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Partial Observation in Distributed Supervisory Control of Discrete-Event Systems

V. Saeidi, A. Afzalian, D. Gharavian

Dept. of Electrical Engineering, Abbaspour School of Engineering, Shahid Beheshti University, Tehran, Iran

ABSTRACT: Distributed supervisory control is a method to synthesize local controllers in discrete-event systems with a systematic observation of the plant. Some works were reported on extending this method by which local controllers are constructed so that observation properties are preserved from monolithic to distributed supervisory control, in an up-down approach. In this paper, we find circumstances in which observation properties are preserved from monolithic to distributed supervisory control. Local observation properties, i.e. local normality and local relative observability are employed for investigating observation properties of each local controller, which are constructed by any localization algorithm that preserves control equivalency to the monolithic supervisor with respect to the plant. These properties enable us to investigate the observation properties from monolithic to distributed supervisory control. Moreover, observation equivalence property is defined according to the control equivalence in a distributed supervisory control with partial observation. It is proved that with preserving observation equivalence of the local controllers to the monolithic supervisor, the control equivalence is satisfied, if and only if the intersection of local event sets is a subset of or equal to the global observable event set.

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1- Introduction

The supervisory control theory handles small scale discrete event systems (DES) [1], Since, in the large scale DES the number of states grows with the number of components. Supervisory control synthesis for the monolithic specification encounters with computational complexity. In order to overcome the computational complexity, modular and decentralized [2, 3], hierarchical [4,5] and heterarchical [6-9] approaches have been proposed in the supervisory control of DES. The decentralized supervisory control scheme reduces the computational complexity in large scale DES [10, 11]. Since a decentralized supervisor observes the plant partially, he does not have enough information about the other supervisors, and their decisions may be in conflict with each other. In [9], a method was introduced for synthesizing the optimal non-blocking decentralized supervisory control using L_m-observer and output control consistency (OCC) properties. In [11], decomposability and strong decomposability (conormality) were defined to construct the decentralized supervisory control in a top-down approach. Other accessible properties are co-observability and relative co-observability which were defined in [11] and [12], respectively. In order to remove conflict between decentralized supervisors, construction of a coordinator was proposed in the literature [13, 14]. In [15], a supervisor localization procedure was proposed to guarantee the optimality and the non-conflicting of the local controllers and the monolithic supervisor.

Observation properties e.g. normality [16], observability [16] and relative observability [17] describe the effect of observation on the control behavior.

The corresponding author; Email: v_saeidi@sbu.ac.ir

The authors of [15] developed a Distributed supervisory control based on localization of the monolithic supervisor with full observation. The paper [18] proposed a method for recognizing the conflicts between the supervisors, using the observer property of a natural projection

In [19], an abstraction method was proposed to construct a distributed supervisory control (in a top-down approach), in which the observation properties of monolithic supervisor is preserved in local controllers. In the present study, We find circumstances for preserving observation properties in local controllers, constructed by supervisor localization procedure [15], or constructed by decomposition of the monolithic supervisor [11].

This paper is the extended version of the conference paper [20]. Here, we study the observation properties in the supervisory control of DES using a set algebra approach. Also, local observation properties e.g. local normality and local relative observability [20], are employed to study the observation problem in the distributed supervisory control scheme. Control equivalence [15] describes the equivalency between the control behavior of local controllers and the monolithic supervisor in the plant. Moreover, observation equivalence is defined to investigate the effect of observation properties on the control behavior. Although the observation equivalence has been introduced in [21] for nondeterministic automata, in this paper, observation equivalence is defined to describe the equivalency between observation properties in decentralized (local) controllers and those in the monolithic supervisor. It is shown that having identical observation equivalence in local controllers and the monolithic supervisor, the control equivalence is satisfied if and only if the intersection of local event sets belongs or equals to the global observable event set. Theoretical results can be employed in industrial applications, such as gas transmission networks and power systems [22].

In the rest of the paper, necessary preliminaries are reviewed in section 2. In order to redefine the observation properties by set algebra approach, some formulations are introduced in section 3. Observation properties in distributed supervisory control are redefined by a set algebra approach in section 4. The observation equivalence is introduced and is compared with control equivalence in section 5. In section 6, the extended theorem is illustrated by some examples. Finally, the concluding remarks are presented in section 7.

2- Preliminaries

The set of all finite strings over Σ is denoted Σ^*. The concatenation of two strings $s_1,s_2 \subseteq \Sigma^*$ is written as $s_1 s_2 \subseteq \Sigma^*$ and $s_1 \leq s$, where s_1 is the prefix of s. $\epsilon \in \Sigma^*$ is the empty string, and $s\epsilon=\epsilon s=s$ does hold. A DES is introduced by an automaton $G=(Q,\Sigma,\delta,q_0,Q_m)$ in which Q is a finite set of states, with $q_0 \in Q$ as the initial state and $Q_m \subseteq Q$ being the desired (marked) states. Σ is a finite set of events, and finally δ is a transition mapping $\delta:Q\times\Sigma\rightarrow Q$: $\delta(q,\sigma)=q'$. $L(G):=\{s\in\Sigma^* \mid \delta(q_0,s)!\}$ is the closed behavior of G and $L_m(G):=\{s\in L(G) \mid \delta(q_0,s)\in Q_m\}$ is the marked behavior of G.

The natural projection is a mapping $P:\Sigma^*\rightarrow\Sigma_0^*$ where (1) $P(\epsilon):=\epsilon$,(2) for $s\in\Sigma^*,\sigma\in\Sigma$,$P(s\sigma):=P(s)P(\sigma)$, and (3) $P(\sigma):=\sigma$ if $\sigma\in\Sigma_0$ and $P(\sigma):=\epsilon$ if $\sigma\notin\Sigma_0$. The effect of P on the string $s\in\Sigma^*$ is to erase the events in s that do not belong to the observable event set Σ_0. The natural projection P can be extended and denoted by $P:Pwr(\Sigma^*)\rightarrow Pwr(\Sigma_0^*)$. For any $X\subseteq\Sigma^*$, $P(X):=\{P(s) \mid s\in X\}$. The inverse image function of P is denoted by $P^{-1}:Pwr(\Sigma_0^*)\rightarrow Pwr(\Sigma^*)$ and $P^{-1}(X):=\{s\in\Sigma^* \mid P(s)\in X\}$. The synchronous product of languages $L_1\subseteq\Sigma_1^*$ and $L_2\subseteq\Sigma_2^*$ is defined by $L_1\|L_2=P_1^{-1}(L_1)\cap P_2^{-1}(L_2)\subseteq\Sigma^*$, where $P_i:\Sigma^*\rightarrow\Sigma_i^*$,i=1,2 for the union $\Sigma=\Sigma_1\cup\Sigma_2$ [9,23].

In the supervisory control context, all events in Σ are partitioned as a set of controllable events Σ_c and a set of uncontrollable events Σ_{uc}, where $\Sigma=\Sigma_c\dot{\cup}\Sigma_{uc}$. A control pattern is γ, where $\Sigma_{uc}\subseteq\gamma\subseteq\Sigma$ and the set of all control patterns is denoted by $\Gamma=\{\gamma\in 2^\Sigma \mid \gamma\supseteq\Sigma_{uc}\}$. A supervisor for G is a map $V:L(G)\rightarrow\Gamma$, where V(s) represents the set of enabled events after the occurrence of the string $s\in L(G)$. Namely, a supervisor only disables the controllable events. A pair (G,V) is written as V/G and called "G that is under supervision of V". The closed loop language L(V/G) is defined by: (1) $\epsilon\in L(V/G)$ (2) $s\sigma\in L(V/G)$ iff $s\in L(V/G)$, $\sigma\in V(s)$, and $s\sigma\in L(G)$. The marked strings of V/G is defined as $L_m(V/G)=L(V/G)\cap L_m(G)$. The closed loop system is non-blocking if $\overline{L_m(V/G)}=L(V/G)$. $\overline{L_m(V/G)}$ is the set of all prefixes of traces in $L_m(V/G)$. A language $K\subseteq\Sigma^*$ is controllable with respect to (w.r.t.) L(G) and Σ_{uc}, if $\overline{K}\Sigma_{uc}\cap L(G)\subseteq\overline{K}$. The set of all controllable sublanguages E w.r.t. L(G) and Σ_{uc} is denoted by $C(E)=\{K\subseteq E \mid \overline{K}\Sigma_{uc}\cap L(G)\subseteq\overline{K}\}$, that is nonempty and closed under union. For every specification language E, there exists a supremal controllable sublanguage of E w.r.t. L(G) and Σ_{uc}.

3- Observation Properties

In order to handle the lack of enough observation of the plant, some observation properties have been defined. Observability describes that the natural projection P preserves at least the information required to decide consistently the question of continuing membership in $\overline{K}$ after the occurrence of an event σ and to decide membership in K when membership in $\overline{K}\cap L_m(G)$ is known. $K\subseteq\Sigma^*$ is (G,P)-observable if for $s,s'\in\Sigma^*$ such that $P(s)=P(s')$ the following conditions are satisfied [16]:

(i) $(\forall\sigma\in\Sigma)$ $s\sigma\in\overline{K}$,$s'\in\overline{K}$,$s'\sigma\in L(G)\Rightarrow s'\sigma\in\overline{K}$,

(ii) $s\in K$,$s'\in\overline{K}\cap L_m(G)\Rightarrow s'\in K$.

Normality is another property which is stronger than observability [16]. K is $(L_m(G),P)$-normal if $P^{-1}P(K)\cap L_m(G)=K$. Also, $\overline{K}$ is (L(G),P)-normal if $P^{-1}P(\overline{K})\cap L(G)=\overline{K}$. Normality is the strong property and may not hold in practice. Another property defined is called relative observability [17]. Relative observability is stronger than observability and weaker than normality; it imposes no constraint on the disablement of unobservable controllable events. Let $K\subseteq C\subseteq L_m(G)$. K is relatively observable w.r.t. $\overline{C}$,G and P ($\overline{C}$-observable) if for every pair of strings $s,s'\in\Sigma^*$ such that $P(s)=P(s')$, the following two conditions hold:

(i`) $(\forall\sigma\in\Sigma)$ $s\sigma\in\overline{K}$,$s'\in\overline{C}$,$s'\sigma\in L(G)\Rightarrow s'\sigma\in\overline{K}$,

(ii`) $s\in K$,$s'\in\overline{C}\cap L_m(G)\Rightarrow s'\in K$.

In Proposition 1, relative observability is written by a set algebra in two relationships.

Proposition 1: K is relatively observable w.r.t. $\overline{C}$,G and P if and only if $P(s)=P(s')$ for every pair of strings $s,s'\in\Sigma^*$ and the following two conditions hold:

$$P^{-1}P\overline{K}\cap\overline{C}\Sigma\cap L(G)\subset\overline{K}, \quad (1)$$

$$P^{-1}PK\cap\overline{C}\cap L_m(G)=K. \quad (2)$$

Proof: (If) Assume$P(s)=P(s')$. We can write

$(\forall\sigma\in\Sigma)$ $s\sigma\in\overline{K}$,$s'\in\overline{C}$,$s'\sigma\in L(G)\Rightarrow s'\sigma\in\overline{C}\Sigma$.

Also,

$P(s)=P(s')\Rightarrow P(s\sigma)=P(s'\sigma)\Rightarrow s'\sigma\in P^{-1}P\overline{K}$.

Thus,

$\Rightarrow s'\sigma\in P^{-1}P\overline{K}\cap\overline{C}\Sigma\cap L(G)$.

From (1) we conclude that $s'\sigma\in\overline{K}$ and (i`) is proved.

From (2) we write

$s\in K$,$s'\in\overline{C}\cap L_m(G)$,$P(s)=P(s')\Rightarrow s'\in P^{-1}PK$,

$\Rightarrow s'\in P^{-1}PK\cap\overline{C}\cap L_m(G)$,

$\Rightarrow s'\in K$,

and (ii`) is proved.

(Only if) From (i`) we write

$(\forall\sigma\in\Sigma)s\sigma\in\overline{K}$,$s'\in\overline{C}$, $s'\sigma\in L(G)\Rightarrow s'\sigma\in\overline{K}$,

$\Rightarrow P(s\sigma)\in P\overline{K}\Rightarrow P(s'\sigma)\in P\overline{K}$,

$\Rightarrow s'\sigma\in P^{-1}P\overline{K}$.

Then,

$s'\sigma\in P^{-1}P\overline{K}$,$s'\in\overline{C}$, $s'\sigma\in L(G)\Rightarrow s'\sigma\in\overline{K}$,

$\Rightarrow P^{-1}P\overline{K}\cap\overline{C}\Sigma\cap L(G)\subseteq\overline{K}$.

Also,

$\epsilon\notin\overline{C}\Sigma$, $\epsilon\in\overline{K}\Rightarrow P^{-1}P\overline{K}\cap\overline{C}\Sigma\cap L(G)\subset\overline{K}$

From (ii`), we can write,

$s\in K$,$s'\in\overline{C}\cap L_m(G)\Rightarrow s'\in K$,

$\Rightarrow P(s)\in PK\Rightarrow P(s')\in PK$,

$\Rightarrow s'\in P^{-1}PK$.

Thus,

$s'\in P^{-1}PK$,$s'\in\overline{C}$,$s'\in L_m(G)\Rightarrow s'\in K$,

$$\Rightarrow P^{-1}PK\cap\overline{C}\cap L_m(G)\subseteq K. \quad (3)$$

Moreover,

$K\subseteq P^{-1}PK$,$K\subseteq L_m(G)$,$K\subseteq\overline{C}$,

$$\Rightarrow K\subseteq P^{-1}PK\cap\overline{C}\cap L_m(G). \quad (4)$$

From (3) and (4), we have $P^{-1}PK\cap\overline{C}\cap L_m(G)=K$.

If $\overline{C}=\overline{K}$, then the definition of relative observability turns into that of observability property as:

$$P^{-1}P\overline{K}\cap\overline{K}\Sigma\cap L(G)\subset\overline{K}, \quad (5)$$

$$P^{-1}PK\cap\overline{K}\cap L_m(G)=K. \quad (6)$$

If $\overline{C}=L(G)$, then definition of relative observability appears in the following two relationships:

$P^{-1}P\bar{K}\cap L(G)\Sigma\cap L(G)\subset\bar{K}$, (7)

$P^{-1}PK\cap L_m(G)=K$. (8)

Equation (8) guarantees that K is $(L_m(G),P)$-normal. Since $\epsilon\notin L(G)\Sigma$, $L(G)\Sigma\cap L(G)$ consists of all strings in L(G) except for ϵ. Thus, the relative observability may not lead to the normality of $\bar{K}$ w.r.t. (L(G),P) in general.

Although Propositions 2 to 5 were proved in [17]; we prove them again in the appendix by a set algebra approach.

Proposition 2: If $K\subseteq C$ is $\bar{C}$-observable, then K is also observable.

Proposition 3: If $K\subseteq C$ is $(L_m(G),P)$-normal and $\bar{K}$ is (L(G),P)-normal, then K is $\bar{C}$-observable.

Proposition 4: Let $K_i\subseteq C$,$i\in I$, be $\bar{C}$-observable. Then $K=\cup_i\{K_i \mid i\in I\}$ is also $\bar{C}$-observable.

Proposition 5: Relative observability is not closed under intersection.

Properties such as decomposability and strong decomposability (conormality) have been proposed to solve distributed supervisory control problem with a global specification. It means that the monolithic supervisor can be decomposed to more than one local supervisor if the local versions of K (i.e. $P_1(K)$ and $P_2(K)$) contain enough information to reconstruct the global supervisor [11].

Such circumstances are too strict and may not be satisfied in practice. Therefore, control equivalency has been defined as another property [15]. The set of controllers K_i which satisfies the following two properties, are control equivalent to K w.r.t. G,

$L_m(G)\cap[\cap_i P_i^{-1}K_i]=K$, (9)

$L(G)\cap[\cap_i P_i^{-1}\bar{K}_i]=\bar{K}$. (10)

Informally, the synchronization of local controllers with the plant is equivalent to that of the monolithic supervisor.

4- Observation Problem In Distributed Supervisory Control

We employ the proposed method in [9] to investigate the observation properties in decentralized supervisory control (Fig.1). In general case, $\cup_{i\neq j}(\Sigma_i\cap\Sigma_j)\subseteq\Sigma_0$, but for simplicity, assume that $\Sigma_1\cap\Sigma_2\subseteq\Sigma_0\subseteq\Sigma_1\cup\Sigma_2$, and the natural projections are defined as follows,

$P_i:(\Sigma_1\cup\Sigma_2)^*\longrightarrow\Sigma_i^*$,

$P_i^{-1}:Pwr(\Sigma_i^*)\longrightarrow Pwr((\Sigma_1\cup\Sigma_2)^*)$, i=0,1,2

$Q_i: \Sigma_i^*\longrightarrow(\Sigma_i\cap\Sigma_0)^*$,

$Q_i^{-1}:Pwr((\Sigma_i\cap\Sigma_0)^*)\longrightarrow Pwr(\Sigma_i^*)$, i=1,2

$R_i: \Sigma_0^*\longrightarrow(\Sigma_i\cap\Sigma_0)^*$,

$R_i^{-1}:Pwr((\Sigma_i\cap\Sigma_0)^*)\longrightarrow Pwr(\Sigma_0^*)$, i=1,2

$T: \Sigma_0^*\longrightarrow(\Sigma_1\cap\Sigma_2)^*$, $T^{-1}:Pwr((\Sigma_1\cap\Sigma_2)^*)\longrightarrow Pwr(\Sigma_0^*)$.

It has been proved that if $\Sigma_1\cap\Sigma_2\subseteq\Sigma_0$, then $P_0(K_1\|K_2)=Q_1(K_1)\|Q_2(K_2)=R_1^{-1} Q_1(K_1)\cap R_2^{-1}Q_2(K_2)$ [9]. Thus, we defined local observation properties in a distributed supervisory control [20].

4- 1- Local Normality

Normality is a observation property of a language, related to another language and a projection channel.

Based on normality definition, the supervisor K is $(L_m(G),P)$-normal, if synchronization of P(K) and $L_m(G)$ is equal to that of K. We introduced a similar property for the decentralized (distributed) supervisory control.

In a decentralized supervisor, say K_i, the natural projection Q_i is defined as $Q_i: \Sigma_i^*\longrightarrow(\Sigma_i\cap\Sigma_0)^*$ and $Q_i(K_i)$ is the image of K_i. If $Q_i(K_i)$ and $L_m(G)$ is synchronized and the resulted language

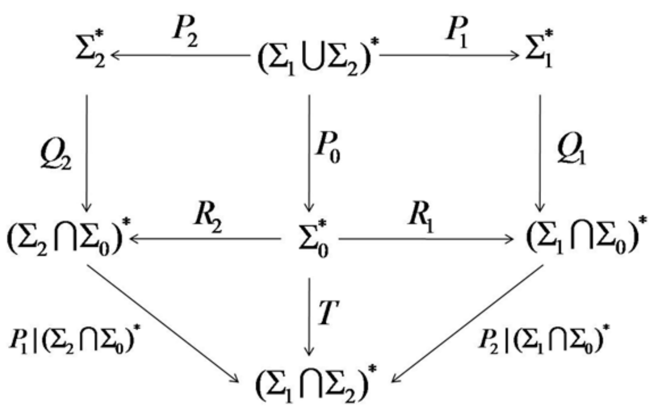


Fig. 1. Natural Projections when $\Sigma_1\cap\Sigma_2\subseteq\Sigma_0$

is equal to K_i, then K_i is normal. Although, K_i is defined in Σ_i^*, $L_m(G)$ is defined in Σ^*. Thus, their reference sets must be the same.

Definition 1 (Local Normality)[20]: Let K_i be a language for i=1,2 and P_i: $\Sigma^*\longrightarrow\Sigma_i^*$,$Q_i$: $\Sigma_i^*\longrightarrow(\Sigma_i\cap\Sigma_0)^*$, i=1,2 and $Q_i(s)=Q_i(s`)$. K_i is called $(L_m(G),P_i^{-1} ,Q_i)$-normal, if

$P_i^{-1}K_i\cap L_m(G)=P_i^{-1}Q_i^{-1}Q_iK_i\cap L_m(G)$.

Also, $\bar{K}_i$ is called $(L(G),P_i^{-1},Q_i)$-Normal, if:

$P_i^{-1}\bar{K}_i\cap L(G)=P_i^{-1}Q_i^{-1}Q_i\bar{K}_i\cap L(G)$.

Comparing to the local normality, we call the normality global normality. In the following theorem, circumstances in which local and global normalities are equivalent are investigated.

Theorem 1: Let K_i's be control equivalent to K w.r.t. G and observable events set be Σ_0, such that $\Sigma_1\cap\Sigma_2\subseteq\Sigma_0$, $P_i:(\Sigma_1\cup\Sigma_2)^*\longrightarrow\Sigma_i^*$,i=0,1,2, and Q_j: $\Sigma_j^*\longrightarrow(\Sigma_j\cap\Sigma_0)^*$,j=1,2. Then local normality implies global normality and vice versa.

Proof: (If) Let K_1, K_2 be two local normal languages. Then,

$P_1^{-1}K_1\cap L_m(G)=P_1^{-1}Q_1^{-1}Q_1K_1\cap L_m(G)$,

$P_2^{-1}K_2\cap L_m(G)=P_2^{-1}Q_2^{-1}Q_2K_2\cap L_m(G)$.

Assume K_1, K_2 are control equivalent to K w.r.t. G. Thus,

$K=P_1^{-1}K_1\cap P_2^{-1}K_2\cap L_m(G)$,

We can write,

$K=P_1^{-1}Q_1^{-1}Q_1K_1\cap P_2^{-1}Q_2^{-1}Q_2K_2\cap L_m(G)$.

From Fig. 1 we have

$P_i^{-1}Q_i^{-1}=P_0^{-1}R_i^{-1}$,i=1,2

and

$K=P_0^{-1}R_1^{-1}Q_1K_1\cap P_0^{-1}R_2^{-1}Q_2K_2\cap L_m(G)$.

From the properties of natural projection [9], we have $P_0^{-1}(R_1^{-1}Q_1K_1\cap R_2^{-1}Q_2K_2)=P_0^{-1}R_1^{-1}Q_1K_1\cap P_0^{-1}R_2^{-1}Q_2K_2$ and $P_0K=R_1^{-1}Q_1K_1\cap R_2^{-1}Q_2K_2$. Thus, $K=P_0^{-1}P_0(K)\cap L_m(G)$. It means that K is $(L_m(G), P_0)$-global normal language. With the same analysis scenario, it can be proved that $\bar{K}$ is $(L(G),P_0)$-global normal if both $\bar{K}_i$'s are $(L(G),P_i^{-1} ,Q_i)$-local normal.

(Only if) Let $K=P_0^{-1}P_0(K)\cap L_m(G)$ and the unobservable events set be $(\Sigma_1\cup\Sigma_2)\backslash\Sigma_0$. We assert that $P_1^{-1}K_1\cap L_m(G)=P_1^{-1}Q_1^{-1}Q_1K_1\cap L_m(G)$ and $P_2^{-1}K_2\cap L_m(G)=P_2^{-1}Q_2^{-1}Q_2K_2\cap L_m(G)$. Assume $P_1^{-1}K_1\cap L_m(G)\neq P_1^{-1}Q_1^{-1}Q_1K_1\cap L_m(G)$ or $P_2^{-1}K_2\cap L_m(G)\neq P_2^{-1}Q_2^{-1}Q_2K_2\cap L_m(G)$. Then,

$P_1^{-1}Q_1^{-1}Q_1K_1\cap P_2^{-1}Q_2^{-1}Q_2K_2\cap L_m(G)\nsubseteq P_1^{-1}K_1\cap P_2^{-1}K_2\cap L_m(G)$

$\Rightarrow P_0^{-1}R_1^{-1}Q_1K_1\cap P_0^{-1}R_2^{-1}Q_2K_2\cap L_m(G)\nsubseteq K$

$\Rightarrow P_0^{-1}(R_1^{-1}Q_1K_1\cap R_2^{-1}Q_2K2)\cap L\_m (G)\nsubseteq K$

$\Rightarrow P_0^{-1}P_0(K)\cap L_m(G)\nsubseteq K$.

However, $K=P_0^{-1}P_0(K)\cap L_m(G)$. Thus, by contradiction the claim is proved.

It is obvious the normality of a language is a strict condition. Hence, relative observability is more achievable than normality in practice.

4- 2- Local Relative Observability

Local relative observability was defined in [20] in the case of unobservable controllable events in decentralized supervisory control. This property makes a larger language than the supremal normal counterpart.

Definition 2 (Local Relative Observability)[20]: Let K_i be a language for i=1,2 and $\bar{K}_i \subseteq \bar{C}_i \subseteq P_i L(G)$. Also, $P_i: \Sigma^* \longrightarrow \Sigma_i^*$, $Q_i: \Sigma_i^* \longrightarrow (\Sigma_i \cap \Sigma_0)^*$ and $Q_i(s)=Q_i(s`)$. K_i is locally relatively observable w.r.t. $(\bar{C}_i, G, P_i^{-1}, Q_i)$ if

(i``) $(\forall \sigma \in \Sigma_i)$ $s\sigma \in \bar{K}_i$, $s` \in \bar{C}_i$, $P_i^{-1}(s`\sigma) \in L(G) \Longrightarrow s`\sigma \in \bar{K}_i$

(ii``) $s \in K_i$, $s` \in \bar{C}_i$, $P_i^{-1}(s`) \in L_m(G) \Longrightarrow s` \in K_i$.

According to Proposition 1, it is easy to show that the statements (i``) and (ii``) can be written as follows, respectively.

$$P_i^{-1}Q_i^{-1}Q_i\bar{K}_i \cap P_i^{-1}\bar{C}_i \Sigma \cap L(G) \subset P_i^{-1}\bar{K}_i \cap L(G), \tag{11}$$

$$P_i^{-1}Q_i^{-1}Q_iK_i \cap P_i^{-1}\bar{C}_i \cap L_m(G) = P_i^{-1}K_i \cap L_m(G). \tag{12}$$

Proposition 6: K_i is locally relatively observable w.r.t. $(\bar{C}_i, G, P_i^{-1}, Q_i)$ if and only if $Q_i(s)=Q_i(s`)$ and (11), (12) hold.

Proof: This proposition is proved in the appendix.

Comparing to the local relative observability, we rename the relative observability global relative observability.

In the following proposition, some circumstances in which local and global relative observability are equivalent will be investigated.

Proposition 7: Let K_i's be control equivalent to K w.r.t. G, and observable event set be Σ_0, which $\Sigma_1 \cap \Sigma_2 \subseteq \Sigma_0$ and $P_i: (\Sigma_1 \cup \Sigma_2)^* \longrightarrow \Sigma_i^*$,i=0,1,2, $Q_j: \Sigma_j^* \longrightarrow (\Sigma_j \cap \Sigma_0)^*$, j=1,2. Then local relative observability guarantees global relative observability, and global relative observability guarantees that at least one of the local controllers is locally relatively observable and the other is local normal.

Proof: This proposition is proved in the appendix.

If $\bar{C}_i = \bar{K}_i$, then local relative observability property turns into local observability property. Thus, it is sufficient to replace $\bar{C}_i$ by $\bar{K}_i$ in the local relative observability definition.

5- Observation-equivalent Versus Control Equivalent

Previously, we showed that if all the shared events between local controllers (which are control equivalent to the monolithic supervisor w.r.t. the plant) are observable, then they have observation properties similar to those of the monolithic supervisor. For example, if K_1, K_2 are local controllers, and K is relatively observable w.r.t. $(\bar{C}, G, P_0)$, then K_1, K_2 are locally relatively observable w.r.t. $(\bar{C}_i, G, P_i^{-1}, Q_i)$,i=1,2 in the partial observation case. We prove that this property has an essential role to keep the local controllers control equivalent to the monolithic supervisor w.r.t. the plant.

Definition 3 (Observation-equivalent): Let K be a monolithic supervisor and K_i's be local controllers with $P_i: \Sigma^* \longrightarrow \Sigma_i^*$,i=0,1,2 and $Q_j: \Sigma_j^* \longrightarrow (\Sigma_j \cap \Sigma_0)^*$,j=1,2. K_i's are observation equivalent to K w.r.t. G, if the following statement is satisfied. "$\forall i$,$\bar{C}i \supseteq \bar{K}_i$, if K_i is locally relatively observable w.r.t. $(\bar{C}_i, G, P_i^{-1}, Q_i)$, then $\exists \bar{C} \supseteq \bar{K}$ such that K is global relative observable w.r.t. $(\bar{C}, G, P_0)$."

However, the control equivalency of a set of local controllers to the monolithic supervisor is guaranteed in full observation case; the observation equivalence of local controllers to the monolithic supervisor may be violated if the observation of local controllers is restricted to observe some events which are not significant for a consistent decision making. Hence, we investigate circumstances in which observation equivalence leads to the control equivalence.

In the following theorem, we prove that control equivalency of local controllers to the monolithic supervisor is satisfied if and only if $\Sigma_1 \cap \Sigma_2 \subseteq \Sigma_0$.

Theorem 2: Let K_i's be observation-equivalent to K w.r.t. G, and observable events set be Σ_0, where $P_i: (\Sigma_1 \cup \Sigma_2)^* \longrightarrow \Sigma_i^*$,i=0,1,2, and $Q_j: \Sigma_j^* \longrightarrow (\Sigma_j \cap \Sigma_0)^*$, j=1,2. Then, K_i's are control equivalent to K w.r.t. G if and only if $\Sigma_1 \cap \Sigma_2 \subseteq \Sigma_0$.

Proof: (If) The proof is similar to that of Proposition 7.

(Only if) Assume that $\Sigma_0 \subseteq \Sigma_1 \cap \Sigma_2$. Then, $P_1^{-1}Q_1^{-1} = P_2^{-1}Q_2^{-1} = P_0^{-1}$, $P_0K \subseteq Q_1K_1 \cap Q_2K_2$, and $P_1^{-1}\bar{C}_1$, $P_2^{-1}\bar{C}_2$ be in conflict. We can write,

$P_1^{-1}K_1 \cap P_2^{-1}K_2 \cap L_m(G) = P_1^{-1}Q_1^{-1}Q_1K_1 \cap P_2^{-1}Q_2^{-1}Q_2K_2 \cap (P_1^{-1}\bar{C}_1 \cap P_2^{-1}\bar{C}_2) \cap L_m(G) = P_0^{-1}(Q_1K_1 \cap Q_2K_2) \cap (P_1^{-1}\bar{C}_1 \cap P_2^{-1}\bar{C}_2) \cap L_m(G)$.

Define $C := P_1^{-1}C_1 \cap P_2^{-1}C_2$. Then, $\bar{C} \subseteq P_1^{-1}\bar{C}_1 \cap P_2^{-1}\bar{C}_2$. Thus, $P_0^{-1}(P_0K) \cap \bar{C} \cap L(G) \subseteq P_0^{-1}(Q_1K_1 \cap Q_2K_2) \cap P_1^{-1}\bar{C}_1 \cap P_2^{-1}\bar{C}_2) \cap L(G) \Longrightarrow K \subseteq P_1^{-1}K_1 \cap P_2^{-1}K_2 \cap L_m(G)$. It means that K_1 and K_2 may be synchronously in conflict. By contradiction $\Sigma_1 \cap \Sigma_2 \subseteq \Sigma_0$.

In Proposition 7, if K_1 is locally observable w.r.t. (G, P_1^{-1}, Q_1) (respectively K_2 is local observable w.r.t.(G, P_2^{-1}) ,Q_2)) then $\bar{C}_1 = \bar{K}_1$ (respectively $\bar{C}_2 = \bar{K}_2$). Therefore, $(\bar{C}` := P_1^{-1}\bar{C}_1 \cap P_2^{-1}\bar{C}_2 \cap L(G) = P_1^{-1}\bar{K}_1 \cap P_2^{-1}\bar{C}_2 \cap L(G)$ and $\bar{K} \subseteq \bar{C}`$. Namely, K is globally relatively observable w.r.t. $(\bar{C}`, G, P_0)$. Moreover, if K_1 is locally normal w.r.t. (G, P_1^{-1}, Q_1) then $\bar{C}`` := P_1^{-1}\bar{C}_1 \cap P_2^{-1}\bar{C}_2 \cap L(G) = P_2^{-1}\bar{C}_2 \cap L(G)$ and $\bar{K} \subseteq \bar{C}``$. Namely, K is globally relatively observable w.r.t. $(\bar{C}``, G, P_0)$.

6- Examples

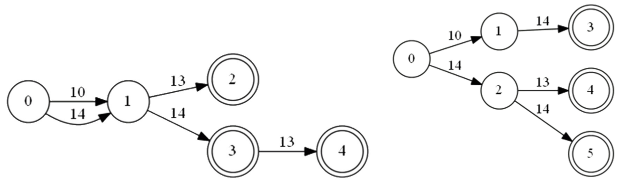
In this section, we consider four examples to illustrate the extended theorem in the previous sections.

Example 1: Consider the plant G and the recognizer of the supervisor K given in Fig. 2. Assume that the set of all possible events is Σ={10,13,14}, the controllable events sets are $\Sigma_{1,c} = \Sigma_{2,c}$={13}, the observable events sets are Σ_{10}={13,14} and Σ_{20}={13}, and the natural projections are as follows,

$P_i: \Sigma^* \longrightarrow \Sigma_i^*$,i=0,1,2, $Q_j: \Sigma_j^* \longrightarrow \Sigma_{j0}^*$,j=1,2

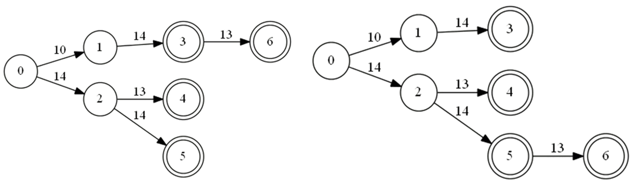
The local controllers which are control equivalent to the supervisor w.r.t. G are shown in Fig.3. The monolithic supervisor and local controllers are constructed by TCT software [25]. Synchronization of local controllers with the plant is the same as that of the monolithic supervisor, shown in Fig. 2 (b). Fig. 4 shows the recognizer of the monolithic supervisor with partial observation. K is globally relatively observable w.r.t. $(\bar{C}, G, P_0)$, where $\bar{C}$={10,1014,101413,14,1413,1414}$\subseteq$L(G). Fig. 5 shows local controller 1 with full observation and local controller 2 with partial observation. Also, the synchronization of local controllers (Fig.5) and the plant is control equivalent to the monolithic supervisor with a partial observation (Fig. 4).

Therefore, when event {10} is not observable for the monolithic supervisor, it can be unobservable for local controller 2, whereas control equivalency between local controllers and



(a) (b)

Fig. 2. (a) Plant G (b) Recognizer of the supervisor K



(a) (b)

Fig. 3. Local controllers with full observation, (a) Local controller1 with full observation Σ_1={13,14} , (b) Local controller 2 with full observation Σ_2={10,13}

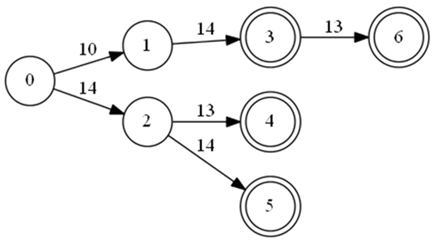
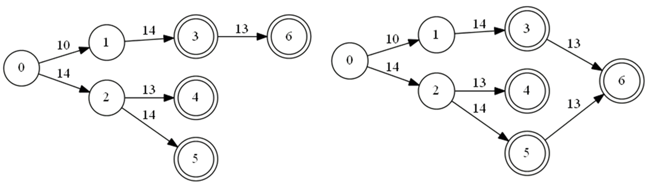


Fig. 4. Recognizer of the monolithic supervisor K with partial observation, Σ_0={13,14}



(a) (b)

Fig. 5. Local controllers with partial observation Σ_0={13,14} , (a) Local controller1 with full observation Σ_{10}={13,14} , (b) Local controller2 with partial observation Σ_{20}={13}

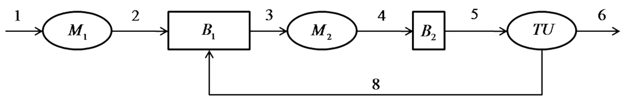


Fig. 6. Transfer Line

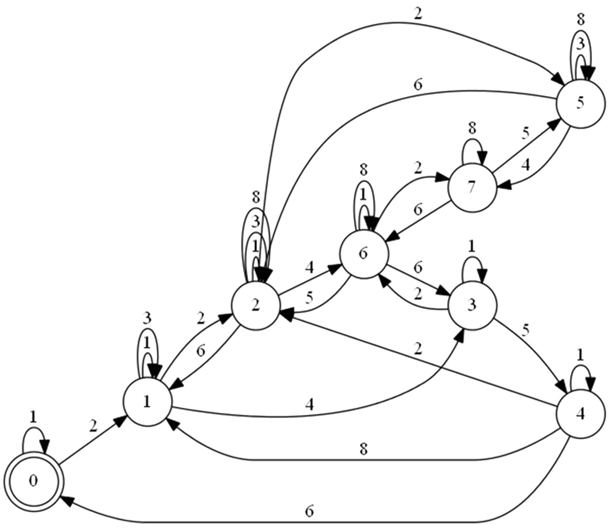


Fig. 7. Reduced supremal relative observable supervisor

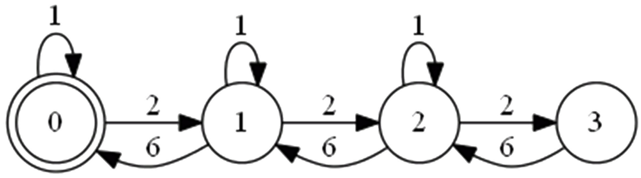


Fig. 8. Local relative observable controller for M1

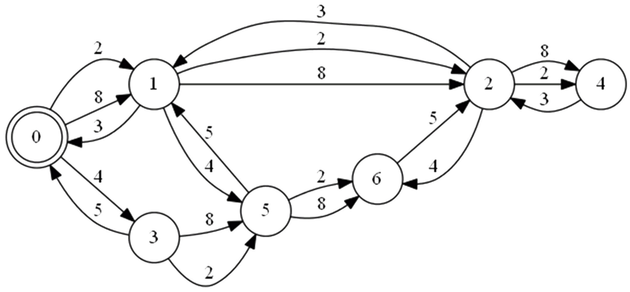


Fig. 9. Local normal controllers for M_2 and TU

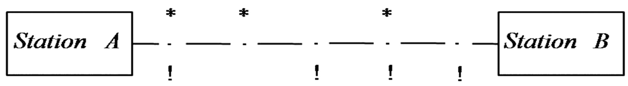


Fig. 10. Schematic of a guide way

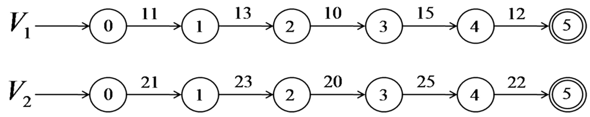


Fig. 11. Discrete-event model of vehicles V_1,V_2

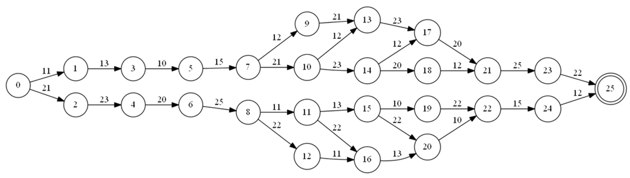


Fig. 12. Recognizer of the supremal normal supervisor (K)

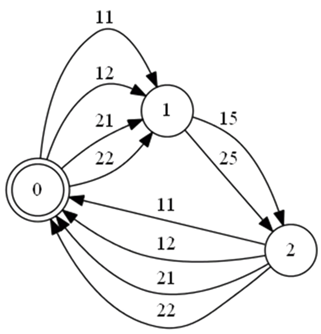


Fig. 13. Recognizer of the reduced supremal normal supervisor

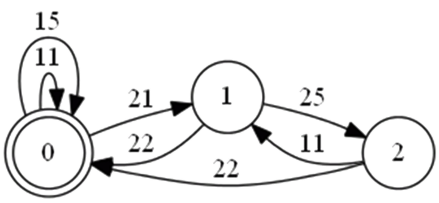


Fig. 14. Local normal controller for V_1

the monolithic supervisor w.r.t. G is preserved. This raises the concept of observation equivalency. It means that, if local controller 2, that is shown in Fig.5 (b), has conflict with the other local controller, then their synchronization with the plant is not control equivalent to the monolithic supervisor (shown in Fig. 4) w.r.t. G.

Example 2 (Supremal relative observable supervisor for Transfer Line): Industrial Transfer Line is a simple model of industrial systems and consists of two machines M1, M2 and a test unit (TU), such that they are linked by buffers B1 and B2 with the capacity of three and one slots, respectively (Fig. 6). If a work piece accepted by TU, it is released from the system; if it is rejected, then it is returned to B1 for reprocessing by M2. The control logic is based on protection B1 and B2 against underflow and overflow [24]. Controllable events are odd-numbered and the unobservable event is {1}. Fig.7 shows the reduced supremal relative observable supervisor synthesized by TCT software [25]. Local controller for the component M1 is shown in Fig. 8 and for M2 and TU in Fig. 9. The local controller of M1 is locally relatively observable and the local controllers of M2 and TU are locally normal. If event {1} is unobservable for local controller of M1, then it becomes self-looped at all states of the local controller (it is not shown). The local controllers of M2 and TU are locally normal because event {1} is self-looped at all states as shown Fig. 9. It means that event {1} does not affect local controllers of M2 and TU, even they are under full observation or under partial observation.

It can be interpreted that event {1} does not belong to local controllers of M_2 and TU. Therefore, the global relative observability of the monolithic supervisor leads to the local relative observability of local controller M_1 and leads to local normality of local controllers M_2 and TU.

Example 3(Supremal normal supervisor for a Guide way)

On a typical guide way, stations A and B are connected by a one-way track from A to B, as shown in Fig. 10. The track consists of four sections, with stoplights which are shown with (*) and with detectors which are shown by (!), installed at various section junctions [23].

Two vehicles V_1 and V_2 use the guide way simultaneously. V_i ,i=1,2, may be at state 0 (at A), state j (while travelling in section j=1,….,4), or state 5 (at B). The discrete-event models of V_i ,i=1,2 are shown in Fig. 11.

The plant to be controlled is G=sync(V_1,V_2). In order to prevent collision, control of the stoplights must ensure that V_1 and V_2 never travel on the same section of the track, simultaneously. In this example, unobservable events are {13,23}. Fig. 12 shows the recognizer of the supremal normal supervisor, and Fig.13 shows the recognizer of the reduced supervisor in which unobservable events {13,23} are not shown, i.e. they are self-looped at all states in the reduced supervisor.

Events {13,23} belong to null space of an arbitrary natural projection P as follows,

$P:\Sigma^*\rightarrow\Sigma_0^*$,$\Sigma_0=\Sigma-\{13,23\}$

The local normal controllers for components V1 and V2, are shown in Fig. 14 and Fig. 15. They are locally normal; because local controllers do not regard the events {13,23}. Therefore, the global normality leads to local normality and vice versa.

Example 4 (Supervisory control synthesis for balancing the pressure of parallel gas trunk lines)

The main sector of a long-distance gas transmission system is a gas trunk line. A gas trunk line is a pipeline which is designed for natural gas transmission from production to market areas. It is similar to trunk of a tree in which the gas processing plants deliver the natural gas through several roots, and consumers receive the gas from some branches. Natural gas is pressurized so that it travels through a pipeline to transport the flow of gas. To keep the minimum pressure for flowing natural gas through each pipeline, compression of the natural gas occurs periodically along the pipe. This is accomplished by compressor stations, placed according to the land topography along the pipeline. Natural gas pipelines include a great number of valves along their entire length. They are divided into two categories: 1. Line Break Valves (LBV's), usually open and allow natural gas to flow freely, but they can be used to stop gas flow along a section of pipe. There are many reasons why a pipeline may need to restrict gas flow in certain circumstances, namely emergency shutdown and maintenance. 2. Control Valves placed on connection pipes between two trunk lines. They are called connection valves each of which connects two trunk lines through a certain connection line.

Since the consumption of natural gas shall be distributed across the trunk lines, several branches from a trunk line are taken to provide the consumption. Hence, the pressure of each trunk line may fluctuate by gas consumption throughout a trunk line. In order to balance the pressure of two or more parallel gas trunk lines, each connection valve between a pair of trunk lines segments can be set to open (Figs. 16, 17, 18). The supervisory control problem for a discrete-event system is formulated by modeling the plant and its control logic (specification) as finite automata [24].

The discrete-event modeling of parallel gas trunk lines, description of specifications, supervisor synthesis and state reduction of the supervisor is carried out in TCT software [25]. We construct a DES model for each pair of parallel gas trunk lines and associated connection valves as follows,

PV_{ij}= Sync(P_i, P_j, V_i) (18, 66) Blocked_events = None

P_i, P_j and V_i are DES models of each pair of parallel gas trunk lines and the connection valve which is placed on the connection pipe between the two trunk lines (Fig. 19). PV_{ij} has 18 states and 66 transitions. Each pair of parallel trunk lines has the same structure with different events.

The control logic for opening and closing the valve is designed as several "If-Then" rules in Table 1, and the corresponding DES model is shown in Fig. 20. It is provided by a designer for balancing the pressure of two parallel gas trunk lines 1 and 2.

The continuous time dynamics of the pressures in a segment of trunk lines 1 and 2, influenced by open/close actuations of the connection valve (Fig. 16) is shown in Fig. 21. The minimum

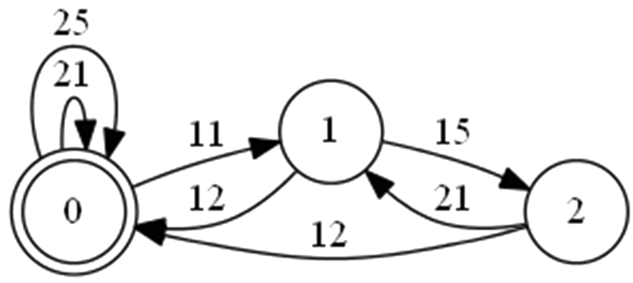


Fig. 15. Local normal controller for V_2

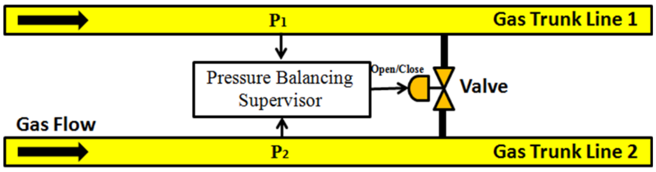


Fig. 16. Schematic diagram of a supervisory control for two parallel gas trunk lines

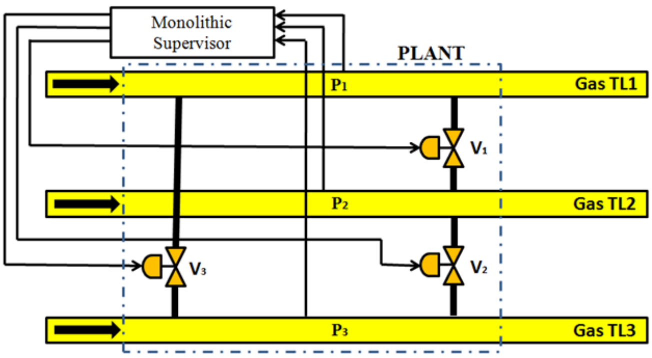


Fig. 17. Schematic diagram of a Monolithic (centralized) supervisory control for three parallel gas trunk lines

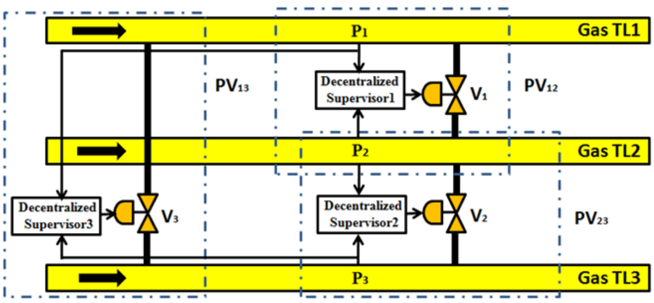
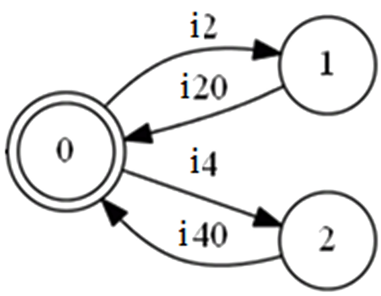
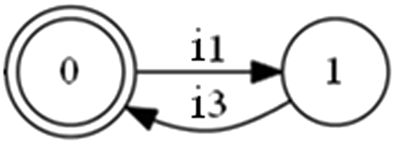


Fig. 18. Schematic diagram of decentralized supervisory control for three parallel gas trunk lines



(a)



(b)

Fig. 19. DES model for each trunk line and the connection valve, i=1,2,3
(a) Trunk Lines (Pi), (b) Connection Valve (Vi)

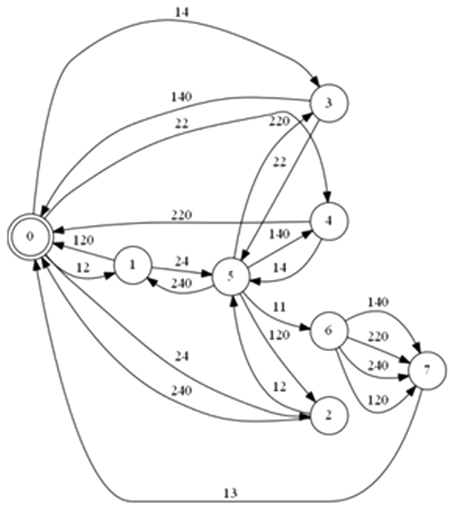


Fig. 20. DES model of the specification for balancing the pressure of two parallel gas trunk lines, E_1

Table 1. Specifications as "If-Then" rules

Rule no.	If	Then
1	The connection valve is closed- ev .13, and the pressure of trunk line 1 is Low- ev.14 (High- ev.12), and the pressure of trunk line 2 is High- ev.22 (Low- ev.24)	Open the connection valve (ev.11)
2	The connection valve is open- ev.11 and the pressure of trunk line 1 returns to the permissible range (from min.)- ev.140	Close the connection valve (ev.13)
3	The connection valve is open- ev.11 and the pressure of trunk line 1 returns to the permissible range (from max.)- ev.120	Close the connection valve (ev.13)
4	The connection valve is open- ev.11 and the pressure of trunk line 2 returns to the permissible range (from min.)- ev.240	Close the connection valve (ev.13)
5	The connection valve is open- ev.11 and the pressure of trunk line 2 returns to the permissible range (from max.)- ev.220	Close the connection valve (ev.13)

and the maximum permissible pressures are assumed to be 50 bar and 60 bar, respectively for each trunk line. Dashed lines show the variations of pressures in one segment of trunk lines 1 and 2, influenced by inlet and outlet gas flows (Fig. 21). This simulation is carried out by state flow toolbox in Matlab. In the case of three parallel gas trunk lines, we obtain two other control logics, structurally the same as E_1 with different events for the other two pairs of parallel trunk lines (2,3) and (1,3). Each decentralized supervisor can be synthesized using supcon procedure as follows,

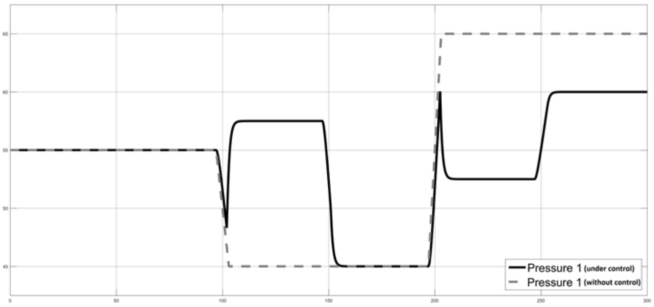
SUP_{ij} = Supcon (PV_{ij}, E_i) (34,95)

Each decentralized supervisor has 34 states and 95 transitions. We can show that SUP_{12}, SUP_{23} and SUP_{13} are synchronously non-conflicting. Now, assume that events {11,21} are

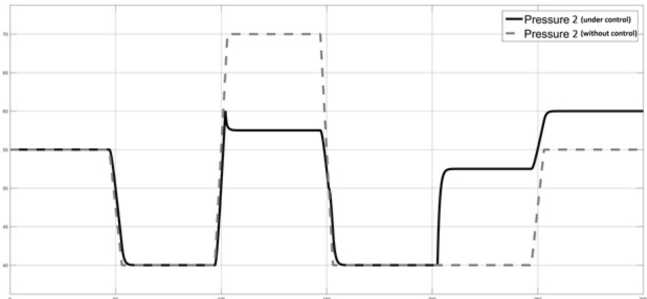
unobservable. Two supremal relative observable supervisors ROSUP$_{12}$ and ROSUP$_{23}$ can be synthesized using supconrobs procedure as follows,
ROSUP$_{12}$= Supconrobs(PV$_{12}$, E$_1$, Null[11]) (65, 181)
ROSUP$_{23}$= Supconrobs(PV$_{23}$, E$_2$, Null[21]) (65, 181)
Moreover,
SUP$_{13}$ = Supcon(PV$_{13}$, E$_3$) (34,95)
SUP$_{13}$ is synthesized based on full observation of PV$_{13}$.
The desired behavior (specification) of three parallel gas trunk lines can be obtained as follows,
E= Meet(E$_1$, E$_2$, E$_3$) (512, 6528)
Assume controllable events {11,21} are unobservable. The supremal relative observable monolithic supervisor can be synthesized by supconrobs procedure as follows,
ROSUP= Supconrobs(PLANT, E, Null[11,21]) (1648, 7272)
ROSUP is the relative observable supervisor with 1648 states and 7272 transitions.
On the other hand, the conjunctive behavior of ROSUP$_{12}$, ROSUP$_{23}$ and SUP$_{13}$, can be obtained as follows,
SROSUP$_{12}$= Selfloop (ROSUP$_{12}$, [32, 34, 320, 340, 21, 23, 31, 33]) (65,367)
SROSUP$_{23}$= Selfloop(ROSUP23, [12, 14, 120, 140, 11, 13, 31, 33]) (65,367)
SSUP$_{13}$= Selfloop(SUP23, [12, 14, 120, 140, 11, 13, 21, 23]) (34,367)
DSUP = Meet(SROSUP$_{12}$, SROSUP$_{23}$, SSUP$_{13}$) (1648, 7272)
We can check the identity of DSUP and ROSUP by isomorph procedure.
true = Isomorph(DSUP, ROSUP; identity)
Since unobservable events {11,21} are not the shared events, local relative observability of decentralized supervisors leads to global relative observability of the monolithic supervisor (ROSUP).
In this example, we clarified that local observation properties lead to the global ones, if the shared events are observable. We know that decentralized supervisory controllers may be in conflict with each other because each decentralized supervisor observes the plant partially. The extended theory in this paper implies that, if the shared events of decentralized supervisors are observable, then unobservable events, in a relative observable supervisor, do not cause the conflict in the plant. In this example {11, 21} are unshared events.

7- CONCLUSIONS

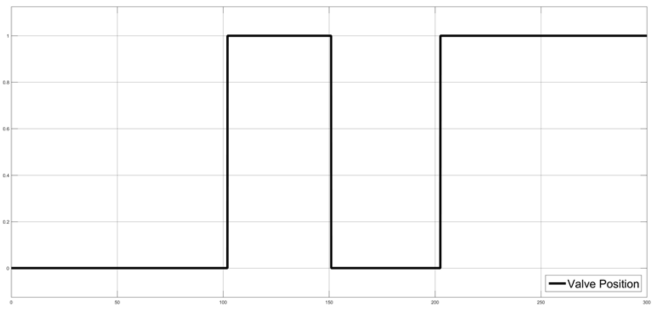
In this paper, a method was introduced to analyze the observation properties such as relative observability, in a set algebra approach . Moreover, the observation properties in distributed supervisory control were investigated. We proved that with having local controllers and the monolithic supervisor, the partial observation properties are preserved from the global supervisor to the local controllers if and only if the intersection of local event sets is a subset of or equal to the global observable event set. Furthermore, the concept of control equivalency was extended for observation problem. Observation equivalence describes the equivalency of observations in the local controllers and the monolithic supervisor in order to have equivalency between control behavior in the monolithic and distributed supervisory control of the plant. It was proved that with having equivalency between local controllers and the monolithic supervisor, the control equivalency is satisfied if and only if the intersection of local event sets is a subset of or equal to the globally observable event set. The extended theory was illustrated by four examples.



a. Variations in pressure of trunk line 1, influenced by open/close actuations in the connection valve



b. Variations in pressure of trunk line 2, influenced by open/close actuations in the connection valve



c. Open/close actuations of the connection valve, influenced by the commands of the supervisor

Appendix 1

In this appendix, we prove some propositions mentioned in this paper.

Proof of Proposition 2: We have

$P^{-1}P\overline{K}\cap\overline{C}\Sigma\cap L(G)\subset\overline{K}$,

$P^{-1}PK\cap\overline{C}\cap L_m(G)=K$.

Then,

$P^{-1}P\overline{K}\cap\overline{C}\Sigma\cap L(G)\cap\overline{K}\Sigma\subset\overline{K}\cap\overline{K}\Sigma$,

$P^{-1}PK\cap\overline{C}\cap L_m(G)\cap\overline{K}=K\cap\overline{K}$.

On the other hand,

$\overline{K}\subseteq\overline{C} \Rightarrow \overline{K}\Sigma\subseteq\overline{C}\Sigma$.

Thus,

$P^{-1}P\overline{K}\cap\overline{K}\Sigma\cap L(G)\subset\overline{K}$,

$P^{-1}PK\cap\overline{K}\cap L_m(G)=K$.

Therefore, K is observable.

Proof of Proposition 3: We have

$P^{-1}P\overline{K}\cap L(G)=\overline{K}$,

$P^{-1}PK\cap L_m(G)=K$.
Then,
$P^{-1}P\overline{K}\cap L(G)\cap\overline{C}\Sigma=\overline{K}\cap\overline{C}\Sigma$,
$P^{-1}PK\cap L_m(G)\cap\overline{C}=K\cap\overline{C}$.
Also $K\subseteq\overline{K}\subseteq\overline{C}$ and $\overline{K}\cap\overline{C}\Sigma\subset\overline{K}$. Thus,
$P^{-1}P\overline{K}\cap L(G)\cap\overline{C}\Sigma\subset\overline{K}$,
$P^{-1}PK\cap L_m(G)\cap\overline{C}=K$.

Proof of Proposition 4: Assume that
$P^{-1}P\overline{K}_i\cap\overline{C}\Sigma\cap L(G)\subset\overline{K}_i$,
$P^{-1}PK_i\cap\overline{C}\cap L_m(G)=K_i$.
We can write
$\cup_i\{P^{-1}P\overline{K}_i\cap\overline{C}\Sigma\cap L(G)\}\subset\cup_i\overline{K}_i$,
$\cup_i\{P^{-1}PK_i\cap\overline{C}\cap L_m(G)\}=\cup_iK_i$.
Thus,
$\cup_i\{P^{-1}P\overline{K}_i\}\cap\overline{C}\Sigma\cap L(G)\subset\cup i\overline{K}_i$,
$\cup_i\{P^{-1}PK_i\}\cap\overline{C}\cap L_m(G)=\cup_iK_i$.
From the properties of natural projection [9], it holds that
$P^{-1}P(\cup_i\{\overline{K}_i\})\cap\overline{C}\Sigma\cap L(G)\subset\cup_i\overline{K}_i$,
$P^{-1}P(\cup_i\{K_i\})\cap\overline{C}\cap L_m(G)=\cup_iK_i$.
Therefore,
$P^{-1}P\overline{K}\cap\overline{C}\Sigma\cap L(G)\subset\overline{K}$,
$P^{-1}PK\cap\overline{C}\cap L_m(G)=K$.

Proof of Proposition 5: Assume that
$P^{-1}P\overline{K}_i\cap\overline{C}\Sigma\cap L(G)\subset\overline{K}_i$,
$P^{-1}PK_i\cap\overline{C}\cap L_m(G)=K_i$.
Then,
$\cap_i\{P^{-1}P\overline{K}_i\cap\overline{C}\Sigma\cap L(G)\}\subset\cap_i\overline{K}_i$,
$\cap_i\{P^{-1}PK_i\cap\overline{C}\cap L_m(G)\}=\cap_iK_i$.
From the properties of natural projection [9],
$\cap_i\{P^{-1}PK_i\}\cap\overline{C}\cap L_m(G)\subseteq P^{-1}P\{\cap_iK_i\}\cap\overline{C}\cap L_m(G)$.
Thus,
$\cap_iK_i\subseteq P^{-1}P\{\cap_iK_i\}\cap\overline{C}\cap L_m(G)$.
Let $K=\cap_iK_i$. We can write $K\subseteq P^{-1}P\{K\}\cap\overline{C}\cap L_m(G)$.
Therefore, the closeness of relative observability, under intersection, is not guaranteed.

Proof of Proposition 6: (If) Assume Q_i (s)=Q_i (s^') and (11), (12) do hold. We can write,
$(\forall\sigma\in\Sigma_i)$ $s\sigma\in\overline{K}_i$,$s`\in\overline{C}_i$,$P_i^{-1}(s`\sigma)\in L(G)\Rightarrow s`\sigma\in\overline{C}_i\,\Sigma_i$.
Also,
$Q_i(s)=Q_i(s`)\Rightarrow Q_i(s\sigma)=Q_i(s`\sigma)\Rightarrow s`\sigma\in Q_i^{-1}Q_i\,\overline{K}_i$
$\Rightarrow P_i^{-1}(s`\sigma)\in P_i^{-1}Q_i^{-1}Q_i\,\overline{K}_i\cap P_i^{-1}\overline{C}_i\,\Sigma\cap L(G)$.
From (11), we can write
$\Rightarrow P_i^{-1}(s`\sigma)\in P_i^{-1}\overline{K}_i\cap L(G)\Rightarrow s`\sigma\in\overline{K}_i\cap P_i\,L(G)$
$\Rightarrow s`\sigma\in\overline{K}_i$.
Thus, (i``) is proved.
From (12), we can write,
$s\in K_i$,$s`\in\overline{C}_i$,$P_i^{-1}(s`)\in L_m(G)\Rightarrow P_i^{-1}(s`)\in P_i^{-1}\overline{C}_i$.
Also,
$Q_i(s)=Q_i(s`)\Rightarrow s`\in Q_i^{-1}Q_i\overline{K}_i\Rightarrow P_i^{-1}(s`)\in P_i^{-1}Q_i^{-1}Q_i\overline{K}_i$
$\Rightarrow P_i^{-1}(s`)\in P_i^{-1}Q_i^{-1}Q_i\,K_i\cap P_i^{-1}\overline{C}_i\cap L_m(G)$.
From (12), we can write,
$\Rightarrow P_i^{-1}(s`)\in P_i^{-1}K_i\cap L_m(G)$
$\Rightarrow s`\in K_i\cap P_i\,L_m(G)$
$\Rightarrow s`\in K_i$.
Thus, (ii``) is proved.
(Only if) Assume $Q_i(s)=Q_i(s`)$ and (i``) hold. we can write
$s\sigma\in\overline{K}_i$,$s`\in\overline{C}_i$, $P_i^{-1}(s`\,\sigma)\in L(G)\Rightarrow s`\,\sigma\in\overline{K}_i$
$\Rightarrow Q_i(s\sigma)\in Q_i\overline{K}_i$,$s`\in\overline{C}_i$, $P_i^{-1}(s`\,\sigma)\in L(G)\Rightarrow s`\,\sigma\in\overline{K}_i$
$\Rightarrow Q_i(s`\sigma)\in Q_i\overline{K}_i$ ,$s`\in\overline{C}_i$, $P_i^{-1}(s`\,\sigma)\in L(G)\Rightarrow s`\,\sigma\in\overline{K}_i$
$\Rightarrow s`\sigma\in Q_i^{-1}Q_i\overline{K}_i$ ,$s`\in\overline{C}_i$,$P_i^{-1}(s`\sigma)\in L(G)\Rightarrow s`\sigma\in\overline{K}_i$
$\Rightarrow P_i^{-1}(s`\sigma)\in P_i^{-1}Q_i^{-1}Q_i\overline{K}_i$ ,$P_i^{-1}(s`\sigma)\in P_i^{-1}\overline{C}_i\Sigma$, $P_i^{-1}(s`\sigma)\in L(G)$
$\Rightarrow s`\sigma\in\overline{K}_i\Rightarrow P_i^{-1}(s`\sigma)\in P_i^{-1}\overline{K}_i$,$P_i^{-1}(s`\sigma)\in L(G)$
$\Rightarrow P_i^{-1}Q_i^{-1}Q_i\overline{K}_i\cap P_i^{-1}\overline{C}_i\Sigma\cap L(G)\subseteq P_i^{-1}\overline{K}_i\cap L(G)$.
On the other hand,
$\epsilon\notin P_i^{-1}\overline{C}_i\,\Sigma$,$\epsilon\in P_i^{-1}\overline{K}_i$ $\Rightarrow$
$P_i^{-1}Q_i^{-1}Q_i\overline{K}_i\cap P_i^{-1}\overline{C}_i\Sigma\cap L(G)\subset P_i^{-1}\overline{K}_i\cap L(G)$.
Moreover, (i``) holds. Then,
$s\in K_i$,$s`\in\overline{C}_i$,$P_i^{-1}(s`)\in L_m(G)\Rightarrow s`\in K_i$
$\Rightarrow Q_i(s)\in Q_i\,K_i$,$s`\in\overline{C}_i$,$P_i^{-1}(s`)\in L_m(G)\Rightarrow s`\in K_i$
$\Rightarrow Q_i(s`)\in Q_iK_i$,$s`\in\overline{C}_i$,$P_i^{-1}(s`)\in L_m(G)\Rightarrow s`\in K_i$
$\Rightarrow s`\in Q_i^{-1}Q_i\,K_i$ ,$s`\in\overline{C}_i$,$P_i^{-1}(s`)\in L_m(G)\Rightarrow s`\in K_i$
$\Rightarrow P_i^{-1}(s`)\in P_i^{-1}Q_i^{-1}Q_iK_i$ ,$P_i^{-1}(s`)\in P_i^{-1}\overline{C}_i$,$P_i^{-1}(s`)\in L_m(G)$
$\Rightarrow s`\in K_i\Rightarrow P_i^{-1}(s`)\in P_i^{-1}K_i$,$P_i^{-1}(s`)\in L_m(G)$
$\Rightarrow P_i^{-1}Q_i^{-1}Q_iK_i\cap P_i^{-1}\overline{C}_i\cap L_m(G)\subseteq P_i^{-1}K_i\cap L_m(G)$.
Moreover, $P_i^{-1}K_i\subseteq P_i^{-1}\overline{C}_i$ and $P_i^{-1}K_i\subseteq P_i^{-1}Q_i^{-1}Q_iK_i$ hold. Thus, $P_i^{-1}K_i\cap L_m(G)\subseteq P_i^{-1}Q_i^{-1}Q_iK_i\cap P_i^{-1}\overline{C}_i\cap L_m(G)$.
Therefore, we can write
$P_i^{-1}Q_i^{-1}Q_iK_i\cap P_i^{-1}\overline{C}_i\cap L_m(G)=P_i^{-1}K_i\cap L_m(G)$.

Proof of Proposition 7: (If) Let K_1, K_2 be two locally relatively observable languages. We can write,
$P_1^{-1}Q_1^{-1}Q_1\overline{K}_1\cap P_1^{-1}\overline{C}_1\Sigma\cap L(G)\subset P_1^{-1}\overline{K}_1\cap L(G)$,
$P_2^{-1}Q_2^{-1}Q_2\overline{K}_2\cap P_2^{-1}\overline{C}_2\Sigma\cap L(G)\subset P_2^{-1}\overline{K}_2\cap L(G)$.
We have K_1, K_2 which are control equivalent to K w.r.t.G. Thus,
$\overline{K}=P_1^{-1}\overline{K}_1\cap P_2^{-1}\overline{K}_2\cap L(G)$,
$\Rightarrow P_1^{-1}Q_1^{-1}Q_1\overline{K}_1\cap P_2^{-1}Q_2^{-1}Q_2\overline{K}_2\cap(P_1^{-1})\overline{C}_1\cap P_2^{-1}\overline{C}_2)\Sigma\cap L(G)\subset\overline{K}$.
By defining $\overline{C}:=P_1^{-1}\overline{C}_1\cap P_2^{-1}\overline{C}_2\cap L(G)$, we have $\overline{K}\subseteq\overline{C}\subseteq L(G)$, and $P_0^{-1}R_1^{-1}Q_1\overline{K}_1\cap P_0^{-1}R_2^{-1}Q_2\overline{K}_2\cap\overline{C}\Sigma\cap L(G)\subset P_0^{-1}R_1^{-1}\,Q_1\overline{K}_1\cap P_0^{-1}\,R_2^{-1}Q_2\overline{K}_2\cap(P_1^{-1}\overline{C}_1\cap P_2^{-1}\overline{C}_2)\Sigma\cap L(G)$. Thus, $P_0^{-1}(P_0\overline{K})\cap\overline{C}\Sigma\cap L(G)\subset\overline{K}$.
Similarly, it can be proved that $K=P_0^{-1}(P_0K)\cap\overline{C}\cap L_m(G)$. Therefore, K is $(\overline{C},G,P_0)$-global relative observable.
(Only if) Let $P_0^{-1}(P_0\overline{K})\cap\overline{C}\Sigma\cap L(G)\subset\overline{K}$,$\overline{K}\subseteq\overline{C}\subseteq L(G)$, and unobservable events set be $(\Sigma_1\cup\Sigma_2)\backslash\Sigma_0$. Assume K is global

relativey observable. Thus,

$P_0^{-1}(P_0\bar{K})\cap\bar{C}\Sigma\cap L(G)\subset\bar{K}$

$\Rightarrow P_0^{-1}(R_1^{-1}Q_1\bar{K}_1\cap R_2^{-1}Q_2\bar{K}_2)\cap\bar{C}\Sigma\cap L(G)\subset\bar{K}$

$\Rightarrow P_0^{-1}R_1^{-1}Q_1\bar{K}_1\cap P_0^{-1}R_2^{-1}Q_2\bar{K}_2\cap\bar{C}\Sigma\cap L(G)\subset\bar{K}$.

We claim that there exists $\bar{K}_1\subseteq\bar{C}_1$, where $P_1^{-1}Q_1^{-1}Q_1\ \bar{K}_1\cap P_1^{-1}\bar{C}_1\Sigma\cap L(G)\subset P_1^{-1}\bar{K}_1\cap L(G)$ and $\bar{K}_2\subseteq\bar{C}_2$, where $P_2^{-1}Q_2^{-1}Q_2\bar{K}_2\cap P_2^{-1}\bar{C}_2\Sigma\cap L(G)\subset P_2^{-1}\ \bar{K}_2\cap L(G)$ (or $P_2^{-1}Q_2^{-1}Q_2\ \bar{K}_2\cap L(G)=P_2^{-1}\bar{K}_2\cap L(G)$). Otherwise,

$$P_1^{-1}\bar{K}_1\cap L(G)\subseteq P_1^{-1}\ Q_1^{-1}Q_1\bar{K}_1\cap P_1^{-1}\bar{C}_1\Sigma\cap L(G), \quad (13)$$

or

$$P_2^{-1}\bar{K}_2\cap L(G)\subseteq P_2^{-1}Q_2^{-1}Q_2\bar{K}_2\cap P_2^{-1}\bar{C}_2\Sigma\cap L(G). \quad (14)$$

If both (13), (14) are satisfied, then $\bar{K}=P_1^{-1}\bar{K}_1\cap P_2^{-1}\bar{K}_2\cap L(G)\subseteq P_0^{-1}P_0\bar{K}\cap\bar{C}\Sigma\cap L(G)$; But $P_0^{-1}P_0\bar{K}\cap\bar{C}\Sigma\cap L(G)\subset\bar{K}$. If only one of the two relationships (13) or (14) holds, then $Q_1^{-1}Q_1=Q_2^{-1}Q_2=1$. It means that the projection channels Q_i ,i=1,2 allow passing all the events from their reference events set. By contradiction, there should be $P_1^{-1}Q_1^{-1}Q_1\bar{K}_1\cap P_1^{-1}\bar{C}_1\Sigma\cap L(G)\subset P_1^{-1}\bar{K}_1\cap L(G)$ and $P_2^{-1}\ Q_2^{-1}\ Q_2\bar{K}_2\cap P_2^{-1}\bar{C}_2\Sigma\cap L(G)\subset P_2^{-1}\bar{K}_2\cap L(G)$. Furthermore, it is easy to prove that if one of the local controllers is locally relatively observable and the other one is locally normal, then K is globally relatively observable. The following relationships can be proved, similarly.

$P_1^{-1}K_1\cap L_m(G)=P_1^{-1}Q_1^{-1}Q_1K_1\cap P_1^{-1}\bar{C}_1\cap L_m(G)$,

$P_2^{-1}K_2\cap L_m(G)=P_2^{-1}Q_2^{-1}Q_2K_2\cap P_2^{-1}\bar{C}_2\cap L_m(G)$.

Appendix 2

A quick review of TCT commands is presented.

DES3=**supcon**(DES1, DES2) for a controlled generator DES1, forms a trim recognizer for the supremal controllable sublanguage of the marked ("legal") language generated by DES2 to create DES3. This structure provides a proper supervisor for DES1.

DES = **sync**(DES1,DES2,: : : ,DESk)
is the (reachable) synchronous product of DES1,DES2,: : : ,DESk.

DAT3= **condat**(DES1, DES2) returns control data DAT3 for the supervisor DES2 of the controlled system DES1. If DES2 represents a controllable language (with respect to DES1),as when DES2 has been previously computed with supcon, then condat will display the events that are disabled at each state of DES2. In general, condat can be used to test whether a given language DES2 is controllable: just check that the disabled events tabled by condat are themselves controllable (have odd-numbered labels).

DES3=**supreduce**(DES1, DES2, DAT2) is a reduced supervisor for plant DES1 which is control-equivalent to DES2, where DES2 and control data DAT2 were previously computed using supcon and condat. Also, returned is an estimated lower bound slb for the state size of a strictly state-minimal reduced supervisor. DES3 is strictly minimal if its reported state size equals the slb.

{LOC1,LOC2,...,LOCm} = **localize**(PLANT,{PLANT1,… ,PLANTm},SUPER)is the set of localizations of SUPER to the m independent components PLANT1,...,PLANTm of PLANT. Independence means that the alphabets of PLANT1,...,PLANTm must be pair wise disjoint. Optionally, correctness of localization is verified and reported as ControlEqu(...). Localize is mainly for use when SUPER is a decentralized supervisor with authority over PLANT1,...,PLANTm, and PLANT is their synchronous product.

DES2=**project**(DES1, NULL/IMAGE EVENTS) is a generator of the projected closed and marked languages of DES1, under the natural projection specified by the listed Null or Image events.

True/False= **isomorph**(DES1, DES2) tests whether DES1 and DES2 are identical up to renumbering of states; if so, their state correspondence is displayed.

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