Amirkabir University of Technology
(Tehran Polytechnic)
Vol. 47 - No. 2 - Fall 2015, pp. 45- 56

Amirkabir International Journal of Science & Research
(Modeling, Identification, Simulation & Control)
(AIJ-MISC)

Robust Sliding Mode Controller for Trajectory Tracking and Attitude Control of a Nonholonomic Spherical Mobile Robot

Jafar Keighobadi [1*] and Mahmood Reza Azizi [1]

1- Faculty of mechanical engineering, University of Tabriz, Tabriz, Iran

ABSTRACT

Based on dynamic modeling, robust trajectory tracking control of attitude and position of a spherical mobile robot is proposed. In this paper, the spherical robot is composed of a spherical shell and three independent rotors which act as the inner driver mechanism. Owing to rolling without slipping assumption, the robot is subjected to two nonholonomic constraints. The state space representation of the system is developed using dynamical equations of the robot's motion. As the main contribution, a dynamical model based SMC (sliding mode controller) is designed for position and attitude control of the robot under parameters uncertainty and unmodeled dynamics. To decrease the chattering phenomena originated by the sign function, the well-known boundary layer technique is imposed on the SMC. The control gains are determined through using Lyapunov's direct method in such a way that the robustness and to zero convergence of the controller's tracking error are guaranteed. Computer simulations are performed to show the significant tracking performance of the proposed SMC in particular against parameters uncertainty and white Gaussian noises. The simulation results show the significant performance of the designed nonlinear control system in trajectory tracking control as well as in attitude control of the spherical robot even in the presence of parameters uncertainty and measurement noises.

KEYWORDS

Spherical robot; Nonholonomic System; Sliding mode control; Boundary Layer; Parameters' uncertainty.

*Corresponding Author, Email: keighobadi@tabrizu.ac.ir

1. INTRODUCTION

A spherical mobile robot which is composed of a spherical shell and an inner driver mechanism is a new prototype of mobile robots which its applications have increased in past two decades. The driver mechanism is installed inside the spherical shell to generate the robot's motion. This structure provides a stable locomotion and some other advantages rather than the traditional types of mobile robots [1].

Several types of the inner driver mechanism have been proposed for spherical robots in recently documented researches which all of them generate the robot's motion either by changing the gravity center of the spherical shell or by changing the angular momentum of the robot. A mobile vehicle [2], a wheeled mass [3], four unbalanced masses [4] and a two DOF pendulum [5] are examples of proposed driver mechanisms which generate the robot motion by changing the gravity center of the robot. On the other hand, two rotors [6], three DOF gyro [7] and three perpendicular rotors [8] have been designed as inner driver mechanism in preceding studies that produce the robot's motion based on the angular momentum conservation principle.

Although, some investigations have been developed on spherical robots recently, motion control of these robots is still one of the major problem in robotic researches. Since the spherical shell is assumed to roll over the ground surface without any slipping, its motion is subjected to two nonholonomic constraints. On the other hand, according to Brocket's theorem, the stabilization of the equilibrium points of the nonholonomic systems through time invariant state-feedback is not possible [9]. By the way, for the control of the spherical robot, Zhao et all have derived the dynamical model of the spherical robot merely for straight line motions and a PID controller has been proposed for the robot's motion on straight line trajectories [10]. The control of the spherical robot by uses of a pendulum as a control actuator has been developed by feedback linearization method in straight line trajectories [11]. Furthermore, trajectory tracking control of the spherical robot on straight paths has been investigated using SMC method [12], adaptive hierarchical sliding mode approach [13]; and also using combined adaptive neuro-fuzzy and SMC method [14].

The control problem of the spherical robot on curvilinear trajectories has been studied using kinematical and simplified dynamical model of the robot and some simplifying assumptions. Cai et al have designed a two-state trajectory tracking control system for the kinematical model of the spherical robot based on shunting model of neurodynamics and Lyapunov's direct method [15]. A SMC has been developed for the linearized model of the spherical robot without considering the dynamical effects of the inner mechanism by Lui et al [16]. Fuzzy control approach has been used to control the spherical robot's motion based on decoupled dynamical model and by neglecting the transversal and longitudinal rotation of the robot [17]. Through neglecting the rotation of the spherical shell around the vertical axis and based on the dynamical model, simplified tracking control of the spherical robot in horizontal plane has been investigated using: real-time fuzzy guidance method [18], back-stepping based trajectory tracking [19] and constant velocity PD sliding mode controller [20].

According to the above mentioned literature review, the trajectory tracking and attitude control in 2-dimentional plane is a major problem with the spherical kind mobile robots that should be solved completely. Although, several linear and nonlinear control strategies based on the kinematic model, linearized model or simplified dynamical model of the spherical robot have been introduced in the literature, these methods are not practically feasible considering high complicated nonlinear structure of the robot's mathematical model [14]. Indeed, the control system for the spherical robot has been designed in previous researches either based on kinematic model or based on no-spin model of the robot. However, as shown by Svinin et al [21], the kinematical model and no-spin model of the robot is not always dynamically realizable and therefore, trajectory planning and control system design must be performed based on the dynamical model of the robot. In addition, the robustness of the designed nonlinear controller against parameters' uncertainty and noisy measurements is a significant issue in practical applications which have not been considered in preceding research works.

Therefore, in this paper the spherical robot comprising of three independent rotors is investigated. Using the dynamical model of the spherical robot [8] and without any simplifying assumptions, the second order mathematical model of the robot is obtained in the standard affine form. A nonlinear SMC is designed for trajectory tracking and attitude control of the robot separately. Furthermore, the boundary layer technique is used to remove the chattering phenomena which is originated by discontinuous switching control term in the neighbor of the sliding surface. The convergence of tracking error to zero and the robustness of the proposed controller are proved by Lyapunov's direct method. Besides, computer simulations are performed to assess the tracking performance and robustness of SMC against parameters' uncertainty and noisy measurements.

The rest of the paper is organized as follows. Section 2 describes the kinematical and dynamical modeling and the state space representation of the spherical robot. In section 3, robust trajectory tracking and attitude controller (SMC) are designed. Section 4 and 5 present simulation results and concluding remarks, respectively.

2. Spherical Robot Modeling

The schematic model of the spherical robot with three independent rotors as the inner driver mechanism is shown in Fig. 1. (a). The robot is composed of a spherical shell, three rotors and three counter weights to balance the rotors' weight. The driving rotors are connected to the inner surface of the spherical shell and rotate by use of three revolute actuators. Furthermore, it is assumed that the counter weights and other instruments are installed inside the spherical shell in such a way that the gravity center of the robot coincides with the geometric center of the spherical shell. Therefore, the resultant of all moment vectors about the contact point is vanished and the angular momentum vector about this point will conserve. Consequently, the rotation of the rotors leads to rotation of the spherical shell and the robot motion is generated based on angular momentum conservation principle. In this case, the robot's motion could be controlled by three revolute actuators.

A. Kinematic Model

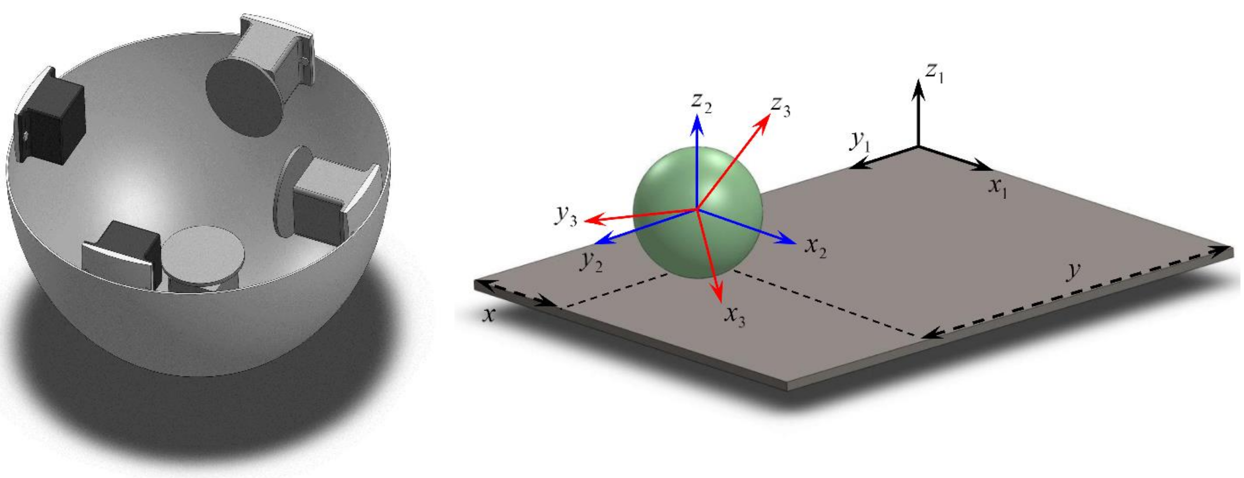
To determine the robot's position and configuration, three coordinate frames are considered as shown in Fig. 1. (b). Frame {1} is an inertial reference frame. The origin of frame {2} is fixed to the geometric center of the spherical shell and its axes remain parallel to the axes of frame {1}. The body frame {3} fixed to the spherical shell and coincides to frame {2} if the rotation angels of the robot are zero values. By defining x and y as the components of 2-dimentional position vector of the spherical shell with respect to frame {1} and q_1, q_2, q_3, q_4 as the components of a unit quaternion to represent the orientation of the frame {3} with respect to frame {2}, the kinematical equations of the robot are obtained as follow [8]:

$$ {}_3^2q = {}_3^1q = \begin{bmatrix} q_0 \\ \vec{q} \end{bmatrix} = q_0 + q_1\mathbf{i} + q_2\mathbf{j} + q_3\mathbf{k} \tag{1} $$

$$ \left\| {}_3^2q \right\|^2 = q_0^2 + q_1^2 + q_2^2 + q_3^2 = 1 \tag{2} $$

$$ \begin{aligned} {}^2\vec{\omega}_3 &= Vector\left(2\, {}_3^2\dot{q}\, {}_3^2\bar{q}^T\right) = \begin{bmatrix} {}^1\omega_{3x} \\ {}^1\omega_{3y} \\ {}^1\omega_{3z} \end{bmatrix} \\ &= \begin{bmatrix} -2q_1 & 2q_0 & -2q_3 & 2q_2 \\ -2q_2 & 2q_3 & 2q_0 & -2q_1 \\ -2q_3 & -2q_2 & 2q_1 & 2q_0 \end{bmatrix} \begin{bmatrix} \dot{q}_0 \\ \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} \end{aligned} \tag{3} $$

$$ \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = 2R_s \begin{bmatrix} -q_2 & q_3 & q_0 & -q_1 \\ q_1 & -q_0 & q_3 & -q_2 \end{bmatrix} \begin{bmatrix} \dot{q}_0 \\ \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} \tag{4} $$



(a) (b)

Fig. 1. (a) The construction of the spherical robot, (b) Spherical shell and coordinate frames

where ${}^{1}\vec{\omega}_3$ is the angular velocity vector of frame {3} with respect to frame {1}. Equations (2) and (4) denote one algebraic and two differential constraints between the considered position and orientation variables. These two differential constraints (4) are indeed non-integrable equations which make the robot a nonholonomic system.

B. Kinematic Model

Considering the dynamical effects of the inner mechanism and rotation of the spherical shell around the vertical axis, the robot differential equations of motion have been obtained using Kane's method and are rewritten in the following form (see [8] for details).

$$\begin{aligned}
&f_i\,(q_0,q_1,q_2,q_3,\dot q_0,\dot q_1,\dot q_2,\dot q_3,\ddot q_0,\ddot q_1,\ddot q_2,\ddot q_3,\Omega_x,\\
&\Omega_y,\Omega_z,\dot\Omega_x,\dot\Omega_y,\dot\Omega_z)=0 \qquad (i=1,2,3)\\
&f_4\left(q_0,q_1,q_2,q_3,\ddot q_0,\ddot q_1,\ddot q_2,\ddot q_3,\dot\Omega_x,T_x\right)=0\\
&f_5\left(q_0,q_1,q_2,q_3,\ddot q_0,\ddot q_1,\ddot q_2,\ddot q_3,\dot\Omega_y,T_y\right)=0\\
&f_6\left(q_0,q_1,q_2,q_3,\ddot q_0,\ddot q_1,\ddot q_2,\ddot q_3,\dot\Omega_z,T_z\right)=0
\end{aligned} \tag{5}$$

where $\Omega_x,\Omega_y,\Omega_z$ are angular velocities of the rotors with respect to the spherical shell; T_x,T_y,T_z are actuators' torque and $f_1(\cdot)$ to $f_6(\cdot)$ are nonlinear scalar terms. It should be noted that, the components of the position vector of the spherical shell (x and y) do not appear in (5). Therefore, using (5), second time derivative of (2) and first time derivative of (4), the mathematical model of the spherical robot in terms of all the kinematical variables, $x,y,q_0,q_1,q_2,q_3,\theta_x,\theta_y,\theta_z$ could be written in the following standard form.

$$\begin{aligned}
&\ddot x=-2R_S\left(\ddot q_0q_2-\ddot q_1q_3-\ddot q_2q_0+\ddot q_3q_1\right)\\
&\ddot y=2R_S\left(\ddot q_0q_1-\ddot q_1q_0+\ddot q_2q_3-\ddot q_3q_2\right)\\
&\left\|{}_3^2\dot q\right\|^2+q_0\ddot q_0+q_1\ddot q_1+q_2\ddot q_2+q_3\ddot q_3=0
\end{aligned} \tag{6}$$

$$\mathbf{q}=\left[x,y,q_0,q_1,q_2,q_3,\theta_x,\theta_y,\theta_z\right]^T \tag{7}$$

$$\begin{aligned}
&\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}}+\mathbf{V}(\mathbf{q},\dot{\mathbf{q}})+\mathbf{G}(\mathbf{q})=\boldsymbol{\tau}\\
&\boldsymbol{\tau}=\left[0,0,0,0,0,T_x,T_y,T_z,0\right]^T
\end{aligned} \tag{8}$$

where $\theta_x,\theta_y,\theta_z$ are rotors' angular displacement with respect to the spherical shell, $\mathbf{q}\in R^9$ denote the state variable vector, $\mathbf{M}(\mathbf{q})\in R^{9\times9}$ is the mass matrix of the robot, $\mathbf{V}(\mathbf{q},\dot{\mathbf{q}})\in R^9$ is the vector of Coriolis and centrifugal forces, $\mathbf{G}(\mathbf{q})\in R^9$ is the vector of gravitational effects and $\boldsymbol{\tau}\in R^9$ is the vector of impressed torque. Since $\mathbf{M}(\mathbf{q})$ is a nonsingular matrix, the robot mathematical model could be rewritten in the following affine form by multiplying both side of (8) by $\mathbf{M}^{-1}$.

$$\ddot{\mathbf{q}}=\mathbf{f}(\mathbf{q},\dot{\mathbf{q}})+\mathbf{g}(\mathbf{q})\mathbf{u} \tag{9}$$

where $\mathbf{f}(\mathbf{q},\dot{\mathbf{q}})$ is a (9×1) vector, $\mathbf{g}(\mathbf{q})$ is a (9×3) distribution matrix and $\mathbf{u}$ is the 3-dimentional torque vector.

3. Control System Design

Considering the parameters uncertainty, measurement noises and disturbance torques which affect a real spherical robot, (8) is modified in the following form [22].

$$\begin{aligned}
&(\mathbf{M}+\Delta\mathbf{M})\ddot{\mathbf{q}}+\mathbf{V}(\mathbf{q},\dot{\mathbf{q}})+\Delta\mathbf{V}(\mathbf{q},\dot{\mathbf{q}})\\
&+\mathbf{G}(\mathbf{q})+\Delta\mathbf{G}(\mathbf{q})+\mathbf{D}=\boldsymbol{\tau}
\end{aligned} \tag{10}$$

where $\mathbf{M},\mathbf{V}$ and $\mathbf{G}$ stand for the nominal values of mass matrix, centrifugal and Coriolis force vector and gravitational force vector, respectively. $\Delta\mathbf{M},\Delta\mathbf{V}$ and $\Delta\mathbf{G}$ denote the effects of parameters uncertainty and unmodeled dynamics which are considered unknown and bounded values and the exogenous input vector, $\mathbf{D}$ stands for disturbances, clearances and measuring noise effects. Therefore, the affine model (9) could also be completed according to (10) as

$$\ddot{\mathbf{q}}=\mathbf{f}(\mathbf{q},\dot{\mathbf{q}})+\Delta\mathbf{f}(\mathbf{q},\dot{\mathbf{q}})+\left(\mathbf{g}(\mathbf{q})+\Delta\mathbf{g}(\mathbf{q})\right).\mathbf{u} \tag{11}$$

where $\Delta\mathbf{f}(\mathbf{q},\dot{\mathbf{q}})$ and $\Delta\mathbf{g}(\mathbf{q})$ are unknown but 2-norm bounded terms of state variables.

A. Trajectory tracking control system

To design a robust trajectory tracking control system of the spherical mobile robot, the output vector is considered as

$$\mathbf{H}=\begin{bmatrix}x & y & \varphi\end{bmatrix}^T=\begin{bmatrix}x & y & \int {}_3^2\omega_z\,dt\end{bmatrix}^T \tag{12}$$

where φ denotes time integral of the robot's angular velocity around the vertical axis, z. The second time derivative of the output vector is calculated as

$$\begin{aligned}
\ddot{\mathbf{H}}&=\begin{bmatrix}\ddot x & \ddot y & {}_3^2\alpha_z\end{bmatrix}^T\\
&=\begin{bmatrix}\ddot x & \ddot y & -\ddot q_0q_3-\ddot q_1q_2+\ddot q_2q_1+\ddot q_3q_0\end{bmatrix}^T
\end{aligned} \tag{13}$$

Through substituting $\ddot{x}, \ddot{y}$ and ${}_3^2\alpha_z$ from (11) in (13), the control input vector $\mathbf{u}$ appears. Therefore, the system (9) is of relative degree two. So, Equation (13) could be written in the following form:

$$\ddot{\mathbf{H}} = \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ {}_3^2\alpha_z \end{bmatrix} = \mathbf{H_1}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{H_2}(\mathbf{q})\mathbf{u} \tag{14}$$

where $\mathbf{H_1} \in R^3$ and $\mathbf{H}_2 \in R^{3\times 3}$ are obtained by substituting (9) in (13). To design the SMC, the sliding surfaces $\mathbf{S}(t) = 0$ are defined as

$$\mathbf{S}(t) = \begin{bmatrix} \dot{\tilde{x}} + \lambda_1 \tilde{x} \\ \dot{\tilde{y}} + \lambda_2 \tilde{y} \\ \dot{\tilde{\varphi}} + \lambda_3 \tilde{\varphi} \end{bmatrix} \tag{15}$$

where $\tilde{\ }$ stands for the tracking error of the corresponding output variable and the sliding surface parameters, $\lambda_1, \lambda_2, \lambda_3$ are strictly positive and fixed values. The time derivative of the sliding surface vector is obtained as

$$\mathbf{S}(t) = \begin{bmatrix} \dot{\tilde{x}} + \lambda_1 \tilde{x} \\ \dot{\tilde{y}} + \lambda_2 \tilde{y} \\ \dot{\tilde{\varphi}} + \lambda_3 \tilde{\varphi} \end{bmatrix} \dot{\mathbf{S}}(t) = \begin{bmatrix} \ddot{\tilde{x}} + \lambda_1 \dot{\tilde{x}} \\ \ddot{\tilde{y}} + \lambda_2 \dot{\tilde{y}} \\ \ddot{\tilde{\varphi}} + \lambda_3 \dot{\tilde{\varphi}} \end{bmatrix}$$
$$= \begin{bmatrix} \ddot{x} - \ddot{x}_d + \lambda_1 \dot{\tilde{x}} \\ \ddot{y} - \ddot{y}_d + \lambda_2 \dot{\tilde{y}} \\ {}_3^2\alpha_z - {}_3^2\alpha_{z\,d} + \lambda_3 \dot{\tilde{\varphi}} \end{bmatrix} = \ddot{\mathbf{H}}\text{-}\ddot{\mathbf{H}}_\mathbf{d} + \mathbf{\Lambda}\dot{\tilde{\mathbf{H}}} \tag{16}$$

where $\mathbf{H}_d$ is the vector of the desired outputs and $\mathbf{\Lambda}$ is a diagonal matrix of $\lambda_1, \lambda_2, \lambda_3$. Substituting (14) in (16) leads to

$$\dot{\mathbf{S}}(t) = \mathbf{H_1}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{H_2}(\mathbf{q})\mathbf{u} - \ddot{\mathbf{H}}_\mathbf{d} + \mathbf{\Lambda}\dot{\tilde{\mathbf{H}}} \tag{17}$$

To achieve prefect trajectory tracking, $\dot{\mathbf{S}}(t)$ should be remained zero during the robot's motion. Therefore, considering $\dot{\mathbf{S}}(t) = 0$ in (17) and using the nominal model of the robot (9), the following equivalent control law is obtained.

$$\mathbf{u_{eq}} = \mathbf{H_2^{-1}}(\mathbf{q})\left[\ \ddot{\mathbf{H}}_\mathbf{d} - \mathbf{\Lambda}\dot{\tilde{\mathbf{H}}}\text{-}\mathbf{H_1}(\mathbf{q}, \dot{\mathbf{q}})\ \right] \tag{18}$$

Furthermore, the tracking error should reach the sliding surface in finite time and move along it to the origin under parameters uncertainty and unmodeled dynamics. Therefore, the whole control input vector is assumed as

$$\mathbf{u} = \mathbf{u_{eq}} + \mathbf{u_s} \tag{19}$$

where, $\mathbf{u_s}$ is added to guarantee the global stability of the SMC against the uncertainties and unmodeled dynamics. Therefore, the following positive definite function is considered as a Lyapunov function candidate.

$$V(t) = \frac{1}{2}\mathbf{S^T S} \tag{20}$$

The time derivative of (20) along the system trajectories yields

$$\begin{aligned} \dot{V}(t) &= \mathbf{S^T}\dot{\mathbf{S}} = \mathbf{S^T}[\mathbf{H_1}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{\Delta H_1}(\mathbf{q}, \dot{\mathbf{q}}) \\ &+ \left(\mathbf{H_2}(\mathbf{q}) + \mathbf{\Delta H_2}(\mathbf{q})\right)\left(\mathbf{u_{eq}} + \mathbf{u_s}\right) - \ddot{\mathbf{H}}_\mathbf{d} + \mathbf{\Lambda}\dot{\tilde{\mathbf{H}}}] \end{aligned} \tag{21}$$

where $\Delta\mathbf{H_1}(\mathbf{q}, \dot{\mathbf{q}})$, $\Delta\mathbf{H_2}(\mathbf{q})$ are determined according to (11). Substituting $\mathbf{u_{eq}}$ from (18) in (21) leads to:

$$\begin{aligned} \dot{V}(t) &= \mathbf{S^T}\dot{\mathbf{S}} = \mathbf{S^T}[\mathbf{H_1}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{\Delta H_1}(\mathbf{q}, \dot{\mathbf{q}}) \\ &+ \left(\mathbf{H_2}(\mathbf{q}) + \mathbf{\Delta H_2}(\mathbf{q})\right)\left(\mathbf{u_{eq}} + \mathbf{u_s}\right) - \ddot{\mathbf{H}}_\mathbf{d} + \mathbf{\Lambda}\dot{\tilde{\mathbf{H}}}] \end{aligned} \tag{22}$$

Using 2-norm operator $\|\cdot\|$ on (22) results in.

$$\begin{aligned} \dot{V}(t) &= \mathbf{S^T}\dot{\mathbf{S}} \le \|\mathbf{S^T}\| \cdot \|\Delta\mathbf{H_1}(\mathbf{q}, \dot{\mathbf{q}})\| \\ &+ \|\mathbf{S^T}\| \cdot \|\Delta\mathbf{H_2}(\mathbf{q})\| \cdot \|\mathbf{H_2^{-1}}(\mathbf{q})\| \|\ddot{\mathbf{H}}_\mathbf{d}\| \\ &+ \|\mathbf{S^T}\| \cdot \|\Delta\mathbf{H_2}(\mathbf{q})\| \cdot \|\mathbf{H_2^{-1}}(\mathbf{q})\| \cdot \left\|\mathbf{\Lambda}\dot{\tilde{\mathbf{H}}}\right\| \\ &+ \|\mathbf{S^T}\| \cdot \|\Delta\mathbf{H_2}(\mathbf{q})\| \cdot \|\mathbf{H_2^{-1}}(\mathbf{q})\| \cdot \|\mathbf{H_1}(\mathbf{q}, \dot{\mathbf{q}})\| \\ &+ \mathbf{S^T}\left(\mathbf{H_2}(\mathbf{q}) + \Delta\mathbf{H_2}(\mathbf{q})\right)\mathbf{u_s} \end{aligned} \tag{23}$$

To achieve the global asymptotic stability, $\dot{V}(t)$ must be negative. Therefore $\mathbf{u_s}$ is proposed as:

$$\mathbf{u_s} = \left(\mathbf{H_2}(\mathbf{q}) + \Delta\mathbf{H_2}(\mathbf{q})\right)^{-1} \begin{bmatrix} -K_1 \operatorname{sgn}(S_1) \\ -K_2 \operatorname{sgn}(S_2) \\ -K_3 \operatorname{sgn}(S_3) \end{bmatrix} \tag{24}$$

where K_1, K_2 and K_3 are positive gains which are determined in such a way that $\dot{V}(t)$ become negative. Using (24) in (23) leads to

$$\begin{aligned}\dot{V}(t) &= \mathbf{S^T\dot{S}} \le \left\|\mathbf{S^T}\right\|\cdot\left\|\Delta\mathbf{H_1}(\mathbf{q},\dot{\mathbf{q}})\right\| \\ &+\left\|\mathbf{S^T}\right\|\cdot\left\|\Delta\mathbf{H_2}(\mathbf{q})\right\|\cdot\left\|\mathbf{H_2^{-1}}(\mathbf{q})\right\|\left\|\ddot{\mathbf{H}}_\mathbf{d}\right\| \\ &+\left\|\mathbf{S^T}\right\|\cdot\left\|\Delta\mathbf{H_2}(\mathbf{q})\right\|\cdot\left\|\mathbf{H_2^{-1}}(\mathbf{q})\right\|\cdot\left\|\mathbf{\Lambda}\dot{\tilde{\mathbf{H}}}\right\| \\ &+\left\|\mathbf{S^T}\right\|\cdot\left\|\Delta\mathbf{H_2}(\mathbf{q})\right\|\cdot\left\|\mathbf{H_2^{-1}}(\mathbf{q})\right\|\cdot\left\|\mathbf{H_1}(\mathbf{q},\dot{\mathbf{q}})\right\| \\ &-K_1|S_1|-K_2|S_2|-K_3|S_3|<0\end{aligned} \tag{25}$$

Using the fact that $\|\mathbf{S}\|\le|S_1|+|S_2|+|S_3|$ in (25), K_1, K_2 and K_3 are obtained according to the upper bounds of the parameters' uncertainty and unmodeled dynamics. Therefore, the time derivative of Lyapunov's function V along the system trajectories will be negative and thus the global stability of the proposed control system is guaranteed.

B. Attitude Control

The complete attitude controller for the spherical robot is designed in this section using dynamical model (11). Therefore, considering the governing constraint (2) on unit quaternion, to control the attitude of the spherical robot only three components of the unit quaternion should be controlled. Therefore, output vector is defined as

$$\mathbf{H}=\begin{bmatrix}q_1 & q_2 & q_3\end{bmatrix}^T \tag{26}$$

By calculating the second time derivative of (26), the control input vector $\mathbf{u}$ appears.

$$\ddot{\mathbf{H}}=\begin{bmatrix}\ddot{q}_1\\ \ddot{q}_2\\ \ddot{q}_3\end{bmatrix}=\mathbf{H_1}(\mathbf{q},\dot{\mathbf{q}})+\mathbf{H_2}(\mathbf{q})\mathbf{u} \tag{27}$$

Here $\mathbf{H_1}(\mathbf{q},\dot{\mathbf{q}})$ and $\mathbf{H_2}(\mathbf{q})$ are different from (14) and could be obtained from the third through fifth rows of $\mathbf{f}(\mathbf{q},\dot{\mathbf{q}})$ and $\mathbf{g}(\mathbf{q})$ in (9), respectively. Now, 3-dimentional sliding surface vector is defined as

$$\mathbf{S}(t)=\begin{bmatrix}\dot{\tilde{q}}_1+\lambda_1\tilde{q}_1\\ \dot{\tilde{q}}_2+\lambda_2\tilde{q}_2\\ \dot{\tilde{q}}_3+\lambda_3\tilde{q}_3\end{bmatrix} \tag{28}$$

Taking the time derivative of the sliding surface vector leads to:

$$\begin{aligned}\dot{\mathbf{S}}(t)&=\begin{bmatrix}\ddot{\tilde{q}}_1+\lambda_1\dot{\tilde{q}}_1\\ \ddot{\tilde{q}}_2+\lambda_2\dot{\tilde{q}}_2\\ \ddot{\tilde{q}}_3+\lambda_3\dot{\tilde{q}}_3\end{bmatrix}=\begin{bmatrix}\ddot{q}_1-\ddot{q}_{1d}+\lambda_1\dot{\tilde{q}}_1\\ \ddot{q}_2-\ddot{q}_{2d}+\lambda_2\dot{\tilde{q}}_2\\ \ddot{q}_3-\ddot{q}_{3d}+\lambda_3\dot{\tilde{q}}_3\end{bmatrix}\\ &=\ddot{\mathbf{H}}\text{-}\ddot{\mathbf{H}}_\mathbf{d}+\mathbf{\Lambda}\dot{\tilde{\mathbf{H}}}\end{aligned} \tag{29}$$

Therefore, the equivalent control law is calculated from $\dot{\mathbf{S}}(t)=0$ similar to the preceding section.

$$\mathbf{u_{eq}}=\mathbf{H_2^{-1}}(\mathbf{q})\left[\ddot{\mathbf{H}}_\mathbf{d}-\mathbf{\Lambda}\dot{\tilde{\mathbf{H}}}\text{-}\mathbf{H_1}(\mathbf{q},\dot{\mathbf{q}})\right] \tag{30}$$

In the above, $\mathbf{H_1}(\mathbf{q},\dot{\mathbf{q}})$ and $\mathbf{H_2}(\mathbf{q})$ are defined in (27). Now, the attitude control law in sliding mode is designed as

$$\mathbf{u}=\mathbf{u_{eq}}+\left(\mathbf{H_2}(\mathbf{q})+\Delta\mathbf{H_2}(\mathbf{q})\right)^{-1}\begin{bmatrix}-K_1\,\mathrm{sgn}(S_1)\\ -K_2\,\mathrm{sgn}(S_2)\\ -K_3\,\mathrm{sgn}(S_3)\end{bmatrix} \tag{31}$$

where K_1, K_2 and K_3 are designed considering the upper bound of the parameter's uncertainty and unmodeled dynamics such that the convergence of the errors to zero is guaranteed.

C. Smoothing Control Inputs

Although the switching control law (24) and (31) are robust against the parameters uncertainty and unmodeled dynamics, the chattering phenomena may also occur due to using discontinuous sign function in the sliding control. Chattering phenomenon may lead to high frequency control inputs and resonance the vibrations of the system's components and therefore should be removed. To remove the chattering phenomena the boundary layer technique is used in this paper. Therefore, in the boundary layer of thickness ε around the origin, the sign terms are replaced by linear continuous terms as demonstrated in Fig. 2. In this way, the smoother control inputs are obtained and the chattering phenomena is reduced. The thickness of the boundary layer affects the tracking performance and the robustness of the controller against uncertainties. Indeed, the greater the boundary layer thickness used, the less the robustness may be achieved. On the other hand, using the small value for the boundary layer's thickness may lead to chattering again. Therefore, the thickness of the boundary layer is designed in such a way that the robustness of the SMC is considerable and the control torques are sufficiently smooth.

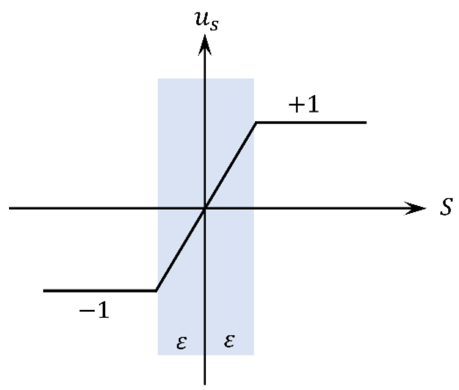


Fig. 2. Interpolated control inputs inside the boundary layer

4. Computer Simulations

The performance of the designed trajectory tracking and attitude control system against the parameters' uncertainty and unmodeled dynamics are investigated by computer simulations. The nominal values of the robot's physical parameters are used according to Table 1. Furthermore, it is assumed that the mass, the radius and the moment of inertia of the spherical shell are not exactly known and therefore, the corresponding nominal values together with upper bound of their uncertainties are used in the SMC system.

TABLE 1. NOMINAL VALUES OF THE ROBOT'S PHYSICAL PARAMETERS

Robot's Parameter	*Nominal Value*	*Uncertainty*
M_S	$9\,Kg$	$\Delta M_S = 10\% M_S$
R_S	$0.55\,m$	$\Delta R_S = 20\% R_S$
$l_i \quad (i=1,2,3)$	$0.35\,m$	-
I_S	$\begin{bmatrix} 1.815 & 0 & 0 \\ 0 & 1.815 & 0 \\ 0 & 0 & 1.815 \end{bmatrix} Kg.m^2$	$\Delta I_S = 10\% I_S$
$M_{ri} \; (i=1,2,3)$	$1\,kg$	-
I_r	$\begin{bmatrix} 0.05 & 0 & 0 \\ 0 & 0.05 & 0 \\ 0 & 0 & 0.1 \end{bmatrix} Kg.m^2$	-

A. Trajectory Tracking Control

To assess the performance and robustness of the designed trajectory tracking controller under parameter uncertainty, a circular reference trajectory in x-y plane is considered as

$$
\begin{aligned}
x_d &= 2\cos(0.2\pi t) \\
y_d &= 2\sin(0.2\pi t)
\end{aligned}
\tag{32}
$$

It is assumed that in trajectory tracking mode, the middle ring of the spherical shell should roll along the desired reference path. Therefore, the desired trajectory of angular velocity of the spherical shell around the vertical axis is considered as

$$
\begin{aligned}
{}^2_3\omega_{z\,d} &= \frac{V_S}{R_C} = \frac{\sqrt{\dot{x}_d^2 + \dot{y}_d^2}}{R_C} = 0.2\pi \\
{}^2_3\alpha_{z\,d} &= 0
\end{aligned}
\tag{33}
$$

where V_S and R_C denote the velocity of geometric center of the spherical shell and the curvature of trajectory, respectively. Considering the following initial off-tracks in the state variables and using suitable controller gains as $[K_1 \quad K_2 \quad K_3] = [2 \quad 2 \quad 2]$, the thickness of boundary layer as, $\varepsilon = 0.2$ and $\mathbf{\Lambda} = diag(2,2,2)$ which are selected by trial and error, the simulation results are obtained as shown in Figs. 3 to 10.

$$
\begin{aligned}
&[x, y, r_1, r_2, r_3, r_4, \theta_x, \theta_y, \theta_z] \\
&= [0,0,0,0,0,1,0,0,0] \quad , \quad at \quad t=0 \\
&[\dot{x}, \dot{y}, \dot{r}_1, \dot{r}_2, \dot{r}_3, \dot{r}_4, \Omega_x, \Omega_y, \Omega_z] \\
&= [0,0,0,0,0,0,0,0,0] \quad , \quad at \quad t=0
\end{aligned}
\tag{34}
$$

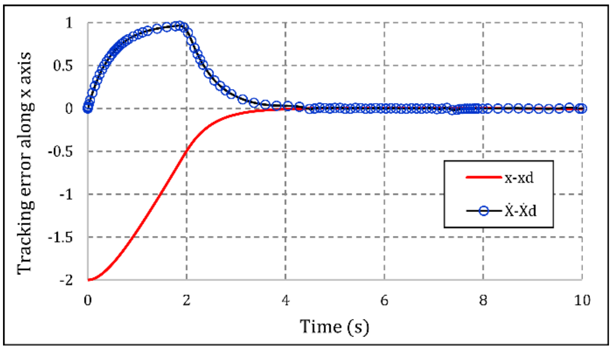


Fig. 3. Tracking error along x direction

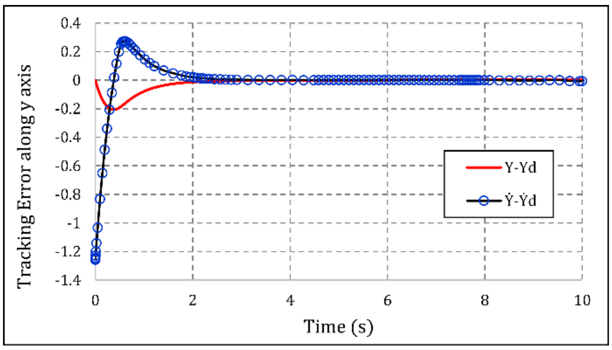


Fig. 4. Tracking error along y direction

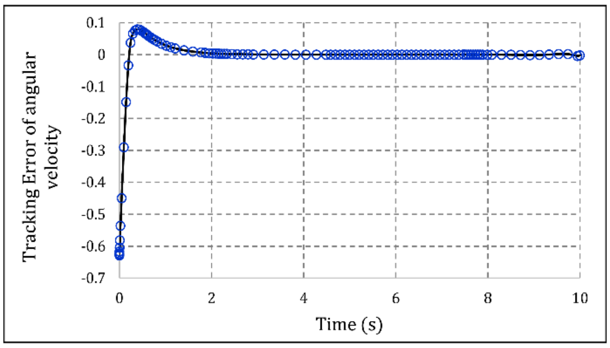


Fig. 5. Tracking error of angular velocity around the vertical axis

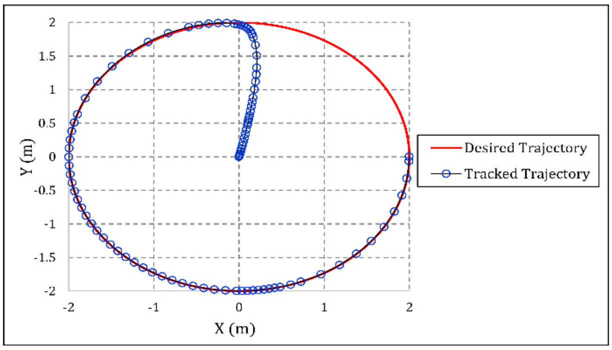


Fig. 6. Robot circular trajectory in x-y plane

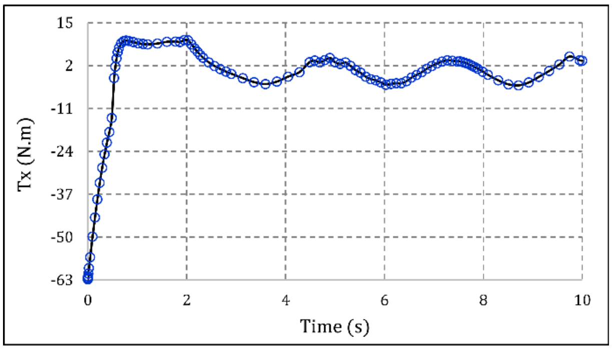


Fig. 7. Control torque, T_x for tracking of circular trajectory

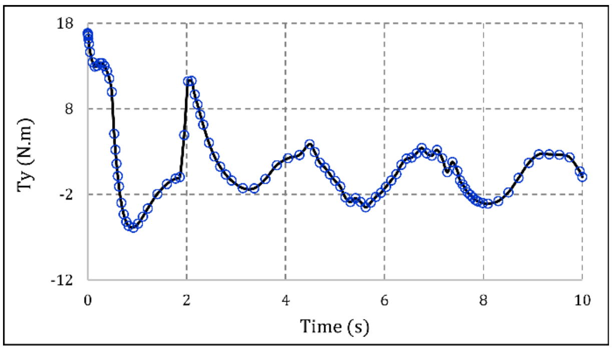


Fig. 8. Control torque, T_y for tracking of circular trajectory

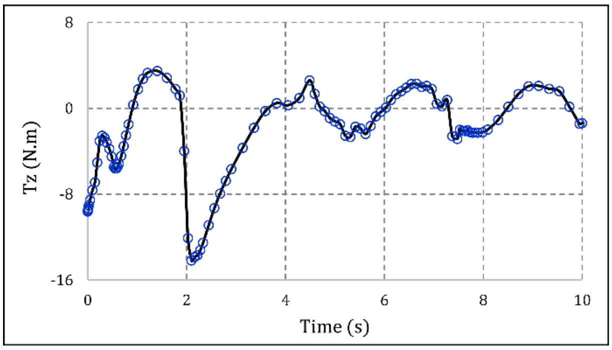


Fig. 9. Control torque, T_z for tracking of circular trajectory

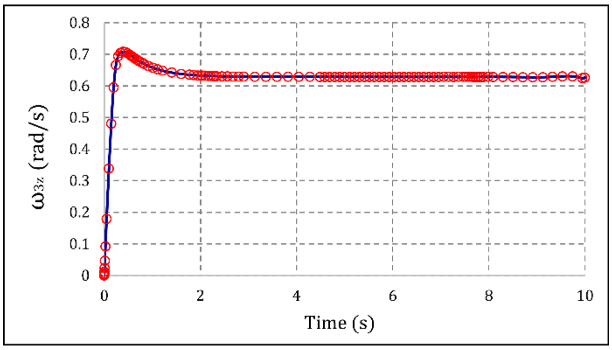


Fig. 10. Angular velocity of robot around the vertical axis

B. Trajectory Tracking Control

For attitude control, the robot should move from its initial configuration (34) to the final desired attitude as, $[q_1, q_2, q_3, q_4] = [0.5, 0.5, 0.5, 0.5]$ and stop at it. Using the nominal and uncertain parameters according to Table 1 and the same control gains as the trajectory tracking control of preceding section, the simulation results are presented in Figs. 11 to 16.

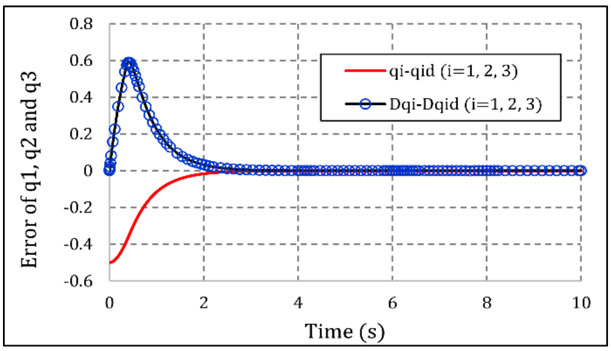


Fig. 11. Tracking error of q_1, q_2, q_3 from desired values 0.5

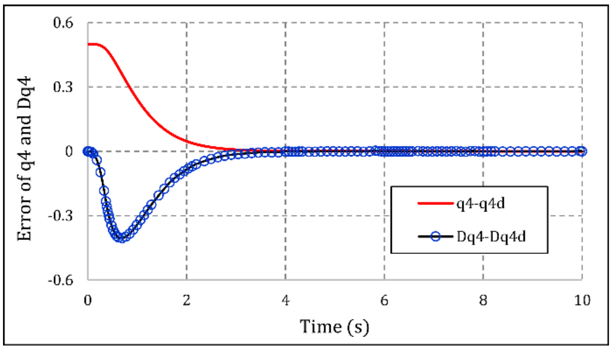


Fig. 12. Tracking error of q_4 from its desired value 0.5

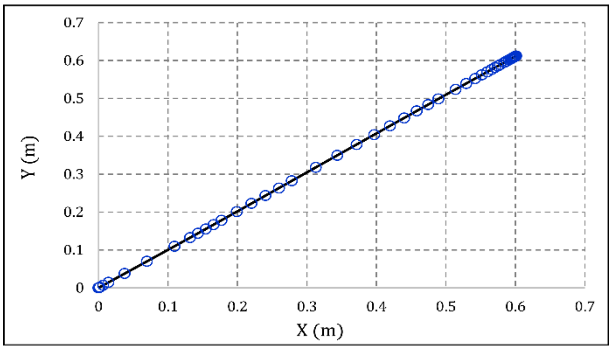


Fig. 13. Robot trajectory during attitude control

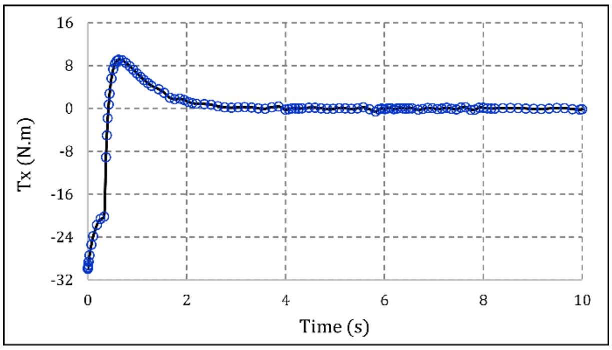


Fig. 14. Control torque, T_x of attitude control

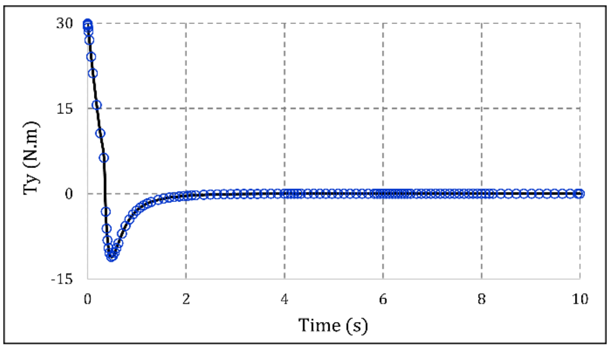


Fig. 15. Control torque, T_y of attitude control

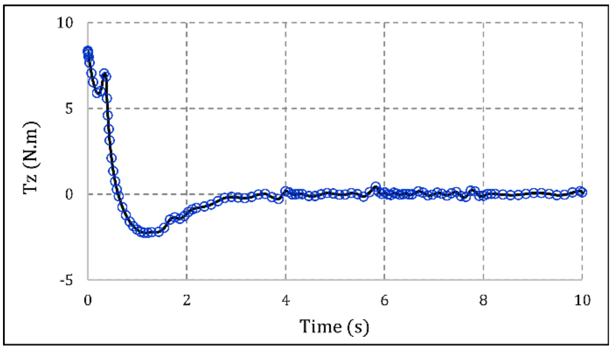


Fig. 16. Control torque, T_z of attitude control

C. Trajectory tracking in the presence of white noise

Besides assessing the robustness of the designed SMC against the parameters uncertainty, the performance and robustness of the control system is evaluated when the robot system is affected by Gaussian white measurement noises. Therefore, the circular trajectory (32) in x-y plane and the angular velocity (33) are considered once again as the reference trajectories. While the measured values of the simulated state variables are gathered with Gaussian white noise, the nominal values of the robot physical parameters and uncertain values are used according to Table 1. Considering the following initial off-tracks in the state variables and using suitable controller gains as $[K_1 \quad K_2 \quad K_3]=[3 \quad 3 \quad 3]$, the thickness of boundary layer as, $\varepsilon=0.4$ and $\boldsymbol{\Lambda}=diag(2,2,2)$, the simulation results of Figs. 17 to 24 are obtained.

$$\begin{aligned}
&[x, y, r_1, r_2, r_3, r_4, \theta_x, \theta_y, \theta_z] \\
&=[0,-1,0,0,0,1,0,0,0] \qquad , \quad at \quad t=0 \\
&[\dot{x}, \dot{y}, \dot{r}_1, \dot{r}_2, \dot{r}_3, \dot{r}_4, \Omega_x, \Omega_y, \Omega_z] \\
&=[0,0,0,0,0,0,0,0,0] \qquad , \quad at \quad t=0
\end{aligned} \tag{35}$$

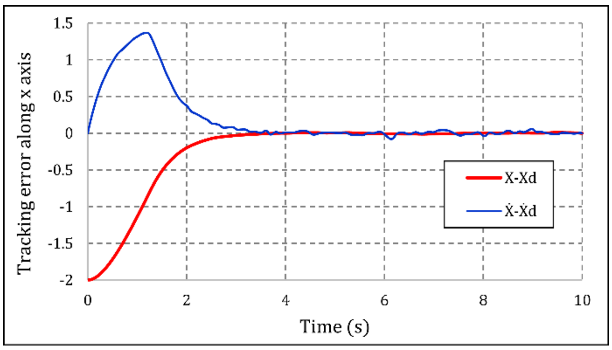


Fig. 17. Tracking error along x direction in the presence of white noise

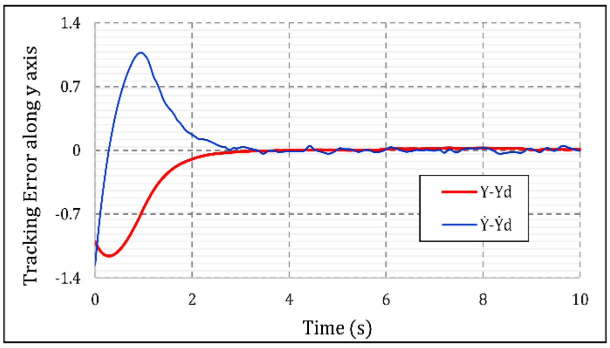


Fig. 18. Tracking error along y direction in the presence of white noise

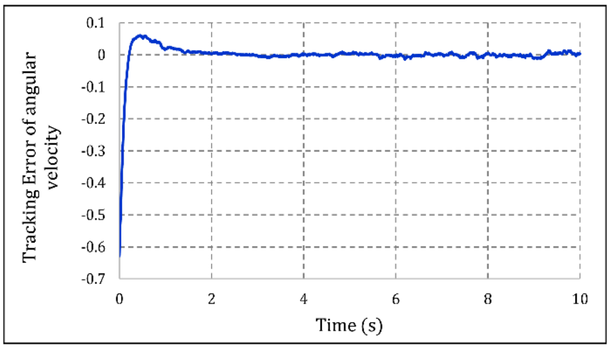


Fig. 19. Tracking error of angular velocity around the vertical axis in the presence of white noise

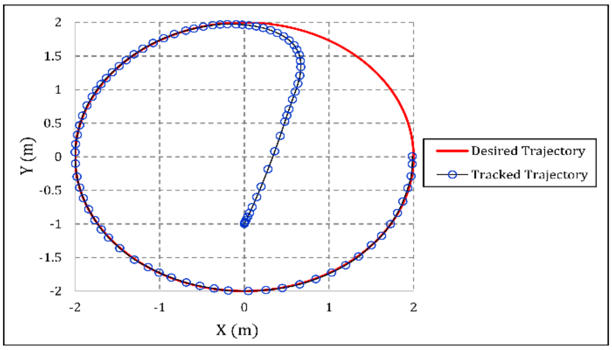


Fig. 20. Robot circular trajectory in x-y plane in the presence of white noise

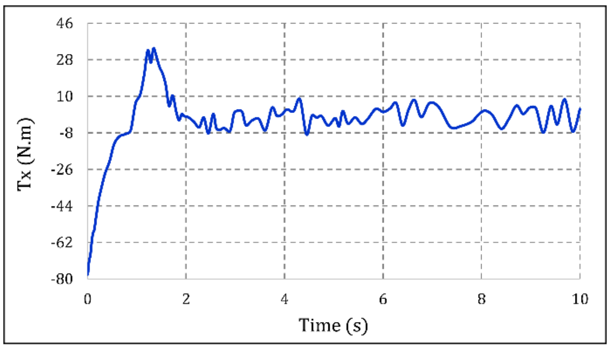


Fig. 21. Control torque, T_x for tracking of circular trajectory with noisy measurements

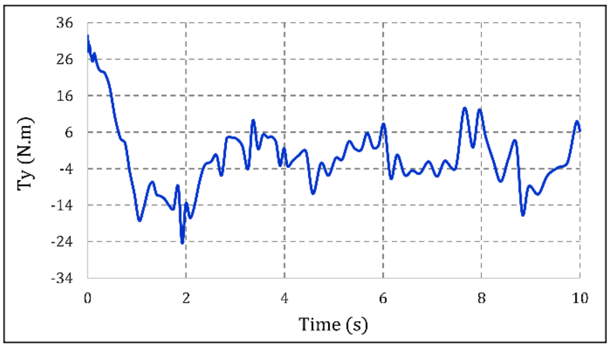


Fig. 22. Control torque, T_y for tracking of circular trajectory with noisy measurements

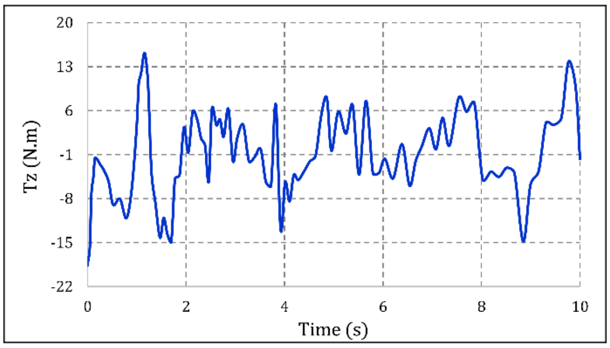


Fig. 23. . Control torque, T_z for tracking of circular trajectory with noisy measurements

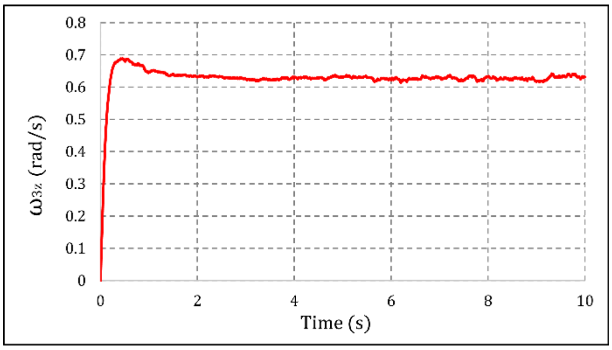


Fig. 24. Angular velocity of robot around the vertical axis in the presence of white noise

5. CONCLUSION

In this paper, robust trajectory tracking and attitude control of a nonholonomic spherical mobile robot with three independent actuators have been investigated. The state space model of the robot has been obtained using the dynamical equations of robot and the two nonholonomic kinematical constraints without using of any simplifying assumptions. For the purpose of trajectory tracking and attitude control of the robot, two nonlinear SMCs have been designed. Using the nonlinear dynamical modeling as the design base of the nonlinear SMCs, all the inertial, Coriolis and centrifugal effects are considered in the control actions. Therefore, unlike kinematical model based designs, the required control torques by driver motors have been smooth, accurate and small. Using Lyapunov's direct method in design and analysis of SMCs, the global stability of the control systems and the convergence conclusion of tracking errors to zero have been obtained. Furthermore, by use of boundary layer technique, the discontinuous switching terms of the SMC have been replaced by linear continuous terms to decrease the chattering effects. Through wide range computer simulations, the trajectory tracking performance of the proposed SMCs has been given. Owing the global stability and the robust nature of the SMCs, the accurate tracking performance has been obtained through considering large position, velocity and attitude off-tracks of the robot. Owing to the robust design of the SMCs against modeling uncertainties, the tracking performance of the robot along all position, velocity and attitude trajectories have been significantly accurate through considering large parameters uncertainty. By using the attitude control system, starting from every desired initial configuration, the spherical robot could reach accurately the desired attitude very fast. As an unmodeled dynamic, 3% white Gaussian noises have been gathered on the measured output state variables to show the robustness of the proposed SMCs. The results show that the robot could track the desired position and attitude trajectory as well as the angular velocity of shell around the vertical axis in the existence of parameters' uncertainty and noisy measurements. Therefore, due to the stability and the robustness, the proposed SMCs systems could be implemented on the spherical robot in real world applications. Furthermore, the capability of the designed SMC in control of the angular velocity of the robot around the vertical axis could be extended to achieve full trajectory tracking together with complete attitude control of the robot in feature studies.

REFERENCES

[1] J. Suomela, T. Ylikorpi, "Ball shaped robots: An historical overview and recent," *Field and Service Robotics*, vol. 25, no. 6, pp. 343– 354, 2006.

[2] A. Bicchi, A. Balluchi, D. Prattichizzo, A. Gorelli, "ntroducing the “SPHERICLE”: an experimental testbed for research and teaching in nonholonomy," in *IEEE International Conference on Robotics and Automation*, 1997.

[3] A. Halme, T. Schönberg, Y. Wang, "Motion control of a spherical mobile robot," in *International Workshop on Advanced Motion Control (AMC)*, 1996.

[4] A.H. Javadi, P. Mojabi, "Introducing august: a novel strategy for an omnidirectional spherical rolling robot," in *IEEE International Conference on Robotics and Automation*, 2002.

[5] Q. Zhan, T. Zhou, M. Chen, S. Cai, "Dynamic trajectory planning of a spherical mobile robot," in *IEEE International Conference on Robotics, Automation & Mechatronics (RAM)*, 2006.

[6] S. Bhattacharya, S.K. Agrawal, "Spherical rolling robot: A design and motion planning studies," in *IEEE Transactions on Robotics and Automation*, 2000.

[7] T. Otani, T. Urakubo, S. Maekawa, H. Tamaki, Y. Tada, "Position and Attitude Control of a Spherical Rolling Robot Equipped with a Gyro," in *IEEE International Workshop on Advanced Motion Control (AMC)*, 2006.

[8] M.R. Azizi, D. Naderi, "Dynamic Modeling and Trajectory Planning for a Mobile Spherical Robot with a 3DOF Inner Mechanism," *Mechanism and Machine theory*, vol. 64, pp. 251-261, 2013.

[9] R.W. Brockett, in Brockett, R.W., Milman, R.S., Sussmann, H.J. (Eds.), Asymptotic stability and feedback stabilization, Boston, Dzflerential, pp. 181-191, 1983.

[10] B. Zhao, M. Li, H. Yu, H. Hu, L. Sun, "Dynamics and motion control of a two pendulums driven spherical robot," in *IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*, 2010.

[11] D. Liu, H. Sun, Q. Jia, "A family of spherical mobile robot: Driving ahead motion control by feedback linearization," in *IEEE International Symposium on Systems and Control in Aerospace and Astronautics (ISSCAA)*, 2008.

[12] B. Liu, M. Yue, R. Liu, "Motion control of an underactuated spherical robot: A hierarchical sliding-mode approach with disturbance estimation," in *IEEE International Conference on Mechatronics and Automation (ICMA)*, 2012.

[13] M. Yue, B. Liu, "Adaptive control of an underactuated spherical robot with a dynamic stable equilibrium point using hierarchical sliding mode approach," *International journal of adaptive control and signal processing*, vol. DOI: 10.1002/acs.2413, 2013.

[14] E. Kayacan, E. Kayacan, H. Ramon, W. Saeys, "Adaptive Neuro-Fuzzy Control of a Spherical Rolling Robot Using Sliding-Mode-Control-Theory-Based Online Learning Algorithm," *IEEE TRANSACTIONS ON CYBERNETICS*, vol. 43, no. 1, pp. 170-179, 2013.

[15] Y. Cai, Q. Zhan, C. Yan, "Two-state trajectory tracking control of a spherical robot using

neurodynamics," *Robotica*, vol. 30, no. 2, pp. 195-203, 2012.

[16] D. Liu, H. Sun, Q. Jia, "Stabilization and path following of a spherical robot," in *IEEE International Conference on Robotics, Automation and Mechatronics (RAM)*, 2008.

[17] E. Kayacan, Z.Y. Bayraktaroglu, W. Saeys, "Modeling and control of a spherical rolling robot: a decoupled dynamics approach," *Robotica*, vol. 30, no. 4, pp. 671-680, 2012.

[18] Y. Cai, Q. Zhan, X. Xi, "Path tracking control of a spherical mobile robot," *Mechanism and Machine Theory*, vol. 51, pp. 58- 73, 2012.

[19] Q. Zhan, Z. Liu, Y. Cai, "A back-stepping based trajectory tracking controller for a non-chained nonholonomic spherical robot," *Chinese Journal of Aeronautics*, vol. 21, no. 5, pp. 472-480, 2008.

[20] M. Zheng, Q. Zhan, J. Liu, Y. Cai, "Control of a spherical robot: Path following based on nonholonomic kinematics and dynamics," *Chinese Journal of Aeronautics*, vol. 24, no. 3, pp. 337-345, 2011.

[21] M. Svinin, A. Morinaga, M. Yamamoto, "On the dynamic model and motion planning for a spherical rolling robot actuated by orthogonal internal rotors," *Regular and Chaotic Dynamics*, vol. 18, no. 1-2, pp. 126-143, 2013.

[22] J. Keighobadi, Y. Mohamadi, "Fuzzy Robust Trajectory Tracking Control of WMRs," in *S. I. Ao, O. Castillo, X. Huang, Eds., Intelligent Control and Innovative Computing*, Singapore, Springer, 2012, pp. 77-90.