_{12(1,1),(3,3)}$	-0.227	0.291
$\gamma_{21(1,1),(3,1)}$	-0.008	0.006	$\gamma_{21(1,1),(3,2)}$	-0.005	0.007	$\gamma_{21(1,1),(3,3)}$	-0.009	0.004
$\gamma_{22(1,1),(3,1)}$	0.451	0.129	$\gamma_{22(1,1),(3,2)}$	0.264	0.462	$\gamma_{22(1,1),(3,3)}$	0.412	0.243
$\gamma_{31(1,1),(3,1)}$	0.056	0.023	$\gamma_{31(1,1),(3,2)}$	0.061	0.024	$\gamma_{31(1,1),(3,3)}$	0.054	0.012
$\gamma_{32(1,1),(3,1)}$	0.012	0.169	$\gamma_{32(1,1),(3,2)}$	0.013	0.174	$\gamma_{32(1,1),(3,3)}$	0.031	0.158
$\gamma_{0(2,1),(1,1)}$	20.447	4.397	$\gamma_{0(2,1),(1,2)}$	22.564	0.398	$\gamma_{0(2,1),(1,3)}$	23.218	4.637
$\gamma_{11(2,1),(1,1)}$	0.038	0.031	$\gamma_{11(2,1),(1,2)}$	0.045	0.030	$\gamma_{11(2,1),(1,3)}$	0.048	0.021
$\gamma_{12(2,1),(1,1)}$	-0.231	0.451	$\gamma_{12(2,1),(1,2)}$	-0.321	0.427	$\gamma_{12(2,1),(1,3)}$	-0.351	0.541
$\gamma_{21(2,1),(1,1)}$	-0.003	0.010	$\gamma_{21(2,1),(1,2)}$	-0.008	0.006	$\gamma_{21(2,1),(1,3)}$	-0.009	0.007
$\gamma_{22(2,1),(1,1)}$	0.324	0.129	$\gamma_{22(2,1),(1,2)}$	0.218	0.146	$\gamma_{22(2,1),(1,3)}$	0.371	0.112
$\gamma_{31(2,1),(1,1)}$	0.053	0.023	$\gamma_{31(2,1),(1,2)}$	0.056	0.032	$\gamma_{31(2,1),(1,3)}$	0.047	0.032
$\gamma_{32(2,1),(1,1)}$	0.011	0.163	$\gamma_{32(2,1),(1,2)}$	0.018	0.210	$\gamma_{32(2,1),(1,3)}$	0.017	0.198
$\gamma_{0(2,1),(2,1)}$	22.567	5.432	$\gamma_{0(2,1),(2,2)}$	20.645	4.931	$\gamma_{0(2,1),(2,3)}$	24.108	3.651
$\gamma_{11(2,1),(2,1)}$	0.034	0.026	$\gamma_{11(2,1),(2,2)}$	0.045	0.063	$\gamma_{11(2,1),(2,3)}$	0.057	0.031
$\gamma_{12(2,1),(2,1)}$	-0.231	0.445	$\gamma_{12(2,1),(2,2)}$	-0.341	0.411	$\gamma_{12(2,1),(2,3)}$	-0.451	0.601
$\gamma_{21(2,1),(2,1)}$	-0.004	0.007	$\gamma_{21(2,1),(2,2)}$	-0.009	0.008	$\gamma_{21(2,1),(2,3)}$	-0.006	0.005
$\gamma_{22(2,1),(2,1)}$	0.234	0.122	$\gamma_{22(2,1),(2,2)}$	0.346	0.212	$\gamma_{22(2,1),(2,3)}$	0.234	0.156
$\gamma_{31(2,1),(2,1)}$	0.045	0.023	$\gamma_{31(2,1),(2,2)}$	0.067	0.029	$\gamma_{31(2,1),(2,3)}$	0.049	0.026
$\gamma_{32(2,1),(2,1)}$	0.012	0.153	$\gamma_{32(2,1),(2,2)}$	0.025	0.160	$\gamma_{32(2,1),(2,3)}$	0.028	0.142
$\gamma_{0(2,1),(3,1)}$	23.500	5.112	$\gamma_{0(2,1),(3,2)}$	22.423	4.876	$\gamma_{0(2,1),(3,3)}$	21.567	4.438
$\gamma_{11(2,1),(3,1)}$	0.041	0.026	$\gamma_{11(2,1),(3,2)}$	0.043	0.021	$\gamma_{11(2,1),(3,3)}$	0.051	0.065
$\gamma_{12(2,1),(3,1)}$	-0.247	0.418	$\gamma_{12(2,1),(3,2)}$	-0.325	0.570	$\gamma_{12(2,1),(3,3)}$	-0.289	0.612
$\gamma_{21(2,1),(3,1)}$	-0.004	0.003	$\gamma_{21(2,1),(3,2)}$	-0.006	0.008	$\gamma_{21(2,1),(3,3)}$	-0.007	0.005
$\gamma_{22(2,1),(3,1)}$	0.242	0.117	$\gamma_{22(2,1),(3,2)}$	0.278	0.159	$\gamma_{22(2,1),(3,3)}$	0.293	0.178
$\gamma_{31(2,1),(3,1)}$	0.056	0.027	$\gamma_{31(2,1),(3,2)}$	0.062	0.029	$\gamma_{31(2,1),(3,3)}$	0.071	0.025
$\gamma_{32(2,1),(3,1)}$	0.012	0.193	$\gamma_{32(2,1),(3,2)}$	0.023	0.189	$\gamma_{32(2,1),(3,3)}$	0.053	0.187
$\gamma_{0(1,2),(1,1)}$	23.560	4.948	$\gamma_{0(1,2),(1,2)}$	22.782	4.650	$\gamma_{0(1,2),(1,3)}$	21.830	4.216
$\gamma_{11(1,2),(1,1)}$	0.039	0.041	$\gamma_{11(1,2),(1,2)}$	0.032	0.021	$\gamma_{11(1,2),(1,3)}$	0.043	0.031
$\gamma_{12(1,2),(1,1)}$	-0.267	0.417	$\gamma_{12(1,2),(1,2)}$	-0.341	0.587	$\gamma_{12(1,2),(1,3)}$	-0.296	0.663
$\gamma_{21(1,2),(1,1)}$	-0.008	0.006	$\gamma_{21(1,2),(1,2)}$	-0.003	0.006	$\gamma_{21(1,2),(1,3)}$	-0.001	0.005
$\gamma_{22(1,2),(1,1)}$	0.221	0.124	$\gamma_{22(1,2),(1,2)}$	0.247	0.118	$\gamma_{22(1,2),(1,3)}$	0.276	0.165
$\gamma_{31(1,2),(1,1)}$	0.034	0.024	$\gamma_{31(1,2),(1,2)}$	0.042	0.033	$\gamma_{31(1,2),(1,3)}$	0.041	0.031
$\gamma_{32(1,2),(1,1)}$	0.018	0.187	$\gamma_{32(1,2),(1,2)}$	0.021	0.154	$\gamma_{32(1,2),(1,3)}$	0.014	0.192
$\gamma_{0(1,2),(2,1)}$	21.487	4.560	$\gamma_{0(1,2),(2,2)}$	22.639	4.612	$\gamma_{0(1,2),(2,3)}$	21.598	3.879
$\gamma_{11(1,2),(2,1)}$	0.041	0.026	$\gamma_{11(1,2),(2,2)}$	0.053	0.031	$\gamma_{11(1,2),(2,3)}$	0.045	0.039
$\gamma_{12(1,2),(2,1)}$	-0.243	0.621	$\gamma_{12(1,2),(2,2)}$	-0.276	0.523	$\gamma_{12(1,1),(2,3)}$	-0.321	0.469
$\gamma_{21(1,2),(2,1)}$	-0.011	0.009	$\gamma_{21(1,2),(2,2)}$	-0.006	0.008	$\gamma_{21(1,2),(2,3)}$	-0.010	0.004

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Table 4: Results of parameters estimations of our model (Continued)

$\gamma_{22(1,2),(2,1)}$	0.211	0.142	$\gamma_{22(1,2),(2,2)}$	0.321	0.144	$\gamma_{22(1,1),(2,3)}$	0.278	0.153
$\gamma_{31(1,2),(2,1)}$	0.045	0.021	$\gamma_{31(1,2),(2,2)}$	0.065	0.028	$\gamma_{31(1,2),(2,3)}$	0.048	0.072
$\gamma_{32(1,2),(2,1)}$	0.019	0.184	$\gamma_{32(1,2),(2,2)}$	0.019	0.148	$\gamma_{32(1,1),(2,3)}$	0.012	0.153
$\gamma_{0(1,2),(3,1)}$	22.980	3.899	$\gamma_{0(1,2),(3,2)}$	21.839	4.923	$\gamma_{0(1,2),(3,3)}$	22.912	4.519
$\gamma_{11(1,2),(3,1)}$	0.041	0.028	$\gamma_{11(1,2),(3,2)}$	0.045	0.031	$\gamma_{11(1,2),(3,3)}$	0.031	0.025
$\gamma_{12(1,2),(3,1)}$	-0.268	0.523	$\gamma_{12(1,2),(3,2)}$	-0.329	0.568	$\gamma_{12(1,2),(3,3)}$	-0.241	0.421
$\gamma_{21(1,2),(3,1)}$	-0.004	0.006	$\gamma_{21(1,2),(3,2)}$	-0.011	0.009	$\gamma_{21(1,2),(3,3)}$	-0.009	0.007
$\gamma_{22(1,2),(3,1)}$	0.312	0.152	$\gamma_{22(1,2),(3,2)}$	0.412	0.142	$\gamma_{22(1,2),(3,3)}$	0.211	0.112
$\gamma_{31(1,2),(3,1)}$	0.045	0.023	$\gamma_{31(1,2),(3,2)}$	0.043	0.031	$\gamma_{31(1,2),(3,3)}$	0.034	0.021
$\gamma_{32(1,2),(3,1)}$	0.012	0.183	$\gamma_{32(1,2),(3,2)}$	0.011	0.187	$\gamma_{32(1,2),(3,3)}$	0.021	0.201
$\gamma_{0(2,2),(1,1)}$	22.036	4.310	$\gamma_{0(2,2),(1,2)}$	21.567	4.670	$\gamma_{0(2,2),(1,3)}$	22.312	3.245
$\gamma_{11(2,2),(1,1)}$	0.031	0.022	$\gamma_{11(2,2),(1,2)}$	0.033	0.041	$\gamma_{11(2,2),(1,3)}$	0.039	0.027
$\gamma_{12(2,2),(1,1)}$	-0.352	0.415	$\gamma_{12(2,2),(1,2)}$	-0.254	0.612	$\gamma_{12(2,2),(1,3)}$	-0.219	0.452
$\gamma_{21(2,2),(1,1)}$	-0.002	0.005	$\gamma_{21(2,2),(1,2)}$	-0.004	0.013	$\gamma_{21(2,2),(1,3)}$	-0.008	0.009
$\gamma_{22(2,2),(1,1)}$	0.243	0.126	$\gamma_{22(2,2),(1,2)}$	0.321	0.118	$\gamma_{22(2,2),(1,3)}$	0.411	0.301
$\gamma_{31(2,2),(1,1)}$	0.045	0.021	$\gamma_{31(2,2),(1,2)}$	0.056	0.042	$\gamma_{31(2,2),(1,3)}$	0.031	0.022
$\gamma_{32(2,2),(1,1)}$	0.021	0.201	$\gamma_{32(2,2),(1,2)}$	0.032	0.189	$\gamma_{32(2,2),(1,3)}$	0.021	0.191
$\gamma_{0(2,2),(2,1)}$	21.312	4.231	$\gamma_{0(2,2),(2,2)}$	22.519	4.189	$\gamma_{0(2,2),(2,3)}$	20.396	3.987
$\gamma_{11(2,2),(2,1)}$	0.032	0.021	$\gamma_{11(2,2),(2,2)}$	0.034	0.024	$\gamma_{11(2,2),(2,3)}$	0.054	0.038
$\gamma_{12(2,2),(2,1)}$	-0.261	0.521	$\gamma_{12(2,2),(2,2)}$	-0.129	0.451	$\gamma_{12(2,2),(2,3)}$	-0.312	0.587
$\gamma_{21(2,2),(2,1)}$	-0.017	0.010	$\gamma_{21(2,2),(2,2)}$	-0.008	0.006	$\gamma_{21(2,2),(2,3)}$	-0.009	0.005
$\gamma_{22(2,2),(2,1)}$	0.312	0.221	$\gamma_{22(2,2),(2,2)}$	0.198	0.201	$\gamma_{22(2,2),(2,3)}$	0.274	0.283
$\gamma_{31(2,2),(2,1)}$	0.047	0.032	$\gamma_{31(2,2),(2,2)}$	0.054	0.021	$\gamma_{31(2,2),(2,3)}$	0.039	0.041
$\gamma_{32(2,2),(2,1)}$	0.013	0.162	$\gamma_{32(2,2),(2,2)}$	0.022	0.184	$\gamma_{32(2,2),(2,3)}$	0.032	0.173
$\gamma_{0(2,2),(3,1)}$	22.452	3.987	$\gamma_{0(2,2),(3,2)}$	21.549	4.658	$\gamma_{0(2,2),(3,3)}$	22.999	4.872
$\gamma_{11(2,2),(3,1)}$	0.043	0.021	$\gamma_{11(2,2),(3,2)}$	0.035	0.029	$\gamma_{11(2,2),(3,3)}$	0.039	0.028
$\gamma_{12(2,2),(3,1)}$	-0.264	0.511	$\gamma_{12(2,2),(3,2)}$	-0.232	0.419	$\gamma_{12(2,2),(3,3)}$	-0.321	0.621
$\gamma_{21(2,2),(3,1)}$	-0.003	0.007	$\gamma_{21(2,2),(3,2)}$	-0.009	0.004	$\gamma_{21(2,2),(3,3)}$	-0.018	0.005
$\gamma_{22(2,2),(3,1)}$	0.231	0.122	$\gamma_{22(2,2),(3,2)}$	0.242	0.115	$\gamma_{22(2,2),(3,3)}$	0.262	0.217
$\gamma_{31(2,2),(3,1)}$	0.054	0.029	$\gamma_{31(2,2),(3,2)}$	0.069	0.042	$\gamma_{31(2,2),(3,3)}$	0.046	0.025
$\gamma_{32(2,2),(3,1)}$	0.013	0.143	$\gamma_{32(2,2),(3,2)}$	0.019	0.189	$\gamma_{32(2,2),(3,3)}$	0.010	0.158
$\pi_{(1,1),(1,1)}$	0.031	0.013	$\pi_{(1,1),(1,2)}$	0.025	0.012	$\pi_{(1,1),(1,3)}$	0.039	0.015
$\pi_{(1,1),(2,1)}$	0.030	0.013	$\pi_{(1,1),(2,2)}$	0.031	0.013	$\pi_{(1,1),(2,3)}$	0.034	0.014
$\pi_{(1,1),(3,1)}$	0.035	0.014	$\pi_{(1,1),(3,2)}$	0.044	0.012	$\pi_{(1,1),(3,3)}$	0.40	0.017
$\pi_{(1,2),(1,1)}$	0.035	0.014	$\pi_{(1,2),(1,2)}$	0.028	0.012	$\pi_{(1,2),(1,3)}$	0.033	0.014
$\pi_{(1,2),(2,1)}$	0.031	0.013	$\pi_{(1,2),(2,2)}$	0.028	0.013	$\pi_{(1,2),(2,3)}$	0.029	0.013
$\pi_{(1,2),(3,1)}$	0.033	0.0139	$\pi_{(1,2),(3,2)}$	0.025	0.011	$\pi_{(1,2),(3,3)}$	0.033	0.014
$\pi_{(2,1),(1,1)}$	0.029	0.013	$\pi_{(2,1),(1,2)}$	0.030	0.013	$\pi_{(2,1),(1,3)}$	0.028	0.012
$\pi_{(2,1),(2,1)}$	0.027	0.011	$\pi_{(2,1),(2,2)}$	0.029	0.012	$\pi_{(2,1),(2,3)}$	0.029	0.010
$\pi_{(2,1),(3,1)}$	0.026	0.012	$\pi_{(2,1),(3,2)}$	0.029	0.014	$\pi_{(2,1),(3,3)}$	0.028	0.013
$\pi_{(2,2),(1,1)}$	0.023	0.006	$\pi_{(2,2),(1,2)}$	0.020	0.004	$\pi_{(2,2),(1,3)}$	0.023	0.006
$\pi_{(2,2),(2,1)}$	0.018	0.005	$\pi_{(2,2),(2,2)}$	0.019	0.005	$\pi_{(2,2),(2,3)}$	0.013	0.004
$\pi_{(2,2),(3,1)}$	0.012	0.004	$\pi_{(2,2),(3,2)}$	0.021	0.005	$\pi_{(2,2),(3,3)}$	0.011	0.004

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Please cite this article using:

Ehsan Bahrami Samani, New general location models for mixed responses, AUT J. Math. Comput., 7(2) (2026) 189-200
https://doi.org/10.22060/AJMC.2025.23674.1284