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Electromagnetic Field Due to Lightning Strikes to Mountainous Ground

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ABSTRACT

The produced electric and magnetic fields due to lightning strikes to mountainous ground are determined in this paper. For the sake of simplicity a cone-shaped ground with finite conductivity is assumed to represent a natural nonflat ground. By this assumption, we deal with an axillary symmetrical structure so we use the cylindrical 2D-FDTD to save the simulation memory and time, dramatically. The return stroke channel is modeled using the antenna theory model with fixed inductive loading (ATIL-F) which is appropriately incorporated into the FDTD algorithm. We have derived the updating equations of 2D-FDTD for distributed resistance and inductance in ATIL-F model. Both the first and the subsequent return strokes are considered and their related radiated electromagnetic fields are determined and compared with each other. The fields are calculated at an intermediate horizontal distance from the cone-axis.

The calculated results show that the presence of the cone-shape ground introduces an enhancement in electric and magnetic fields, for both first and subsequent return strokes. Since sharper lossy cone means larger current density in the ground, the increment in the amplitude of the fields is inversely proportional to the cone-angle. A hump is observed in electromagnetic field waveforms because of the current reflection form joint point of the cone-shape ground and the flat ground beneath it. It is more specific in the subsequent-stroke fields. simulation results for different cone-height are presented. It shows that for small values of height, the results approaches to those of the flat ground.

KEYWORDS

Lightning, Mountainous ground, Return stroke, ATIL model, Cylindrical 2D-FDTD

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1- INTRODUCTION

Lightning is known as a major natural source of the electromagnetic radiation with significant effects on electric, electronic, and telecommunication instruments and systems. These effects can be evaluated by measuring or determining the electromagnetic fields that are generated by the lightning discharge. In previous studies that have been concerned the lightning effects, different analysis have been presented (see [1] for a review). Specifically, theoretical and experimental investigations of lightning discharges to flat and lossy ground (e.g., [2-4]), inhomogeneous and frequency-dependent ground (e.g., [5-6]), and lightning strikes to towers (or in general terms, "tall structures") (e.g., [7-8]) have been considered in recent studies.

Undoubtedly, the expense of maintenance and replacement of the electrical instruments and systems increases when they are installed on a mountainous terrain. Then it will be worthy to study and determine how the nonflatness of the ground influences the radiated electromagnetic fields produced by lightning.

Among various approaches for the evaluation of produced electromagnetic fields due to lightning, the finite-difference time-domain (FDTD) method [9] has the advantage of being easily implemented in computer codes. In our study the nonflat profile of the ground and the finite-ground conductivity has been taken into account in a straight forward way in the FDTD method.

In this paper, the so-called ATIL-F model [10] is used to present the lightning channel as a monopole antenna. A current source has been assumed to exite the antenna. Both the first and the subsequent return stroke waveforms has been considered for the channel-base current source and the their related horizontal electric, azimuthal magnetic, and vertical electric fields are calculated at an intermediate horizontal distance from the cone-axis.

This paper is organized as follows. In Section 2 the ATIL-F model will be reviewed. In Section 3, the cylindrical 2D-FDTD method and its related considerations are described. The simulation results and discussions are presented in Section 4. Finally, Section 5 deals with conclusions.

2 - LIGHTNING CHANNEL MODELING

The electromagnetic return stroke models allow a self-consistent full-wave solution for both lightning-current distribution and resultant electromagnetic fields [11-12]. In Antenna Theory (AT) model [13] which belongs to the category of EM models, the lightning channel is represented as a lossy monopole antenna above a perfectly conducting ground. The ATIL [10] model, which is a modified AT (antenna theory) model, introduces inductive loading along the channel to control the propagation speed of the upward traveling current wave. The typical measured values of return stroke wavefront speed are $c/3$ to $2c/3$, where c is the speed of light [14].

TABLE I
PARAMETERS OF THE TWO HEIDLER'S FUNCTIONS USED TO PRODUCE THE CHANNEL-BASE CURRENT WAVESHAPE

	I_{01} (kA)	τ_{11} (μS)	τ_{21} (μS)	I_{02} (KA)	τ_{12} (μS)	τ_{22} (μS)	N_1, N_2
First Stroke	28	1.8	95	-	-	-	2
Subsequent Stroke	10.7	0.25	2.5	6.5	2	230	2

A current source is assumed to energize the monopole antenna that represents the lightning channel. The channel-base current is reproduced by means of a sum of two Heidler’s functions [15]. Its terms are presented here in (1).

$$I(0,t)=\sum_{k=1}^{2}\frac{I_{0k}}{\eta_k}\left(\frac{t}{\tau_{1k}}\right)^{N_k}\frac{e^{-t/\tau_{2k}}}{1+\left(t/\tau_{1k}\right)^{N_k}}$$
$$\eta_k=\exp\left[-\left(\frac{\tau_{1k}}{\tau_{2k}}\right).\left(N_k\frac{\tau_{2k}}{\tau_{1k}}\right)^{\frac{1}{N_k}}\right] \quad (1)$$

The first return-stroke channel-base current is characterized by a peak value of 30 kA and maximum steepness of 12 kA/μs, whereas the subsequent return stroke current has a peak value of 12 kA and maximum steepness of 40 kA/μs. The parameters of the Heidler's functions are given in Table I (see [16]). Fig. 1 presents both channel-base current waveforms.

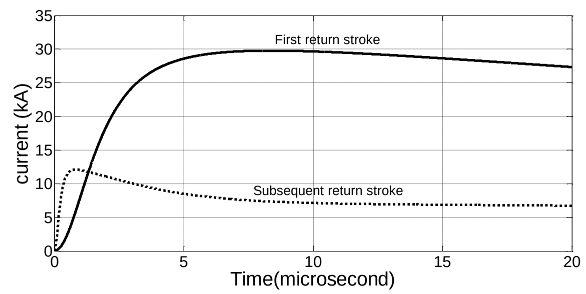


Fig.1. Channel-base current waveshapes for typical first and subsequent return strokes

The adopted values for the first and the subsequent return-stroke speed within the lightning channel, are, respectively, 1.7×10^8 m/s and 1.9×10^8 m/s. In the ATIL model, these values can be achieved by choosing proper amount of resistance, r_d, and inductance, L_d, distributed on the lossy monopole antenna that presents the lightning channel.

3 – TWO-DIMENSIONAL FDTD METHOD

By assuming a cone-shape ground that represent a nonflat ground we will deal with an axially symmetrical structure that let us apply the cylindrical 2D-FDTD to save the simulation memory/time, dramatically, compared to the 3D-FDTD method. The two dimension view of the structure in presented in Fig. 2.

For the case of a mountainous ground, on account of its profile, we deal with a curved surface in FDTD computation domain which is the boundary between the dielectric and air. Here, the Conformal FDTD (CFDTD), which is a simple, yet accurate technique is used [17].

Since the ATIL model is chosen in this paper to represent the lightning channel, we have derived the updating equations in cylindrical 2D-FDTD algorithm considering the distributed inductance and resistance along the antenna. The updating equations and the validity of our code are presented in Appendices A and B, respectively.

For all simulations in Section 4, the first-order Mur (Mur 1) absorbing boundary condition [18] is applied in order to simulate unbounded space (Fig. 2).

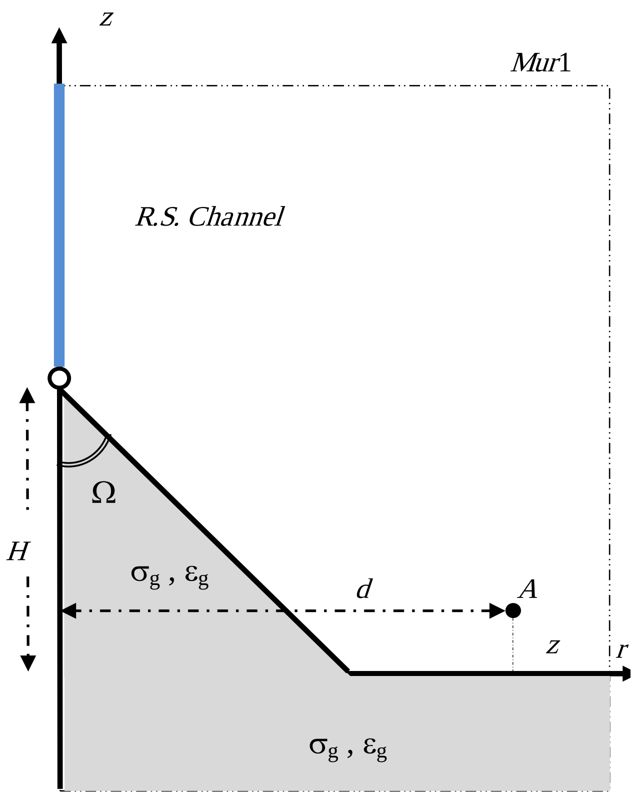


Fig. 2. Lightning strikes to a cone-shaped ground in two dimension view

4 - SIMULATION RESULTS AND DISCUSSIONS

In this section, the simulation results are presented. The cone-shape ground and the flat ground beneath it are assumed to have the same finite conductivity of σ_g = 0.001 S/m. This value for the ground conductivity is typical of mountainous terrains. The horizontal electric (E_r), the azimuthal magnetic (H_φ), and the vertical electric (E_z) fields are calculated at a horizontal distance d = 1 km from the cone-axis (z-axis in Fig. 2) and at a height of z = 10 m from the flat ground (point A in Fig. 2).

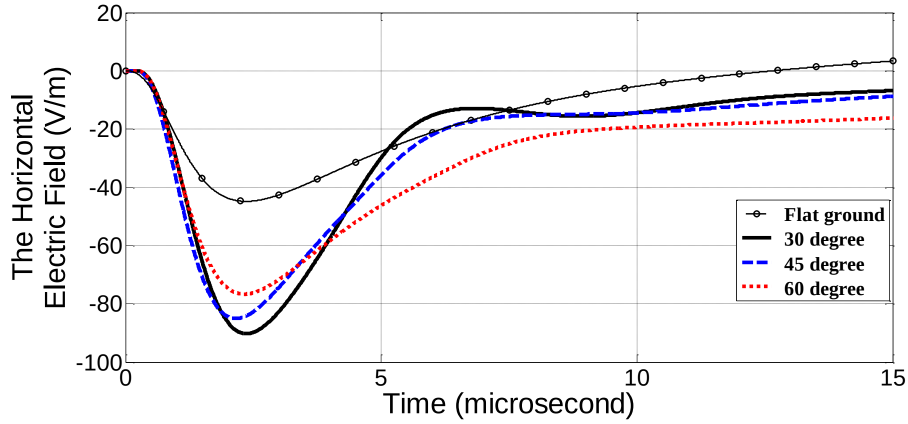
Figs. 3 presents the first-stroke radiated field waveforms for three cone angles: Ω = 30°, 45°, and 60°. The height of the cone is taken constant, H = 400 m, to only the effect of the cone angle on the produced electromagnetic field can be determined. Although a sharp mountainous terrain resembling a cone with angle Ω = 30° is rarely found in nature, we have considered this value to observe the effect of the cone angel on the calculated fields better.

Fig. 3(a) shows that the presence of the cone-shape ground increases the magnitude of negative peak in E_r, significantly, comparing with the case of flat ground. For smaller value of Ω we deal with sharper lossy cone which means larger current density in the ground. Consequently the horizontal electric field enhances inversely proportional to Ω. The hump, subsequent the negative peak, in the waveform of E_r is due to the current reflection from the junction between the cone-shaped ground and the flat ground beneath it.

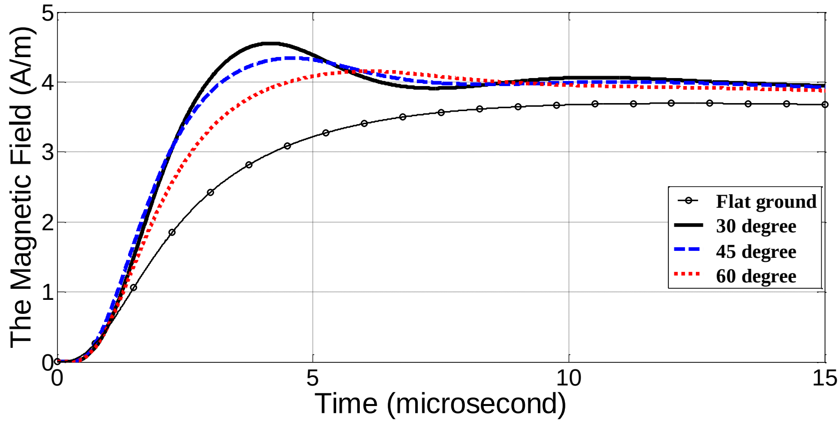
The enhancement of H_φ and E_z is inversely proportional to Ω as can be seen in Fig. 3(b) and Fig. 3(c). Their waveforms change as well, and an intial peak can be observed. It can be seen that the risetime of H_φ and E_z, decreases proportional to the cone angle.

Similar results for subsequent return stroke are presented in Fig. 4. The negative peak amplitude of E_r increases significantly at the presence of the cone-shape ground but not clearly proportional to Ω. The subsequent-stroke magnetic field waveform, same as Fig. 3(b), have a peak that increases inversely proportional to Ω. But in comparison, its peak enhancement is more pronounced and occurs at an earlier instant of time.

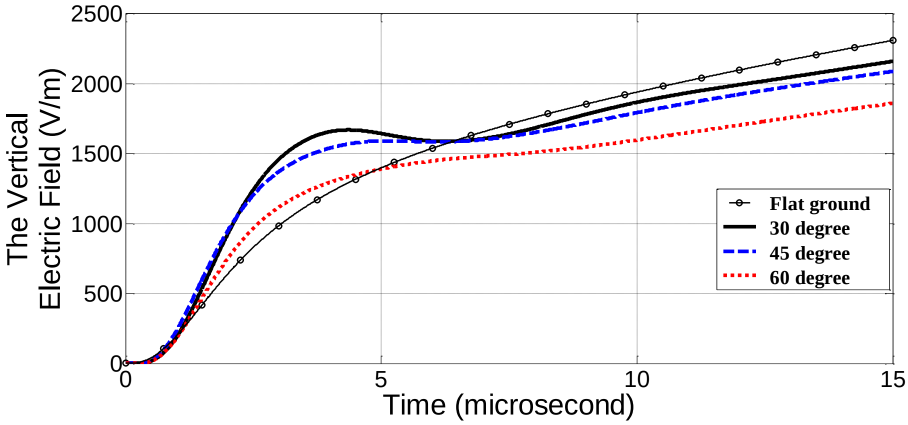
Fig. 5 depicts the first-stroke fields for three cone heights: H = 200 m, 400 m, and 600 m where cone-ange is taken to be Ω = 45°. Fig. 6 depicts similar results as in Fig. 5 but for subsequent return stroke.



(a)

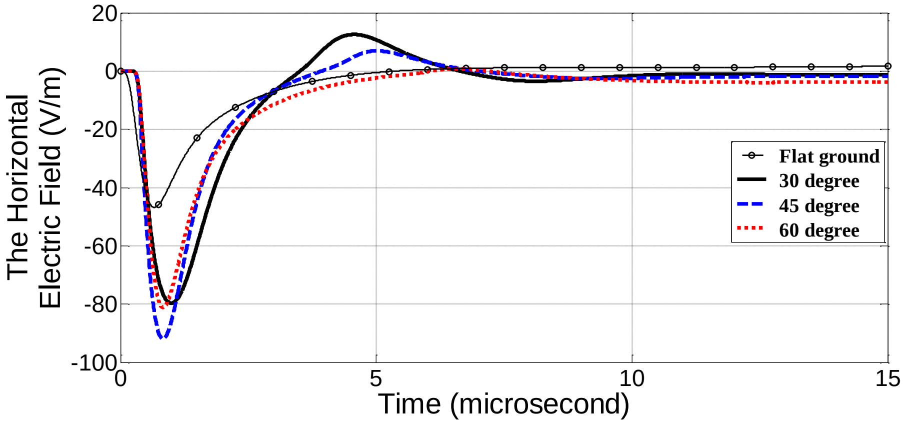


(b)

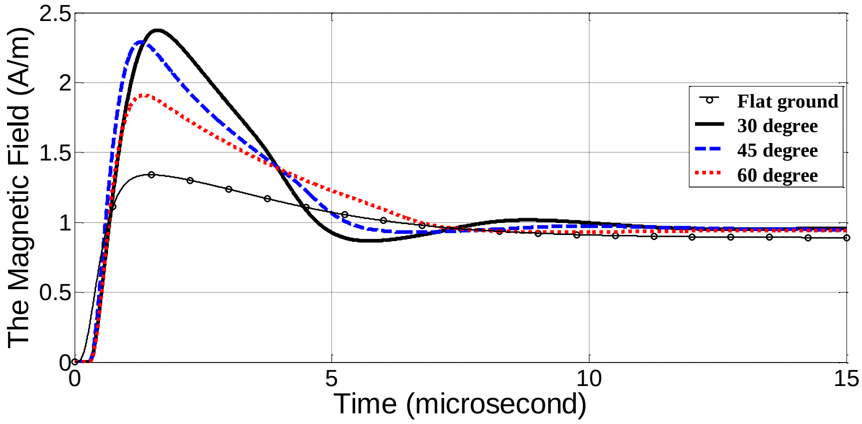


(c)

Fig. 3. The first-stroke fields: (a) Er, (b) Hφ, and (c) Ez, at d = 1000 m from the z-axis (Fig. 2) for angles, Ω = 30° (black line), Ω = 45° (blue dash), and Ω = 60° (red dot). The height of the cone-shaped ground is H = 400 m.



(a)



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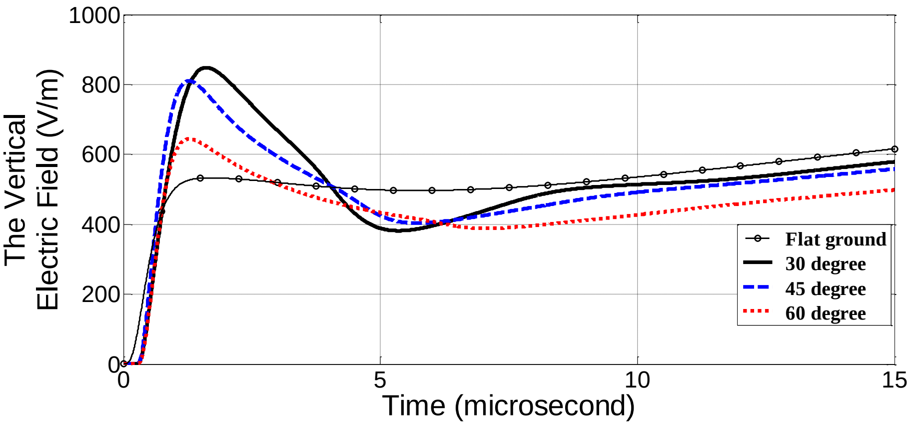
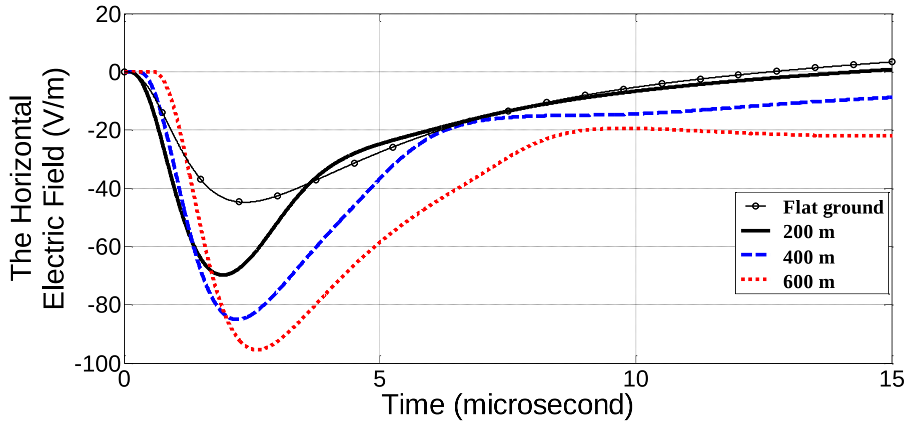
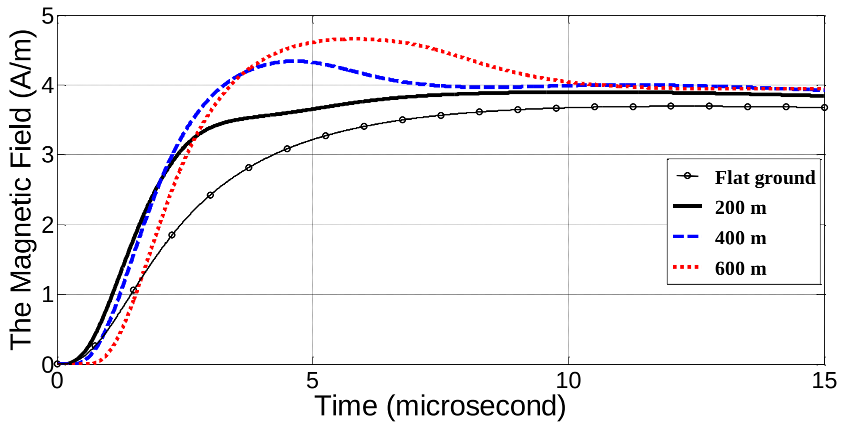


(c)

Fig. 4. The subsequent-stroke fields: (a) Er, (b) Hφ, and (c) Ez, at d = 1000 m from the z-axis (Fig. 2) for angles, Ω = 30° (black line), Ω = 45° (blue dash), and Ω = 60° (red dot). The height of the cone-shaped ground is H = 400 m.



(a)



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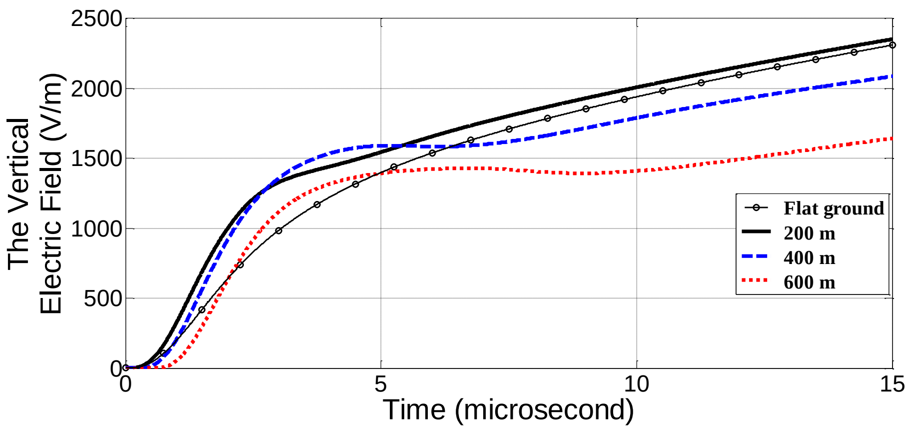
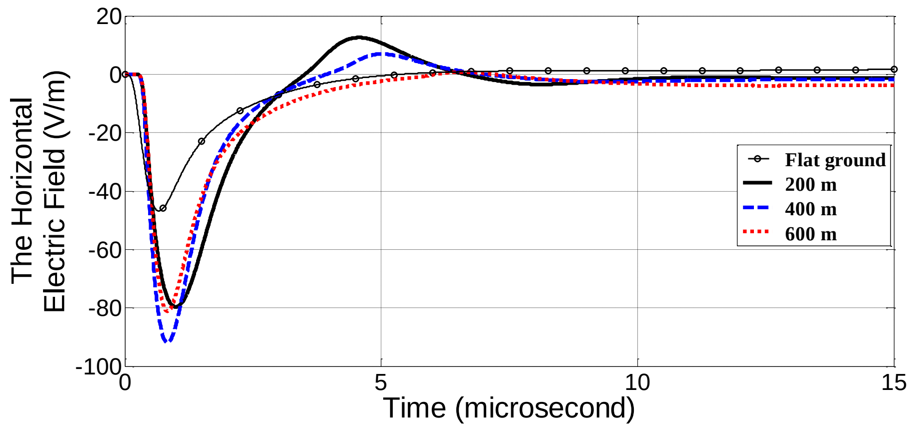
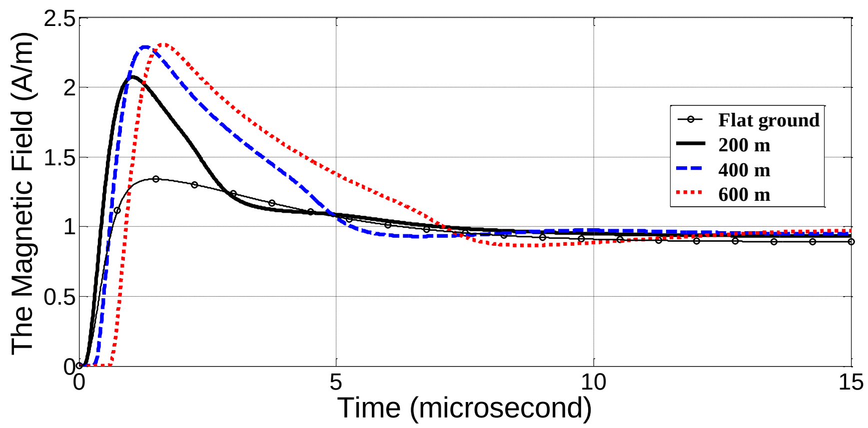


(c)

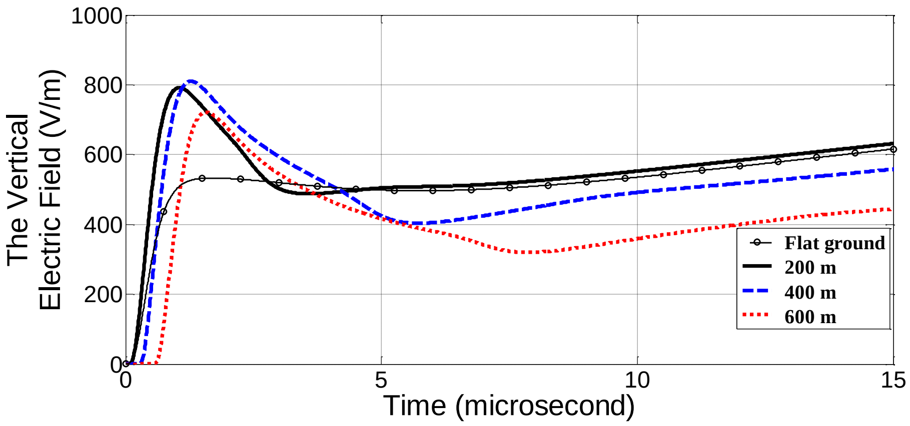
Fig. 5. The first-stroke fields: (a) Er, (b) Hφ, and (c) Ez, at d = 1000 m from the z-axis (Fig. 2) for H = 200 m (red dot), H = 400 (blue dash), and H = 600 (black line) when Ω = 45°



(a)



(b)



(c)

Fig. 6. The subsequent-stroke fields: (a) Er, (b) Hφ, and (c) Ez, at d = 1000 m from the z-axis (Fig. 2) for H = 200 m (red dot), H = 400 (blue dash), and H = 600 (black line) when Ω = 45°.

Fig. 5(a) shows that the negative peak amplitude of the first-stroke horizontal electric field increases by cone height increment. Time spread of the V-shape waveform depends on the H and increase for higher cone-shape ground. The H_φ and E_z, enhanced slightly when as we assume higher cone-shape ground.

5 - CONCLUSIONS

In this paper, an analysis of the electric and magnetic fields radiated by lightning strikes to a cone-shape ground with finite conductivity has been presented. The EM field is calculated at an intermediate distance from the cone-axis. The horizontal electric, azimuthal magnetic and vertical electric field waveforms corresponding to typical first and subsequent strokes have been computed and compared. The ATIL model, that is a modified antenna theory model, has been chosen to present the lightning channel. The related cylindrical 2D-FDTD updating equations for distributed resistance and inductance, distributed on the channel, have been derived.

It has been shown that when lightning strikes to a cone-shape ground, the radiated electric and magnetic fields experience an enhancement in their amplitude in comparision with the case of lightning strikes to a flat ground. This is true for both first and subsequent return strokes.

The negative peak amplitude of the first-stroke horizontal electric field increases inversely proportional to the cone-angle. For the subsequent-stroke a clear relation could not be observed.

The waveform of the first-stroke magnetic field changes when considering cone-shape ground and a peak in initial time can be observed. Besides, by decreasing the cone-angle, the risetime of the magnetic field decreases. Similar results can be considered for the subsequent-stroke magnetic field but its risetime changes slightly.

Simulation results for three different values of H, shows that for small cone-height, the results approaches to those for flat ground. The considerable point is that the V-shape waveform of the first-stroke horizontal electric field spreads in time by increasing the cone-height.

APPENDIX A
UPDATING EQUATIONS IN CYLINDRICAL 2D-FDTD

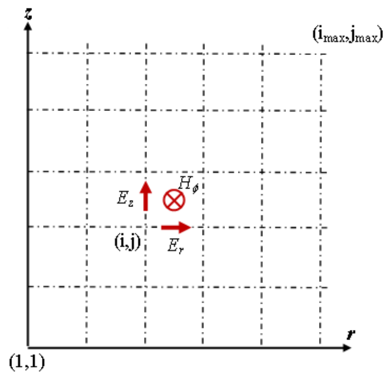


Fig. A1. The two dimensional cylindrical coordinate

The updating electric and magnetic field equations in the cylindrical two dimensional FDTD are reviewed in here.

A. Horizontal electric, magnetic and vertical electric field updating equations

$$B.E_r^{n+1}(i,j+1)=A.E_r^n(i,j+1)-\left[H_\phi^{n+1}(i,j+1)-H_\phi^{n+1}(i,j)\right] \quad \text{(A1)}$$

$$H_\phi^{n+1}(i,j)=H_\phi^n(i,j)+\frac{\Delta t}{\mu_0\Delta r}\left[E_z^n(i+1,j)-E_z^n(i,j)\right]-\frac{\Delta t}{\mu_0\Delta z}\left[E_r^n(i,j+1)-E_r^n(i,j)\right] \quad \text{(A2)}$$

$$B.E_z^{n+1}(i+1,j)=A.E_z^n(i+1,j)+\left[r_{i+0.5}H_\phi^{n+0.5}(i+1,j)-r_{i-0.5}H_\phi^n(i,j)\right] \quad \text{(A3)}$$

where,

$$A=\frac{2\varepsilon-\sigma\Delta t}{2\Delta t} \quad , \quad B=\frac{2\varepsilon+\sigma\Delta t}{2\Delta t}$$

To overcome the singularity in on-axis (r = 0) electric field updating equation, $E_z(1, j)$, the integral form of the Maxwell curl equation in H (i.e., Ampere's law) is applied.

$$\iint_S\left(\nabla\times\vec{H}\right)d\vec{s}=\oint_C\vec{H}.d\vec{l}=\iint_S\left(\varepsilon\frac{\partial\vec{E}}{\partial t}+\sigma\vec{E}+\vec{J}\right).d\vec{s} \quad \text{(A4)}$$

C in (A4), is a small loop of radius r centered at r = 0 and perpendicular to the z axis. $\vec{J}$ is the current density vector. Doing a simple mathematical calculation, one can come up with the following equation:

$$B.E_z^{n+1}(1,j)=A.E_z^n(1,j)+\frac{4H_\phi^{n+1}(1,j)}{\Delta r}-J_z(1,j) \quad \text{(A5)}$$

In section II, it was mentioned that we have chosen the ATIL model in this paper where a monopole lossy antenna with distributed inductance on it presents the lightning channel. The antenna is excited by a current source with a waveform of that in (1).

In cylindrical 2D-FDTD, the antenna places along the z-axis (with r = 0). Then we should apply the Ampere's law for three cases of current source in the antenna base, and resistance and inductance distributed along the antenna.

B. The current source

For the general form, we assume a current source, I_s, with internal parallel resistance, R_s, located on the z-axis.

$$E_z^{n+1}(1,j_{source})=\frac{\left(A-\frac{\Delta z}{2\pi R_S(0.5\Delta r)^2}\right)}{\left(B+\frac{\Delta z}{2\pi R_S(0.5\Delta r)^2}\right)}E_z^n(1,j_{source}) + \frac{4}{\left(B+\frac{\Delta z}{2\pi R_S(0.5\Delta r)^2}\right)\Delta r}H_\varphi^{n+\frac{1}{2}}(1,j_{source}) - \frac{4}{\left(B+\frac{\Delta z}{2\pi R_S(0.5\Delta r)^2}\right)\pi\Delta r^2}I_S \quad \text{(A6)}$$

By assuming $R_s \neq 0$, we can reduce the effect of the reflections on the current source.

C. The distributed resistance on the antenna

By taking I_s = 0 and replacing R_s with r_d (the distributed resistance (Ω/m)) in (A6), the updating equation for electric field yields.

$$E_z^{n+1}(1,j_{source})=\frac{\left(A-\frac{\Delta z}{2r_d(0.5\Delta r)^2}\right)}{\left(B+\frac{\Delta z}{2\pi r_d(0.5\Delta r)^2}\right)}E_z^n(1,j_{source}) + \frac{4}{\left(B+\frac{\Delta z}{2r_d(0.5\Delta r)^2}\right)\Delta r}H_\varphi^{n+\frac{1}{2}}(1,j_{source}) \quad \text{(A7)}$$

D. The distributed inductance on the antenna

The relation between the voltage and the current of an inductance can be easily presented as (A8).

$$v(t)=L\frac{di(t)}{dt} \quad \text{(A8)}$$

Discretizing the above equation, leads to (A9).

$$j_{iz}^{n+1}(i,j)=j_{iz}^n(i,j)+\frac{\Delta t\Delta z}{L\pi(0.5\Delta r)^2}E_z^n(i,j) \quad \text{(A9)}$$

L is the distributed inductance (H/m) along the antenna. Substituting A8 into A5 will easily present the updating electrical field for distributed inductance, L (H/m).

APPENDIX B

VALIDATION OF CYLINDRICAL 2D-FDTD

In order to validate the cylindrical 2D-FDTD code used in this paper, we simulate the case of lightning strikes to a flat ground with finite conductivity, and determine the horizontal electric field that can be compare with the result of the analytic equation derived in [4]. In Equ. (B1) the horizontal electric field above the ground with conductivity of σ_g and relative permittivity ε_{rg} is calculated analytically by considering the horizontal magnetic field.

$$e_{r,\sigma}(t,r,z)=\int_0^t h_{\phi,\sigma}(t,r,0)k(t-\tau)d\tau+e_{rp}(t,r,z) - \sqrt{\frac{\mu_0}{\varepsilon_0\varepsilon_{rg}}}h_{\phi,\sigma}(t,r,0) \quad \text{(B1)}$$

where

$$k(t)=-\sqrt{\frac{\mu_0}{\varepsilon_0\varepsilon_{rg}}}\frac{a}{2}e^{-\frac{at}{2}}\left[I_1\left(\frac{at}{2}\right)-I_0\left(\frac{at}{2}\right)\right]$$

in which, $a=(\sigma_g/\varepsilon_0\,\varepsilon_{rg})$, I_n is the modified Bessel function of first type and order n.

For both numerically simulation and analytically calculation, we assume, σg = 0.001 S/m and εr = 10. The determined horizontal electric field at a distance of 1000 m from the lightning channel base and 10 m above the lossy ground are depicted in Fig. B1 and show a good agreement.

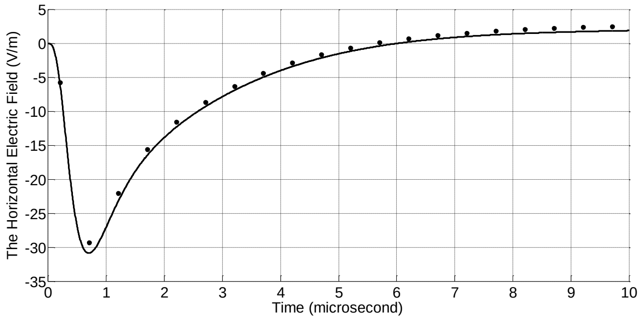


Fig. B1. The determined horizontal electric field using cylindrical 2D-FDTD method (solid line) and Equ. (B1) (dot marker)

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